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A STUDY ON THE SENSITIVITY AND SIMULTANEOUS ADJUSTMENT OF A HOOP-COLUMN ANTENNA SURFACE

KYONG BEEN LIM
JER-NAN JUANG
PEIMAN MAGHAMI

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National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665-5225

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K.B. Lim, J-N. Juang and P. Maghami

1. Introduction

In this report, the results of some recent analysis of the 15-meter hoop-column space antenna are presented. A model and some test data of the antenna are reported in reference 1 and 2. Although a detailed description of the antenna can be found in the above references, a brief description of the antenna is given for completeness of this paper. Figure 1 shows the 15-meter diameter Hoop-Column antenna. The primary structural elements of the antenna are the telescoping column and the hoop. The hoop is connected to the column by cables that emanate from the upper and lower regions of the column. The mesh surface is shaped by a network of cables that emanate from the lower section of the column. The shape adjustment refers to the adjustment of the mesh surface by means of a subset of control cables attached directly to the surface.

The work reported in this manuscript consists of three parts. First, the original finite-element model of the antenna given by the EAL code has been converted to a COSMIC/NASTRAN finite-element model. The purpose for the above conversion was to enhance portability of the model, particularly for use by engineers in industry. This conversion required some remodeling of a few components in the antenna. The NASTRAN model and the original EAL model are compared using predicted responses.

Secondly, the least-squares differential procedure for shape adjustment as outlined in reference 1 has been modified by using singular value decomposition to avoid introducing unnecessary instabilities into the problem. In addition, whereas reference 1 neglected lateral motion in these calculations, it has been included here because of the high accuracy requirement of the antenna. Furthermore, the set of cable length changes which serves as the tuning parameters of the least-squares procedure is expanded to include parameters of the perfect parabola of the four quadrants of the antenna surface.

The final part of the study involves a sensitivity analysis of the RMS error of a nominal antenna shape. An estimate of the error bound due to the uncertainty in the cable lengths have been obtained. This analysis provides some quantitative measure of the needed accuracy of the cable adjustments in the laboratory.

2. *NASTRAN model*

An initial NASTRAN model of the antenna was obtained by converting the existing EAL model via the PATRAN program. After updating the pretension values in some of the control cables, the NASTRAN model was placed into an iteration loop to obtain the global stiffness matrix of the antenna assembly. The need for an iterative procedure arises from the fact that the hoop-column antenna is kinematically unstable without the differential stiffness provided by the hoop cables and control cables. Thus, the global stiffness matrix which contains a nominal component (from rigid format 1) and a differential component (from rigid format 4) cannot be determined directly by using a single NASTRAN rigid format.

After computing the nominal component, the iteration procedure can be summarized as follows:

(a) With the initial pretension values of the cables and the mesh elements, and with almost all degrees of freedom constrained, NASTRAN rigid format 4 (static analysis with differential stiffness) is executed to yield the initial estimate of the differential stiffness matrix.

(b) The estimate of the differential stiffness matrix is then added (via NASTRAN Alter commands) to the nominal stiffness matrix to obtain an approximation to the global stiffness.

(c) The approximate global stiffness along the true grid point constraints is used in another rigid format 4 execution, to obtain a better estimate of the differential stiffness matrix.

(d) Step (b) and (c) are repeated until desired accuracy is achieved.

(e) The combination of the stiffness matrix and the most recently updated differential stiffness matrix referred to as the global stiffness matrix is used in the NASTRAN static analysis (rigid format 1) to generate the influence coefficients that are utilized in the least-squares adjustment procedure.

The DMAP Alters utilized in the NASTRAN runs of steps (a)–(e) are summarized in Appendix A.

3. SVD Solution to Least-Squares Problem

The singular value decomposition (SVD) solution to the least-squares problem is well established and an excellent discussion can be found in reference 3. What follows next is a summary of the essential equations used in this study.

The error vector for an overdetermined set of equations is defined by

$$e \equiv Ax - b \tag{1}$$

The $(m \times n)$ matrix A and the $(m \times 1)$ measurement vector e is assumed to be given. For the antenna shape adjustment problem considered here, $m = 888$ and $n = 96$. A quadratic weighted sum of the error is defined by

$$J \equiv e^T W^2 e \tag{2}$$

where W represents a positive semi-definite set of measurement weights. The weights used in this study are taken to be represented by an identity matrix. Besides the trivial but standard identity matrix weighting, a set of measurement weights based on the electromagnetic energy distribution on the surface is given in reference 1.

The SVD solution to the least-squares problem of minimizing J with respect to x is given by

$$x_{LS} = V\Sigma^{-1}U_n^T W b \tag{3}$$

where

$$WA = [\,U_n \quad U_{m-n}\,] \begin{bmatrix} \Sigma \\ 0 \end{bmatrix} V^T \tag{4}$$

It should be mentioned that the above form of the least squares solution is the most numerically reliable.[3]

4. Inclusion of the (X,Y) Sensitivity

The original least-squares formulation of the antenna[1] is based on the RMS error given by

$$RMS_1 \equiv \|z^I - \left(z_o^M + \frac{\partial z^M}{\partial u}|_o \Delta u\right)\|_2 \tag{5}$$

where z^I is the given "ideal" z coordinate of the parabola corresponding to the measured x and y coordinates, x^M and y^M. Note that x^M, y^M and z^I are vectors of dimension

888×1 while Δu represents the 96×1 vector of cable adjustments. The subscript "o" denotes the values at the nominal point. In Eq.(5), $\|\cdot\|_2$ denotes vector 2-norm. Figure 2 illustrates the problem with the cost function given by RMS_1. The cable adjustment, Δu, affects not only the z coordinate but the x and y as well. Since the objective of the adjustment is to approach the perfect parabola as close as possible, ignoring the x and y deflections may produce small error with respect to unadjusted z coordinates but large errors with respect to the corrected z corresponding to the corrected x and y coordinates.

Thus the modified RMS error to reflect the x and y dependence of z^I is

$$RMS_2 \equiv \|z_o^I + \frac{\partial z^I}{\partial x}|_o \Delta x^M + \frac{\partial z^I}{\partial y}|_o \Delta y^M - \left(z_o^M + \frac{\partial z^M}{\partial u}|_o \Delta u\right)\|_2 \tag{6}$$

The predicted changes in x and y coordinates of the target points are given by

$$\begin{aligned} \Delta x^M &\doteq \frac{\partial x^M}{\partial u}|_o \Delta u \\ \Delta y^M &\doteq \frac{\partial y^M}{\partial u}|_o \Delta u \end{aligned} \tag{7}$$

where the x and y sensitivity matrices, $\frac{\partial x^M}{\partial u}|_o$ and $\frac{\partial y^M}{\partial u}|_o$, are available from the NASTRAN model.

5. *Simultaneous Adjustment of Cable and Parabola*

Previously, the desired parabola is assumed known and the cable adjustments were made to minimize the RMS fit error with respect to the given parabola. Consequently, the design freedom in the variables parameterizing a parabola have not been utilized although a desired or an optimum parabola is not known a priori to the adjustments; recall that a least squares solution depends on the choice of the desired parabola. In this section, the least-squares differential correction algorithm is extended to include, in addition to cable length adjustments, the focal lengths, f, and the vertex offsets, V_x and V_y, of each quadrant.

We begin by defining the fit error,

$$e \equiv z^I(x^M, y^M, p) - z^M(u) \tag{8}$$

where the parameter vector p is defined by

$$p \equiv (f_1, V_{x1}, V_{y1}, f_2, V_{x2}, V_{y2}, f_3, V_{x3}, V_{y3}, f_4, V_{x4}, V_{y4})^T \tag{9}$$

By linear expansions of z^I and z^M in Eq.(8) about the nominal point, x_o^M, y_o^M and p_o,

$$z^I(x^M, y^M, p) = z^I(x_o^M, y_o^M, p_o) + \frac{\partial z^I}{\partial x^M}|_o \Delta x^M + \frac{\partial z^I}{\partial y^M}|_o \Delta y^M + \frac{\partial z^I}{\partial p}|_o \Delta p + \ldots$$
$$z^M = z_o^M + \frac{\partial z^M}{\partial u}|_o \Delta u + \ldots \tag{10}$$

where Δx^M and Δy^M are given in Eq.(7). By substituting the linear expansions of Eqs.(10) into Eq.(8), the linearized error vector takes the form

$$\epsilon = z^I(x_o^M, y_o^M, p_o) - z_o^M - G \begin{Bmatrix} \Delta u \\ \Delta p \end{Bmatrix} + O\left(\Delta u^2, \Delta p^2\right) \tag{11}$$

where the gradient matrix is given by

$$G \equiv \left[-\frac{\partial z^I}{\partial x^M}\frac{\partial x^M}{\partial u} - \frac{\partial z^I}{\partial y^M}\frac{\partial y^M}{\partial u} + \frac{\partial z^M}{\partial u} \quad -\frac{\partial z^I}{\partial p} \right] \tag{12}$$

In the above equations, the terms, $\frac{\partial z^I}{\partial x^M}$ and $\frac{\partial z^I}{\partial y^M}$ can easily be obtained from the parabolic equations for the j-th target in the k-th quadrant

$$z_j = \frac{1}{f_k}\left[(x_j - V_{xk})^2 + (y_j - V_{yk})^2\right] \tag{13}$$

The terms, $\frac{\partial x^M}{\partial u}$, $\frac{\partial y^M}{\partial u}$, and $\frac{\partial z^M}{\partial u}$ are of course obtained from the NASTRAN model. The least-squares solutions can be computed by SVD as mentionted earlier. The resulting equations represent a simultaneous update for both cable lengths and several parameters of the perfect parabola. For improved results, a few iterations can be carried out provided the gradients can be updated.

The Table below shows the predicted surface RMS errors for several cases. Case 1 represents the initial unadjusted surface conditions. The initial errors from both EAL and NASTRAN finite-element models were almost identical. Case 2 is the reference case obtained from reference 1 after the adjustment, which uses an EAL model. Case 3 uses the same least-squares adjustment as case 2 but with an equivalent NASTRAN model obtained in this study. Clearly, the EAL and NASTRAN models produces very similar results. The least-squares extension to include x and y sensitivities as given by RMS_2 in Eq.(6) are shown in case 4. Only .5 mils improvement is indicated. The results of a simultaneous adjustment of the cables and a set of parabola parameters are shown in case 5. It can be seen that 6.3 mils of improvement is obtained over case 3 after the second iteration. The table below shows average cable length changes corresponding to the various sets of adjustments. Note that the average cable length change given in the table is the deviation from case 2 cable lengths in mils. It can be observed that the simultaneous tuning gives an improvement which is not insignificant and corresponds to cable length changes which are implementable.

TABLE: Predicted Surface RMS Error

Case	RMS Error (mils)	Average Length Change (mils)
1. Initial	158.7	–
2. Belvin (EAL)	82.0	0
3. Belvin (NASTRAN)	82.8	3
4. With (x,y) Sensitivity	82.3	2
5. Simultaneous Tuning	76.7 (1^{st} iter)	12
	76.5 (2^{nd} iter)	12

Figure 3 shows the effect of initial RMS error distribution on the parabola parameter changes. It is interesting to observe that quadrants 3 and 4 having larger initial RMS errors result in larger parameter changes.

6. Error Bounds Due to Uncertainty in Cable Adjustments

Among various concepts and idealizations, "exact" adjustments of the cables constitute wishful thinking and is painfully evident in the laboratory. In this section, an attempt is made to address the uncertainties in the cable adjustment and its influence on the surface RMS error. By matrix manipulations, upper bounds of the surface RMS error are obtained as a function of the nominal sensitivity and the magnitude of the cable adjustment error or uncertainty.

First, let the 2-norm of the cable adjustment error vector be bounded by ϵ, i.e.,

$$\|\Delta u\|_2 \equiv \left(\Delta u^T \Delta u\right)^{1/2} \leq \epsilon \tag{14}$$

By writing the linear perturbations in z as

$$\Delta z \doteq S \Delta u; \quad \text{where } S \equiv \left[\tfrac{\partial z}{\partial u}\right] \tag{15}$$

the surface $RMS_{\Delta z}$ error in z can be written as

$$RMS_{\Delta z} \equiv \left(\frac{\Delta z^T \Delta z}{m}\right)^{1/2} \doteq \left(\frac{\Delta u^T S^T S \Delta u}{m}\right)^{1/2} \tag{16}$$

The sensitivity matrix, S, is of dimension $(m \times n)$. The RMS is bounded by the Rayleigh's quotient, namely,

$$\max_{\|\Delta u\|_2=1} \quad \Delta u^T S^T S \Delta u = \lambda_{max}[S^T S] = \bar{\sigma}^2[S] \tag{17}$$

Clearly then,

$$\max_{\|\Delta u\|_2=\epsilon} \quad RMS_{\Delta z} = \frac{\epsilon}{\sqrt{m}}\bar{\sigma}[S] \tag{18}$$

Therefore, the $RMS_{\Delta z}$ corresponding to any Δu satisfying

$$\|\Delta u\|_2 \leq \epsilon \tag{19}$$

can be bounded by μ_2 where

$$\mu_2 = \frac{\epsilon}{\sqrt{m}}\bar{\sigma}[S] \tag{20}$$

A more convenient bound for the RMS error than the 2-vector norm or the Euclidean norm is the ∞-norm defined by

$$\|\Delta u\|_\infty \equiv \max\left(|\Delta u_i|; \quad i = 1, \ldots, n\right) \tag{21}$$

From the matrix identity

$$\sqrt{n}\|\Delta u\|_\infty \geq \|\Delta u\|_2 \tag{22}$$

it follows that if Δu satisfies

$$\sqrt{n}\|\Delta u\|_\infty \leq \sqrt{n}\epsilon \tag{23}$$

then, Δu must also satisfy

$$\|\Delta u\|_2 \leq \sqrt{n}\epsilon \tag{24}$$

In summary, for all Δu satisfying

$$\|u\|_\infty \leq \epsilon \tag{25}$$

the corresponding $RMS_{\Delta z}$ is bounded by μ_∞ where

$$\mu_\infty = \sqrt{\frac{n}{m}}\epsilon\bar{\sigma}[S] \tag{26}$$

For the given antenna structure,

$$\bar{\sigma}\left[\frac{\partial z}{\partial u}\right] = 15.4$$

$$m = 888 \quad \text{(number of targets)}$$

$$n = 96 \quad \text{(number of cables)}$$

Based on the above numerical values, two conditions bounding $RMS_{\Delta z}$ error can be written as follows:

CONDITION 1: For all $\|\Delta u\|_2 \leq \epsilon$: $RMS_{\Delta z} \leq 0.52\epsilon$

CONDITION 2: For all $\|\Delta u\|_\infty \leq \epsilon$: $RMS_{\Delta z} \leq 5.06\epsilon$

Note that the above bounds are dependent on the choice of the norm of the uncertainty in the cables. In general, it is anticipated that the 2-norm should give a tighter bound, as compared to the ∞-bound, due to its measure in an average sense. As an example in using the above conditions, if the average error (in the 2-norm sense) in the cable adjustment is given as 2 mils, then, the $RMS_{\Delta z}$ error must be less than 1.04 mils. On the other hand, if each cable uncertainty is known only to be bounded by 2 mils, then, the $RMS_{\Delta z}$ error can be guaranteed to be less than 10.12 mils.

7. Concluding Remarks and Recomendations

Further improvements in addition to the numerical results given in this study appears inevitable if additional antenna structural data is available. In particular, the gradients are not updated after each least-squares correction due to computational costs; the gradients are computed by a finite-difference procedure using the approximately 10,000 degrees of freedom finite-element model from NASTRAN. Hence the gradients are assumed fixed during the least-squares differential correction iterations. It is necessary to include NASTRAN model in the least-squares loop.

Perhaps a more apparent problem in the general solution strategy is the lack of correlation of the RMS shape error from the measurements and its predicted values. Specifically, the RMS error from the finite-element model is only approximately 7 mils whereas the measured RMS values ranged approximately from 70 to 150 mils. The large difference in the RMS values seem to indicate that the mixture of sensitivity matrix obtained from the NASTRAN model and the measured coordinates of the target points of the real structure is severely limiting. Note that the sensitivity matrix at the target points are genenrated from the finite-element model which requires the coordinates of the target points. Clearly an accuracy problem will result if the measured and predicted coordinates do not correlate, as evidenced by the differing RMS error. As noted by earlier studies, the above modeling problem originates because the finite-element model does not contain any information on fabrication errors.

The RMS error basically is a function of the accuracy of the target point coordinates and the analytical model of the system. Imperfection of assemblying, material stiffness, joints, etc, will significantly introduce the modeling error which can be measured through laboratory testing. Thus, to further improve the RMS error, laboratory tests are required to measure the real sensitivity matrix in order to further improve the RMS error prediction capability. However, experience has shown that testing is usually a time consuming and costly task which requires considerable expertise in different disciplines. Furthermore, exact estimation of the real sensitivity matrix is not feasible.

The following are recomended to further enhance the shape adjustments: (1) Human errors in assembling and imperfections in manufacturing should be minimized through the establishment of stricter guidelines, (2) the adherence of the test articles to manufacturer specifications and the validity of the analytical model should be verified via laboratory measurements, (3) the errors in modeling and the sensitivity matrix should be statistically quantified to obtain more realistic estimates of the RMS error bounds. It is believed that these recommendations, while painful (if one were to actually carry them out), will further improve the antenna shape.

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Reference

[1] Belvin, W.K., Edighoffer, H.H., and Herstrom, C.L., "Quasi-Static Shape Adjustment of a 15 Meter Diameter Space Antenna," AIAA Paper No. 87-0869-CP, presented at the 28-th Structures, Structural Dynamics and Materials Conference, Monterey, CA, April 6-8, 1987.

[2] "Develpoment of the 15 meter Diameter Hoop Column Antenna," NASA CR 4038, Harris Corporation, Melbourne, Florida, 1986.

[3] Golub, G.H. and VanLoan C.F., *Matrix Computations*, The John Hopkins University Press, Baltimore, MD, 1983, Chapter 6.

Appendix A

(a) Executive Deck data and Dmap Alters used to generate and save the intial differential stiffness matrix:

```
ID ANTENNA,STATIC
SOL 4,0
TIME 30
APP DISPLACEMENT
ALTER 87 $
OUTPUT1 KDGG,,,,//-1/6 $
EXIT $
ENDALTER
CEND
```

(b) Executive Deck data and Dmap Alters used in the iterations of step-(b) to generate the updated differential stiffness matrices:

```
ID ANTENNA,STATIC
SOL 1,0
TIME 300
APP DISPLACEMENT
ALTER 37 $
INPUTT1 /KGGG,,,, /C,N,-1/6 $
ADD KGGG,KGGX/KKK1 $
EQUIV KKK1,KGGX $
ALTER 87 $
OUTPUT1 KDGG,,,,//-1/7 $
EXIT $
ENDALTER
CEND
```

(c) Executive Deck data and Dmap Alters used to generate the global stiffness matrix and obtain the displacement solution:

```
ID ANTENNA,STATIC
SOL 1,0
TIME 300
APP DISPLACEMENT
ALTER 49 $
INPUTT1 /KDGG,,,, /C,N,-1/7 $
ADD KDGG,KGGX/KKK1 $
EQUIV KKK1,KGGX $
ENDALTER
CEND
```

FIGURE 1: Finite element model of Hoop-Column antenna

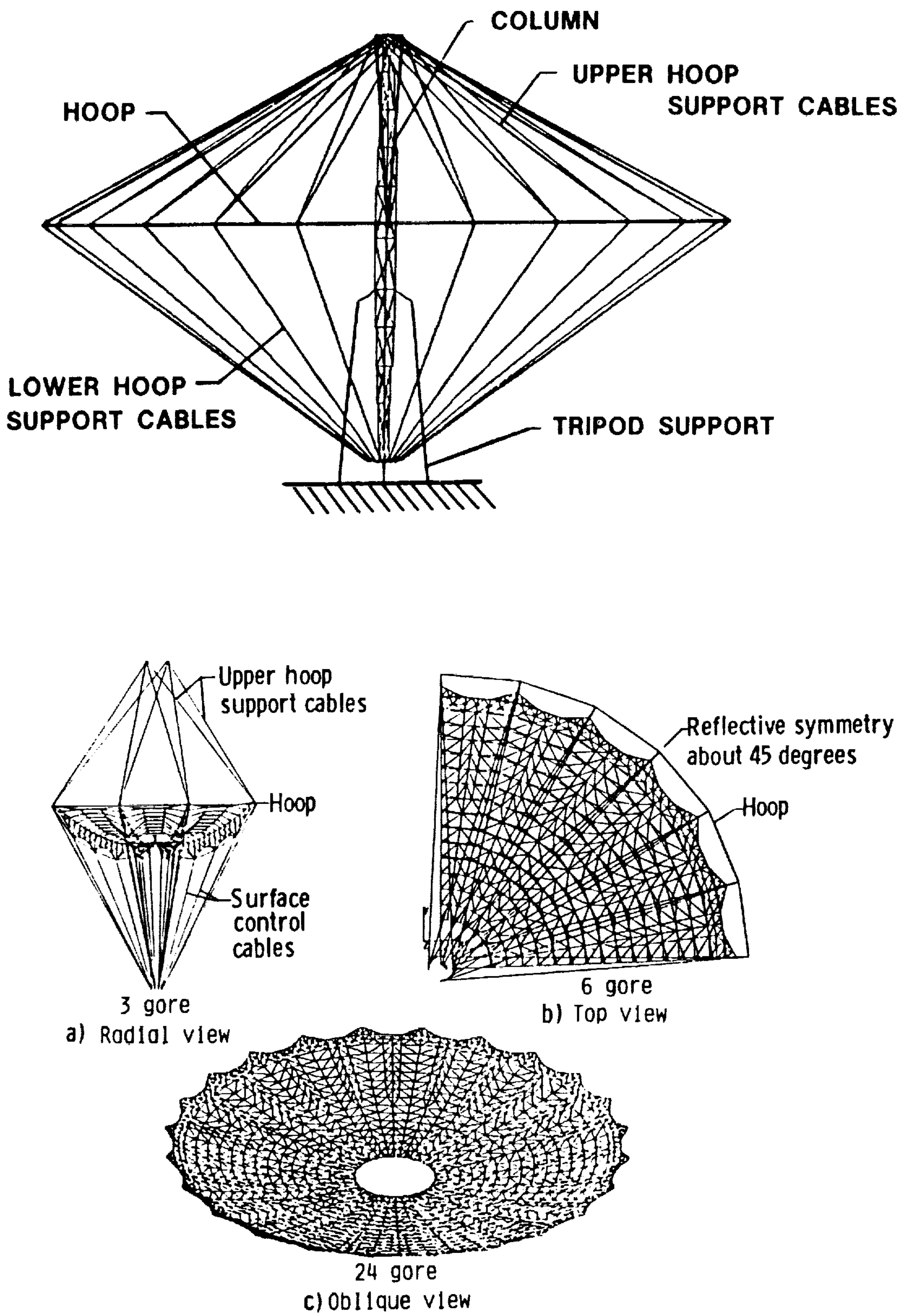

FIGURE 2: Effect of (x, y) sensitivity on RMS error

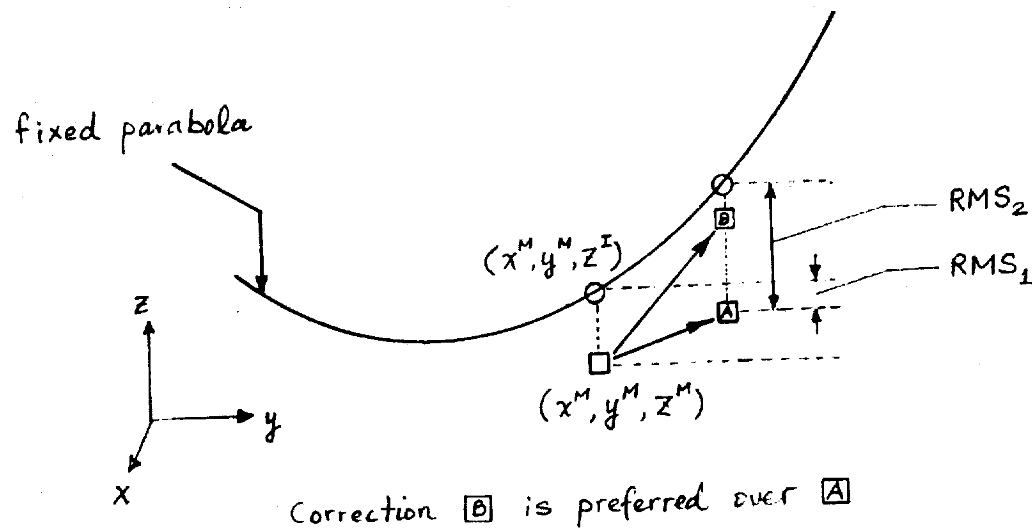

FIGURE 3: The effect of error distribution on parabola changes

		$\Delta f_2 = .5$	$\Delta f_1 = .1$
.107	.120	$\Delta V_{x_2} = 4.2$	$\Delta V_{x_1} = .1$
		$\Delta V_{y_2} = 7.1$	$\Delta V_{y_1} = 4.1$
		$\Delta f_3 = 2.4$	$\Delta f_4 = 1.1$
.142	.132	$\Delta V_{x_3} = 11.3$	$\Delta V_{x_4} = 8.8$
		$\Delta V_{y_3} = 16.2$	$\Delta V_{y_4} = 8.7$

Initial RMS Error | % Parameter Change

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15. Supplementary Notes

Kyong Been Lim: Planning Research Corporation, Hampton, Virginia.
Jer-Nan Juang: Langley Research Center, Hampton, Virginia.
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16. Abstract

In this report, the results of recent surface adjustment of the 15-meter diameter hoop-column antenna are presented. A least-squares differential alogrithm is used to adjust the surface shape as close as possible to a perfect parabola. Since the desired perfect parabola is not uniquely known a priori, parameters of the perfect parabola are included in the design vector along with the cable length changes. As an extension to an earlier study, lateral sensitivity is included in the least-squares adjustment procedure. In addition, the effect of cable length uncertainties on the surface RMS error is considered and an error bound is derived. The results in this study indicate an improvement over earlier studies. The sensitivity analysis provided a quantatative measure of the needed accuracy of the cable adjustments in the laboratory. Recommendations are included to further enhance shape adjustment.

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