TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 41

INFLUENCE OF SPAN AND LOAD PER SQUARE METER ON THE AIR FORCES OF THE SUPPORTING SURFACE.

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A. Betz.

Translated from Technische Berichte, Vol. I, Section 4,
By
Lt. Walter S. Diehl,
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It should be clear that in order to obtain a lift it is necessary that the air which flows past an aerofoil be given a downward acceleration; indeed the lift can be only the reaction produced by the downward acceleration of the flowing air. The motion of the air in the neighborhood of an aerofoil may be followed theoretically with great exactness. In the following, it will be undertaken to make understood, through the simplest possible considerations, the effect of span and loading on the air force on an aerofoil, and while these do not form a strong proof of the correctness of the formulae developed, yet they explain the essential features of the phenomena.

One may obtain a practical mental picture of the flow in the region of an aerofoil by imagining that at a given instant a horizontal surface behind the wing is moving downward with a velocity w. This surface has a breadth equal to the span b and extends to an infinite distance to the rear of the aerofoil (infinite in relation to the point from which the flight of the aerofoil began.) The qualitative course of such a flow requires no special difficulty in presentation. The entire air column above and below the surface partakes of the downward motion, although the velocity will be less and less as the distance from the surface increases. In order to further simplify our investigation let us pass from this infinite air mass, with its velocity decreasing to zero, to an air mass of finite boundaries which is in uniform motion with the velocity w. Let the breadth of this air column be b and its height be h.

The lift A is equal to the vertical momentum imparted to the air per second, or to the product of the mass by the vertical velocity. Since in each second, a mass equal to $(\frac{\gamma}{g} bhv)$ is affected by the aerofoil and is influenced anew (the hypothetical board becomes v m. longer each second), the imparted vertical momentum and therefore the lift, is

$$A = \frac{\gamma}{g} bhvw$$

where

$\frac{\gamma}{g}$ = air density,

v = velocity of flight.

Concerning the magnitude of the effective height h, we can not be certain without experiment. Yet it is clear that it may be expressed in multiples of the breadth of the board, i.e., of the space b. We can therefore say that

$$h = \kappa b$$

The vertical velocity behind the aerofoil is then given by

$$w = \frac{A}{\frac{\gamma}{g} \cdot \kappa b^2 v}$$

If we introduce the impact pressure

$$q = \frac{1}{2} \frac{\gamma}{g} v^2$$

and the surface F, it is

$$w = \frac{v}{2 \kappa} \cdot \frac{A}{qF} \cdot \frac{F}{b^2}$$

where $\frac{A}{qF} = c_a$ the lift coefficient and b^2/F is the mean aspect ratio of the aerofoil. (If the chord t is constant $F = bt$ and $\frac{b^2}{F} = \frac{b}{t}$). The velocity w prevails some distance to the rear of the aerofoil in the path of flight, at some distance forward the air is yet undisturbed. It is therefore permissible to assume that at the aerofoil a vertical velocity of w' prevails and that its value is between o and w. Considerations of similarity lead to the conclusion that this velocity may be taken proportional to w so that we obtain

$$\frac{w'}{v} = \frac{1}{2 \kappa} \cdot c_a \cdot \frac{F}{b^2}$$

if $\kappa' = \kappa \frac{w}{w'}$

Ordinarily the vertical velocity w' will not be constant over the entire aerofoil. Its variation along the span will depend upon the distribution of lift along the span. The more exact consideration* of the actual course of the streamlines and their influence upon the coefficients shows that in the best case the vertical velocity is constant along the span; κ' therefore has the value of $\pi/2$. This occurs when the distribution of lift along the span is proportional to the ordinates of a half ellipse (see Fig. 1). From the usual performance of aerofoils it appears the mean value of κ' previously derived is not essentially different from this best value. We can therefore see that the formula

$$\frac{w'}{v} = \frac{1}{\pi} \cdot c_a \cdot \frac{F}{b^2}$$

using the best value for $\frac{w'}{v}$ which is possible for the given proportions (lift and span) gives very accurate approximations for the normal aerofoil. Pronounced deviations are to be expected if the aerofoil is strongly washed out or if the sections at tips are essentially different from those at the center (Taube).

The aerofoil is in an airstream deflected downwards by its motion. The tangent of the angle of inclination of this airstream to the horizontal has the mean value $\frac{w'}{v}$. It is on one hand proportional to the lift coefficient $c_a = \frac{A}{qF}$

* Originating in the thorough research of Prof. Prandtl that whenever the lift varies along the span vortices are always formed especially in the proximity of the wing tips and are dirtributed in an approximate straight line behind the aerofoil. It is assumed that the "field" set up by these vortices give the actual course or direction of the air motion.

and on the other hand approximately proportional to the mean aspect ratio $\frac{b^2}{F}$. The inclination will be zero if there is no lift, and also for all lifts in the case where the aspect ratio is infinite. As a result of this latter condition the aerofoil of infinite aspect ratio has special significance because after a fashion it supplies a normal form to which the force on a chosen aerofoil in an undistorted airstream, without the disturbing influence of a deflected air stream, may be compared. The force on an aerofoil of finite span is therefore the same as that obtained on an aerofoil of infinite span placed in an airstream which has the inclination of $\frac{w'}{v}$. The calculation is made according to the following outline, in which the angle of inclination of the airstream is denoted by φ ($\tan \varphi = \frac{w'}{v}$) and the angle of the chord of the aerofoil by α. The values for the aerofoil of infinite span are denoted by the subscript ∞.

The aerofoil of infinite span at the angle of attack α_∞ has a lift coefficient of c_{a_∞} and a resistance coefficient of c_{w_∞}; the resultant is therefore c_{r_∞} (Fig. 2). In order to find the force on an aerofoil of finite span one must next obtain the angle of inclination φ from the equation

$$\tan \varphi = \frac{1}{\pi} \cdot c_a \cdot \frac{F}{b^2}$$

(Since the lift coefficient c_a of the aerofoil of finite span is very nearly equal to c_{a_∞}, the value c_{a_∞} may be used to calculate φ).

If we now consider the entire figure to be turned through the angle φ (Fig. 3) we obtain the picture for the aerofoil of finite span in a horizontal airstream (Fig.4). In order to obtain the lift and drag coefficients for the aerofoil of finite span, we now need but resolve the resultant c_r which has been given the additional inclination φ into its vertical and horizontal components c_a and c_w. From the customary small angle approximations ($\cos\epsilon = 1$, $\sin\epsilon = \tan\epsilon = \epsilon$) there result the following simple formulae

$$\left.\begin{aligned} \alpha &= \alpha_\infty + \varphi \\ c_a &= c_{a\infty} \\ c_w &= c_{w\infty} + c_a \cdot \varphi \\ \frac{W}{A} &= \frac{c_w}{c_a} = \left(\frac{W}{A}\right)_\infty + \varphi \end{aligned}\right\} \quad \text{in which} \quad \varphi = \frac{1}{\pi} \cdot c_a \cdot \frac{F}{b^2}$$

There is one fact which is not taken into account in these formulae, but it is practically of slight importance: the vertical velocity w' of the deflected airstream at the leading edge of the aerofoil is somewhat less than that at the trailing edge, the airstream is therefore somewhat curved. Accordingly it is necessary that the aerofoil of finite span be more deeply cambered than the aerofoil of infinite span in order that they may have the same resultant force.

2. Practical Formulae.

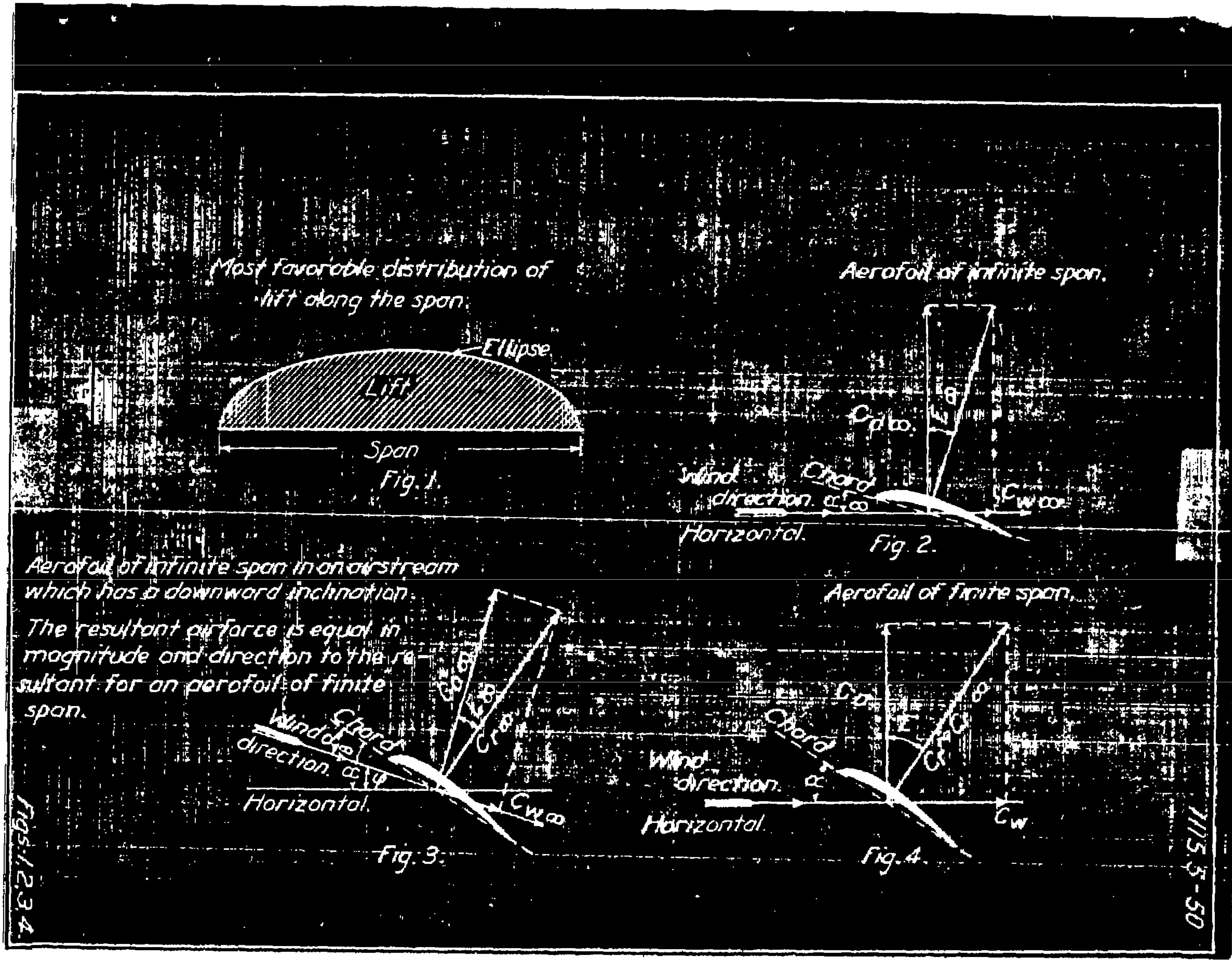
It quite frequently happens in practice that it is desired to obtain the characteristics of an aerofoil of given aspect ratio when we have the data on the same profile for another aspect ratio. Denoting the coefficients for one aerofoil by the subscript 1 and those for the other by 2, we have the following equatiôns which give the desired result if we compare each surface with the aerofoil of infinite length, using the above formulae, and take the difference of the results.

$$\left.\begin{aligned} \alpha_2 &= \alpha_1 + \psi \\ c_{a2} &= c_{a1} = c_a \\ c_{w2} &= c_{w1} = c_a \cdot \psi \\ \left(\frac{W}{A}\right)_2 &= \left(\frac{W}{A}\right)_1 + \psi \end{aligned}\right\} \quad \text{in which} \quad \psi = \frac{1}{\pi} \cdot c_a \left(\frac{F_2}{b_2^2} - \frac{F_1}{b_1^2} \right)$$

The results of these calculations do not give the air-force on the two aerofoils for the same angles of incidence. Ordinarily this is no objection since the values may be plotted in a curve of air force against angle of attack, which may be extended to cover the entire range of angles for the new aerofoil.

Note. The angles are measured in radians. They may be converted to degrees by the use of the factor 57.3 for example

$$\alpha^\circ = \alpha^\circ_\infty + 57.3^\circ \ \varphi .$$


Most favorable distribution of
lift along the span.
Ellipse
Lift
Span
Fig. 1.
Aerofoil of infinite span.
θ
$C_{a\infty}$
Wind
direction.
Chord
α_∞
$C_{w\infty}$
Horizontal.
Fig. 2.
Aerofoil of infinite span in an airstream
which has a downward inclination.
The resultant airforce is equal in
magnitude and direction to the re-
sultant for an aerofoil of finite
span.
Chord
Wind
direction.
α
φ
θ
C_r
Horizontal.
$C_{w\infty}$
Fig. 3.
Aerofoil of finite span.
C_a
Chord
θ
ε
C_r
Wind
direction.
α
Horizontal.
C_w
Fig. 4.
Figs. 1, 2, 3, 4.
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