in K decay. The interference between the two contributions can produce rate differences between the decay and its CP conjugate. These effects are of particular interest because they do not depend upon strong phases and hence the measured asymmetries can be directly related to the CKM phases. In any given channel there can also be direct CP violation in addition to the CP violation due to mixing, this complicates the relationship between the measured asymmetry and the CKM parameters. We will briefly discuss later techniques to separate such contributions.

A third type of CP violation, referred to as indirect CP violation, would arise from any difference in the widths $\Delta\Gamma$ of the two mass eigenstates, or more precisely from complex mixing effects that would also give a nonvanishing lifetime difference for the two B mass eigenstates. Such effects are expected to be tiny in the B system. In the particular case of semileptonic decays there are no penguin diagram contributions, and hence, in the approximations used throughout the discussion above, the CP-conjugate decay rates are equal. However there is a possible indirect CP-violating asymmetry in these decays, which would be seen as an charge asymmetry in the same-sign dilepton events produced via mixing from an incoherent state that initially contains a $B^0\overline{B}^0$ pair. This asymmetry vanishes with $\Delta\Gamma$; it is expected to be no larger than 1% [7].

A simple way to distinguish the three types of CP violation is to note that direct CP violation occurs when $|\mathcal{A}/\overline{\mathcal{A}}| \neq 1$, indirect CP violation requires $|q/p| \neq 1$, but CP violation due the interference between decays with and without mixing can occur when both quantities have unit absolute value; it requires only that the ratio of them has a nonzero weak phase [8].

Neutral B decays to CP eigenstates: The decays of neutral B's into a CP eigenstates is of particular interest because many of these decays allow clean theoretical interpretation in terms of the parameters of the Standard Model [9]. We denote such a state by f_{CP}, for example $f_{CP} = J/\psi(1S)K_S$ or $f_{CP} = \pi\pi$, and define the amplitudes

$$\mathcal{A}_{f_{CP}} \equiv \langle f_{CP}|B^0\rangle , \quad \overline{\mathcal{A}}_{f_{CP}} \equiv \left\langle f_{CP}|\overline{B}^0\right\rangle \ . \tag{12}$$

For convenience let us introduce the quantity $r_{f_{CP}}$

$$r_{f_{CP}} \equiv \frac{q}{p}\,\frac{\overline{\mathcal{A}}}{\mathcal{A}} \ . \tag{13}$$

In the limit of no CP violation, $r_{f_{CP}} = \pm 1$ where the sign is given by the CP eigenvalue of the particular state f_{CP}. (Note that in the literature the quantity $r_{f_{CP}}$ is frequently denoted by λ, but we have chosen to avoid this notation as it introduces a confusion with the $\lambda = \sin(\theta_{\text{Cabibbo}})$ in the Wolfenstein parameterization of the CKM matrix.)

The time-dependent rates for initially pure B^0 or $\overline{B}^0$ states to decay into a final state f_{CP} at time t is then given by:

$$\begin{aligned}
&\langle f_{CP}|B^0_{\text{phys}}(t)\rangle = \\
&\quad \mathcal{A}\exp(-\Gamma t/2)\exp(-iMt) \\
&\qquad \times [\cos(\Delta M t/2) + i r_{f_{CP}}\sin(\Delta M t/2)] \ , \\
&\left\langle f_{CP}|\overline{B}^0_{\text{phys}}(t)\right\rangle = \\
&\quad \mathcal{A}\exp(-\Gamma t/2)\exp(-iMt)(p/q) \\
&\qquad \times [i\sin(\Delta M t/2) + r_{f_{CP}}\cos(\Delta M t/2)] \ .
\end{aligned} \tag{14}$$

Thus

$$\begin{aligned}
&\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) = \\
&\quad |\mathcal{A}|^2 e^{-\Gamma t}\Bigg[\frac{1+|r_{f_{CP}}|^2}{2} + \frac{1-|r_{f_{CP}}|^2}{2} \\
&\qquad \times \cos(\Delta M t) - \text{Im}\ r_{f_{CP}}\sin(\Delta M t)\Bigg], \\
&\Gamma(\overline{B}^0_{\text{phys}}(t) \to f_{CP}) = \\
&\quad |\mathcal{A}|^2 e^{-\Gamma t}\Bigg[\frac{1+|r_{f_{CP}}|^2}{2} - \frac{1-|r_{f_{CP}}|^2}{2} \\
&\qquad \times \cos(\Delta M t) + \text{Im}\ r_{f_{CP}}\sin(\Delta M t)\Bigg] \ .
\end{aligned} \tag{15}$$

The time-dependent CP asymmetry is

$$a_{f_{CP}}(t) \equiv \frac{\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) - \Gamma(\overline{B}^0_{\text{phys}}(t) \to f_{CP})}{\Gamma(B^0_{\text{phys}}(t) \to f_{CP}) + \Gamma(\overline{B}^0_{\text{phys}}(t) \to f_{CP})} \tag{16}$$

and thus

$$a_{f_{CP}}(t) = \frac{(1-|r_{f_{CP}}|^2)\cos(\Delta M t) - 2\text{Im}\ (r_{f_{CP}})\sin(\Delta M t)}{1+|r_{f_{CP}}|^2} \tag{17}$$

When the small difference in width of the two B states is ignored we can write

$$(q/p)_{B_d} = \frac{(V^*_{tb}V_{td})}{V_{tb}V^*_{td})} \ , \qquad (q/p)_{B_s} = \frac{(V^*_{tb}V_{ts})}{V_{tb}V^*_{ts})} \ .$$

and thus

$$q/p = e^{-2i\phi_M} \ , \tag{18}$$

where $2\phi_M$ denotes the CKM phase of the B–$\overline{B}$ mixing diagram. (Note, Eq. (9) allows us to write the sum of all contributions to the mixing in this form, up to small corrections of order $(m_c^2 - m_u^2)/m_W^2$.) Further, when there is no direct CP violation in a channel that is when all amplitudes that contribute to the decay have the same CKM phase, ϕ_D, then $|\mathcal{A}/\overline{\mathcal{A}}| = 1$. In that case $r_{f_{CP}}$ depends on CKM-matrix parameters only, without hadronic uncertainties, and can be written $r_{f_{CP}} = \pm e^{-2i(\phi_D+\phi_M)}$. Then Eq. (17) simplifies to

$$\begin{aligned}
a_{f_{CP}}(t) &= \mp\text{Im}\ (r_{f_{CP}})\sin(\Delta M t) \\
&= \pm\sin(2(\phi_M+\phi_D))\sin(\Delta M t) \ .
\end{aligned} \tag{19}$$