The relevance of positivity in spin physics

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Outline

- **Exclusive Reactions**
  - Spin 0 + Spin 1/2 $\longrightarrow$ Spin 0 + Spin 1/2
  - Spin 1/2 + Spin 1/2 $\longrightarrow$ Spin 1/2 + Spin 1/2
  - Exclusive pseudoscalar meson photoproduction
- **Inclusive Reactions**
  - Total cross sections
  - One-particle inclusive hadronic reactions
  - Deep inelastic scattering
  - Parton distributions
  - Single spin asymmetries and hyperon production

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SPIN OBSERVABLES AND SPIN STRUCTURE FUNCTIONS: INEQUALITIES AND DYNAMICS

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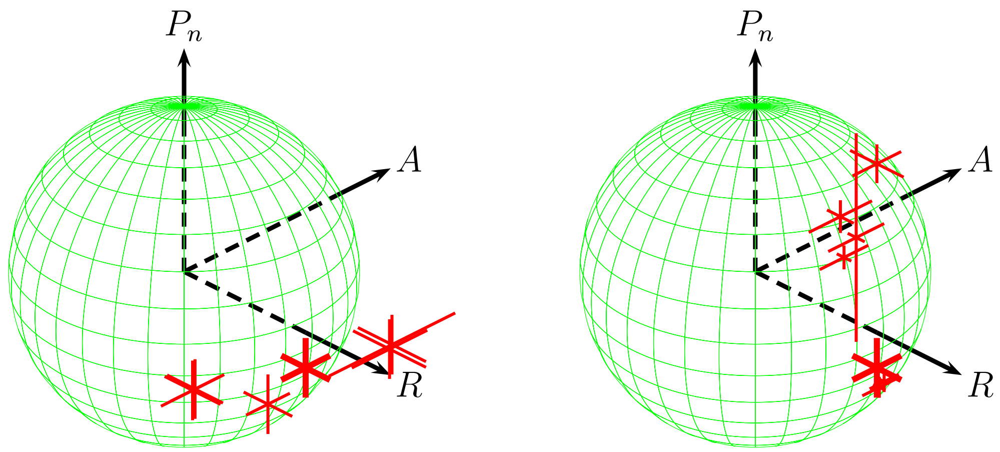
Spin 0 + Spin 1/2 $\longrightarrow$ Spin 0 + Spin 1/2

A scattering process involving spinning particles and described by n *complex* amplitudes is fully determined by $(2n-1)$ *real* functions, up to an over-all phase. Since there are n^2 possible measurements for this reaction, we must have $(n-1)^2$ independent quadratic relations between the n^2 observables.

First consider the simplest case of an exclusive two-body reaction with a spin-0 and a spin-1/2 particle, namely $0+1/2 \rightarrow 0+1/2$. As an example $\pi N \rightarrow \pi N$ is described in terms of 2 amplitudes, the non-flip f_+ and the flip f_-, so we have 4 observables, the cross section $d\sigma/dt$, the polarization P_n and two rotation parameters R and A. There is one well known quadratic relation, that is $P_n^2 + A^2 + R^2 = 1$.

$\pi N \to \pi N$



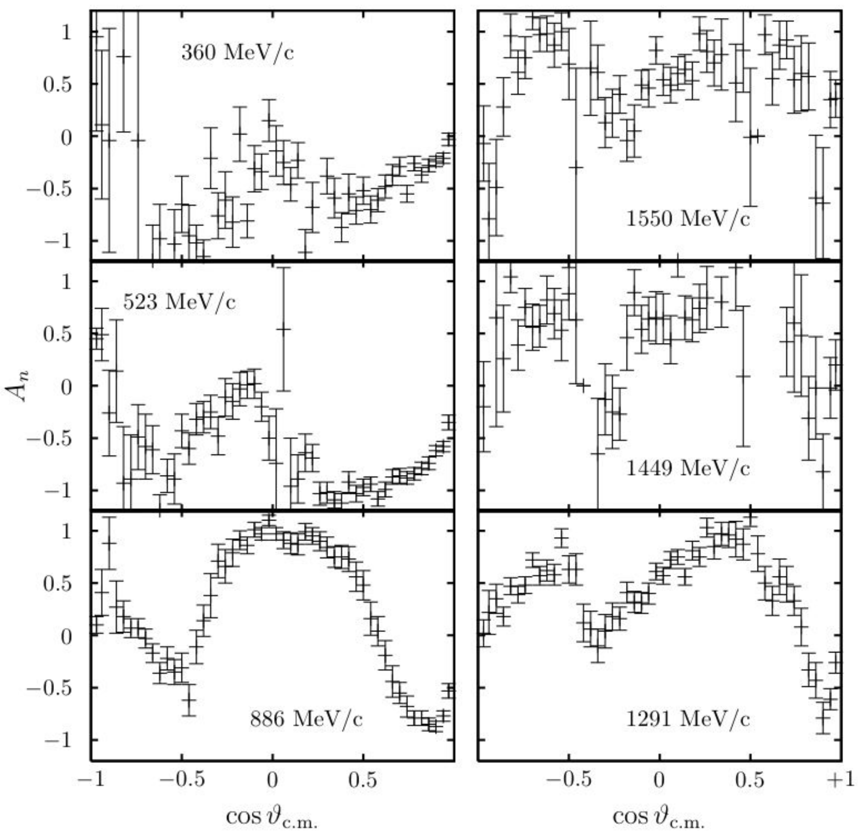
When all three are measured the constraint is reasonably satisfied by the data

$\bar{p}p \longrightarrow \pi^+\pi^-$

Large spin effects observed in the crossed reaction (LEAR)

One has a similar identity as for πN $A_n^2 + A_{mm}^2 + A_{ml}^2 = 1$

so if $A_n = 1$ then $A_{mm} = A_{ml} = 0$



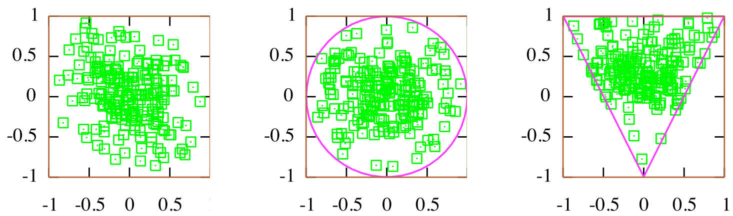
Spin 1/2 + Spin 1/2 $\rightarrow$ Spin 1/2 + Spin 1/2 (C.Bourrely, J.S., Phys. Rev. D12,2932 (1975))

For the reaction $1/2 + 1/2 \rightarrow 1/2 + 1/2$, e.g. nucleon-nucleon scattering, we have 5 amplitudes, therefore 25 observables and 16 quadratic relations between them. For the derivation one considers a 5x5 matrix of the observables which is positive Hermitian and the final results can be found in B.S. These relations are very valuable to check the data and when one observable is not measured, it is set to zero and the equality becomes an inequality.

The reaction $\bar{p}p \rightarrow \bar{\Lambda}\Lambda$, is described in terms of 6 amplitudes and in this case one finds the following very simple condition $C_{LL}^2 + D_{NN}^2 \leq 1$, between the two-spin correlation parameters C_{LL} and D_{NN}. This condition turns out to be very usefull because if one of the parameters is close to one, the other one is bounded to be near zero. It was found, in a certain kinematic region $C_{LL} = -1$, so before making any measurement, with a polarized target, one can conclude that $D_{NN} \sim 0$, which is also a no-go theorem for some theoretical considerations (J.M. Richard, Phys. Lett. B369,358 (1996)).

Empirical Method for Pairs of Spin Observables

Consists of generating randomly 6 complex amplitudes
to compute the observables and look at constraints
P_n vs A_n (square), A_n vs D_{mm} (disk), P_n vs C_{mm} (triangle)



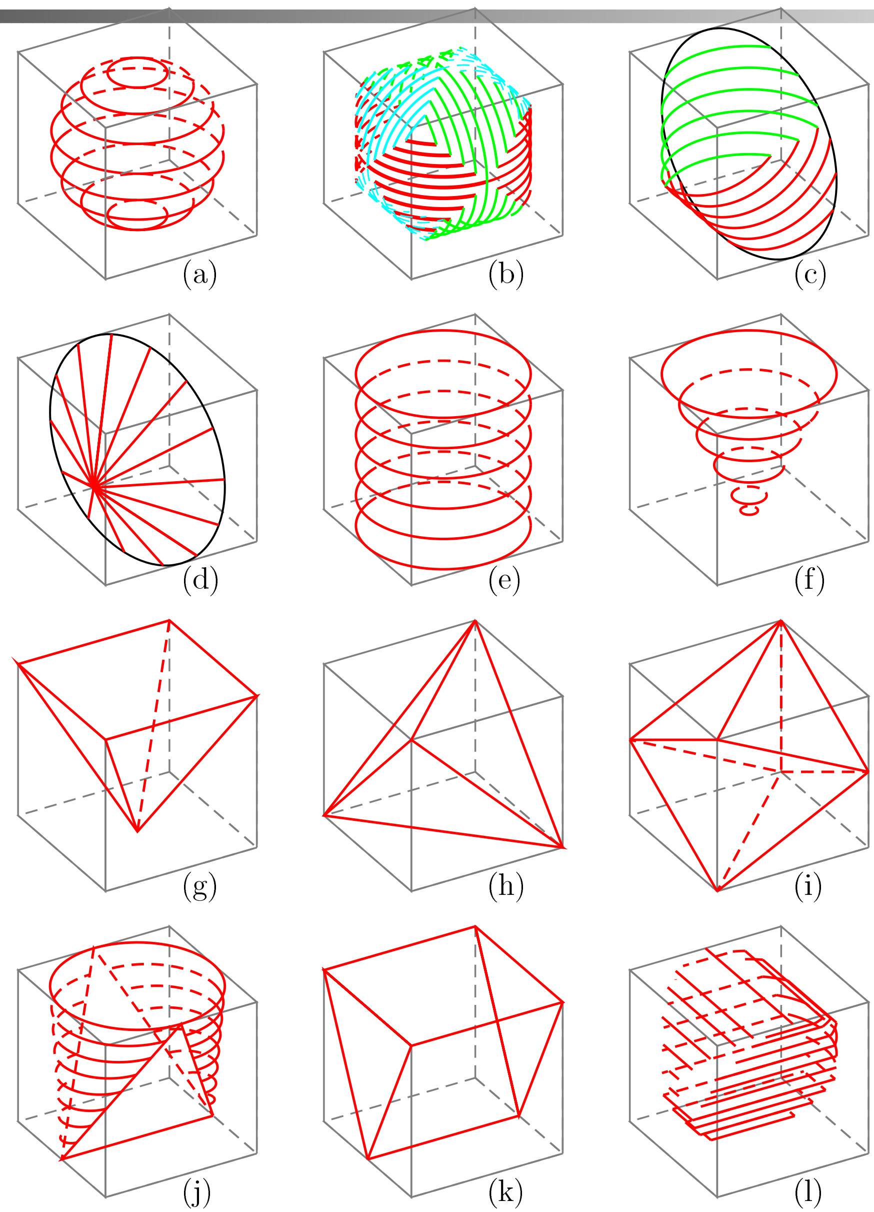
Case of three observables

The empirical method was extended to the case of three simultaneous observables.
Several shapes observed for the allowed domain:
sphere, cone, pyramid, tetrahedron, octahedron, intersection of two cylinders, intersection of three cylinders, coffee filter, etc...
In particular:

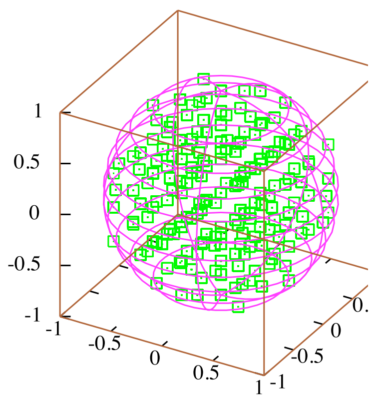
any triple is constrained,

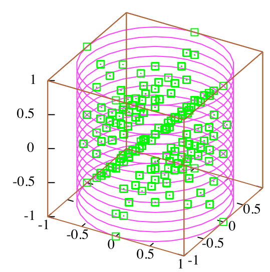
constraint even if all pairs are unconstrained.
For instance the tetrahedron (next slide) has $[-1, +1]^2$ projection on all faces.

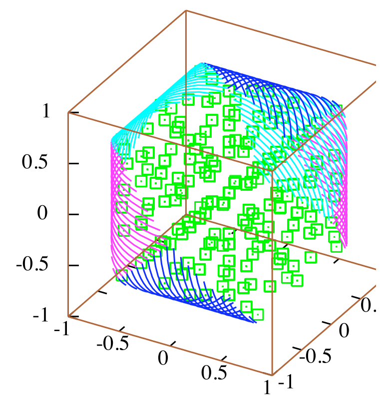
Case of three observables

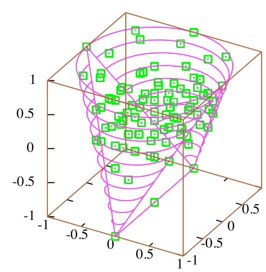


Case of three observables: Some remarkable shapes







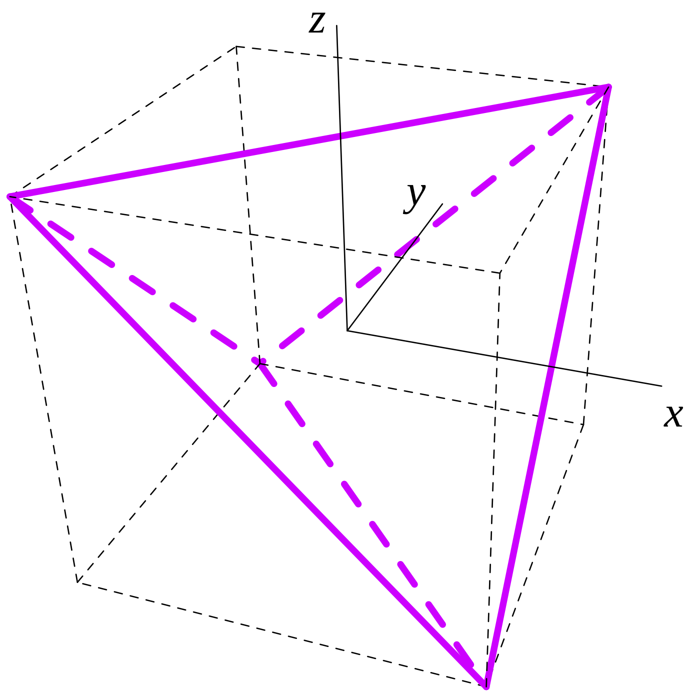


Exclusive pseudoscalar meson photoproduction

Consider the particular reactions $\gamma N \to KY$, with $Y = \Lambda, \Sigma$, where the incoming photon beam is polarized, the nucleon target is polarized and the polarization of the outgoing Λ, Σ is measured. For these parity conserving exclusive reactions, one can define 4 helicity amplitudes denoted $f_{K,Y;\gamma,N}$, one non-flip $f_{0,-1/2;+1,+1/2}$, two single-flip $f_{0,-1/2;+1,-1/2}$ and $f_{0,+1/2;+1,+1/2}$ and one double-flip $f_{0,+1/2;+1,-1/2}$.
So the complete knowledge of these reactions requires, up to an overall phase, the determination of 7 real functions. Moreover, one can perform 16 different experiments, which are the number of bilinear products of the 4 amplitudes. In addition to the unpolarized differential cross section, there are 3 single spin asymmetries, the linearly polarized photon asymmetry Σ^γ, the polarized target asymmetry A_N and the recoil baryon polarization P_Y. Finally, there are 4 double correlations between the target and the recoil baryon spins in the scattering plane with unpolarized photons, 4 double spin correlations with linearly polarized photons and 4 double spin correlations with circularly polarized photons. Clearly, these 16 measurements which can be expressed in terms of 7 real functions, cannot be independent and they must be constrained by 9 quadratic relations among these parameters. This implies that the allowed domain of 2 observables is not the square $[-1,+1]^2$, but rather a disk $O_i^2 + O_j^2 \leq 1$ (or for 3 a sphere instead of a cube). Futhermore, we have the linear relations $|A_N \pm P_Y| \leq 1 \pm \Sigma^\gamma$, which give a tetrahedron.

Exclusive pseudoscalar meson photoproduction

$x = A_N, y = P_Y, z = \Sigma^\gamma$. The allowed volume is 1/3 of the entire cube



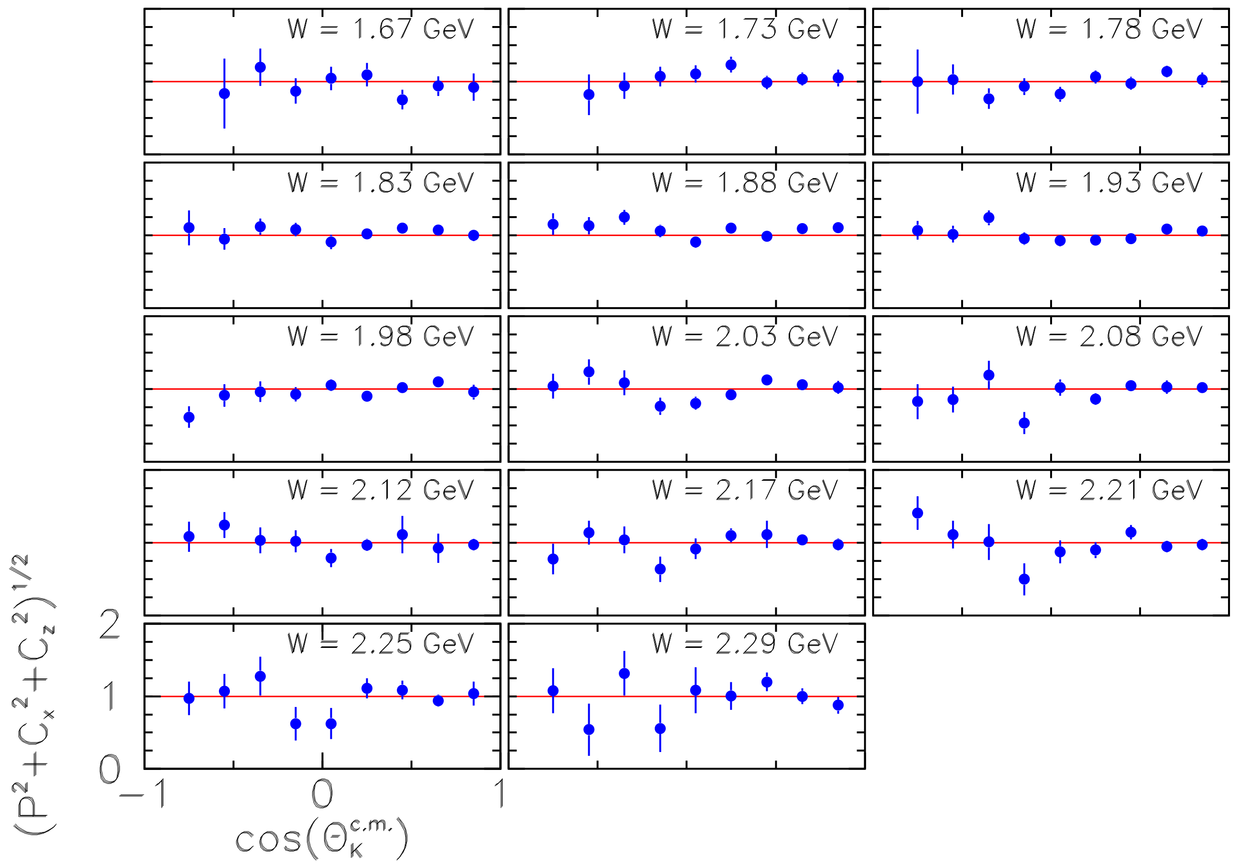
Exclusive pseudoscalar meson photoproduction

Let us consider 3 spin observables, namely P_Y and the double correlation parameters between the circularly polarized photon and the recoil baryon spin along the directions $\hat{x}$ and $\hat{z}$ in the scattering plane, C_x^Y and C_z^Y. In such a case, it can be shown that one has the following rigorous inequality $(P_Y)^2 + (C_x^Y)^2 + (C_z^Y)^2 \leq 1$,
a constraint which must be satisfied for any values of the energy and of the scattering angle of the reactions $\gamma N \to KY$, with $Y = \Lambda, \Sigma$.
This constraint involves three spin parameters, whose values are strongly correlated since they are restricted to lie in a certain domain. For instance, if one of the parameters has the maximum allowed values ± 1, the other two parameters must be zero. In addition to the differential cross section for $\gamma p \to K^+ Y$ measured for center-of-mass energies W between 1.6 and 2.53 GeV and for $-0.85 <$ cos$\theta_K^{c.m} < +0.95$, the measurement of these three parameters has also been performed using the CLAS spectrometer in Hall B at Jefferson Lab.

The constraint is saturated

Exclusive pseudoscalar meson photoproduction

This implies the saturation of other quadratic constraints with three observables



Total cross sections (J. S and D. Wray, Phys. Lett. B43, 514 (1973))

Let us consider the spin-dependent total cross sections for the scattering of two spin-1/2 particles

$$a_{s=1/2} + b_{s=1/2} \rightarrow anything\ ,$$

which are, for example, $pp, \bar{p}p, p\Lambda, pn, etc....$ The reaction can be described in terms of three independent observables which are the unpolarized cross section

$$\sigma_{tot} = 1/2[\sigma_{tot}(+,+) + \sigma_{tot}(+,-)] = 1/2[\sigma_{tot}(\uparrow,\uparrow) + \sigma_{tot}(\uparrow,\downarrow)]\ ,$$

and the two asymmetries

$$\Delta\sigma_L = \sigma_{tot}(+,+) - \sigma_{tot}(+,-) \text{ and } \Delta\sigma_T = \sigma_{tot}(\uparrow,\downarrow) - \sigma_{tot}(\uparrow,\uparrow)\ .$$

Here +(-) denote the longitudinally polarized (or helicity) states of a and b and $\uparrow$ $(\downarrow)$ their transversely polarized states.
If $\mathbf{P_a}$ and $\mathbf{P_b}$ are the polarization unit vectors of a and b, the polarized total cross sections corresponding to the reaction

$$\sigma_{tot}(\mathbf{P_a}, \mathbf{P_b}) = \mathbf{Tr}(\mathcal{M}\rho)\ .$$

Total cross sections (J. S and D. Wray, Phys. Lett. B43, 514 (1973))

$\mathcal{M}$ denotes the forward scattering amplitude for the elastic reaction

$$a + b \rightarrow a + b\ ,$$

and ρ is the 4x4 density matrix $\rho = 1/4(\mathbf{1}_2 + \mathbf{P_a}\sigma) \otimes (\mathbf{1_2} + \mathbf{P_b}\sigma)$, where $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ stands for the three 2x2 Pauli matrices and $\mathbf{1}_2$ is the 2x2 unit matrix. $\mathcal{M}$ is the following 4x4 matrix

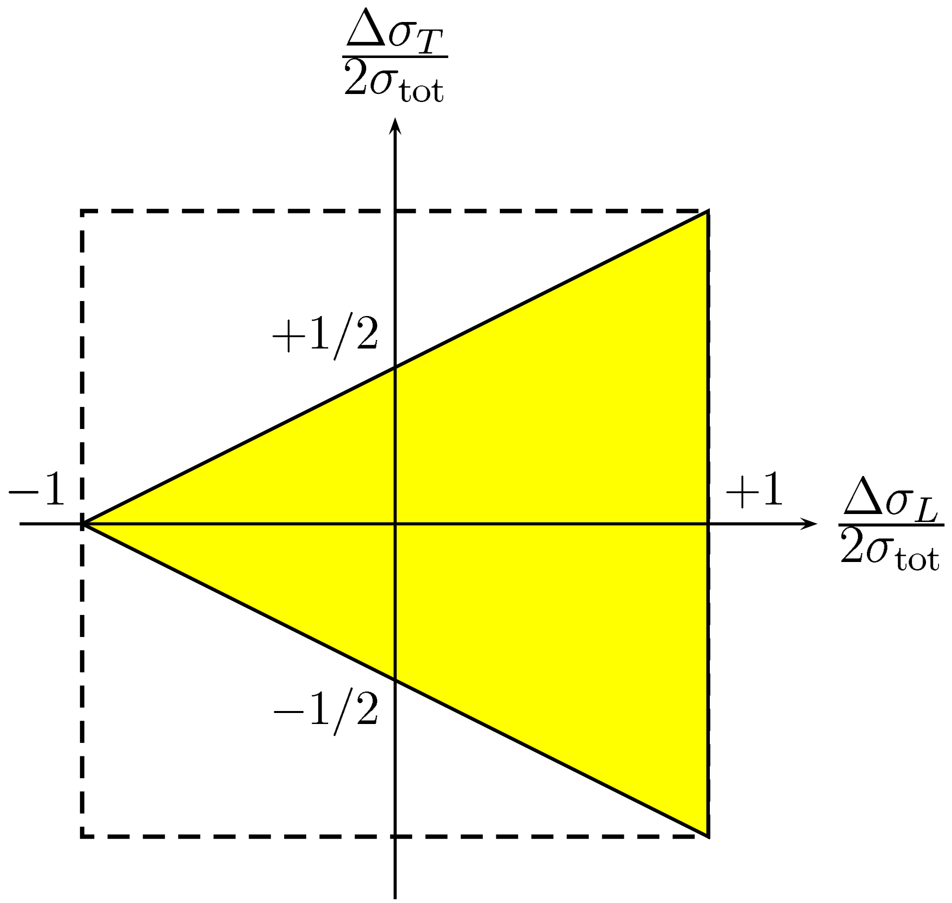
$$\mathcal{M} = [2\sigma_{tot}\mathbf{1}_4 + \Delta\sigma_L\sigma_{1z} \otimes \sigma_{2z} + \Delta\sigma_T(\sigma_{1x} \otimes \sigma_{2x} + \sigma_{1y} \otimes \sigma_{2y})]\ ,$$

where $\mathbf{1}_4$ is the 4x4 unit matrix. $\mathcal{M}$ must be Hermitian and *positive*, which implies that all its principal minors (*i.e.* subdeterminants of $\mathcal{M}$ with diagonal elements) must be *positive*. In terms of the three observables defined above, we get two trivial conditions, *i.e.* $|\Delta\sigma_i| \leq 2\sigma_{tot}, (i = L, T)$ and one non-trivial positivity bound, namely

$$|\Delta\sigma_T| \leq \sigma_{tot} + \Delta\sigma_L/2\ .$$

For $p_{lab} \leq 1$GeV/c, σ_{tot} can be as small as 25mb or so and $\Delta\sigma_L$ can reach the value -30mb, so one gets $|\Delta\sigma_T| \leq 10$mb, with some errors.

Total cross sections (J. S and D. Wray, Phys. Lett. B43, 514 (1973))



Spin transfer observables (M.Doncel, A.Mendez, Phys. Lett. B41, 83 (1972))

For a parity conserving inclusive reaction of the type, (e.g. $p^{\uparrow}p \rightarrow \Lambda^{\uparrow}X$)

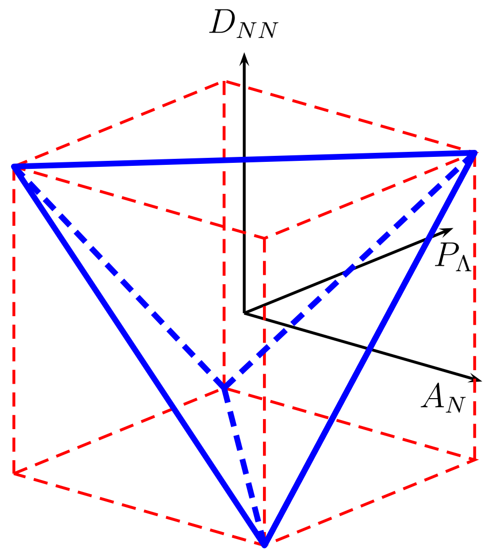
$$spin 1/2 + unpolarized \rightarrow spin 1/2 + anything,$$

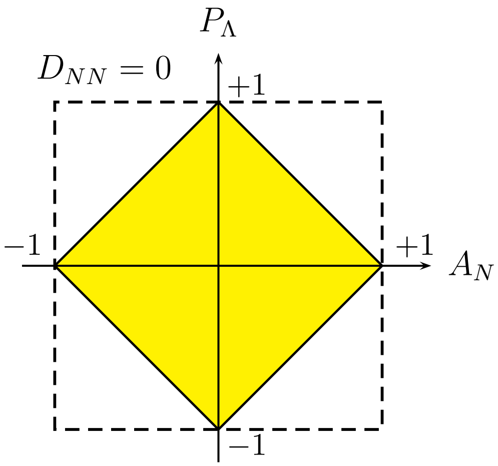
one can define eight observables, namely, the unpolarized differential cross section, the spin parameters P,A,R, for the initial and final spin 1/2 particles and the depolarization parameter D, which corresponds to a spin transfer between the proton and the Λ. Each spin parameter depends on the two particles polarization either transverse or longitudinal with respect to the scattering plane. Several years ago, very general inequalities constraints on these parameters were derived. If we concentrate for the moment, on the case where the particle spins are normal to the scattering plane, we obtain

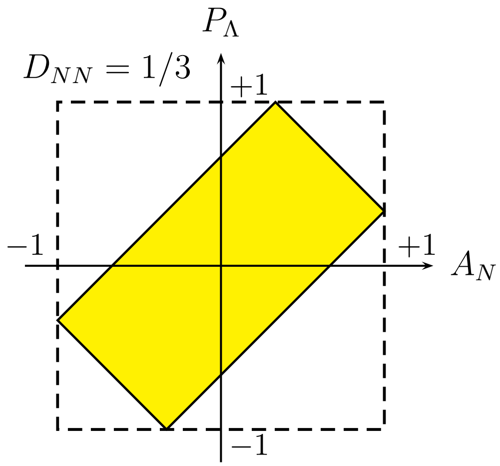
$$1 + D_{NN} \geq |P_{\Lambda} + A_N| \text{ and } 1 - D_{NN} \geq |P_{\Lambda} - A_N|,$$

these constraints must be satisfied for any kinematic values of the variables $x_F, p_T, \sqrt{s}$. For a given D_{NN} one has a domain for P_{Λ} and A_N. $D_{NN} = \pm 1, P_{\Lambda} = \pm A_N$.
NOTE: The eight TMD quark distributions obey the same constraints since they are related to $nucleon(p, S) \rightarrow quark(k, S') + X$

Spin transfer observables (M.Doncel, A.Mendez, Phys. Lett. B41, 83 (1972))







Spin transfer observables (M.Doncel, A.Mendez, Phys. Lett. B41, 83 (1972))

At FNAL, the experiment E-704 has performed the measurement of $P_\Lambda,\ A_N$ and D_{NN} with transversely polarized proton beam at 200GeV/c, in the kinematic range $0.2 \leq x_F \leq 1.0$ and $0.1 \leq p_T \leq 1.5$GeV/c. Their data indicate a negative A_N and a value of D_{NN} up to 30%. If we take for instance $p_T \sim 1$GeV/c, $x_F \sim 0.8$, then $D_{NN} \sim 30\%$, for these kinematical values, they have $A_N \sim -10\%$ and $P_\Lambda \sim -30\%$, the inequalities are well satisfied. These constraints can be very useful to check the experimental data and errors, if they lie outside the allow domain of positivity. When the polarization is not normal to the scattering plane, one has new transfer parameters. In the case where both p and Λ are longitudinally polarized, one can measure D_{LL} and when each particle has a transverse polarization in the scattering plane, we define the depolarization D_{SS}. In that case, one get the non-trivial constraint

$$1 \pm D_{NN} \geq |D_{LL} \pm D_{SS}|,$$

Note that the two depolarizations in the right hand side have not yet been measured. One can see that from the knowledge of some measured observables, we can deduce from the positivity constraints, bounds on other unknown observables. (RHIC-BNL, see below)

Initial spin observables (J. S., Phys. Rev. Lett. 91, 092005 (2003))

Let us consider an inclusive reaction of the type

$$A_1(\text{spin}1/2) + A_2(\text{spin}1/2) \to B + X \ ,$$

where the spins of both initial spin1/2 particles can be in any possible directions and no polarization is observed in the final state. The observables of this reaction, which are the spin-dependent differential cross sections with respect to the momentum of B, can be expressed in terms of the discontinuities (with respect to the invariant mass squared of X) of the amplitudes for the forward three-body scattering $A_1 + A_2 + \bar{B} \to A_1 + A_2 + \bar{B}$, as given by the generalized optical theorem. We assume parity conservation, so the complete knowledge of this reaction requires the determination of eight real functions, which is the number of independent spin observables. In order to define these observables, we recall the standard notation $(A_1 A_2|BX)$, by which the spin directions of A_1, A_2, B and X are specified in one of the three possible directions L, N, S. We have in fact $(A_1 A_2|00)$ and $\mathbf{L}, \mathbf{N}, \mathbf{S}$ are unit vectors, in the center-of-mass system, along the incident momentum, along the normal to the scattering plane which contains A_1, A_2 and B, and along $\mathbf{N} \times \mathbf{L}$, respectively.

Initial spin observables (J. S., Phys. Rev. Lett. 91, 092005 (2003))

In addition to the unpolarized cross section $\sigma_0 = (00|00)$, there are seven spin dependent observables, two single transverse spin asymmetries

$$A_{1N} = (N0|00) \quad \text{and} \quad A_{2N} = (0N|00) \ ,$$

and five double-spin asymmetries

$$\begin{aligned} A_{LL} &= (LL|00) \ , \ A_{SS} = (SS|00) \ , \ A_{NN} = (NN|00) \ , \\ & A_{LS} = (LS|00) \ \text{ and } \ A_{SL} = (SL|00) \ . \end{aligned} \tag{1}$$

The state of polarization of the two spin1/2 particles A_1 and A_2 is characterized by the 2×2 density matrices ρ_1 and ρ_2 defined as

$$\rho_i = 1/2(\mathbf{1}_2 + \mathbf{P_i} \cdot \sigma) \qquad \text{i = 1,2} \quad ,$$

where $\mathbf{P_1}$ and $\mathbf{P_2}$ are the polarization unit vectors of A_1 and A_2, $\sigma = (\sigma_x, \sigma_y, \sigma_z)$ stands for the three 2×2 Pauli matrices and $\mathbf{1}_2$ is the 2×2 unit matrix.

Initial spin observables (J. S., Phys. Rev. Lett. 91, 092005 (2003))

The state of polarization of the incoming system in the reaction is described by the 4×4 density matrix ρ, which is the direct product $\rho = \rho_1 \otimes \rho_2$. The spin-dependent cross section corresponding to this reaction is

$$\sigma(\mathbf{P_1}, \mathbf{P_2}) = \mathrm{Tr}(\mathcal{M}\rho) \ ,$$

where $\mathcal{M}$ denotes the 4×4 cross section matrix which we shall parametrize in the following way

$$\begin{aligned} \mathcal{M} \quad &= \quad \sigma_0[\mathbf{1}_4 + A_{1N}\sigma_{1z} \otimes \mathbf{1}_2 + A_{2N}\mathbf{1}_2 \otimes \sigma_{2z} + A_{NN}\sigma_{1z} \otimes \sigma_{2z} + A_{LL}\sigma_{1x} \otimes \sigma_{2x} \\ & \quad\quad + A_{SS}\sigma_{1y} \otimes \sigma_{2y} + A_{LS}\sigma_{1x} \otimes \sigma_{2y} + A_{SL}\sigma_{1y} \otimes \sigma_{2x}]. \end{aligned}$$

Here $\mathbf{1}_4$ is the 4×4 unit matrix and σ_0 stands for the spin-averaged cross section. This expression is fully justified, since we have explicitly

$$\begin{aligned} \sigma(\mathbf{e_1}, \mathbf{e_2}) \quad &= \quad \sigma_0[1 + A_{1N}e_{1z} + A_{2N}e_{2z} + A_{NN}e_{1z}e_{2z} + \\ & \quad\quad A_{LL}e_{1x}e_{2x} + A_{SS}e_{1y}e_{2y} + A_{LS}e_{1x}e_{2y} + A_{SL}e_{1y}e_{2x}] \ . \end{aligned} \tag{1}$$

Initial spin observables (J. S., Phys. Rev. Lett. 91, 092005 (2003))

The crucial point is that $\mathcal{M}$ is a Hermitian and *positive* matrix and in order to derive the positivity conditions one should write the explicit expression of $\mathcal{M}$. Then one observes that by permuting two rows and two columns, it reduces to the simple form $\left(\begin{array}{c|c} \mathcal{M}_+ & 0 \\ \hline 0 & \mathcal{M}_- \end{array}\right)$, where $\mathcal{M}_\pm$ are 2×2 Hermitian matrices which must be positive, leading to the following *two* strongest constraints

$$(1 \pm A_{NN})^2 \geq (A_{1N} \pm A_{2N})^2 + (A_{LL} \mp A_{SS})^2 + (A_{LS} \pm A_{SL})^2 \ .$$

As special cases of the above equation, we have the *six* weaker constraints

$$1 \pm A_{NN} \geq |A_{1N} \pm A_{2N}| \ ,$$

$$1 \pm A_{NN} \geq |A_{LL} \mp A_{SS}| \ ,$$

$$1 \pm A_{NN} \geq |A_{LS} \pm A_{SL}| \ .$$

Initial spin observables (J. S., Phys. Rev. Lett. 91, 092005 (2003))

These constraints are very general and must hold in any kinematical region and for many different situations such as electron–proton scattering, electron–positron scattering or quark–quark scattering, but we now turn to a specific case, which is of direct relevance to the spin programme at the BNL-RHIC polarized pp collider. Now let us consider a proton–proton collision and let us call y the rapidity of the outgoing particle B. In this case since the initial particles are identical, we have $A_{1N}(y) = -A_{2N}(-y)$ and $A_{LS}(y) = -A_{SL}(-y)$. In this case we get two constraints among five independent spin observables

$$(1 \pm A_{NN}(y))^2 \geq (A_{1N}(y) \mp A_{1N}(-y))^2 + (A_{LL}(y) \mp A_{SS}(y))^2 \\ +(A_{LS}(y) \mp A_{LS}(-y))^2 \ . \quad (2)$$

This implies for $y = 0$, $1 - A_{NN}(0) \geq 2|A_N(0)|$ and $1 - A_{NN}(0) \geq 2|A_{SL}(0)|$, so that, the allowed range of A_N and A_{SL} is strongly reduced, if A_{NN} turns out to be large and positive. Conversely if $A_{NN} \simeq -1$, these constraints are useless. The direct consequence of these estimates is that $|A_N|$ and $|A_{SL}|$, for these processes are essentially bounded by 1/2. In addition, there are two other non-trivial constraints: $1 \geq |A_{LL} \pm A_{SS}|$.

Deep inelastic scattering (J. S. and O. Teryaev, Phys. Lett. B490, 106 (2000))

As we have seen above positivity is playing an important role in constraining spin-dependent observables, in particular by providing a bound for the transverse asymmetry in polarized deep inelastic scattering (DIS). It is a well-known condition established long time ago and based on an extensive study written in the form

$$|A_2| \leq \sqrt{R}\,,$$

where A_2 is the usual transverse asymmetry and $R = \sigma_L/\sigma_T$ is the standard ratio in DIS of the cross section of longitudinally to transversely polarized off-shell photons. It reflects a non-trivial positivity condition one has on the photon-nucleon forward helicity amplitudes. However a stronger bound was established more recently,

$$|A_2| \leq \sqrt{R(1+A_1)/2}\,,$$

where A_1 denotes the asymmetry with longitudinally polarized nucleon. It is worth noting that if A_1 and A_2 are measured with a good precision, can be used as a lower bound for R, a rather non-trivial constraint.

Neutrino deep inelastic scattering (C. Bourrely, O.Teryaev, J. S, PRD69,114019,(2004))

Consider the following reaction

$$\nu_\tau(\bar{\nu}_\tau) + N \to \tau^-(\tau^+) + X \quad ,$$

The corresponding hadronic tensor is expressed in terms of 5 structure functions

$$\begin{aligned} W_{\mu\nu}(p,q) \quad &= \quad -g_{\mu\nu}W_1(\nu,q^2) + \frac{p_\mu p_\nu}{M^2}W_2(\nu,q^2) - i\epsilon_{\mu\nu\alpha\beta}\frac{p^\alpha q^\beta}{2M^2}W_3(\nu,q^2) \\ &+\frac{q_\mu q_\nu}{M^2}W_4(\nu,q^2) + \frac{p_\mu q_\nu + q_\mu p_\nu}{2M^2}W_5(\nu,q^2)\ . \end{aligned}$$

The unpolarized cross section reads

$$\frac{d\sigma^\pm}{dE_\tau d\cos\theta} = \frac{G_F^2}{2\pi}\frac{M_W^4 p_\tau}{(Q^2+M_W^2)^2}R_\pm\ ,$$

Neutrino deep inelastic scattering (C. Bourrely, O.Teryaev, J. S, PRD69,114019,(2004))

$$R_\pm = \frac{1}{M}\left\{\left(2W_1 + \frac{m_\tau^2}{M^2}W_4\right)(E_\tau - p_\tau\cos\theta) + W_2\,(E_\tau + p_\tau\cos\theta)\right.$$
$$\left.\pm\frac{W_3}{M}\left(E_\nu E_\tau + p_\tau^2 - (E_\nu + E_\tau)p_\tau\cos\theta\right) - \frac{m_\tau^2}{M}W_5\right\},$$

Because of time reversal invariance, the polarization vector $\overrightarrow{P}$ of the τ in its rest frame, lies in the scattering plane defined by the momenta of the incident neutrino and the produced τ. It has a component P_L along the direction of $\overrightarrow{p_\tau}$ and a component P_P perpendicular to $\overrightarrow{p_\tau}$, whose expressions are, in the laboratory frame,

$$P_P = \mp\frac{m_\tau\sin\theta}{MR_\pm}\left(2W_1 - W_2 \pm \frac{E_\nu}{M}W_3 - \frac{m_\tau^2}{M^2}W_4 + \frac{E_\tau}{M}W_5\right),$$
$$P_L = \mp\frac{1}{MR_\pm}\left\{\left(2W_1 - \frac{m_\tau^2}{M^2}W_4\right)(p_\tau - E_\tau\cos\theta) + W_2\,(p_\tau + E_\tau\cos\theta)\right.$$
$$\left.\pm\frac{W_3}{M}\left((E_\nu + E_\tau)p_\tau - (E_\nu E_\tau + p_\tau^2)\cos\theta\right) - \frac{m_\tau^2}{M}W_5\cos\theta\right\}.$$

If $W_3 = 0$, $R_+ = R_-$ and τ^+ and τ^- have opposite polarizations. If $m_\tau = 0$, $P_P = 0$, so such a lepton is purely left-handed or purely right-handed.

Neutrino deep inelastic scattering (C. Bourrely, O.Teryaev, J. S, PRD69,114019,(2004))

The hadronic tensor $W_{\mu\nu}(p,q)$ is Hermitian and semi-positive which implies that

$$a^*_\mu W_{\mu\nu}(p,q)a_\nu \geq 0 \ ,$$

for *any* complex 4-vector a_μ. The 4x4 matrix representation of $W_{\mu\nu}(p,q)$ in the laboratory frame where $p=(M,0,0,0)$ and $q=(\nu,\sqrt{\nu^2+Q^2},0,0)$ reads $\begin{pmatrix} M_1 & 0 \\ 0 & M_0 \end{pmatrix}$ where M_1 and M_0 are the following 2x2 Hermitian matrices

$$M_1 = \begin{pmatrix} -W_1+W_2+\frac{\nu^2}{M^2}W_4+\frac{\nu}{M}W_5 & \frac{\sqrt{\nu^2+Q^2}}{M}(\frac{\nu}{M}W_4+\frac{1}{2}W_5) \\ \frac{\sqrt{\nu^2+Q^2}}{M}(\frac{\nu}{M}W_4+\frac{1}{2}W_5) & W_1+\frac{\nu^2+Q^2}{M^2}W_4 \end{pmatrix} ,$$

$$M_0 = \begin{pmatrix} W_1 & \frac{-i\sqrt{\nu^2+Q^2}}{2M}W_3 \\ \frac{+i\sqrt{\nu^2+Q^2}}{2M}W_3 & W_1 \end{pmatrix} .$$

Neutrino deep inelastic scattering (C. Bourrely, O.Teryaev, J. S, PRD69,114019,(2004))

The necessary and sufficient conditions for $W_{\mu\nu}(p,q)$ to satisfy positivity are: all principal minors of M_1 and M_0 must be positive definite. For the diagonal elements we get three inequalities linear in the W_i's

$$W_1 \geq 0 \ , -W_1 + W_2 + \frac{\nu^2}{M^2} W_4 + \frac{\nu}{M} W_5 \geq 0 \ , W_1 + \frac{\nu^2 + Q^2}{M^2} W_4 \geq 0 \, ,$$

from the 2x2 determinants of M_0 and M_1 we get two inequalities quadratic in the W_i's

$$W_1^2 \geq \frac{\nu^2 + Q^2}{4M^2} W_3^2 \ , \text{or equivalently,} \quad W_1 \geq \frac{\sqrt{\nu^2 + Q^2}}{2M} |W_3| \ , \text{and}$$

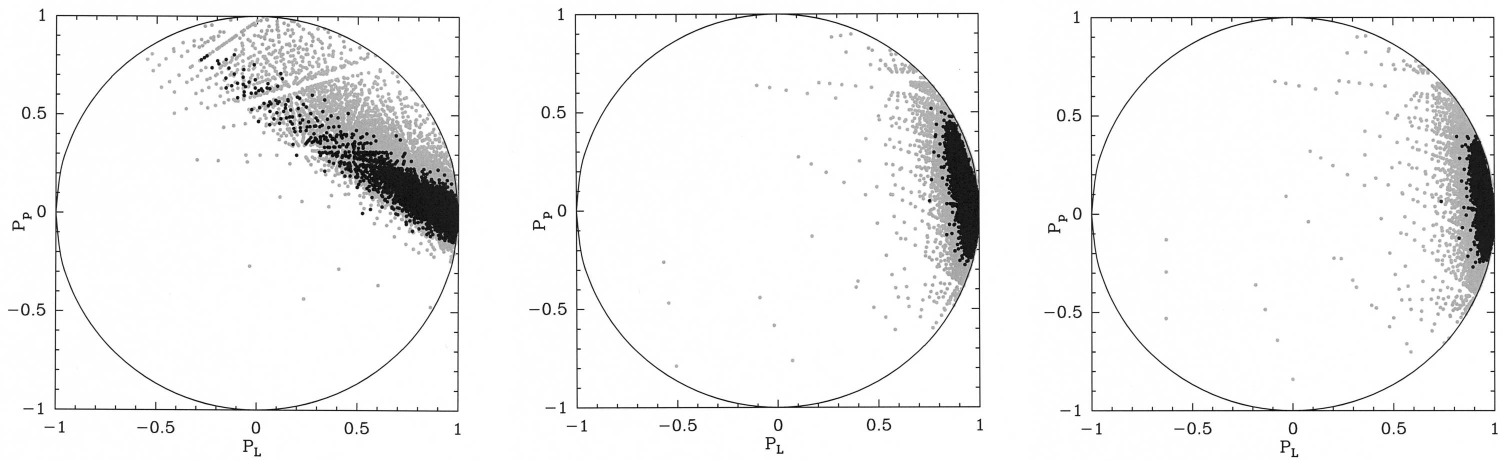
$$\left(-W_1 + W_2 + \frac{\nu^2}{M^2} W_4 + \frac{\nu}{M} W_5 \right) \left(W_1 + \frac{\nu^2 + Q^2}{M^2} W_4 \right) \geq \frac{\nu^2 + Q^2}{M^2} \left(\frac{\nu}{M} W_4 + \frac{1}{2} W_5 \right)^2 \ .$$

Neutrino deep inelastic scattering (C. Bourrely, O.Teryaev, J. S, PRD69,114019,(2004))

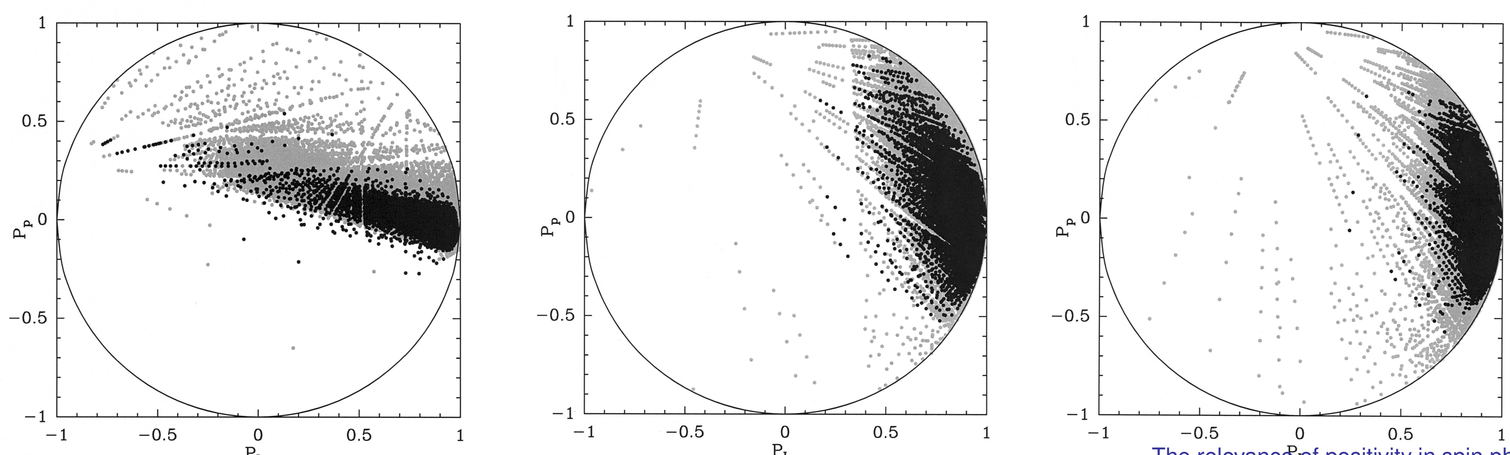
To see how these constraints restrict the allowed domains for P_P and P_L, we proceed by the following method, without a specific model for the W_i's. We generate randomly the values of the W_i's, in the ranges [0,+1] for W_1 and W_2, which are clearly positive and [-1,+1] for $i = 3, 4, 5$. The most trivial constraints are $R_\pm \geq 0$, but in fact they are too weak and do not imply the obvious requirements $|P_L| \leq 1$ and $|P_P| \leq 1$ or $P \leq 1$. So we first impose $R_\pm \geq 0$ and $P \leq 1$ for different values of E_ν, Q^2 and x, for τ^+ production, the points which satisfy these constraints are represented by grey dots inside the disk, $P_L^2 + P_P^2 \leq 1$. If we now add the non trivial positivity constraints, which also garantee that $P \leq 1$, we get the black dots, a much smaller area. The black allowed area increases with Q^2 and gets smaller for increasing incident energy and increasing x. For τ^- production, the corresponding areas are obtained by symmetry with respect to the center of the disk. For increasing x, since P_L is more and more restricted to values close to +1 for τ^+ (-1 for τ^-), it is striking to observe that the non trivial positivity constraints lead to the τ^+ (τ^-) almost purely right-handed (left-handed), although it has a non zero mass. There is another way to present our results, which shows the upper and lower bounds from the non trivial positivity constraints for different x values, versus Q^2. These bounds are obtained by selecting the larger and smaller allowed values of P_L and P_P, when the W_i's are varied for a fixed bin of E_ν and x. We also indicate the scattering angle which increases with Q^2 and we recall that for $\theta = 0$ we have $P_P = 0$.

Neutrino deep inelastic scattering (C. Bourrely, O.Teryaev, J. S, PRD69,114019,(2004))

$$E_\nu = 10\text{GeV},\ Q^2 = 1\text{GeV}^2,\ x = 0.25, 0.6, 0.9$$

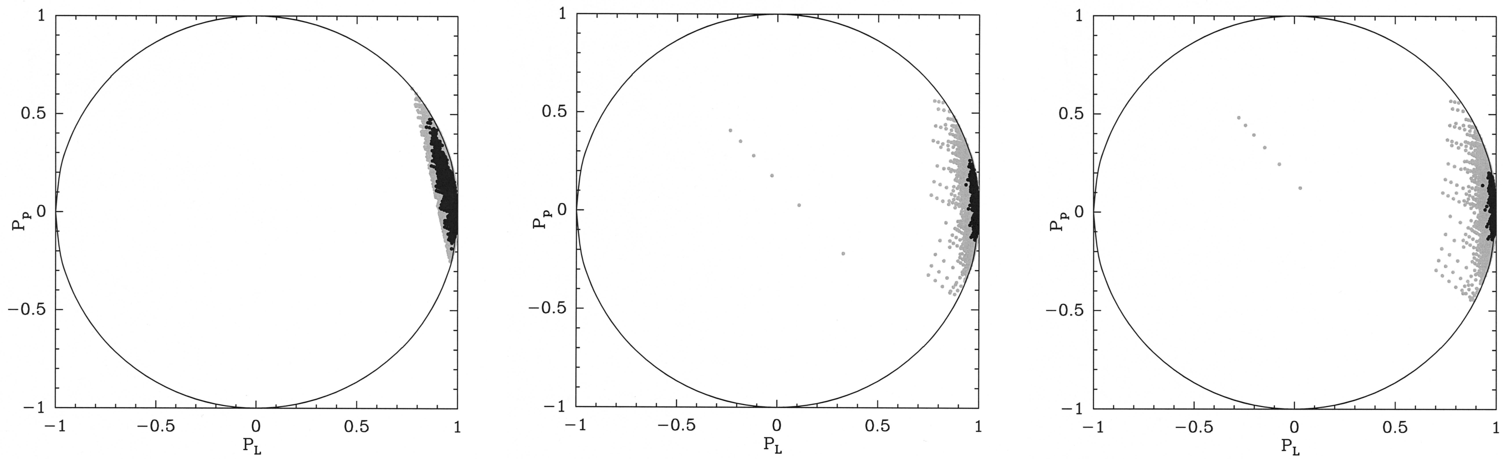


$$E_\nu = 10\text{GeV},\ Q^2 = 4\text{GeV}^2,\ x = 0.4, 0.6, 0.9$$

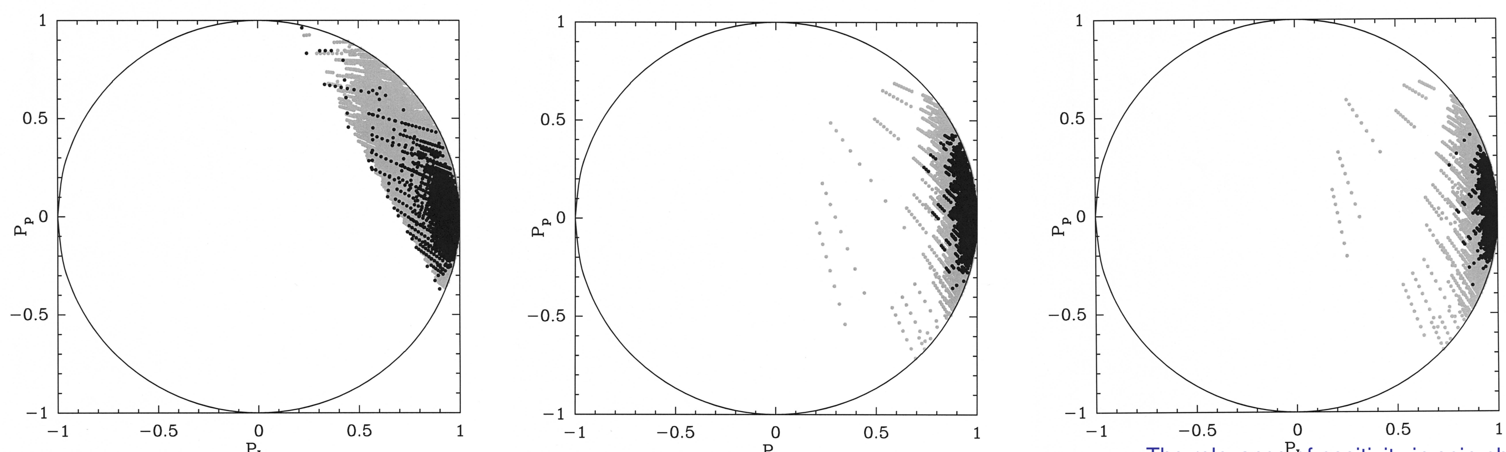


Neutrino deep inelastic scattering (C. Bourrely, O.Teryaev, J. S, PRD69,114019,(2004))

$$E_\nu = 20\text{GeV},\ Q^2 = 1\text{GeV}^2,\ x = 0.25, 0.6, 0.9$$

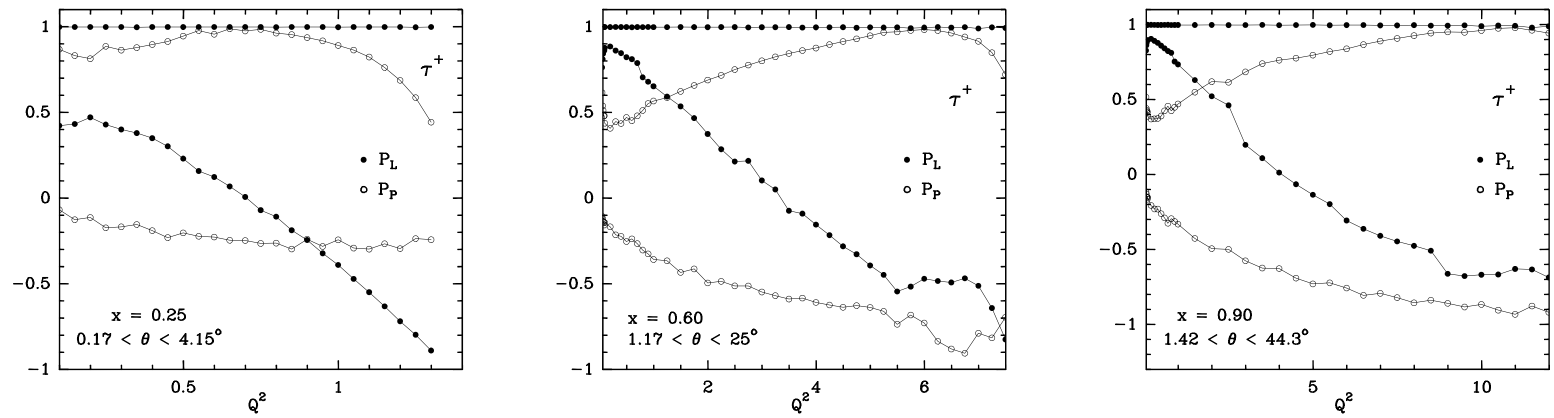


$$E_\nu = 20\text{GeV},\ Q^2 = 4\text{GeV}^2,\ x = 0.25, 0.6, 0.9$$



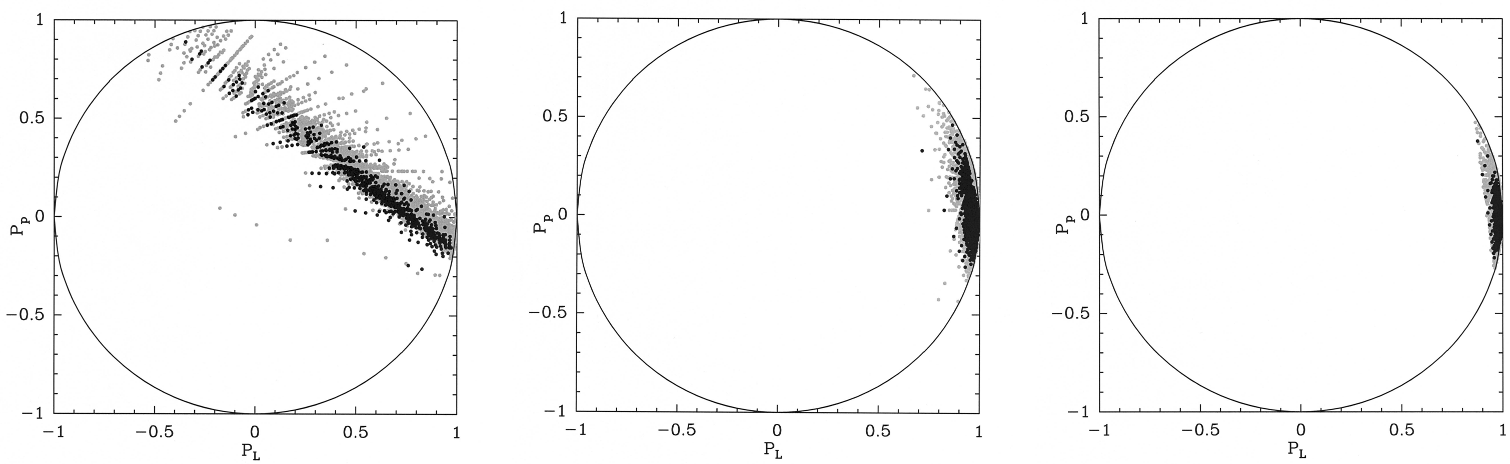
Neutrino deep inelastic scattering (C. Bourrely, O.Teryaev, J. S, PRD69,114019,(2004))

$$E_\nu = 10\text{GeV},\ x = 0.25, 0.6, 0.9$$

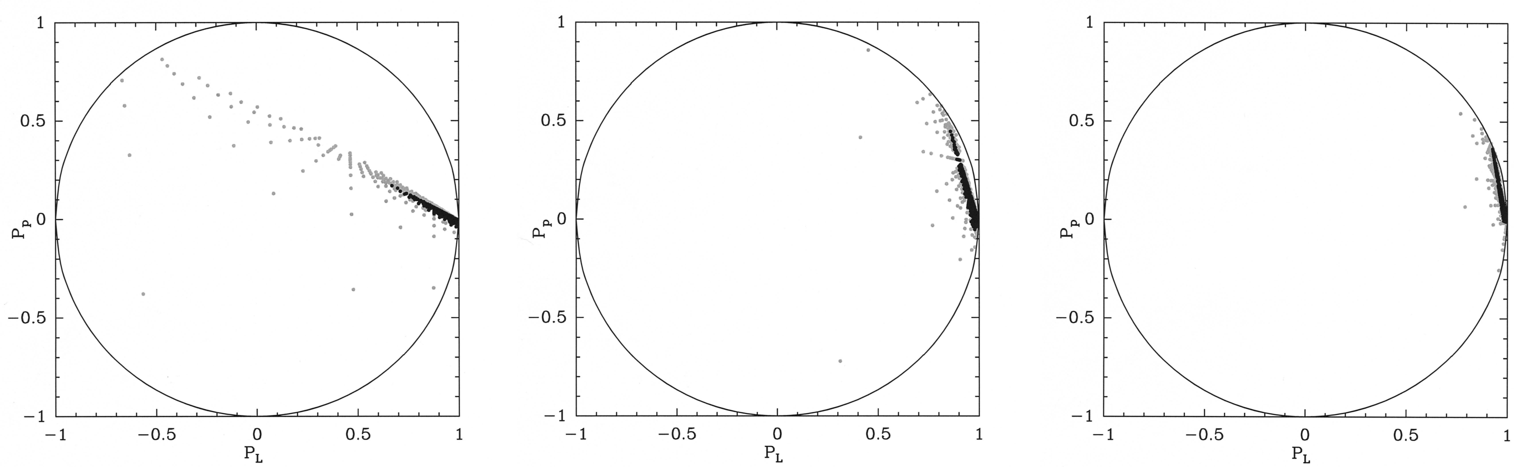


Neutrino deep inelastic scattering (C. Bourrely, O.Teryaev, J. S, PRD69,114019,(2004))

$$E_\nu = 10\text{GeV},\ Q^2 = 1\text{GeV}^2,\ x = 0.25, 0.6, 0.9$$



Up Callan-Gross ($Q^2 W_1 = \nu^2 W_2$), Down Albright-Jarlskog ($MW_1 = \nu W_5$ and $W_4 = 0$)

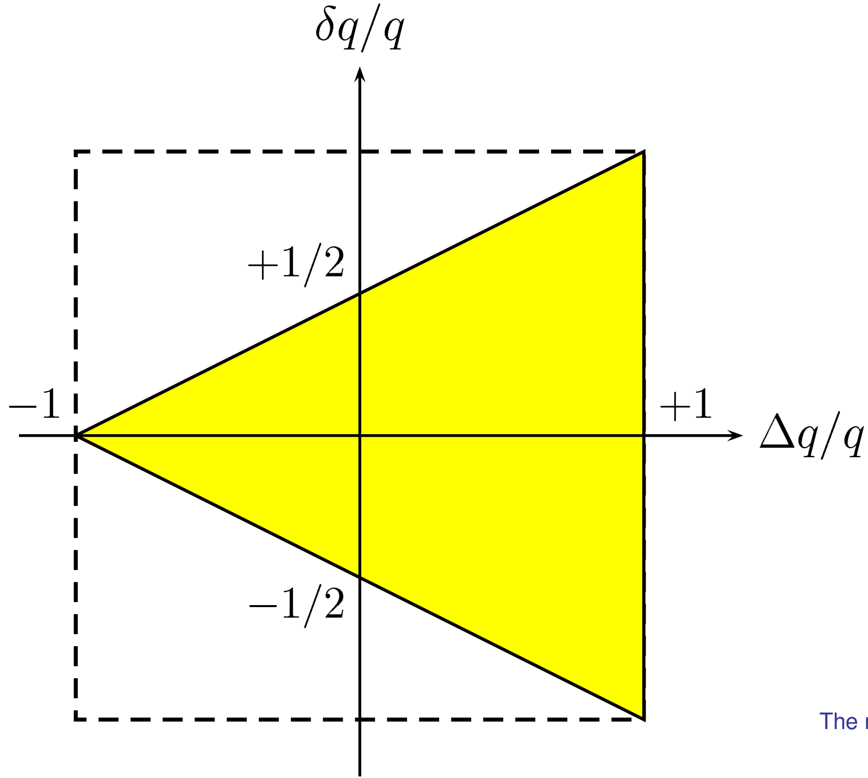


Quark Transversity $\delta q(x, Q^2)$

In addition to $q(x, Q^2)$ and $\Delta q(x, Q^2)$, there is a new distribution function $\delta q(x, Q^2)$, chiral odd, leading twist, which decouples from DIS,and was never measured.
It must satisfy the following positivity bound (J. S., PRL 74,1292,1995)

$$q(x, Q^2) + \Delta q(x, Q^2) \geq 2|\delta q(x, Q^2)|$$

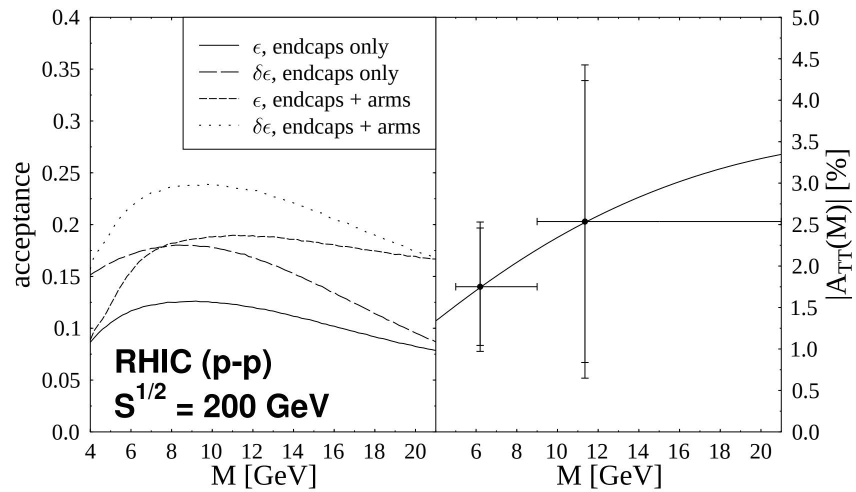
which survives up to NLO corrections



A_{TT} in Drell-Yan dimuon production at RHIC

It can be measured in Drell-Yan dimuon production

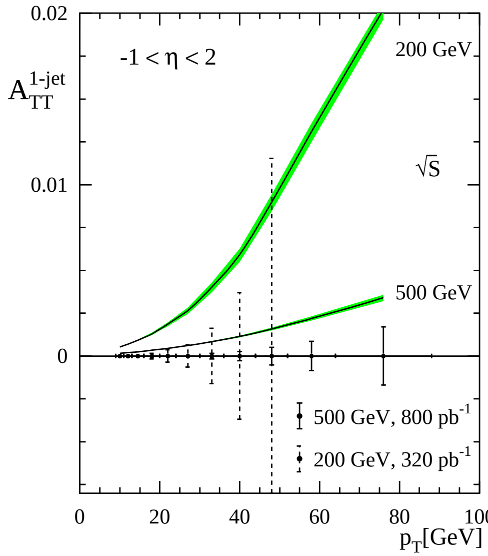
$$A_{TT} = \widehat{a}_{TT} \frac{\sum_q e_q^2 \delta q(x_1, M^2) \delta \bar{q}(x_2, M^2) + (1 \leftrightarrow 2)}{\sum_q e_q^2 q(x_1, M^2) \bar{q}(x_2, M^2) + (1 \leftrightarrow 2)}.$$

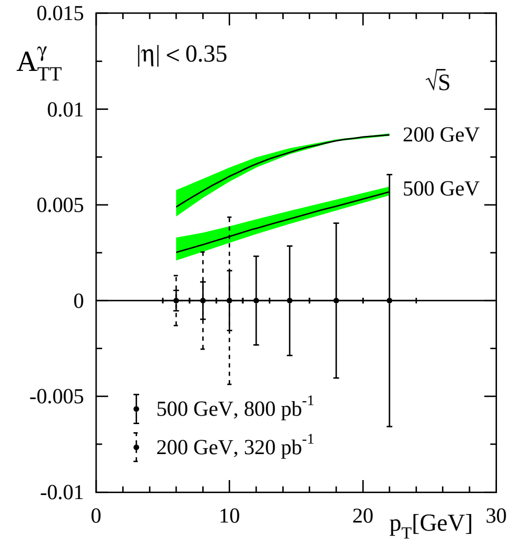


Upper bounds on A_{TT} in Jet and prompt photon productions

Sensitivity to δq only in the high p_T region, dominated by $qq \to qq$, because no transversity for the gluons. Use positivity bound. Expect $|A_{TT}| << |A_{LL}|$

It is important to verify this theoretical expectation





Single spin asymmetries A_N

What is a single spin asymmetry (SSA)?

Consider the collision of a proton of momentum $\vec{p}$, carrying a transverse spin $\vec{s_T}$ and producing an outgoing hadron with transverse momentum $\vec{k_T}$. The SSA defined as

$$A_N = \frac{d\sigma(\vec{s_T}) - d\sigma(-\vec{s_T})}{d\sigma(\vec{s_T}) + d\sigma(-\vec{s_T})}$$

is zero, unless the cross section contains a term $\vec{s_T} \cdot (\vec{p} \times \vec{k_T})$

It can be shown that this requires the existence of an helicity flip and final state interactions, which generate a phase difference between the flip and the non-flip amplitudes, to avoid violation of time reversal invariance.

In the naive parton model one expects very small SSA, because of the double suppression $\alpha_s m_q/Q$.

Actually a large SSA has been discovered 32 years ago (We should celebrate!!!) at FNAL with a 300 GeV proton beam in $pBe \rightarrow \vec{\Lambda} X$ (G.Bunce et al., PRL 36, 1113 (1976)) and many more SSA have been observed later. In particular large SSA in $p^\uparrow p \rightarrow \pi X$ and $\bar{p}^\uparrow p \rightarrow \pi X$ observed at FNAL by E704 in 1990.

Single spin asymmetries in QCD

Two QCD mechanisms

- Introduce Transverse Momentum Dependence (TMD)
  - TMD parton distributions $\Rightarrow$ Sivers effect 1990
  - TMD fragmentation distributions $\Rightarrow$ Collins effect 1993
- Consider higher twist operators
  - In collinear approach introduce quark-gluon correlators (Efremov-Teryaev 1982 Qiu-Sterman 1991)

The gauge-invariance properties of the TMD PDF have been first clarified (S. Brodsky, D.S. Hwang, I. Schmidt, Phys. Lett.B 530,99 (2002)) for DIS and Drell-Yan processes. In general both Sivers and Collins effects contribute to a specific reaction, although there are some cases in which only one of them contributes. For example in SIDIS, the Collins effect is the only mechanism that can lead to asymmetries A_{UT} and A_{UL}. On the other hand, it does not appear in some electroweak interaction processes, where there is only the Sivers effect. In direct photon production in pp collisions, which is dominated by $qg \to q\gamma$, the SSA is sensitive to both the quark and the gluon Sivers functions, according to x_F of the photon. (I. Schmidt, J. S. and J. J. Yang, Phys. Lett.B 612, 258 (2005)).

What positivity can bring into this game?

We have seen above, the existence of new general positivity bounds, among the spin observables in a single particle inclusive reaction, where the two initial particles carry a spin-1/2. In particular for $y = 0$ one has

$$1 - A_{NN} \geq 2|A_N|$$

for any value of $\sqrt{s}$ and p_T. From the previous results $A_{NN} \sim 0$ for $pp \rightarrow \gamma X$ and $pp \rightarrow jetX$, we get a strong bound e.g. $|A_N| \leq 1/2$.

Hyperon production at RHIC

First, we are interested in the reaction $\overrightarrow{p}p \rightarrow \overrightarrow{\Lambda}X$, where both the initial proton and the produced Λ are longitudinaly polarized and we consider the helicity transfer parameter D^{Λ}_{LL} defined as follows

$$D^{\Lambda}_{LL}(\eta, p_T) = (d\sigma_{++} - d\sigma_{+-})/(d\sigma_{++} + d\sigma_{+-}) \ ,$$

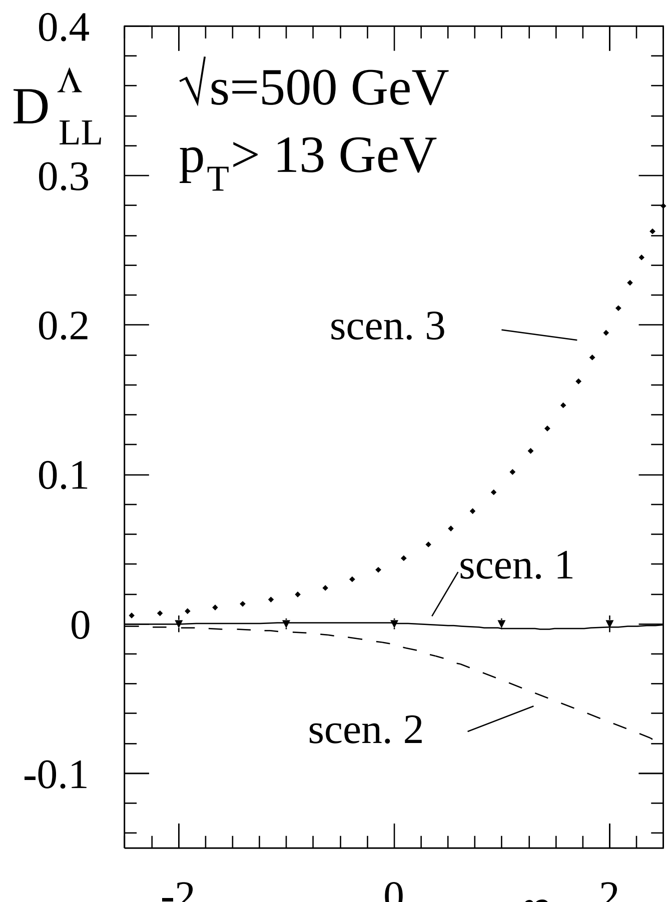
where η and p_T are the rapidity and the transverse momentum of the outgoing Λ. $d\sigma_{++}$ [$d\sigma_{+-}$] denotes the cross section where the proton and the Λ helicities have the same [opposite] sign. When η is positive the Λ has the direction of $\overrightarrow{p}$ and p_T is always assumed to be large. D^{Λ}_{LL} is directly related to the $\Delta D^{\Lambda}_{f}\ 's$, the analogue of Δq for the fragmentation functions $D^{\Lambda}_{f}\ 's$. Although these FF are reasonably known, we have to make models for ΔD^{Λ}_{f}, and also for the other hyperons.

$$\Delta D^{\Lambda}_{u} = \Delta D^{\Lambda}_{d} = N_u \Delta D^{\Lambda}_{s} \ \text{ and } \ \Delta D^{\Lambda}_{s} = z^{\alpha} D_s \ ,$$

in terms of *two* new parameters α and N_u at a given scale. To cover a wide range of plausible models, they consider three different scenarios:

Helicity transfer D^Y_{LL} (A^Y)

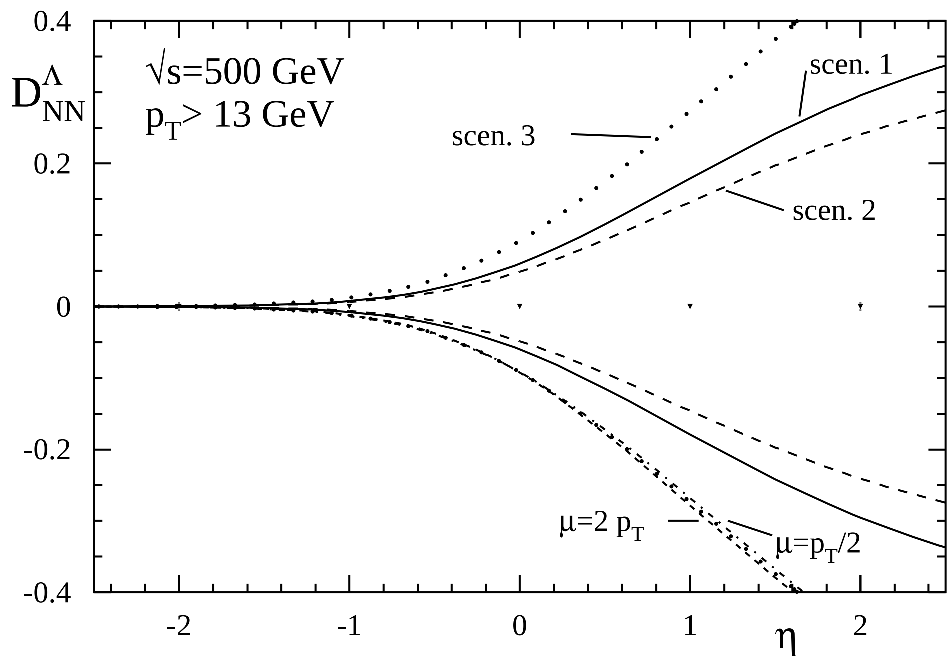
1) In the non-relativistic quark model, only strange quarks contribute, so $N_u = 0$.
2) By transposing an estimate for the polarized u and d quark distributions in a Λ, to the polarized fragmentation functions, one gets $N_u = -0.2$.
3) By assuming that all polarized fragmentation functions are equal, which is an extreme case, one gets $N_u = 1$.



Bounds for the Spin transfer D^{Λ}_{NN}

Use the analogue of the quark transversity positivity bound

$$2|\Delta_T D_q^H(z,Q^2)| \leq D_q^H(z,Q^2) + \Delta D_q^H(z,Q^2)\ .$$



POSITIVITY IS VERY USEFUL TO GIVE NON-TRIVIAL BOUNDS ON SPIN OBSERVABLES

DON'T FORGET IT !!!

We all look forward to new exciting data!!

THANK YOU