MISSION ANALYSIS FOR LASER INTERFEROMETER SPACE ANTENNA (LISA)

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ABSTRACT

The interplanetary orbits of three pairs of spaceprobes carrying laser interferometer antennae are designed such that their mutual distances, i.e. the lengths of the interferometer arms, remain nearly constant. The pairs move relative to each other in an equilateral triangle. Feasible probe masses are computed for a scenario with an Ariane 5 launch into a 'Geostationary' Transfer Orbit and a fuel optimum three-burn transfer from this GTO to the triangular motion. The relative motion is perturbed by planetary gravity, However, the arm rate differences degrading the interferometer accuracy can be kept below certain limits by choosing optimum initial conditions and/or by controlling them in a fuel optimum way. Finally, the achievable orbit determination accuracy is given for systems processing two-way range and Doppler data collected on ground and/or Laser data gained on board the probes,

INTRODUCTION

The LISA project is basically a pair of Michelson interferometers mounted on 3 pairs of' spaceprobes flying in orbits such that their relative motion forms an equilateral triangle. In that way gravitational waves emerging from different sources in our galaxy might be detected by observing, their Influence on the interferometer arms to a sub-Ångstrom precision in the frequency band 10^{-4} to 10^{-1} Hz. It became clear during the LISA assessment study (ESA SCI(94)6, 1994) that long enough interferometer arms in a sufficiently quiet environment can only be realised in deep space and under the condition that non-gravitational forces are compensated for instance by a Field Emission Electric Propulsion System (FEEPS) exhausting caesium with a speed of 60 km/s. For details on the experiment we refer to the LISA Pre-Phase A Report (Bender et al., 1996).

BASIC ORBITAL CONFIGURATION

The three pairs of the 6 LISA probes shall move in orbits in which their mutual distances (d), i.e. the arm lengths of the interferometer, are kept as constant as possible. The distance between the probes of a pair is 200-300 km. The distances to the Earth and the orbital configuration shall be such that the design of the attitude control and of the Earth-spacecraft communication becomes feasible and that the perturbations of the arm lengths stay below tolerable limits.

The above requirements basically are met by putting the pairs in heliocentric orbits with diameter $D = 2AU$, eccentricity $e = d/(D\sqrt{3})$ and inclination w.r.t. the ecliptic $i = d/D$ (see also LISA assessment study report, 1994), The 3 pairs will form an equilateral triangle with a mean side length $d = 2e\sqrt{3}$ AU if the orbital nodes are separated by 120° and if the true anomalies and arguments of perihelion are chosen such that each spacecraft has its maximum distance from the ecliptic when it is at perihelion (2 solutions!).

This triangle rotates once per year about its centre which is moving in the ecliptic plane at a longitude λ behind the mean position of the Earth. The plane formed by the triangle is inclined by 60° to the ecliptic. Figure 1 depicts the orbital configuration for the case with perihelia above the ecliptic plane.

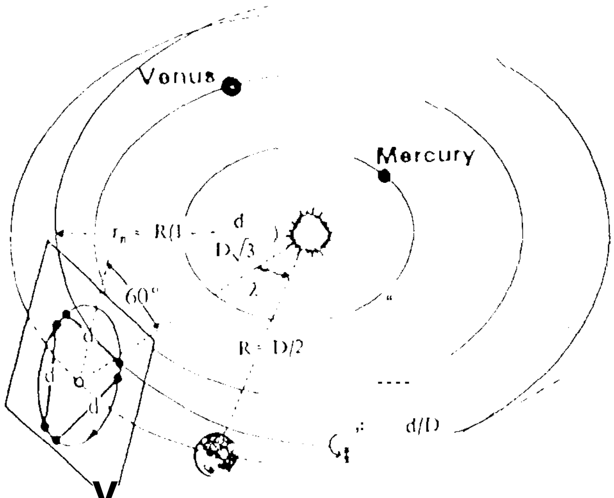

Fig. 1. The orbital configuration

ARIANE 5 LAUNCH, GTO TO TRIANGLE TRANSFER AND BASELINE ORBITS

The probe pairs are supposed to be put by an Ariane 5 into a common orbit from which they are manoeuvred by means of 3 Propulsion Modules ($I_{sp} = 312$ s) into the interplanetary target orbits.

This common orbit could be an interplanetary trajectory or an Earth orbit. A direct launch into an interplanetary trajectory is not always more attractive from a mass point of view than a transfer via a special Earth orbit, namely the 'Geostationary' Transfer Orbit (GTO) (Hechler, 1993). This is a consequence of the Ariane 5 specific design and of the constraints imposed on its ascent trajectory.

LISA shall use an Ariane 5 in a triple launch configuration in which a usable mass of 4880 kg can be delivered into the GTO (Cornelisse, 1994). The achievable probe masses after arrival in the 6 individual orbits, i.e. at Begin Of Mission (BOM), are then be calculated by minimizing the ΔV-requirement for the GTO to triangle transfer taking into account that about 65 m/s are needed for the annihilation of the navigation uncertainties, for the attitude control manoeuvres and for orbit manoeuvres during the final delivery into the individual mission orbits. The maximum masses are functions of the launch date. Some results of above calculations are shown in the following figures.

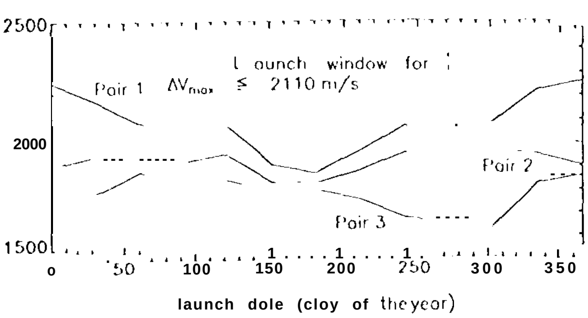

Fig. 2. ΔV as function of launch date for $\lambda = 20°$, $d = 5 \times 10^6$ km

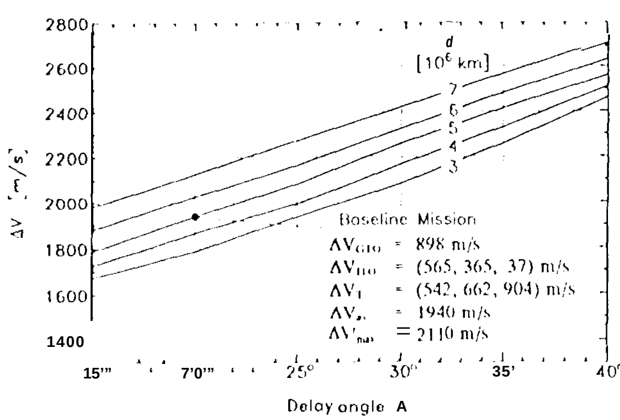

Fig. 3. Average ΔV-requirement for the GTO to BOM transfer as function of arm length d and of delay angle λ

Figures 2 and 3 visualise the following facts.

- The fuel needed for the transfer is quite different for the three different pairs (Figure 2). Notice that this necessitates a pair specific design of the PMs. Although the total achievable mass at BOM varies by less then 10% between its minimum for launches around New Year and its maximum for launches in the middle of the year the seasonal launch window has been constrained to April - October in order to keep the maximum ΔV-requirement for a single pair below 2110 m/s.
- The average ΔV-requirement for the transfer is mainly a function of the arm length d and the delay angle λ (Figure 3). For $\lambda = 15°$, $d = 3 \times 10^6$ km the average probe mass at BOM would become 471 kg and it would be 335 kg yet for $\lambda = 40°$, $d = 7 \times 10^6$ km.

The orbits about the Sun are perturbed by the gravity of other bodies in the solar system. The lengths of the interferometer arms do not remain constant. The most unwelcome perturbations are due to the Earth/Moon gravity. They decrease with increasing λ and with decreasing d. Large delay angles and small triangles would be desirable from the stability point of view.

However, the experiment requires an arm length of d = 5 10⁶ km, This and the feasible masses at BOM even would permit delay angles above 40° (Figure 3). Unfortunately, the study of the communications problem (distance Earth - probe, ESA and S/C antenna size(s) and power) reveals that delay angles $\lambda > 20°$ are not feasible (Bender et al., 1996).

Hence, d = 5106 km for $\lambda = 20°$ define the feasible orbits of the three corners of the triangle, i.e. our baseline orbits. These orbits will be used throughout the following considerations

ARM RATE DIFFERENCES AND THEIR CONTROL

Perturbations and higher order effects of the orbital eccentricity. change the arm lengths d_i and also their rates $\dot{d}_i$, i = 1,2,3. In particular the resulting 'natural' Arm Rate Differences (ARD) $v_{ij} = \dot{d}_i - \dot{d}_j$, $i \neq j$ = 1,2,3 set limits to the performance of the interferometer. The following two types of configuration stability are to be considered.

Case I: Only one single ARD, e.g. $|v_{12}|$, must be constrained.

Case II: The extreme value of the ARDs between all three arms, i.e. Max. $\{|v_{12}|, |v_{13}|, |v_{23}|\}$, has to be constrained.

For a given observation period, T, the natural ARDs can be minimised by an appropriate choice of the initial states of the probes. If the natural ARDs are not tolerable they must be controlled by means of the FEEPS. The experiment is interrupted by this control since the probes cannot be kept 'drag-free' anymore. We thus seek after a fuel minimum control of the motion of the corners of the triangle that keeps the ARDs $|v_{ij}|$ below a specified tolerance for all $t \in (0,T)$.

The attitude control must not be interrupted during the ARD-control manoeuvres since the link between the spacecraft must not get lost. Hence the pointing directions of the thrusters are prescribed. The spacecraft accelerations along these directions are almost constant because the probe mass will remain nearly constant: the high specific impulse $I_{sp} \approx 6000$ s of the FEEPS allows to realise the control by a few grammes of caesium. Furthermore, the required velocity corrections are 4 orders of magnitudes smaller than the spacecaft velocity. Under above conditions, the underlying low thrust control problem can be converted into a linear optimisation problem (Hechler, 1981). The determination of the optimum initial states can easily be included in the optimisation process (Hechler, 1993).

Figure 4 shows the ΔV-requirements for such a fuel optimum ARD-control for both the cases I and II. Since the results were computed for a worst case thruster configuration with only 6 nozzles per probe the resulting control may need up to 50% more fuel than a control with a thruster system allowing omni-directional burns.

Case II, i.e. the complete control of all the ARDs in the triangle, will demand much more fuel then the control of a single arm-rate difference. But the fuel consumption is not our prime concern in this case. This rather is the weak FEEP-thrust level of 100 μN, because it is much to small for producing the required velocity changes in a sufficiently short time interval.

The situation is more promising in the case I: a small fuel consumption goes along with a tolerable amount of manoeuvres. For the worst case, i.e. for a tolerance of 0,05 m/s of the single ARD, the ΔV-requirement per year is only 1.3 m/s. Suppose a probe mass is 300 kg then the yearly average of the caesium consumption per probe will stay below 2.2 gr. The detailed results show that the longest total burn time of any of the 100μN-thrusters and thus the total experiment interruption time will be 18 days.

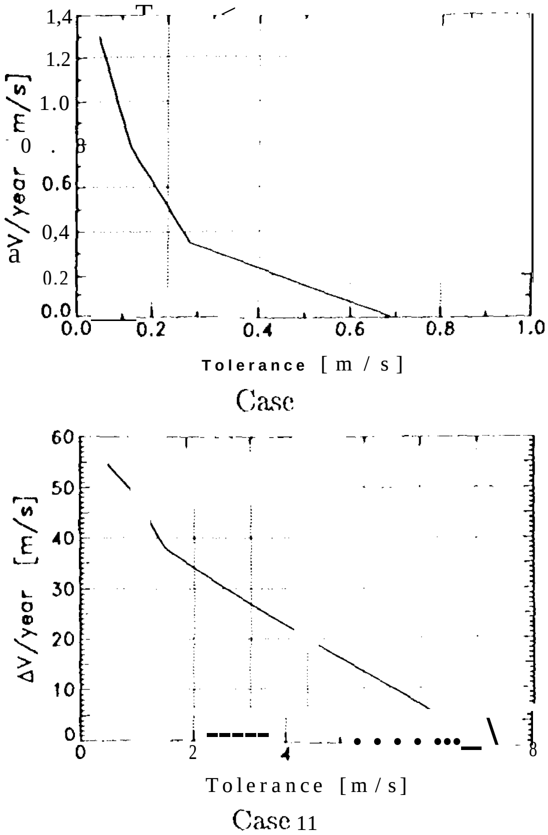

Fig. 4. ΔV-requirement for the control of arm rate differences

If the ARDs are not controlled but the inital states are chosen in an optimum way the following results were obtained for above baseline mission. In the case I, the natural ARDs can be constraint to 3.6 m/s for an observation period of five years. The optimum rates drop below 0.7 m/s for observation periods below 2 years, and below 0.3 m/s for observation periods below 1 year. In the case II, the natural ARDs will exceed 7 m/s for observation periods as short as 1 year. Recall, that larger delay angles, λ, essentially could improve the situation.

ORBIT DETERMINATION REQUIREMENTS AND ACCURACY

The orbit determination requirements are different for the three phases of the mission, i.e. the transfer phase (around 13 months, PM) between launch and arrival at the triangle corners, the delivery phase (3 months, PM + possibly FEEPS) during which the probes are manoeuvred into their individual states and attitudes at BOM and the operational phase (up to 5 years, FEEPS) with the arm control manoeuvres. Table 1 gives a summary of the orbit determination requirements as they were worked out during the assessment phase.

The next Table 2 shows the achievable orbit determination accuracies using the following tracking Systems,

I. Ground based radio tracking system with following properties,

One X-band station with position errors below 3 cm; Two-way range data (noise: < 2m (1 σ); bias: < 10m) -1 two-way Doppler data (max. error: < 0.1 mm/s for 60 s averaging) scheduled every 30 minutes; 1 onosphere zenith delay after calibration by means of GPS signals: ≤ 3 cm; Troposphere zenith delay after modelling: ≤4 cm; Earth orbit orientation error: ≤25 nrad and position error: ≤10 km,

1 t is important to notice that above assumptions on Ionosphere-errors and radio data noise and biases only could be met by the ESA Multi Purpose Tracking System after a few enhancements and/or modifications (X-band, GPS-calibration, highly stable frequency standards).

II. On board Laser tracking system

Provides relative distances from roundtrip laser phase for each arm and between the collocated pairs (noise < 0.1 mm; phase bias estimated; schedule: every 30 minutes)

Table 1: **Required** orbit **determination** accuracies

Phase	Accuracy (σ-rms)	Considered
Transfer	position: 100 km velocity: 1 m/s	manoeuvre dispersions
Delivery	position: 100" km → < 10 km velocity: 1 m/s → < 10 cm/s	stability of configuration, attitude acquisition, natural ARD
Experiment	position: ≤ 12 km velocity: 2 rim/s arm-length: ≤ 100m	attitude keeping, modelling of known gravity signals

Table 2: Achievable **orbit** determination accuracies
Mode = tracking, mode (R = Radio, L = Laser)
Arc = data arc (days)

Case		Accuracy (σ-rms)			
Mock	Arc	Pos. (km)	Vel. (mm/s)	Arm 1 (m)	Arm2 (m)
R	16	11.5	1.9	1486	5790
R + L	8	10.5	1.9	52	383
R + L	16	11.5	2.1	17	122

The comparison of requirements and achievable accuracies in above tables reveals the following essential facts:

- During transfer and delivery phase the well established orbit determination from ground by means of radio tracking data is accurate enough for navigating the 6 probes into the desired states and attitudes.
- During the experiment the required accurate knowledge of the arm lengths necessitates the incorporation of Laser tracking data collected on board the probes in the orbit determination process.

CONCLUSION

Three pairs of spacecraft can be flown in interplanetary orbits such that they form a rather stable equilateral triangular configuration at relative distances up to a few million kilometres. This allows to build a unique Laser interferometer Space Antenna for the detection of gravitational waves. The distances, i.e. the lengths of interferometer arms, are perturbed by planetary gravity and higher order eccentricity effects. Two of these arm lengths can be controlled to the required level of accuracy by a time minimum orbit control strategy which is tolerable from the experiment and fuel point of view. A complete 3-arm control dots not seem to be feasible.

European facilities, i.e. the Ariane S launcher and slighly enhanced ESA S-band net with its 15 m dishes, would allow to realise a LISA mission with interferometer arm lengths of 5 x 10^6 km and a triangle centre at a mean longitude 20° away from the Earth.

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