On HBT parameters from high energy nuclear collisions - the λ-parameter

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Abstract

Using a microscopic transport model together with a HBT after-burner, we study the behavior of the pion HBT λ-parameter in Au+Au central collisions at $\sqrt{s} = 200\ AGeV$. We argue that this parameter is sensitive to the sequential freeze-out in heavy ion collisions. The resonance decay which causes the pions freeze-out from different sizes is included in this general argument.

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The space-time evolution information of high energy nucleus-nucleus collisions at freeze-out, i.e. when the hadrons cease to interact, can be extracted from the two-particle correlation function. For recent reviews, see Refs. [1–3]. Parameterized in the out-side-longitudinal (osl) coordinate system, the correlation function is given as:

$$C_2(\mathbf{q}) = 1 + \lambda exp(-R_o^2 q_o^2 - R_s^2 q_s^2 - R_l^2 q_l^2), \tag{1}$$

where the components of relative momentum $\mathbf{q} = \mathbf{p}_1 - \mathbf{p}_2$ are defined parallel to the beam (l= longitudinal), parallel to the transverse momentum (o = out), and perpendicular to the transverse momentum (s = side). Usually, a cross-term R_{ol}^2, which is of kinematic origin, appears in Eq. (1) [4]. It vanishes in azimuthally symmetric collisions at mid-rapidity [3] due to the fact that there is no correlation between q_l and q_o. The R_o and R_l parameters mix spatial and temporal information on the emitter in a non-trivial way, and their interpretation depends on the frame in which the relative momenta q are specified.

The phenomenological parameter λ is the chaoticity parameter or strength parameter and has the value of one in the ideal case [5,6]. Experimental results [7–9], as well as model calculations [10], often show that the value of λ is less than unity. A possible interpretation for the effect (of lambda less than 1) is partial quantum coherence [11], in which the correlation function at $q \to 0$ becomes less than 2. Sometimes the final-state interactions, finite resolution in the momentum difference in experiment, and even particle mis-identification are also blamed for the smallness of the λ [12]. More importantly, it is believed that resonances, especially those decaying with long lifetimes, will reduce the value of the λ-parameter [14–17]. A crucial question is what physics can we learn from the correlation measurements, specifically, from the λ-parameter? This is the focal point of the present paper. In this work, pion correlation functions are derived from events generated by a simple thermal model and also by relativistic quantum molecular dynamic model (RQMD) [18] to investigate the physics in the pion HBT λ-parameter.

In the thermal model tests, Gaussian sources are generated for the space coordinates and also for the time durations. The momenta are generated by thermal distributions corresponding to given temperatures. In the produced pion source, the space coordinates and momenta are not correlated. Using events from simulation, the pion-pion correlation functions are calculated here as in Ref. [10,16,19]. Momentum bin width of 5 MeV/c is used in all of the 3 $\mathbf{q}$ components[1]. A simultaneous 3-dimensional fit to the correlation functions is performed by using Eq. (1), and the results are listed in Table 1. Three cases are tested: (1) single Gaussian source $\sigma_x = \sigma_y = \sigma_z = 3$ fm with duration lifetime set to be zero; (2) the duration of lifetime is set to be equal to the space extension; (3) two Gaussian sources with different weights are superimposed on each other, $\sigma_x = \sigma_y = \sigma_z = 3$ fm, with 80% and $\sigma_x = \sigma_y = \sigma_z = 15$ fm, with 20% of weights, respectively. As can be seen from the table, source size parameters of case (1) are well reproduced by the fitting. In case (2), the size parameters in both longitudinal and transverse out directions are increased due to the including of time duration. The side parameter remains to be the same as input value. In both cases, the $\lambda-$parameter is unity - implying the correlation functions in each of 3 dimensions is in Gaussian shape. When one mixes two different size Gaussian sources in case (3), the fitting results of the size parameters are neither equal to the bigger nor the smaller one. Furthermore, the $\lambda-$parameter becomes to be less than unity. Note that in all three cases, the binnings in q_o, q_s and q_l are kept as 5 MeV/c. Although the intercept of the $C_2(q)$ at $q = 0$ is 2, the shape of correlation functions in case (3) is no longer Gaussian leading to non-unity $\lambda-$parameter and the difference between the input and fitted size-parameters. It is worthy noting, due to lack of correlation between space and momentum in the source, the final size-parameters are not sensitive to the temperature of the Gaussian source.

In heavy ion collisions, the gradients of particle density and temperature from the center of the system to the edge lead to expansion. The time span is typically about 10 - 20 fm/c depending on the bombarding energy. Therefore, particles freeze-out is a continuous

[1]The best approached experimental momentum resolution in relative momentum projection was about 5 MeV/c [7,8].

process over a wide period of time - sequential freeze-out. Earlier freeze-out particles come from relatively smaller source than those freeze-out later. In the above thermal model study the sources are static and no expansion is involved. In order to have a more realistic test, in the following we will use RQMD model (version 2.4) to simulate the pion phase-space distributions from the Au+Au central collisions at $\sqrt{s} = 200\ AGeV$. The RQMD model is a microscopic transport approach, based on string, resonance excitation and decay, rescattering of the produced particles. A detailed description can be found elsewhere [18]. Pions are taken from central collision events from a range of impact parameters (0-3 fm). No azimuthal angle restriction is used in the acceptance for them, i.e., only mid-rapidity ($-0.5 < y < 0.5$) and transverse momentum, p_T, are applied to the selection. Then the pion-pion correlation functions are calculated as discussed before. Fig. 1 shows (solid lines) the Gaussian fit parameters as a function of p_T from correlation functions. All the radius parameters, R_o, R_s and R_l, decrease dramatically as p_T goes up, as also observed in Ref. [10] for collisions at the SPS energy. The λ-parameter also changes as a function of transverse momentum.

In Fig. 1, the HBT radius parameters are compared to the variances of the particle spatial-time distributions at freeze-out [2]: $RMS(r_i)$, where the r_i are defined as $r_s = (yp_x - xp_y)/p_T$, $r_o = (xp_x + yp_y)/p_T - tp_T/E$ and $r_l = z - tp_z/E$. Here x, y, z, t are space-time vector of the particle and p_x, p_y, p_z, E are the momentum vector. Distances, which would always be centered on zero, between randomly selected pion pairs are calculated in each projection and fitted with static Gaussian formalism to a range of $\pm$ 1.5 RMS value of the distribution. Fig. 1 shows $1/\sqrt{2}$ sigma of the Gaussian fit two-particle distances (dashed lines) as a function of p_T. Due to the non-Gaussian shape in the distance distributions [20], the Gaussian fit results will depend on the fitting range. In Fig. 1, the dashed-dot lines represent the uncertainties from the fitting range from 1.0 to 2.0 RMS value of the distribution. **Although both**

[2] More detailed studies on this issue can be found in Ref. [20]

methods show a similar behavior of the size parameter as a function of p_t, the variation of the fitting range, is the largest at low p_t - indicating that the effect of the non-Gaussianness is the strongest in the low p_t region. This is exactly the region where the λ-parameter has largest deviation from unity (see below for more discussion).

As can be seen in Fig. 1, while the size-parameters decrease as p_T increases, the λ-parameter increases gradually. The reasons for the decreasing in the size-parameters are several folds: (I) Space-time momentum correlation including the effects of string breaking [21,22] and possibly the collective flow [7,23] will lead to such behavior. This is because the extended space that can be 'seen' within a given momentum window is correlated with the magnitude of the pair averaged momentum. (II) Resonance decayed pions come from relatively large size with low-p_T [24]. (III) In case of thermal equilibrium, the temperature gradient in the source will also lead to the p_T dependence [25].

The p_T dependence of the λ-parameter was attributed mostly to the resonance decays [14,25]. The finite resolution in relative momentum q does also play a role for large size source. However, the hot (high temperature) and dense (high particle density) system in heavy ion collisions will expand and freeze-out continuously. One may imagine that particles located closer to the edge will freeze-out earlier than those near the center of the hot zone. Matter initially at the center of the collision zone where the density is high will have to first expand till the system becomes dilute enough to freeze-out. Then the freeze-out will be continuous and sequential, meaning particles which come out of the system at early time will be from relatively smaller source compared to those at late time. **Indeed, we found that the model source size parameters increase monotonically as time increases.** Assuming the source is Gaussian at each time interval, the total source integrated over the whole time evolution is not necessarily Gaussian due to the change of the source size as a function of collision time. As mentioned above, such non-Gaussian shape will lead to the non-unity in the λ-parameter if it is extracted by using Eq. (1) to fit the correlation functions.

Here we argue that this is a general explanation for the non-unity of the measured λ-parameter. In order to test this idea, we use the RQMD+CRAB calculations. For clarity, we remove those pions from resonance decay and take only directly produced pions [3] to construct the correlation functions. Fig. 2 shows the pion HBT λ-parameters as a function of freeze-out time in Au+Au central collisions at $\sqrt{s} = 200\ AGeV$.

Due to expansion and/or resonance decay, the correlation functions are not in perfect Gaussian shape and the λ-parameter extracted from Eq. (1) is less than unity. However, within the time window of $\Delta t = 10\ fm/c$, the correlation functions are much closer to the Gaussian shape and the λ is unity, see Fig. 2. In the figure, the shaded bar is the value of the total time integrated λ-parameter (≈ 0.8). In a time integrated window, ($\Delta t = 0 - t_i, t_i = 10, 20, 30...\ fm/c$), the λ parameter also decreases and it eventually reaches the total time integrated value. This means that whence sequential freeze-out occurs, whether due to resonance decay or expansion, the summed source is not a Gaussian even it is at a given time integral. As a result, the correlation function is destroyed and the λ-parameter from the HBT measurement becomes less than 1. The sequential freeze-out, at least from the model used here, is more general than either resonance decay or expansion. The experimental measured λ parameter is less than 1 may be interpreted as an evidence of sequential freeze-out in heavy ion collisions.

Up to now, we have used only the directly produced pions to construct the correlation functions and demonstrate that the non-unitiness of the λ-parameter is the result of the sequential freeze-out of the source produced in the heavy ion collisions. We argue that the effect of the resonance decay is part of the story here. In order to show this, we study resonance contributions by investigating the correlation functions of pions from the ω meson decays. With full decay width 8.41 MeV/c^2 and lifetime 23 fm/c, the ω meson is the moderately long-lived resonance. It is expected to be produced in significant numbers in

[3] Direct pions are defined in RQMD as those only from rescattering at the moment of freeze out.

high-energy nucleus-nucleus collision, and the effect on correlation functions has been studied by several authors [26–28]. As shown in Fig. 3, the correlation functions are not perfect Gaussian shape and the λ-parameter is less than 1. However, within the time window, $10 < t < 30\ fm/c$, the correlation functions are much closer to the Gaussian shape and the λ is unity. Similar conclusion was also reached in efs. [26,27] where an analytically parameterized source was used while the authors in Ref. [28] claimed that the ω resonance decay does not distort the Gaussian shape of the correlation functions in S+Pb $\rightarrow \pi + \pi$+X reactions at CERN SPS. Note that in elementary collisions, i.e. $e^+ + e^-$ or $p + p$ collisions, one expects less of an effect from expansion, and resonance decay should account for most of the reduction of the λ-parameter[4]. This is what authors argued in [13]. Again, our study means that whence sequential freeze-out occurs, whatever due to decay or expansion or due to other mechanism, the correlation function is different from a Gaussian and the fitted λ-parameter becomes less than 1. This effect is particularly important if one wants to draw conclusions from the λ-parameter about a possible contribution of a coherent component in multiparticle production.

In summary, we study the behavior of the pion HBT λ-parameter in Au+Au central collisions at $\sqrt{s} = 200\ AGeV$. The HBT parameters R_{side} and R_{long} are consistent with source size values. However, the HBT R_{out} parameter is not consistent with the source size, which is still an open question. Within a given narrow time window, $\Delta t = 10\ fm/c$, the λ-parameter is unity while the total time integrated value is about 0.8. This is also true even for the pions from the ω resonance decays. Within the model, these observations are considered as an evidence of sequential freeze-out. Particles emitted at different times with different size parameters or resonance decays will distort the correlation functions from ideal Gaussian shape and thus reduce the value of the λ. The strength of the effect is somewhat model dependent, but the general feature is true for heavy ion collisions.

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[4]Dr. H. van Hecke made this point clear to us.

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FIGURES

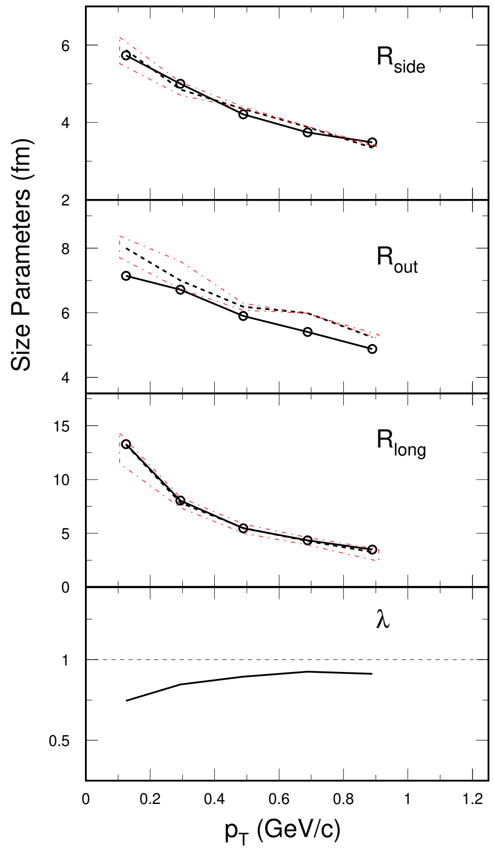

FIG. 1. The pion source parameters as a function of transverse momentum p_T. The dashed and solid lines represent the parameters extracted from Gaussian fit and correlation functions, respectively.

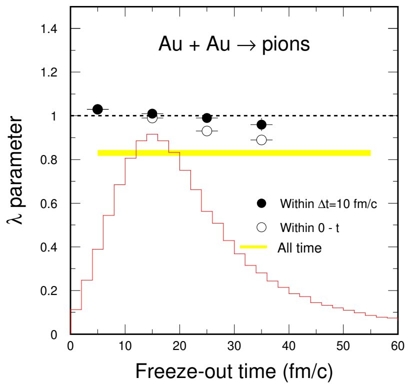

FIG. 2. The λ-parameters as a function of freeze-out time in Au+Au central collisions at the c.m.s. energy 200 $AGeV$. Only direct pions are used to construct the correlation functions. The results calculated within a fixed time window 10 fm/c are shown as solid circles and those calculated within time interval $0 \rightarrow t_i, t_i = 10, 20, 30, 40$ fm/c are shown as open circles. The light histogram shows the pion time profile (arbitrarily normalized).

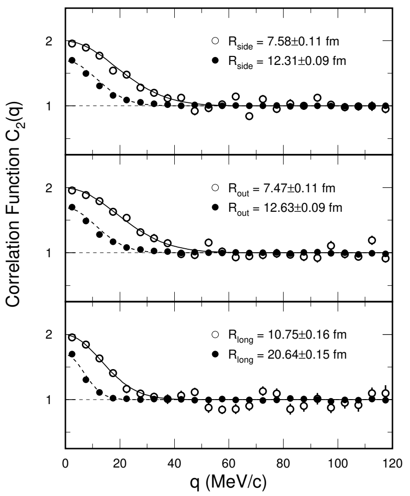

FIG. 3. Projections of 3-dimensional correlation functions constructed from the ω meson decayed pions in Au+Au central collisions at the c.m.s. energy 200 $AGeV$. It should be emphasized that all HBT radii presented here are derived by a simultaneous fit to all three **q** components in C_2. The projections over the lowest 5 MeV/c in the other momentum difference directions are generated for better visibility only. The HBT λ-parameter is 0.69 ± 0.01 (solid circles) when all pions are used in the calculation. 0.99 ± 0.01 is found for pions freeze-out in the time window $10 < t < 30$ fm/c, open circles.

TABLES

TABLE I. HBT parameters from $\pi\pi$ correlation functions in thermal model tests. Statistical errors in the table are negligible with high statistics data. (1) An ideal Gaussian source with T=150 MeV, $\sigma_x = \sigma_y = \sigma_z = 3\ fm$ and $\sigma_\tau = 0$. (2) A spatial Gaussian source with time duration: T=100 MeV, $\sigma_x = \sigma_y = \sigma_z = 3\ fm$, $\sigma_\tau = 3\ fm/c$. (3) Two spatial Gaussian sources superimposed: T=100 MeV, $\sigma_\tau = 0$, $\sigma_x = \sigma_y = \sigma_z = r$ with 80% $r = 3\ fm$ and 20% $r = 15\ fm$.

	(1)	(2)	(3)
R_o (fm)	3.01	3.44	3.25
R_s (fm)	2.99	2.96	3.32
R_l (fm)	3.01	3.29	3.33
λ	0.999	0.998	0.775