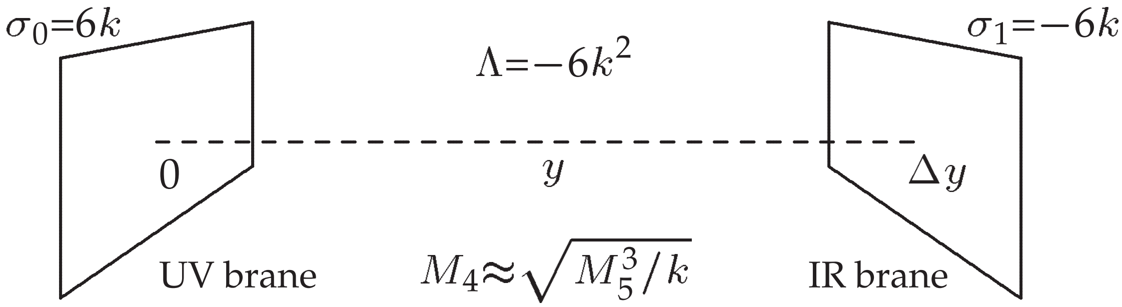
RADION-INDUCED BRANE PREHEATING

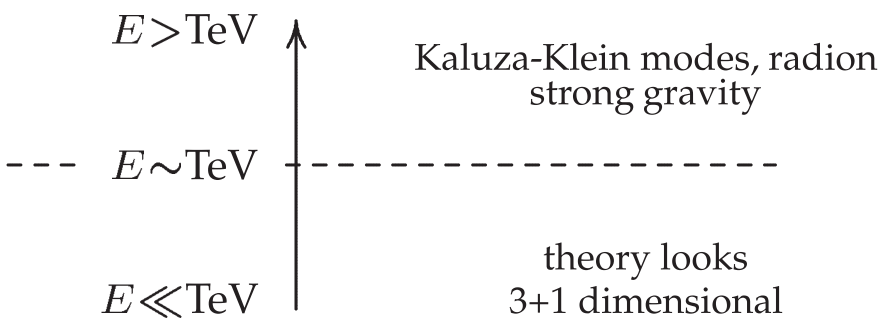
hep-ph/0205240 + work in progress
done in collaboration with Rich Holman and Matthew Martin

The Randall-Sundrum scenario provides an alternative explanation for the exponential hierarchy that exists between the scales of gravity and of the electroweak interactions.



– The interbrane separation is encoded by the *radion*. It is stabilized by the Goldberger-Wise mechanism.

– Since the radion arises from the 5d geometry, it couples directly to the fields on the IR brane.



In this talk, I shall discuss how radion oscillations in the early universe produce an exponential growth in the number of brane particles.

The effective action for the radion

To begin, we need the correct description of the radion

- essentially (until GW) it is a Goldstone mode; the action is invariant under translations of the IR brane
- it can mix with other massless modes present, *e.g.* 4d gravitons

Charmousis, Gregory and Rubakov solved this problem by writing (hep-th/9912160)

$$ds^2 = g_{ab}\, dx^a dx^b = e^{-2k\left[y+f(x)e^{2ky}\right]}\,\hat{g}_{\mu\nu}(x)\, dx^\mu dx^\nu + \left(1+2kf(x)e^{2ky}\right)^2\, dy^2$$

where

$$\begin{aligned} f(x) &= \text{the radion} \\ \hat{g}_{\mu\nu}(x) &= \text{a 4d metric (flat, FRW, \ldots)} \end{aligned}$$

- We only keep the low energy degrees of freedom.

The Einstein equations show no quadratic mixing

$$\begin{aligned} \hat{R}_{\mu\nu} - \tfrac{1}{2}\hat{g}_{\mu\nu}\hat{R} &= \mathcal{O}(f^2) \\ \hat{\nabla}^2 f &= \mathcal{O}(f^2) \end{aligned}$$

(Aside: a source for gravitational radiation)

The size of the extra dimension—and the hierarchy—is stabilized by the Goldberger-Wise mechanism

– $\lesssim$ TeV, we can integrate out the compact dimension
– The bulk Goldberger-Wise field (of mass m) generates an effective potential for the radion:

$$\boxed{\mathcal{L}_{\text{radion}} = -\tfrac{1}{2}\partial_\mu F \partial^\mu F - \tfrac{1}{2} m_F^2 F^2 + \mathcal{O}(F^3)}$$

The physical radion, with a canonically normalized kinetic term is

$$F = 2\sqrt{3} e^{k\Delta y} M_4 k\, f$$

The mass of the radion is

$$\boxed{m_F = \frac{v_1 (1-\eta)^{1/4}}{3} \left(\frac{m}{M_5} \right)^{3/2} \Lambda_F}$$

(η is a free parameter)

(v_1 is the GW field's value on the IR brane)

We have defined a scale associated with the radion,

$$\boxed{\Lambda_F = \sqrt{3} e^{-k\Delta y} M_4 \sim \text{TeV}}$$

Λ_F provides a natural scale for the cut off of the effective theory—above Λ_F the radion becomes strongly interacting

Couplings of the radion to brane fields

Since the radion is part of the geometry, it naturally couples to any fields present—including Standard Model fields on the IR brane.

- A scalar field offers an efficient channel for the radion.
- The coupling between a scalar field Φ, with mass m_Φ, and the radion is given by

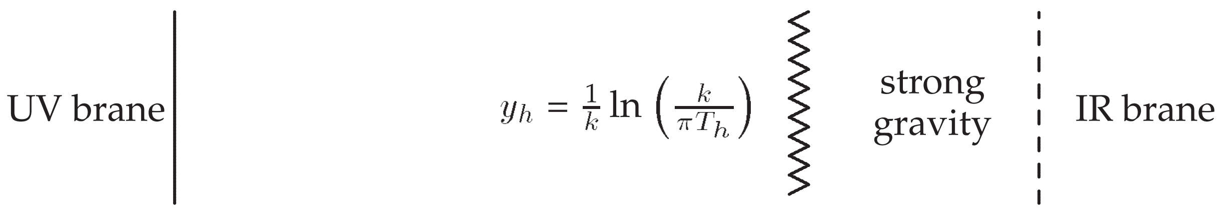
$$\boxed{\mathcal{L}_\Phi = -\frac{1}{2}e^{-F/\Lambda_F}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}m_\Phi^2 e^{-2F/\Lambda_F}\Phi^2 + \cdots}$$

- The effects of the radion-Standard Model couplings, as well as interactions with the Kaluza-Klein modes of the graviton, on collider experiments have been investigated.

We shall instead next examine the effect of small oscillations of the radion in the early universe on the populations of lighter fields.

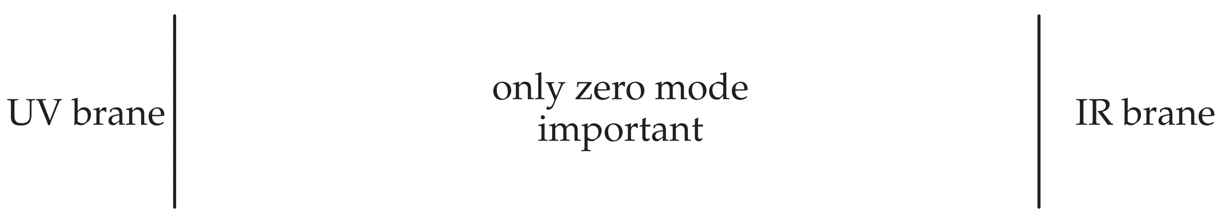
A speculative description of the very early cosmology

– $T \gg$ TeV—the system is unstable to forming a horizon between the branes. Near the IR brane, such energies place us in the strong gravity regime.



(Hebecker and March-Russell, hep-ph/0103214)

– $T <$ TeV—as the temperature decreases, the free energy of the RSI scenario is lower.
At $T_c \lesssim \mathcal{O}(\Lambda_F)$, the universe undergoes a first order phase transition and the IR brane forms.



(Creminelli, Nicolis and Rattazzi, hep-th/0107141)

It is natural, then, after the IR brane forms for the radion to be excited with energy, $\lesssim \mathcal{O}(\Lambda_F)$.

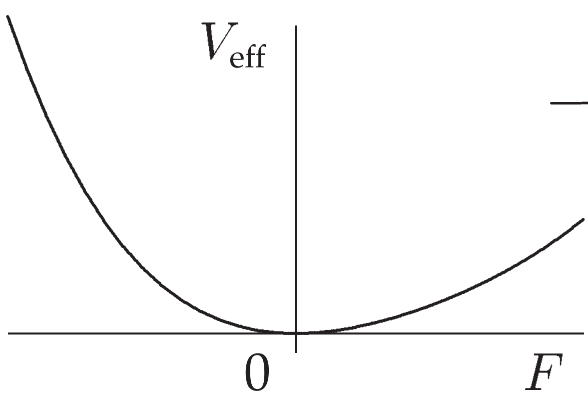
A semi-classical analysis of radion preheating

- Naïvely we might expect a simple decay of the radion, $F \rightarrow \Phi\Phi$, which leads to a population of scalar fields with a typical energy $\sim \frac{1}{2} m_F$.
- **However, the evolution is quite different—it resembles the preheating phase in inflation.**

(Kofman, Linde and Starobinsky)
(Traschen and Brandenberger)

To simplify the calculation,

- we treat the problem semi-classically, quantizing only the scalar field
- the radion oscillations set up a classical background:



$$\longrightarrow F(t) \approx F_0 \Lambda_F \cos(m_F t)$$

- the initial population of Standard Model fields is small
- we neglect the back-reaction on the radion
- we also ignore cosmological expansion

To compute the number of scalar particles produced, we expand Φ in creation and annihilation operators,

$$\Phi(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[a_{\vec{k}}\Phi_{\vec{k}}(t)e^{-i\vec{k}\cdot\vec{x}} + a^\dagger_{\vec{k}}\Phi^*_{\vec{k}}(t)e^{i\vec{k}\cdot\vec{x}}\right]$$

where the $\Phi_{\vec{k}}(t)$ satisfy

$$\frac{d^2\Phi_{\vec{k}}}{dt^2} + F_0 m_F \sin(m_F t)\frac{d\Phi_{\vec{k}}}{dt} + \omega_k^2(t)\Phi_{\vec{k}} = 0$$

with

$$\omega_k^2(t) \equiv |\vec{k}|^2 + m_\Phi^2 e^{-F_0\cos(m_F t)}$$

Initial scalar state

– We define the initial state for the scalar by

$$\partial_t\Phi_{\vec{k}}(0) = -i\omega_k(0)\Phi_{\vec{k}}(0)$$

which defines the vacuum $|0\rangle$ annihilated by $a_{\vec{k}}$

– We can then follow the evolution of the population of scalar particles using

$$N_{\vec{k}}(t) \equiv \langle 0|a^\dagger_{\vec{k}}(t)a_{\vec{k}}(t)|0\rangle$$

Which modes are amplified by the radion oscillations?

Instability bands

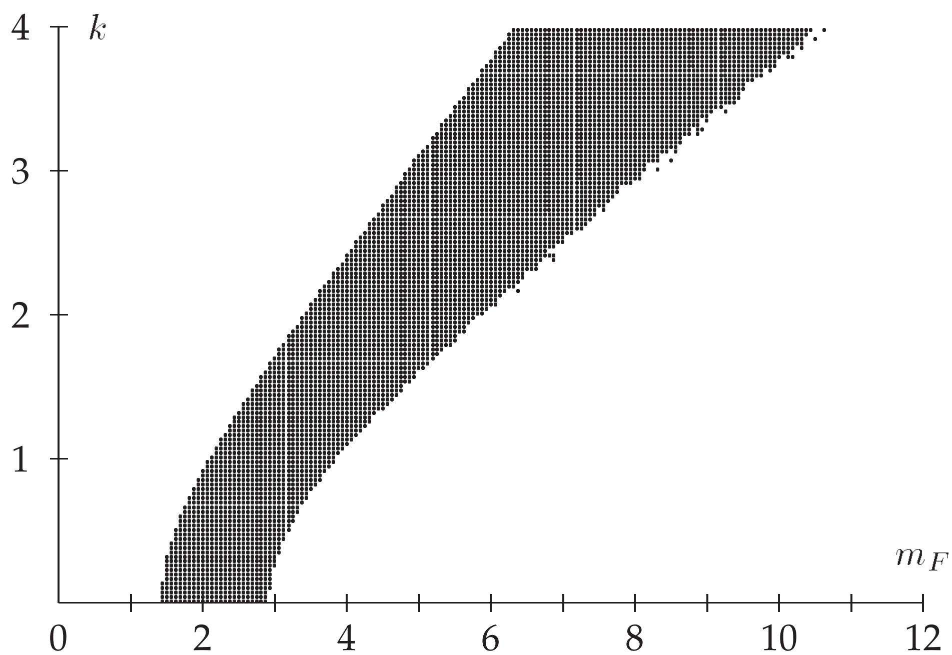
- The coefficient functions in the field equation for $\Phi_{\vec{k}}(t)$ are periodic under $t \to t + 2\pi m_F^{-1}$.
- By Floquet theory, the solution can be written as

$$\Phi_{\vec{k}}(t) = e^{\mu_1(\vec{k})t} g_1(t) + e^{\mu_2(\vec{k})t} g_2(t)$$
$$g_i(t) = g_i(t + 2\pi m_F^{-1})$$

When $\mathrm{Re}\,\mu_i(\vec{k}) > 0$, we obtain an exponential growth of the mode.

- In fact for $F_0 \lesssim \mathcal{O}(1)$, unstable modes exists for a *wide band* of momenta.

e.g. $F_0 = \frac{1}{2}$, exponential growth occurs in dark band



(m_F and $k = |\vec{k}|$ are in units of m_Φ.)

The evolution of the number operator

– In the Heisenberg picture,

$$\frac{\partial}{\partial t} a_{\vec{k}}(t) = i\,[H, a_{\vec{k}}(t)]$$

– It is convenient to write the Hamiltonian in the form

$$H = \int d^3k\ \left[\tfrac{1}{2} e^{F(t)/\Lambda} \hat{\Pi}_{\vec{k}}(t) \hat{\Pi}_{-\vec{k}}(t) + \tfrac{1}{2} e^{-F(t)/\Lambda} \omega_k(t) \hat{\Phi}_{\vec{k}}(t) \hat{\Phi}_{-\vec{k}}(t)\right]$$

where we have introduced the operators

$$\hat{\Phi}_{\vec{k}}(t) = \Phi_{\vec{k}}(t) a_{\vec{k}} + \Phi^*_{-\vec{k}}(t) a^\dagger_{-\vec{k}}$$
$$\hat{\Pi}_{\vec{k}}(t) = e^{-F(t)/\Lambda} \left[\partial_t \Phi_{\vec{k}}(t) a_{\vec{k}} + \partial_t \Phi^*_{-\vec{k}}(t) a^\dagger_{-\vec{k}}\right]$$

and

$$a_{\vec{k}} = a_{\vec{k}}(0) \qquad\qquad a^\dagger_{\vec{k}} = a^\dagger_{\vec{k}}(0)$$

– The solution for $a_{\vec{k}}(t)$ is then

$$a_{\vec{k}}(t) = \frac{i}{\sqrt{2\omega_k(0)}} e^{F(0)/2\Lambda}\, \hat{\Pi}_{\vec{k}}(t) + \sqrt{\frac{\omega_k(0)}{2}} e^{-F(0)/2\Lambda}\, \hat{\Phi}_{\vec{k}}(t)$$

In terms of the creation and annihilation operators of our original vacuum state,

$$a_{\vec{k}}(t) = \alpha_{\vec{k}}(t)a_{\vec{k}} + \beta_{\vec{k}}(t)a^{\dagger}_{-\vec{k}}$$

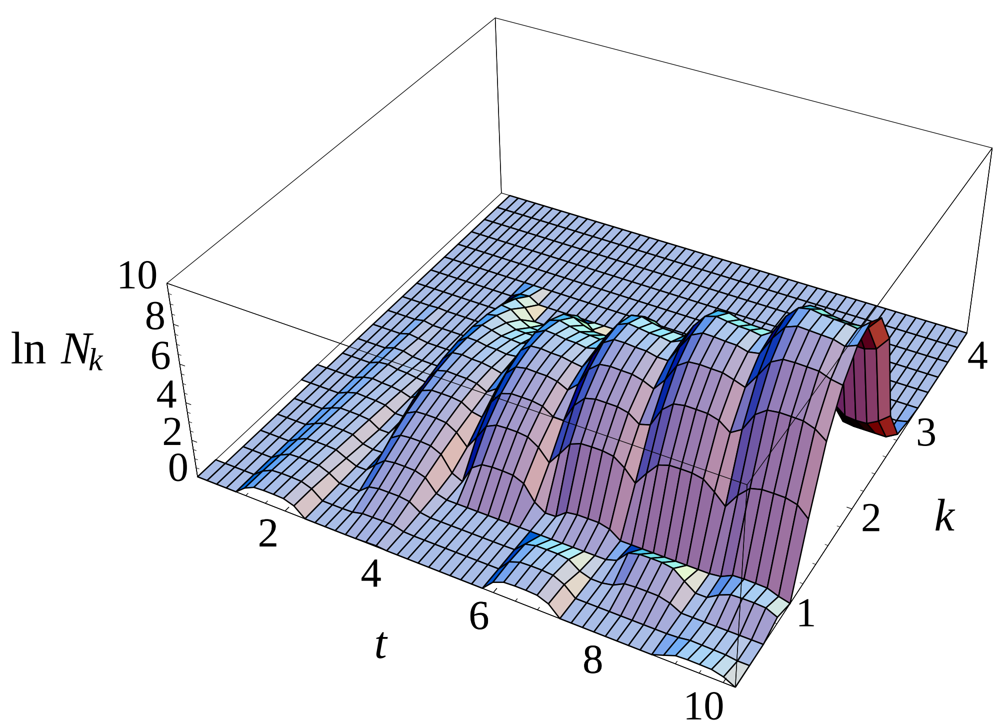
where

$$\beta_{\vec{k}}(t) = \sqrt{\frac{\omega_k(0)}{2}}e^{-F(0)/2\Lambda}\left[\frac{i}{\omega_k(0)}e^{(F(0)-F(t))/\Lambda}\,\partial_t\Phi^*_{-\vec{k}}(t) + \Phi^*_{-\vec{k}}(t)\right]$$

the population of $\vec{k}$-modes then evolves as

$$N_{\vec{k}}(t) = \langle 0|a^{\dagger}_{\vec{k}}(t)a_{\vec{k}}(t)|0\rangle = |\beta_{\vec{k}}(t)|^2$$

Modes within the instability band experience an explosive growth:



Subsequent evolution

- Because the populations of scalar fields in a wide range of momenta are exponentially enhanced, the relaxation of the radion proceeds extremely rapidly.
  - The natural time scale is $\sim m_F$
  - The cosmological expansion during preheating is negligible ($H \sim T^2/M_4 \sim 10^{-16}\,\text{TeV}$)
- This process produces a large non-thermal component of the energy density of the universe at energies $\leq$ TeV.
- An immediate requirement is that the universe should be thermalized by the era of nucleosynthesis.
- Before this—much later—stage, these particles and their equation of state could affect the cosmology and expansion on the IR brane.

A summary of radion-induced preheating

– Quite generically, we expect that in a universe just below the scale associated with the IR brane the radion will be excited.

– Since the radion couples to all brane fields

  its oscillations will parametrically amplify some of the modes of these fields producing an exponential growth in their populations

Since the instability bands are broad, the relaxation of the radion is a rapid, explosive process.

Future work

– A stage of radion-induced preheating is a general feature of a Randall-Sundrum cosmology.
  – How does it affect structure formation, *etc.*?

– A era out of thermal equilibrium at with $T_c \sim v$ could provide a possible source for electroweak baryogenesis.
  – *e.g.* a two Higgs doublet model

– We should relax some of the approximations.
  – include the back-reaction on the radion
    (in the manner of Boyanovsky and Holman)
  – extract the relevant time scales:
    – thermalization time
    – duration of the radion-induced preheating era