ALPHA for the hadron colliders

Workshop on Monte Carlo Generator Physics for Run II at the Tevatron

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Contents

Motivations and Problems

Multiparticle scattering amplitudes (and cross sections) are needed for future colliders

- Multi-jets final states at LHC (backgrounds to new physic searches)
- Six fermions (and more ...) final states at NLC

Analytic approach unpracticle because of algebraic complexity $\Longrightarrow$ authomatic approach required (`SCHOONSHIP, FORM, MADGRAPH, COMPHEP, GRACE`, ...)[a].

- Number of Feynman graphs grows factorially
- Feynman graphs expansion hits a CPU wall

[a] recently appeared `HELAC` which employs the ALPHA algorithm

The ALPHA Algorithm

- The Idea: The Matrix Element 'is' the Legendre Transform Z of the (effective) lagrangian Γ (one-particle-irreducible Green Functions generator) $\rightarrow$ the problem can be recasted as a *minimum problem*, more suitable for a *numerical approach*

1. Z

$$Z(J^{\alpha}) = -\Gamma(\phi^{\alpha}) + J^{\alpha}(x)\phi^{\alpha}(x)$$

where ϕ^{α} are the classical fields defined as the solutions of

$$J^{\alpha} = \frac{\delta\Gamma}{\delta\phi^{\alpha}} \ ,$$

and the J^{α} play the role of classical sources.

2. J^{α}

$$J^{\alpha}(p) = \sum_{j=1}^{n} \epsilon_{\mu}^{\alpha}\delta(p - p_j)$$

notice *finite number* of degrees of freedom

3. $\mathcal{A}$

$$\mathcal{A} \sim \frac{\partial Z}{\partial J_1^{\alpha} \ldots \partial J_n^{\gamma}}|_{J_1^{\alpha}=0,\ldots,J_n^{\gamma}=0}$$

($J_m^{\alpha} = J^{\alpha}(p_m)$, $\sim \rightarrow =$ after truncation)

Working Case: Pure YM theory

- $$\mathcal{L}_{YM}(A) = -1/2 F^a_{\mu\nu} F^a_{\mu\nu} + J_\mu A_\mu$$

  introducing *auxiliary fields* the lagrangian in *momentum space* is

  $$\begin{aligned} \mathcal{L}_{YM}(A) &= -1/2(p_\mu A^a_\nu - p_\nu A^a_\mu)^2 + g f_{abc}(p_\mu A^a_\nu - p_\nu A^a_\mu) A^b_\mu A^c_\nu \\ &\quad -(B^a_{\mu\nu})^2 - 2g f_{abc} B^a_{\mu\nu} A^b_\mu A^c_\nu + J_\mu A_\mu \end{aligned}$$

  (*notice*: at tree-level $\Gamma = \mathcal{L}$):

  1. J standard source terms: $J(p) = \sum_{j=1}^n \epsilon^a_\mu \delta(p - p_j)$, *i.e.* it contains the relevant excitation for the external particles
  2. $A(p)$ are found as solutions of the equation of motion (Feynman Gauge)

     $$\begin{aligned} A_\mu(p) &= \frac{g}{p^2} f_{abc} \left[2(p-k) \cdot A^b(q) A^c_\mu(k) - 2q_\mu A^b(q) \cdot A^c_\mu(k) \right. \\ &\quad \left. - B^b_{\mu\nu}(q) A^c_\nu(k) \right] + \frac{1}{p^2} J_\mu(p) \\ B^a_{\mu\nu}(p) &= -g f_{abc} A^b_\mu(q) A^c_\nu(k) \end{aligned} \tag{1}$$

     (Integration $\int \delta(p+q+k) \mathrm{d}k \mathrm{d}q$ is understood)

- The scattering amplitude is proportional to $\mathcal{L}_{YM}(A)$ with A as in (1)

The (1) equations of motion are solved *iteratively* (expansion in g): the problem is solved with a *loop of matrices multiplication*

- $$J^a_\mu = \sum_{j=1}^n \epsilon^a_\mu(p) \delta(p - p_j)$$

-
$$
\begin{aligned}
A^a_\mu(p_j) &= \epsilon^a_\mu(p_j) \\
B^a_{\mu\nu}(p_j) &= 0
\end{aligned}
$$

  (*notice* truncation)

-
$$
\begin{aligned}
A^a_\mu(p_j+p_k) &= \frac{g f_{abc}}{(p_j+p_k)^2}\left[2(p_j+p_k)\cdot A^b(p_j)A^c_\mu(p_k)\right. \\
&\left. -2p_j\cdot A^b(p_k)A^c_\mu(p_j) - B^b_{\mu\nu}(p_j)A^c_\mu(p_k)\right] \\
B^a_{\mu\nu}(p_j+p_k) &= -g f_{abc} A^b(p_k)A^c_\mu(p_j)
\end{aligned}
$$

- ...
- a few *remarks*
  1. At any stage terms like $A^a_\mu(\cdots + p_k + p_k + \dots)$ (two equal fourier frequencies) are dropped. This would come out from Pauli principle in the case of fermion and this terms do not affect the result.
  2. For a given process like n external gluon, for example, one should compute up to $A^a_\mu(p_{j_1} + \cdots + p_{j_n})$ terms, *i.e.* a *finite* number of terms. In practice, using the equation of motion, only terms up to $A^a_\mu(p_{j_1} + \cdots + p_{j_{n/2}})$ are required (for 6 and 7 external gluons the A^a_μ with up to 3 frequencies are required, for 8 and 9 external gluons up to 4 frequencies).
- The final result is obtained as

$$\mathcal{A}(p_{j_1}, \dots, p_{j_n}) = \mathcal{L}_{YM}(A)$$

  where the A are given in the above steps.
  1. The prescription to drop terms with twice the same frequency is still kept $A(\cdots + p_j + \dots)A(\cdots + p_j + \dots) = 0$. Notice that because of this prescription *no functional derivative is required.*

2. Four momentum conservation is enforced dropping products which do not contains all external fourier frequencies (for $n = 4$, $A(p_1 + p_4)A(p_2)A(p_3)$ is ok $A(p_1)A(p_2)A(p_3)$ is not.
3. In this final step, again because of the equation of motion, only the interaction terms in $\mathcal{L}_{YM}(A)$ are computed and source and kinetic ones are dropped.

The ALPHA code

- The above *algorithm* has been implemented into a `FORTRAN` code `ALPHA` for the *authomatic* calculation of *fully massive, tree level* scattering amplitude, using the standard model lagrangian as input.

  1. authomatic calculation $\rightarrow$ *reliability*
  2. *compact* storing of the information (typical size of the *arrays* $A \sim 2^{next}$)
  3. fully massive matrix element
  4. reasonable CPU time performances: (slow growth, power law like, with number of external particle) at most a few hours (ALPHA dec station) to generate realistic event samples for the problems studied up to now

$$
\begin{aligned}
e^+e^- &\rightarrow 4f(\gamma) \\
e^+e^- &\rightarrow 6f \\
e^+e^- &\rightarrow \bar{\nu}\nu(\gamma\gamma\gamma)
\end{aligned}
$$

QCD calculation

- First of all, interesting final states with larger jet multiplicities will become available with the next generation of colliders (LHC and NLC) [a]

Process	$n = 7$	$n = 8$	$n = 9$	$n = 10$
$g\ g \to n\ g$	559,405	10,525,900	224,449,225	5,348,843,500
$q\bar{q} \to n\ g$	231,280	4,016,775	79,603,720	1,773,172,275

Table 1:

Number of Feynman diagrams corresponding to amplitudes with different numbers of quarks and gluons.

(*notice*: $n = 10$ expected in some R-parity breaking scenarios)

- Secondly, one would like to be able to complement the calculation of parton-level matrix elements with the evaluation of the full hadronic structure of the final state.
  1. Dual amplitudes can be easily evaluated using the `ALPHA` algorithm, by taking N sufficiently large.

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[a] some of the items here are discussed also in P. Draggiotis, R.H.P. Kleiss, C.G. Papadopoulos, Phys.Lett.B439:157-164,1998 e-Print Archive: hep-ph/9807207 employing the ALPHA algorithm

2. Dual amplitudes correspond to planar amplitudes in the $N \to \infty$ limit of QCD $\Longrightarrow$ identification of a specific colour flow $\Longrightarrow$ soft-gluon emission corrections to the hard process (incoherent sum over the emission probabilities from each individual colour-string).

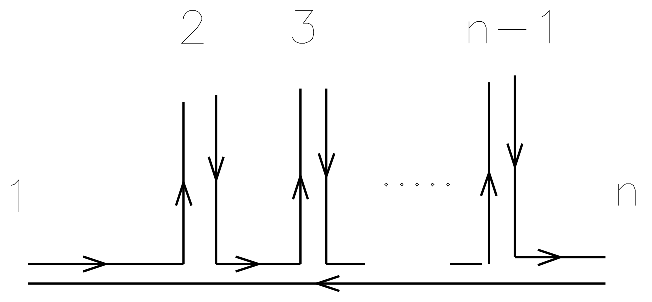

Figure 1:
Colour structure of the n-gluon amplitude in the large-N limit.

3. Sudakov form factors to next-to-leading log (`HERWIG`)

The prescription to correctly generate the parton-shower associated to a given event in the large-N limit is therefore the following:

1. Calculate the $(n-1)!$ dual amplitudes corresponding to all possible planar colour configurations.
2. Extract the *most likely* colour configuration for this event on a statistical basis

Efficient event generation and $1/N$ corrections

- the number of dual amplitudes grows like $n!$
- size of non leading $1/N$ corrections?
- Solution:
  1. choose a standard $SU(3)$ orthonormal basis (Gell-Mann matrices for example)
  2. randomly select a non-vanishing colour assignement for the exernal gluons
  3. *if the event is accepted* choose randomly among the *contributing* dual amplitudes a color flow on the basis of their relative weight

     Two advantages

     - dual amplitudes required only for a small number of phase space points
     - contributing dual amplitudes to a given external coulor assignment $\ll$ than total number.

An example

$$
\begin{aligned}
g\, g &\rightarrow 8\, g \\
q\, \bar{q} &\rightarrow 8\, g\,.
\end{aligned} \tag{2}
$$

$$
\sqrt{\hat{s}} = 1500\,\mathrm{GeV}\,, \quad p_{T_i} > 60\,\mathrm{GeV}\,, \quad |\eta_i| < 2\,, \quad \Delta R_{ij} > 0.7\,. \tag{3}
$$

These values, and the choice of a fixed strong coupling $\alpha_s = 0.12$, only serve for illustrative purposes.

Process	Exact (pb)	`SPHEL` (pb)
$g\ g \rightarrow 8\ g$	0.719 ± 0.019	1.53 ± 0.03
$q\ \bar{q} \rightarrow 8\ g$	$(\,0.901 \pm 0.009\,)\times 10^{-3}$	$(\,1.042 \pm 0.008\,)\times 10^{-3}$

Table 2:

Partonic cross-sections in pb at $\sqrt{\hat{s}} = 1500$ GeV, with the cuts described in the text.

(`SPHEL`: all amplitudes equal to maximally helicity violating ones)

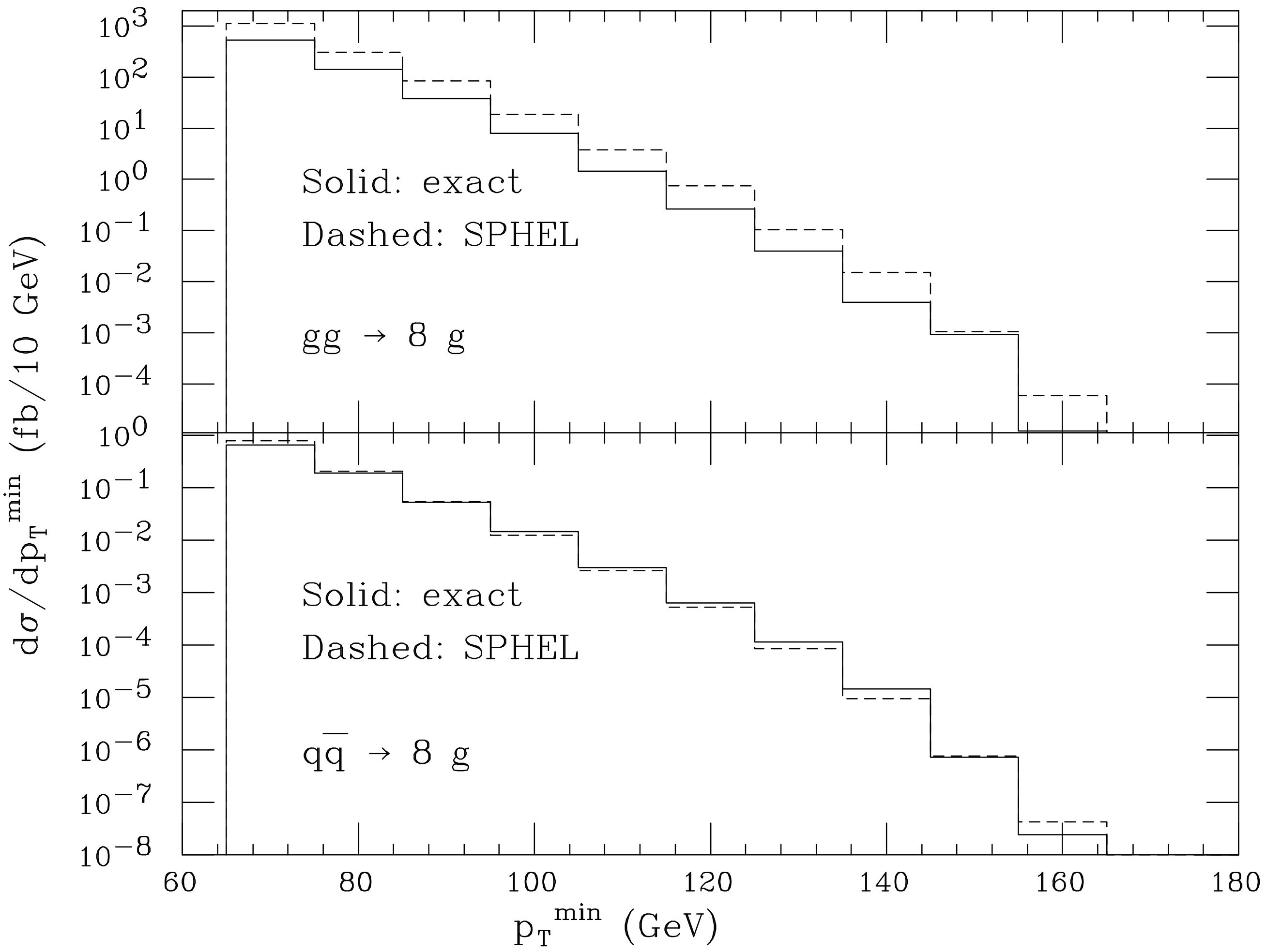

Figure 2:

Differential distributions for the minimum gluon transverse momentum. Exact result vs. `SPHEL`.

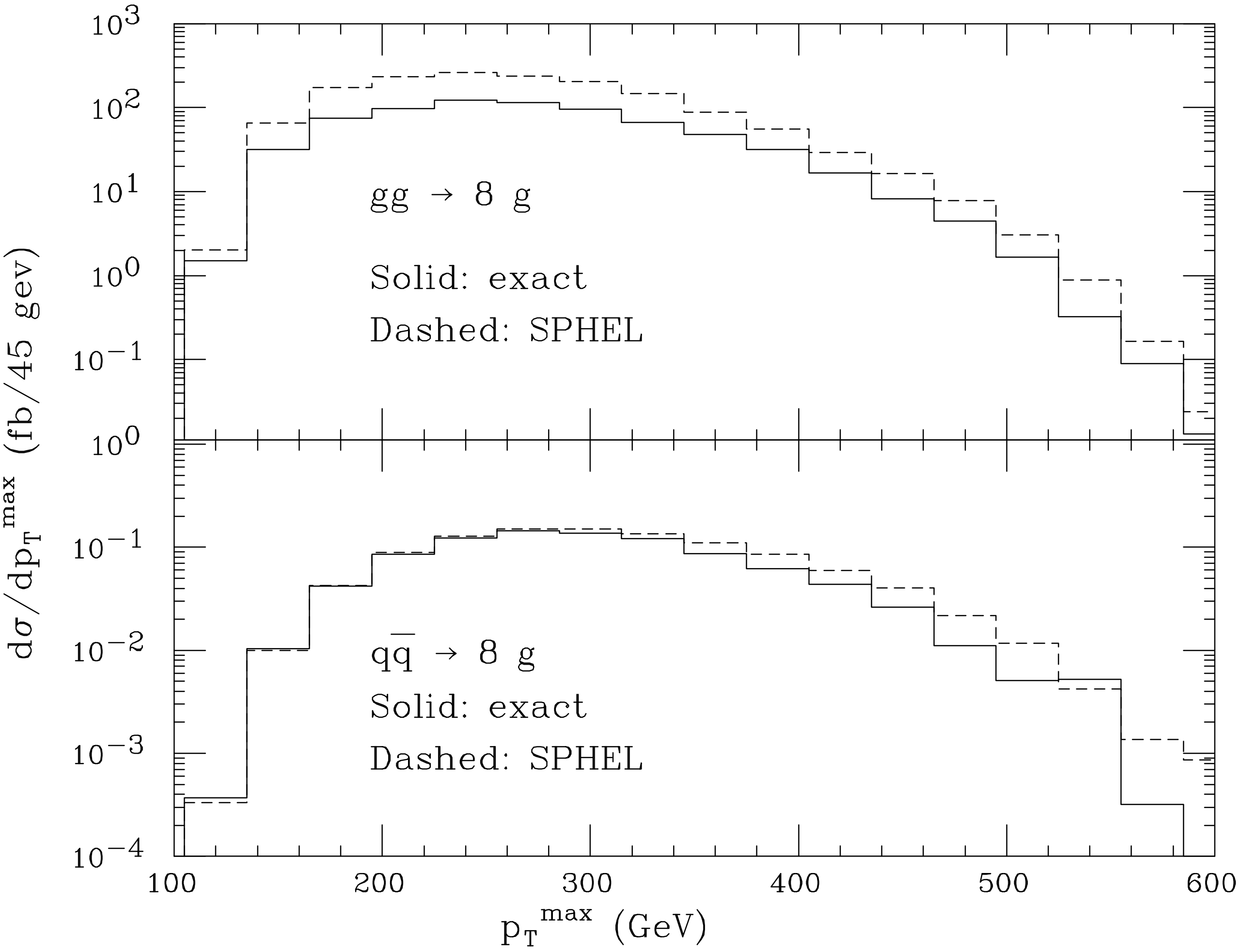

Figure 3:

Differential distributions for the maximun gluon transverse momentum. Exact result vs. `SPHEL`.

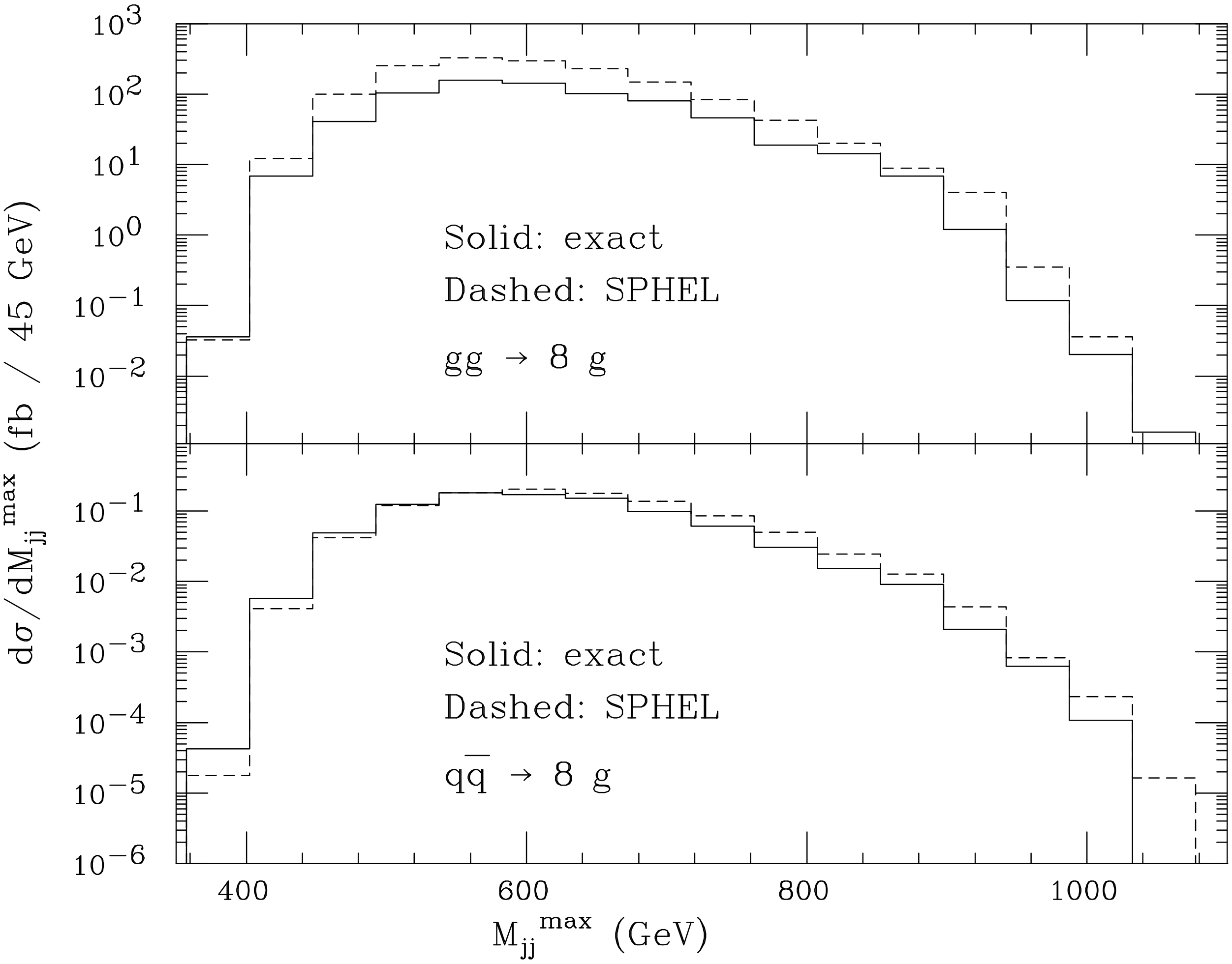

Figure 4:

Differential distributions for the maximum two-gluon invariant mass. Exact result vs. `SPHEL`.

Performance:$g\ g \to 8\ g$

1. 10,525,900 diagrams
2. 1.9×10^6 Monte Carlo points passing the selection cuts (1% of the total)
3. unweighting efficiency $\sim 10^{-4}$
4. 91 secconds CPU to compute 100 (weighted) events (passing the selection cuts) with a 200 MHz processor. (8 times better on a DEC-ALPHA)

A few references

ALPHA

F. Caravaglios and M. Moretti
Phys. Lett. B358 (1995) 332

ALPHA + QCD

F. Caravaglios, M. L. Mangano, M. Moretti and R. Pittau
Nucl. Phys. **B539** (1999) 215

O'Mega

T. Ohl, J. Reuter, C. Schwinn, M.M.

$$\left\{\begin{array}{l}\left\{\begin{array}{l}\text{physics model}\\ \text{target language}\end{array}\right\}\\ \text{external particles}\end{array}\right\} \Longrightarrow \text{source code for amplitude}$$

- the *physics model* is currently specified in *O'Caml*, e. g.:

```
let charged_currents n =
  [ ((L (-n), Wm, N n), FBF (1, Psibar, VL, Psi), G_CC);
    ((N (-n), Wp, L n), FBF (1, Psibar, VL, Psi), G_CC);
    ((D (-n), Wm, U n), FBF (1, Psibar, VL, Psi), G_CC);
    ((U (-n), Wp, D n), FBF (1, Psibar, VL, Psi), G_CC) ]
```

- the *target language* is specified as a O'Caml module.
- the external particles are given on the command line

  `omega.out ep em ep em`

  produces the `FORTRAN` (`C++`, . . .) code

First Results

Radiative corrections to four fermion production, standard model, unitarity gauge:

process	Diagrams		O'Mega	
$e^+e^- \to$	#	vertices	#prop.	vertices
$e^+\bar{\nu}_e d\bar{u}$	20	80	14	44
$e^+\bar{\nu}_e d\bar{u}\gamma$	146	730	36	151
$e^+\bar{\nu}_e d\bar{u}\gamma\gamma$	1112	6672	94	468
$e^+\bar{\nu}_e d\bar{u}\gamma\gamma\gamma$	12420	86940	168	1246
$e^+\bar{\nu}_e d\bar{u}\gamma\gamma\gamma\gamma$	138816	1110528	344	3746

First *realistic application*

- simulation of *six fermion final states* in W^+W^- *scattering* for the *TESLA Technical Design Report*, using `WHIZARD` by *Wolfgang Kilian* as unweighted event generator.

Avaliable Models

- Electroweak Standard Models
- Anomalous gauge couplings
- Graviton and Dilatons couplings (for extra-dimensions fan), in progress
- QCD, in progress
- SUSY, in progress

Avaliable at *ftp://heplix.ikp.physik.tu-darmstadt.de/pub/ohl/omega/*