Summary of workshops on hydrodynamics at BNL

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1 Introduction

One of the most striking discoveries made at RHIC is the presence of large collective flow. Models based on ideal hydrodynamics have been very successful in describing the collective flow observed at RHIC. Which seem to indicate that to a good approximation the matter produced in heavy ion collisions at RHIC is thermalized and behaves like an almost perfect fluid. Estimates of the viscous effects show that the viscosity of the matter produced at RHIC is likely to be smaller than the viscosity of any other matter known so far. There, however, are uncertainties in the hydrodynamic models regarding the initial conditions, the equation of state, and the behavior of the matter at the late hadronic stage. Therefore it is important for heavy ion phenomenology to quantify these uncertainties, and to clarify whether it is possible to constrain or verify the QCD equation of state, and most importantly to determine the value of the viscosity from the experimental data. A series of workshops have been held at Brookhaven National Laboratory with the aim to gather the experts from all around the world and discuss the present status and open challenges in different sub-fields related to the hydrodynamical description of the RHIC data, including the RBRC Workshop "Hydrodynamics in Heavy Ion Collisions and QCD Equation of State", April 21-22, 2008 (http://www.bnl.gov/riken/hhic/), the RBRC Workshop "Understanding QGP through Spectral Functions and Euclidean Correlators", April 23-25, 2008 (http://www.bnl.gov/riken/qgp) and the "Workshop on Viscous Hydrodynamics and Transport Models in Heavy Ion Collisions", April 23-May 2, 2008 (http://quark.phy.bnl.gov/p̃etreczk/viscous_hydro.html)

2 Equation of state

One of the important ingredients in the hydrodynamical description is the QCD equation of state (EOS). Lattice QCD offers the possibility to calculate the EOS from first principles. The current status of the lattice calculation of the EOS has been extensively discussed at the workshop. Results from large scale numerical simulations by MILC, RBC-Bielefeld and HotQCD collaborations, as well as by the Wuppertal group, have been presented. Calculations using improved discretization scheme of QCD (p4, asqtad) appear very successful in calculation of the equation of state, with controlled errors due to finite lattice spacing. The results obtained by different collaborations are in remarkably good agreement with each other. This is illustrated in Figure 1, which shows the trace anomaly or the interaction measure, $\epsilon - 3p$, as a function of temperature (left panel). The speed of sound calculated on the lattice (right panel of Figure 1) shows significant deviation from the ideal gas value of $\sqrt{1/3}$ for energy densities relevant for RHIC. The figure shows results for an improved discretization scheme which are in qualitative agreement with the findings of the Wuppertal group obtained with a standard discretization scheme. It remains to be seen what are the effects of these findings will be on the outcome of the

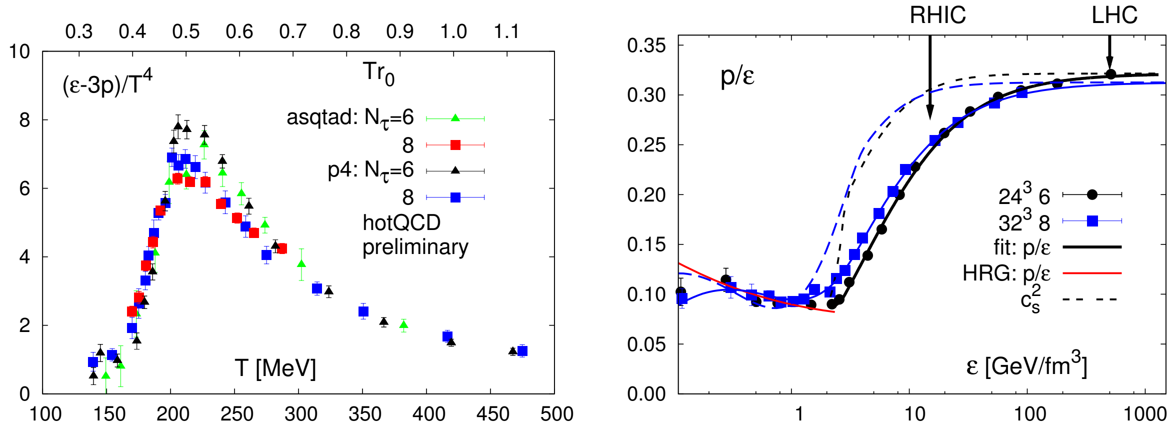

Figure 1: The trace anomaly as the function of the temperature (left) and the speed of sound (right) calculated in lattice QCD by RBC-Bielefeld and HotQCD collaboration.

hydrodynamical modeling.

Current lattice simulations are carried out at quark masses which are still larger by a factor of two than the physical value. This will effect the EOS in the transition region as well as at low temperatures. Furthermore, discetization errors in the low temperature region could be large. It is therefore important to devise a parametrization of the equation of state which relies on the recent lattice data taking into account the uncertainties in the calculationsm which at the same time is easy to use as an input for hydrodynamic codes. This has been identified as a topic for future collaboration among the participants.

3 Ideal hydrodynamics: sensitivity to the initial condition, EOS and hadronic freezout

Although ideal hydrodynamics was successful in describing the RHIC data on collective flow its success depends on the initial conditions, the EOS, and the freeze-out conditions. In hydrodynamic models typically EOS of an ideal gas ($p/\epsilon = 1/3$) and Glauber initial conditions are used. The matter at hadronic stage is either described by hydrodynamics or by hadronic transport theory. The sensitivity of the flow on the EOS has been discussed both in the pure hydrodynamic approach (talk by P. Huovinen) and in the hydrodynamic approach coupled to hadron cascade (talk by T. Hirano). It turns out that if one uses an equation of state without a first order phase transition but with a smooth crossover the hydrodynamic calculation significantly overpredicts the elliptic flow of the protons (talk by Huovinen).

The hybrid approach (hydro + cascade) is significantly better in describing the data on the elliptic flow, including its rapidity dependence if Glauber initial conditions and ideal EOS are used. However, if one uses Color Glass Condensate (CGC) initial conditions a much softer EOS, namely $p/\epsilon = 1/6$ is needed to describe the data (talk by Hirano). It is know that viscous effects will also reduce the elliptic flow. Therefore, in order to use hydrodynamics to constrain the fundamental properties of QCD, such as the EOS and the shear viscosity, one has to have much better understanding of the initial conditions! To leading order in the coupling constant an approach based on the classical fields provides a possibility for an ab initio calculation of the initial conditions. Recently this approach has been extended to next-to-leading order and it has been shown that there exist a factorization formula for particle production in nulceus-nucleus collisions analogous to the usual collinear factorization. It has been discussed that large rapidity correlations in the two particle yield, the ridge, can give an experimental handle on the initial conditions (talk by R. Venugopalan).

4 Transport coefficients, spectral functions and Euclidean correlators

In finite temperature QCD using Kubo formulas different transport coefficients can be calculated from correlation functions of the appropriate field operators (current, energy momentum tensor, etc). More precisely, the transport coefficients are obtained from the zero frequency limit of the corresponding spectral functions. Transport coefficients, in particular bulk and shear viscosity has been calculated in QCD using weak coupling methods. Although weak coupling methods are not reliable in the temperature range relevant for RHIC a straightforward extrapolation of the weak coupling results to the corresponding temperatures (corresponding to $\alpha_s \simeq 0.5$) gives $\eta/s = 0.5$ for the shear viscosity to the entropy ratio. This value is already smaller than the corresponding ratio of all the other known fluids. Transport cascade calculations which include $2 \rightarrow 3$ and $3 \rightarrow 2$ processes give $\eta/s = 0.13 - 0.17$ (talks by A. El and Zhe Xu). QCD calculations of the shear viscosity based on effective models of the Polyakov loop and lattice QCD indicate a significant drop of η/s close to transition temperature (talk by Y. Hidaka). This is shown in Figure 2. Spectral functions are related to correlation functions in Euclidean time, which can be calculated in lattice QCD. Attempts to calculate transport coefficients from lattice QCD using Bayesian analysis and QCD sum rule have been presented during the workshops. It turns out that present lattice data are not precise enough to constrain the values of the shear viscosity and quark diffusion constant. However, progress has been made in extracting bulk viscosity from lattice QCD. Calculations in pure gauge theories, as well as in QCD, show that bulk viscosity strongly increases in the vicinity of the transition temperature as shown in Figure 2

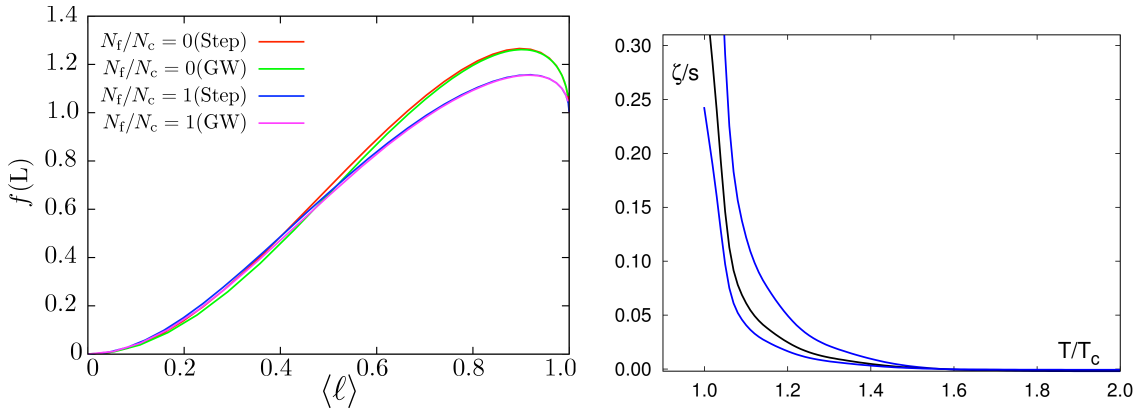

Figure 2: The shear viscosity (left) and the bulk viscosity (right) of QCD above the transition temperature, from talks by Y. Hidaka and D. Kharzeev.

5 Relativistic viscous hydrodynamics and transport models

Since the viscosity cannot be exactly zero it is important to address its effect on the collective flow. In the relativistic case the usual Naiver-Stokes viscous hydrodynamics, which relies on the first order gradient expansion is unstable due to causality violation. This problem can be avoided if second order terms in the gradient expansion are retained. There are several second order viscous hydrodynamics formulations: the Israel-Stewart formalism, the Öttinger-Grmela formalism and the formalism based on AdS/CFT correspondence, i.e. conformal hydrodynamics. While dissipative effects in the usual first order viscous hydrodynamics can be parametrized by a single ratio η/s, in principle many more parameters are present in the second order viscous hydrodynamics. One of the main subjects of the workshop was to understand the relation between different second order formulations and the effects of the additional parameters. It turns out that conformal symmetry implies stringent constraint on the form of the viscous part of the energy-momentum tensor, and the number of the additional parameters. For the situation relevant for heavy ion collisions there only two parameters, the relaxation time τ_π and the self coupling λ_1 of the viscous energy momentum tensor (talks by A. Muronga and P. Romatschke). The Israel - Stewart formalism corresponds to $\lambda_1 = 0$. In the Öttinger Grmela formalism there are two additional parameters and $\lambda_1 \neq 0$. If all the second order terms are included in the calculations the dependence of the viscous effects on the relaxation time τ_π appears to be small (talks by P. Romatschke and H. Song). Thus, in principle, it is possible to quantify η/s. Different groups agree that using the lowest possible value of $\eta/s = 1/(4\pi)$ reduces the elliptic flow v_2 by about $25 - 30\%$ (talks by P. Romatschke, H. Song and J.Y. Ollitrault). This can be seen in Figure 3, where v_2 calculated in the Israel - Stewart hydrodynamics is shown. The comparison to RHIC data seems to provide the following value of $\eta/s = 0.1 \pm 0.1(theory) \pm 0.08(exp)$ with the biggest theory error being due to the uncertainties in the initial conditions (talk by P. Romatschke). Clearly, no description of the experimental data is possible for $\eta/s > 0.4$ (talk by K. Dusling). Since the range of applicability of the second order viscous hydrodynamics is not clear it is important to study collective flow within transport models. It has be demonstrated that for sufficiently large cross section transport models with binary collisions approach the limit of the ideal hydrodynamics (talk by J.Y. Ollitrault). It also has been shown that provided the same initial conditions are used, transport calculations with binary collisions and the Israel-Stewart second order hydrodynamics, provide almost identical results (talk by P. Huovinen). This supports the use of the second order hydrodynamics.

Uncertainties in the experimental determination of the elliptic flow parameter v_2 due to initial state fluctuations and non-flow effects have also been discussed during the workshop (talks by P. Sorensen and P. Steinberg).

A significant part of the workshop was dedicated to comparing different viscous hydrodynamic codes and testing their validity. It seems that different hydrodynamic codes give consistent results.

6 Open problems

To achieve a better understanding of the collective flow at RHIC the following problems have to be addressed in the future:

- A detailed numerical test of the available viscous hydrodynamic codes and verification of their consistency. First steps in this direction have been taken during the workshop.
- A realistic parametrization of the EOS based on lattice QCD. This should be updated on a regular basis as more reliable lattice results become available.
- A more realistic description of the collective flow in the hadronic phase based on viscous hydrodynamics, coupled to hadronic transport model.
- Better understanding of initial conditions and their impact on the hydrodynamic evolution.
- Study of the second order hydrodynamics outside the conformal limit, and in particular the effects of bulk viscosity.

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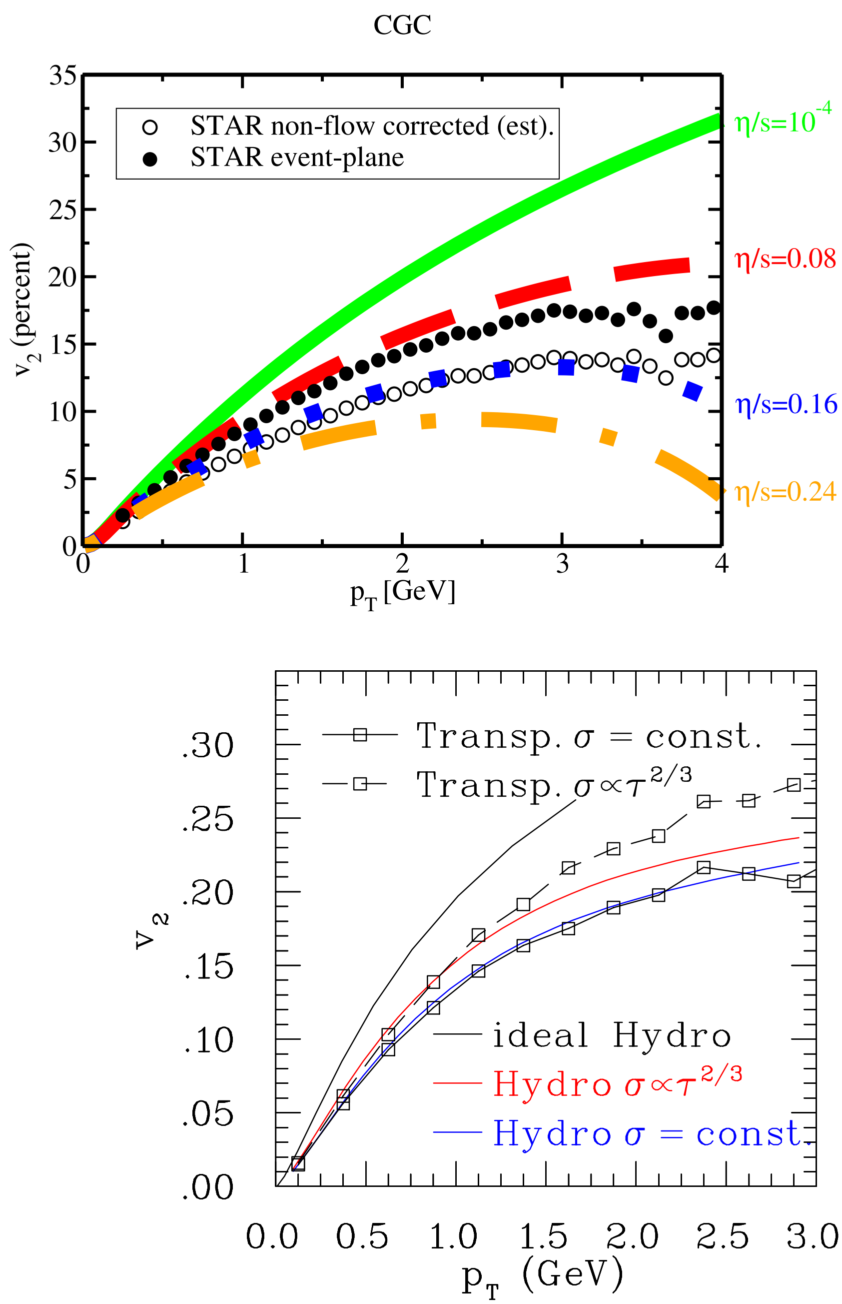

Figure 3: The elliptic flow calculated in Israel-Stewart hydrodynamics for different values of η/s (top) and the comparison to transport calculations with $\eta/s = 1/(4\pi)$ (bottom), from talks by P. Romatschke and P. Huovinen.