Title: Stomach Turner Roller Coaster

Brief Overview:

This unit teaches students to recognize, graph, and write equations for sinusoidal curves with and without the use of technology.

NCTM 2000 Principles for School Mathematics:

- **Equity:** *Excellence in mathematics education requires equity - high expectations and strong support for all students.*
- **Curriculum:** *A curriculum is more than a collection of activities: it must be coherent, focused on important mathematics, and well articulated across the grades.*
- **Teaching:** *Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.*
- **Learning:** *Students must learn mathematics with understanding, actively building new knowledge from experience and prior knowledge.*
- **Assessment:** *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*
- **Technology:** *Technology is essential in teaching and learning mathematics; it influences the mathematics that is taught and enhances students' learning.*

Links to NCTM 2000 Standards:

- **Content Standards**

Algebra
Students will understand patterns and functions as pertaining to a sinusoidal curve; represent and analyze mathematical situations using algebraic symbols; use mathematical models to represent and understand quantitative relationships; and analyze change in various contexts.

Geometry
Students will describe spatial relationships using coordinate geometry; apply transformations and use symmetry to analyze mathematical situations; and use visualization, spatial reasoning, and geometric modeling to solve problems.

Measurement
Students will understand measurable attributes of objects and the units systems and processes of measurement. They will apply appropriate techniques, tools, and formulas to determine measurements.

Data Analysis and Probability
Students will formulate questions that can be addressed with data. They will develop and evaluate inferences and predictions that are based on data.

- **Process Standards**

Mathematics as Problem Solving, Reasoning and Proof, Communication, Connections, and Representation
These five process standards are threads that integrate throughout the unit, although they may not be specifically addressed in the unit. They emphasize the need to help students develop the processes that are the major means for doing mathematics, thinking about mathematics, understanding mathematics, and communicating mathematics.

Students will analyze a sinusoidal curve for the amplitude, the period, and the horizontal and vertical shifts with and without the use of technology. They will use correct mathematical language to justify their conclusions. They also will interpret and analyze sinusoidal functions in order to recognize the connections between the graphical and algebraic representations of a sinusoidal function. Students will use that knowledge to interpret a real-life situation.

Links to Maryland High School Mathematics Core Learning Units:

Functions and Algebra
- **1.1.1**
Students will recognize, describe, and extend patterns and functional relationships that are expressed numerically, algebraically, and geometrically.

- **1.1.2**
Students will represent patterns and functional relationships in a table, as a graph, and/or by mathematical expression.

- **1.1.4**
Students will describe the graph of a non-linear function in terms of the basic concepts of maxima and minima, roots, limits, rate of change, and continuity.

- **1.2.4**
Students will describe how the graphical model of a non-linear function represents a given problem and will estimate the solution.

Geometry, Measurement, and Reasoning
- **2.2.3**
Students will identify and use inductive and deductive reasoning.

- **2.3.1**
Students will use algebraic and geometric properties to measure indirectly.

Data Analysis and Probability

- **3.2.1**
  Students will make informed decisions and predictions based upon the results of simulations and data from research.

Grade/Level:

This learning unit is appropriate for Advanced Mathematics and Pre-Calculus (Grades 11-12).

Duration/Length:

Three class periods of 88 minutes each

Prerequisite Knowledge:

Students should have working knowledge of the following skills:

- General knowledge of using TI-83 or TI-83 Plus
- General knowledge of maximum, minimum, period, vertical shift, and horizontal shift

Student Outcomes:

Students will be able to:

- sketch sinusoidal curves given a function or data.
- analyze a sinusoidal curve for the amplitude, period, horizontal shift and vertical shift with and without the use of technology.
- use the current language of mathematics to justify conclusions.
- interpret and analyze sinusoidal functions.
- recognize the connection between the graphical and algebraic representations of a sinusoidal function.
- use the above knowledge to interpret a real-life situation.

Materials/Resources/Printed Materials:

- Graph paper
- TI-83 or TI-83 Plus
- TI-83 overhead panel/projector
- Activity sheets
- Practice quiz
- Assessment
- Student answer sheet
- Answer keys for all activities

Development/Procedures:

The teacher will introduce the unit by presenting and discussing the equations and graphs for the sine and cosine functions. Students will use the TI-83 or the TI-83 PLUS calculator to investigate and explore transformations of sinusoidal curves. The essential vocabulary associated with sinusoidal curves (period, amplitude, phase shift and vertical shift) will be emphasized. Students will then graph sinusoidal curves in various positions on the grid without the aid of technology. Given a graph of a sinusoidal curve, the students will be able to write the equation to model the graph. Students will then complete Worksheets 1 and 2.

The teacher will provide supplementary materials for review and reinforcement from textbooks and alternate sources, such as the Internet and worksheets. After a knowledge base of sinusoidal curves has been established, the teacher will ask students about their experiences with roller coasters. Students will then complete the Student Activity Sheets.

Assessment:

Students will be given:

a) a formative worksheet, including the vocabulary for and the sketching of sinusoidal curves.
b) a summative assessment, including graphing, analyzing and writing functions for sinusoidal curves. Questions are written in Selected Response, Student Produced Response, Brief Constructed Response, and Extended Constructed Response formats.

Extension/Follow Up:

Students will use the Internet to investigate additional real-life situations that simulate sinusoidal curves. Written and/or oral reports using the language of mathematics and the use of technology will be given.

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WORK SHEET #1

Name _____

Date _____

Part A. Write an equation for the transformations for each of the following:

1. **Given $y = \sin x$**
    a. move 2 units right
    b. move 3 units up
    c. move 2 units right and 3 units up

2. **Given $y = 2 \cos 3x$**
    a. move 1 unit left
    b. move 2 units down
    c. move 1 unit left and 2 units down

Part B. Verify the equations you wrote with your graphing calculator.

WORK SHEET #1

Answer Sheet

Part A. Write an equation for the transformations for each of the following:

1. Given $y = \sin x$

   a. move 2 units right — **$y = \sin (x - 2)$**
   b. move 3 units up — **$y = \sin x + 3$**
   c. move 2 units right and 3 units up — **$y = \sin (x - 2) + 3$**

2. Given $y = 2 \cos 3x$

   a. move 1 unit left — **$y = 2 \cos 3 (x + 1)$**
   b. move 2 units down — **$y = 2 \cos 3x - 2$**
   c. move 1 unit left and 2 units down — **$y = 2 \cos 3 (x + 1) - 2$**

Part B. Verify the equations you wrote with your graphing calculator.

WORK SHEET #2

Name _____

Date _____

Part A. Without the use of a calculator sketch the graph for each of the following sinusoidal equations. You must verify your graph with a calculator.

1. $y = 3 \sin x$
2. $y = - \sin x$
3. $y = \sin x + 2$
4. $y = \sin (x + 2)$
5. $y = 4 \cos x$
6. $y = 4 \cos x – 3$
7. $y = 4 \cos (x + 1)$
8. $y = \cos (\pi/2) x + 3$
9. $y = - 1 + \cos (x – 2)$
10. $y = 3 + 4 \sin 2(x – 1)$

Part B. For <u>each</u> equation in Part A identify the following:

1. amplitude
2. period
3. maximum
4. minimum
5. vertical shift
6. horizontal shift

Activity Sheet – Teacher Copy

A roller coaster called the Stomach Turner has to be designed for the new Mathtique Amusement Park. The path of the roller coaster will be modeled by a sinusoidal curve. (Students are to enter and to graph the following in their calculators:

1. $y = \cos x$
2. $y = \cos 2x$
3. $y = 3 \cos 2 (x - \pi/2)$
4. $y = 3 \cos 2x + 4$
5. $y = 3 \cos 2 (x - \pi/2) + 4$

(**NOTE**: Teachers may use sine or cosine.)

Ask the students which equation they would choose for the path of their roller coaster and to explain why.

(**NOTE**: Students should write their answers in a descriptive paragraph using mathematics vocabulary such as amplitude, period, and shift.)

Student Activity Sheet

Name _____

Date _____

A roller coaster called the Stomach Turner has to be designed for the new Mathtique Amusement Park. The path of the roller coaster will be modeled by a sinusoidal curve. (Students are to enter and to graph the following in their calculators:

1. $y = \cos x$
2. $y = \cos 2x$
3. $y = 3 \cos 2 (x - \pi/2)$
4. $y = 3 \cos 2x + 4$
5. $y = 3 \cos 2 (x - \pi/2) + 4$

Which of the above equations would you choose as a model for the path of a roller coaster? Write a descriptive paragraph using mathematical vocabulary, such as amplitude, period, phase shift, and vertical shift to justify your selection.

Assessment for the Stomach Turner Roller Coaster

Teacher's Guide

Introduction

The purpose of the assessment activity is to determine whether the students mastered the objectives that were taught. The assessment should be given at the end of the unit.

Objectives Covered

Students will be able to:

- sketch sinusoidal curves from a function.
- write a function given a sinusoidal curve.

Tools/Materials Needed for Assessment

- TI-83 or TI-83 Plus
- Graph paper

Administering the Assessment

Students will complete the assessment independently. They will record their answers directly on the answer sheet. The scoring key and rubrics are included.

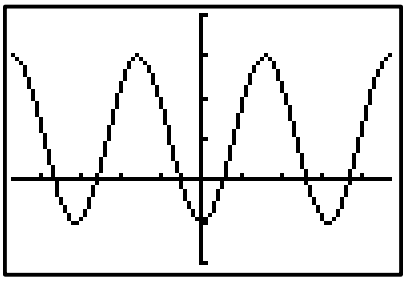
Assessment

Assessment for the Stomach Turner Roller Coaster

1. Given the function $f(x)=3\sin\frac{p}{4}(x-2)+1$, find $f(5)$.

2. Using the function in question #1 above, what is the value of the amplitude and the value of the period?
   A. amplitude = 3 and period = 2
   B. amplitude = 3 and period = 8
   C. amplitude = 8 and period = 3
   D. amplitude = 2 and period = 3

3. Look at the graph at the right.



What is the equation for the graph?

A. $f(x)=\cos 2\left(x-\frac{p}{2}\right)+3$

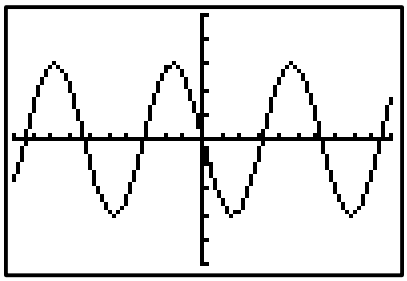
B. $f(x)=2\cos 2\left(x-\frac{p}{2}\right)+1$

C. $f(x)=\cos\frac{p}{2}(x-3)+3$

D. $f(x)=2\cos\frac{p}{2}(x-3)+1$

4. Write the equation of a cosine function with an amplitude of 4, a horizontal shift of $\frac{p}{6}$ to the right, a vertical shift of 1 unit up, and a period of p. Use words, symbols or both to justify your answer.

Look at the graph of the periodic function $f(x)$ below for questions 5 – 9.



5. What is the period of the function?

6. What is the amplitude of the function?

7. Sketch the graph of $y = |f(x)|$.

8. Sketch the graph of $y = f(x-1)$.

9. Sketch the graph of $y = f(x)-3$.

10. The Stomach Turner Roller Coaster modeled after a sinusoidal curve has a minimum height of 4 feet and a maximum height of 50 feet. This section of the roller coaster spans 100 feet along the ground. The ride starts at the minimum point of the curve. People get in the car 10 feet from the entrance gate.

    - From the information given above, draw a sinusoidal graph that represents the path of the roller coaster.

    - Write the equation for the graph. Use words, symbols, or both to justify your answer.

Student Response Sheet
(Stomach Turner Roller Coaster Unit)

Name _____

Date _____

1. _____

2. _____

3. _____

4. _____

   _____

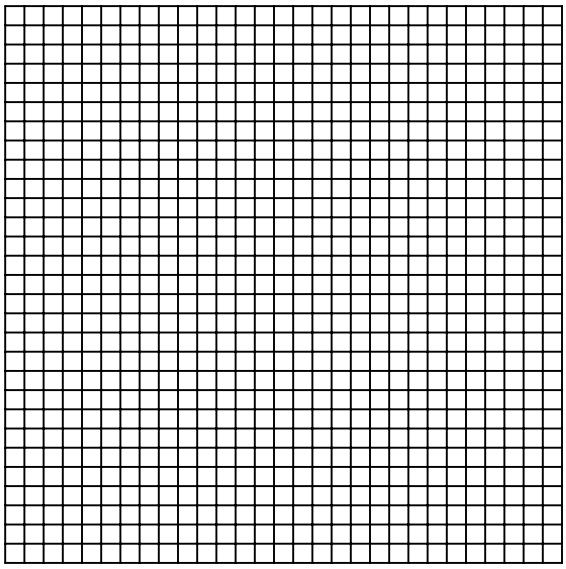
   _____

   _____

5. _____

6. _____

7.



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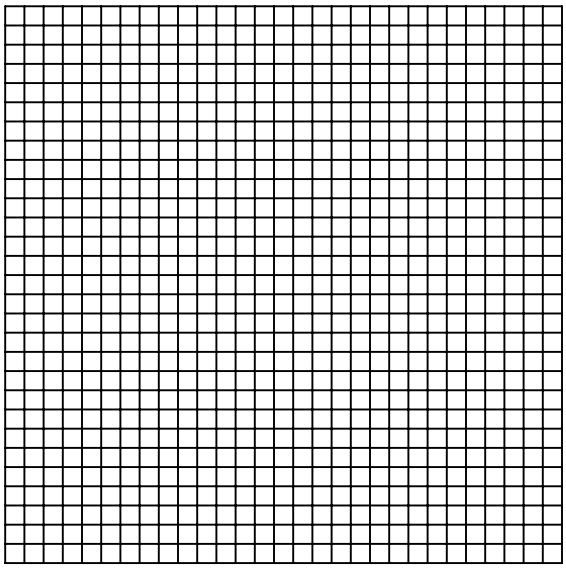

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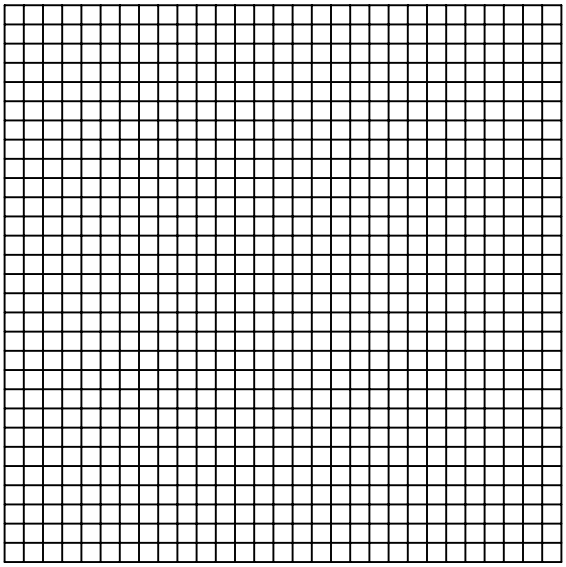

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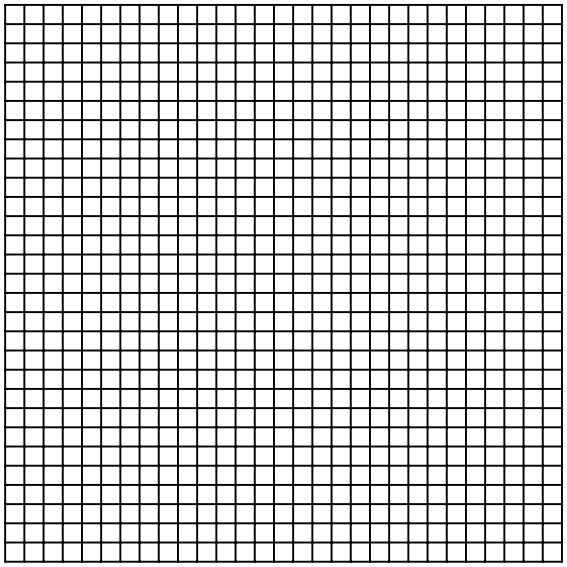


Assessment

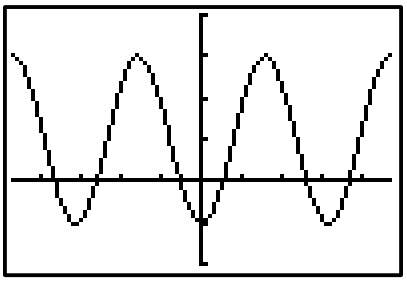
Answer Key and Scoring Guide

1. Given the function $f(x)=3\sin\frac{p}{4}(x-2)+1$, find $f(5)$. **ANSWER** **3.1213** **(8)**

2. Using the function in question #1 above, what is the value of the amplitude and the value of the period?

   A. amplitude = 3 and period = 2
   B. amplitude = 3 and period = 8
   C. amplitude = 8 and period = 3
   D. amplitude = 2 and period = 3 **ANSWER** **B** **(8)**

3. Look at the graph at the right.



What is the equation for the graph?

A. $f(x)=\cos 2\left(x-\frac{p}{2}\right)+3$

B. $f(x)=2\cos 2\left(x-\frac{p}{2}\right)+1$

C. $f(x)=\cos\frac{p}{2}(x-3)+3$

D. $f(x)=2\cos\frac{p}{2}(x-3)+1$ **ANSWER** **B** **(8)**

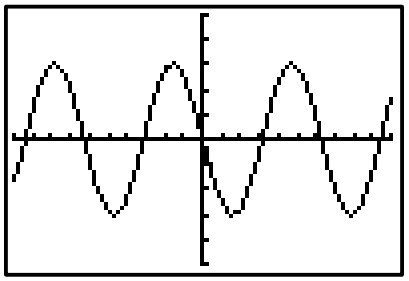
4. Write the equation of a cosine function with an amplitude of 4, a horizontal shift of $\frac{p}{6}$ to the right, a vertical shift of 1 unit up, and a period of p . Use words, symbols or both to justify your answer.

ANSWER **(9)**

$$y = 4\cos 2\left(x - \frac{p}{6} \right) + 1$$

In the equation y = A cos B (x – C) + D, A represents the amplitude, B represents 2 pi divided by the period, C represents the horizontal shift, and D represents the vertical shift. **(9)**

Look at the graph of the periodic function $f(x)$ below for questions 5 – 9.

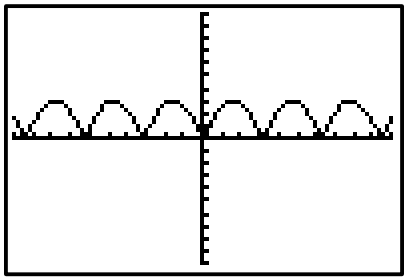


5. What is the period of the function? **ANSWER** **6** **(8)**

6. What is the amplitude of the function? **ANSWER** **3** **(8)**

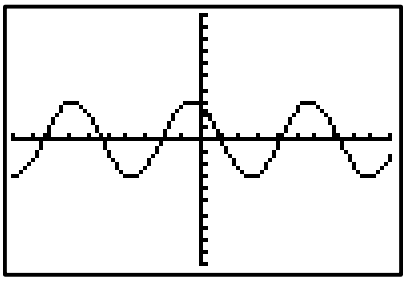
7. Sketch the graph of $y = |f(x)|$.

ANSWER **(8)**



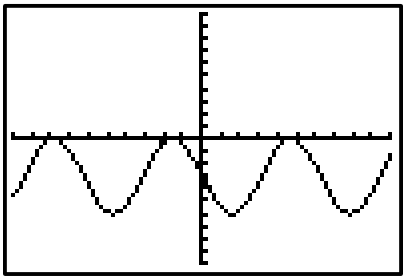
8. Sketch the graph of $y = f(x-1)$.

ANSWER **(8)**



9. Sketch the graph of $y = f(x)-3$.

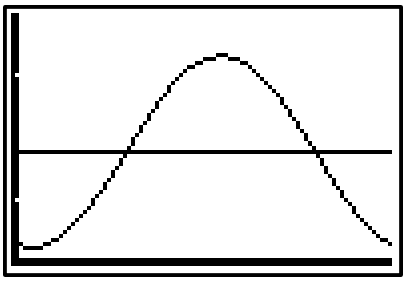
ANSWER **(8)**



10. The Stomach Turner Roller Coaster modeled after a sinusoidal curve has a minimum height of 4 feet and a maximum height of 50 feet. This section of the roller coaster spans 100 feet along the ground. The ride starts at the minimum point of the curve. People get in the car 10 feet from the entrance gate.

- From the information given above, draw a sinusoidal graph that represents the path of the roller coaster.

ANSWER **(9)**



- Write the equation for the graph. Use words, symbols, or both to justify your answer.

ANSWER **(9)**

$$y = 23\sin\frac{p}{100}(x-60)+27$$

or

$$y = 23\cos\frac{p}{100}(x-110)+27$$

In the equation y = A cos B (x – C) + D, A represents the amplitude, B represents 2 pi divided by the period, C represents the horizontal shift, and D represents the vertical shift.