Parton Distributions in Hadrons

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Abstract. We use the statistical model of Zhang et al. [1] to calculate parton distributions in hadrons. The model does reasonably well in predicting the distributions of partons in the proton, including the $(\bar{d}-\bar{u})$ excess in the proton sea. We extend the model to calculate quark and gluon distributions in the pion, kaon, and the pentaquark. The hadrons are described in terms of quark and gluon Fock states. Detailed balance between each pair of states is assumed, from which the coefficients of the Fock state expansion are determined. The parton distribution functions are found in the hadron rest frame from a Monte Carlo calculation. The results are evolved to appropriate QCD scales for comparison with experiment. Despite its simplicity, the model is in good agreement with the experimentally measured distributions of partons in the pion and kaon. This project has included significant participation by undergraduates at Seattle University, made possible by support from the Research in Undergraduate Institutions Program of the National Science Foundation.

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INTRODUCTION

The description of the momentum distribution functions of a hadron's constituent partons is a problem of fundamental interest. Topics of current experimental investigation include the light flavor asymmetry in the proton sea and the valence quark distribution functions of the pion. We have used the statistical model of Zhang et al. [1], which provides a good description of the $(\bar{d}-\bar{u})$ asymmetry in the proton, to calculate other parton distributions in hadrons. Our results include distributions for the pion, the kaon, and the pentaquark, and for the strange sea of the proton.

STATISTICAL MODEL

In its simplest form, the statistical model of Zhang et al. [1] is based on a Fock state expansion of a hadron in terms of its quark and gluon states, together with detailed balance between the states. Transfer between states is assumed to take place through quark-gluon splitting and recombination, quark-antiquark creation and annihilation, and gluon splitting. Transfer rates between Fock states are assumed to be proportional to the number of partons that can split or recombine. For example, the Fock state expansion of the proton, assuming only a light quark sea, is given by

$$|p>=\sum_{i,j,k} c_{ijk}|\{uud\}\{ijk\}>, \tag{1}$$

with i the number of $\bar{u}u$ pairs, j the number of $\bar{d}d$ pairs and k the number of gluons. The states are normalized, so that the sum of the probabilities $\rho_{ijk} = |c_{ijk}|^2$ is unity. From detailed balance, taking into account the two processes, $q \leftrightarrow qg$ and $g \leftrightarrow q\bar{q}$

$$\frac{\rho_{ijk}}{\rho_{000}} = \frac{1}{i!(i+2)!j!(j+1)!k!} \, . \tag{2}$$

This equation, together with normalization, determines all the ρ_{ijk}. It is clear that $u\bar{u}$ states, labelled by i, are suppressed relative to $d\bar{d}$ states, labelled by j. Summing over all states, the light sea asymmetry $(\bar{d} - \bar{u}) \approx 0.124$, remarkably close to the experimental value of 0.118 ± 0.012 [2].

To determine parton distribution functions a Monte Carlo simulation of the distribution of momenta among the n partons in each Fock state was used. The phase space volume f_n^F for n free partons is determined by

$$d f_n^F = \delta^4(P - \sum_{i=1}^{n} p_i) \prod_{i=1}^{n} \frac{d^3 p_i}{(2\pi)^3 2E_i} \, , \tag{3}$$

with P and p_i the 4-momenta of the proton and the i-th parton, respectively. Zhang et al. argue that this free parton phase space distribution should be multiplied by $\prod E_i^{-1}$ because partons with smaller momenta spend more time at the center of the proton where they are almost free; these partons are thus weighted with a higher probability. We found the effect of this weighting function to be small. The calculated distributions are in reasonable agreement with experiment for the light sea asymmetry, $\bar{d}(x) - \bar{u}(x)$, represented in terms of Bjorken-x.

PARTON DISTRIBUTIONS IN THE PION

We have extended this model to calculate parton distributions in the pion [3]. The Fock state expansion for the pion is

$$|\pi^+ >= \sum_{i,j,k} c_{ijk} |\{u\bar{d}\}\{ijk\} >, \tag{4}$$

and the ratio of probabilities for different Fock states is

$$\frac{\rho_{ijk}}{\rho_{000}} = \frac{1}{i!(i+1)!j!(j+1)!k!} \, . \tag{5}$$

It is clear that the π^+ sea is symmetric, i.e. $\bar{u}(x) = d(x)$. Our results for the valence quark distribution of the pion are compared to experiment [4] and to the Dyson-Schwinger calculation of Hecht et al. [5] in Fig. 1. Both theoretical distributions were evolved to $Q^2 = 16$ GeV^2.Our simple statistical model agrees with experiment as well as the covariant, QCD-based model.

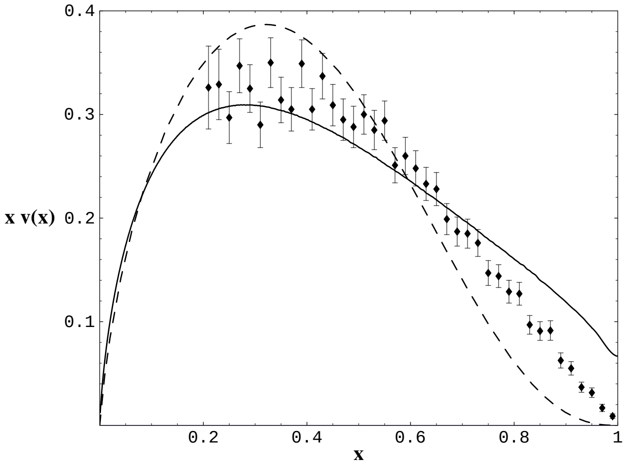

FIGURE 1. Our results (solid curve) for the valence quark distribution $xv(x)$ in the pion, compared to the experimental results of Conway et al. [4] and to the calculation of Hecht et al. [5] (dashed curve).

STUDENT PROJECTS

Student projects have extended the statistical model for the calculation of other parton distributions. Strange quarks have been included by accounting for their mass in the detailed balance rates and in the Monte Carlo calculations of momentum distributions. The results include:

- Determination of the strange quark distributions of the proton
- Calculation of the parton distributions of the pentaquark
- Calculation of the parton distributions of the kaon

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