## APPENDIX L VARIANCE CALCULATION SIMPLIFICATION

## L. 1 Calculation Simplification

The Savannah River Site (SRS) dose for releases in a particular medium (air or water) may be represented by the following sum:

$$
\begin{equation*}
\mathrm{D}=\sum_{1954}^{1992} \sum_{1}^{\mathrm{N}} \sum_{1}^{\mathrm{M}} \mathrm{D}_{\mathrm{ijk}} \tag{L.1}
\end{equation*}
$$

Where,
$\mathrm{D}=$ the dose to a particular receptor;
$\mathrm{D}_{\mathrm{ijk}}=$ the incremental dose (by year, nuclide, and pathway);
i $=$ the index by year, from 1954-1992;
$\mathrm{j}=$ the index by radionuclide, with upper limit N equal to 16 for air and 22 for water;
$\mathrm{k}=$ the index by exposure pathway, with upper limit M equal to 13 for air and 5 for water.
(Note there are no pathways in common for both air and water).
The point estimate for D is calculated by substituting in the point estimates for a number of input variables. This also generates the point estimates for the component doses from each radionuclide and pathway, $\mathrm{D}_{\mathrm{ijk}}$. Denote these point estimates of dose by $\check{\mathrm{D}}$ and $\check{\mathrm{D}}_{\mathrm{ij} \mathrm{k}}$, respectively. Based on these values the fraction of the total dose due to various factors is computed. For example:

$$
\check{\mathrm{E}}_{\mathrm{ijk}}=\check{\mathrm{D}}_{\mathrm{ijk}} \check{\mathrm{D}}
$$

Using shorthand notation, if an index is omitted from a fraction or a dose, then summation over the omitted indexes is assumed. For example,

$$
\begin{equation*}
\check{E}_{\mathrm{j}}=\frac{\sum_{1954}^{19922} \sum_{1}^{1} \check{D}_{i j k}}{\frac{D_{\mathrm{D}}}{}}=\frac{\bar{D}_{\mathrm{D}}}{} \tag{L.2}
\end{equation*}
$$

When the Monte Carlo analysis is performed, an assumption is made that the doses are random variables. A set of realizations is computed leading to a sample of doses; each of these randomly generated doses can be partitioned and indexed as with the point estimate dose (i.e., as indexed in Equation (L.1)). However, an additional index, 1 , representing the realization is needed to completely characterize the set of numbers; the index 1 has a range of 1 to L , where L is the total number of realizations. What is being sought is the population mean and variance. As usual, the population mean and variance is estimated by using the sample mean and variance.

Because the input variable distributions will be chosen to be centered on the point estimate values, it is expected that the sample mean, $\mu$, will be close to the point estimate of dose, $\check{D}$. A similar relationship is expected to hold for any subset of doses or even the $\check{\mathrm{D}}_{\mathrm{ijk}}$. Then by extending the notation used before:

$$
\begin{equation*}
\mu_{\mathrm{ijk}}=\frac{\sum_{1}^{\mathrm{L}} \mathrm{D}_{\mathrm{ikk}}}{\mathrm{~L}} \tag{L.3}
\end{equation*}
$$

That is, $\mu_{\mathrm{ijk}}$ is the mean value of the dose realizations for a particular year, radionuclide, and pathway.

$$
\begin{equation*}
\mathrm{E}_{\mathrm{ijk}}=\mu_{\mathrm{ijk}} \check{\mathrm{D}}=\left[\check{\mathrm{D}}_{\mathrm{ijk}} / \mathrm{D}\right]\left[\mu_{\mathrm{ijk}} / \check{\mathrm{D}}_{\mathrm{ijk}}\right]=\check{\mathrm{E}}_{\mathrm{ijk}} \cdot\left[\mu_{\mathrm{ijk}} / \check{\mathrm{D}}_{\mathrm{ijk}}\right] \tag{L.4}
\end{equation*}
$$

That is, $\mathrm{E}_{\mathrm{ijk}}$ is $\mu_{\mathrm{ijk}}$ divided by the total point-estimate dose, D .
However, for a well-behaved problem in which the point-estimate dose is close to the mean dose, the second factor $\approx 1$. If both sides of (L.4) are summed with respect to all indices, then:

$$
\begin{equation*}
\mathrm{E}=1 \cong \sum_{1954}^{1992} \sum_{1}^{N} \sum_{1}^{\mathrm{M}} \mathrm{E}_{\mathrm{ijk}} \cdot 1 \tag{L.5}
\end{equation*}
$$

In other words, the result is that the estimate of mean dose can be obtained if elements of the sum are omitted, but only if the aggregate sum of the $\mathrm{E}_{\mathrm{ijk}}$ for those elements is a suitable small percentage of unity. However, if the interest is in characterizing the variability, more elements of the sum may be omitted, provided the missing dose is added back in at the end. That is:

$$
\begin{equation*}
\mu=\mu_{\mathrm{v}}+\mu_{\mathrm{f}} \tag{L.6}
\end{equation*}
$$

Where $\mu_{\mathrm{v}}$ is a partial mean calculated based on Monte Carlo sampling and $\mu_{\mathrm{f}}$ is a partial mean calculated based on the point estimate calculation, where the partition is on any basis that is convenient.

To estimate the variance, rather than the mean, the situation is quite different. Although it is expected that the point-estimate total dose will be close to the mean of the total dose from the Monte Carlo sampling, the point estimate provides no information on the variance of the dose distribution. This information must come from the Monte Carlo simulation. To simplify the analysis, consider the sum over time indicated in Equation (L.1). Because the adults and child born in 1955 experience large releases early in the history of the site, the doses for these receptors are dominated by the first 20 years of releases. However, the child born in 1964 misses most of these large doses, so the cumulative releases over a 20 year period are important. If both time ranges are to be studied, then, because of the calculation approach adopted for the project, the doses for the entire 39 year period of the study might as well be calculated. Because simplification by eliminating certain years does not seem feasible, the index related to summing over years will be dropped and the summation will be assumed. Then Equation (L.1) becomes:

$$
\begin{equation*}
\mathrm{D}=\sum_{1}^{N} \sum_{1}^{\mathrm{M}} \mathrm{D}_{\mathrm{jk}} \tag{L.7}
\end{equation*}
$$

where the summation over time is understood, but will not be used to simplify the variance calculation. In other words, $\mathrm{D}_{\mathrm{jk}}$ should be read as the dose for a particular radionuclide and pathway summed over 39 years. Furthermore, it should now be understood that D is a random variable; in practice a sample of this random variable of size L will be created.

To facilitate the analysis, transform both the doses and the independent variables by their point estimate values; i.e.,

$$
\begin{equation*}
Z=X / \breve{X} \tag{L.8}
\end{equation*}
$$

where,
$\mathrm{Z}=$ the normalized input variable or dose;
$\mathrm{X}=$ the lth realization of the input variable or dose;
$\breve{\mathrm{X}}=$ the point estimate value of the input variable or dose.
Then Equation (L.7) becomes:

$$
\begin{equation*}
F=\frac{D}{\bar{D}}=\sum_{1}^{N} \sum_{1}^{M} \frac{D_{j k}}{\stackrel{D}{D}} \tag{L.9}
\end{equation*}
$$

where $\mathrm{D}_{\mathrm{jk}}, \mathrm{D}$, and F are all random variables.
This can be rearranged into a slightly more convenient form (similar to what was done in Equation (L.4)):

$$
\begin{equation*}
F=\sum_{1}^{N} \sum_{1}^{M} \frac{D_{j k}}{\tilde{D}_{j k}} \cdot \frac{\breve{D}_{j k}}{\bar{D}} \tag{L.10}
\end{equation*}
$$

Note the variance of a random variable, X, by $\sigma^{2}\{\mathrm{X}\}$. Then some useful relationships are:
and

$$
\begin{equation*}
\sigma^{2}\{\mathrm{cX}\}=\mathrm{c}^{2} \sigma^{2}\{\mathrm{X}\} \tag{L.11a}
\end{equation*}
$$

$$
\begin{equation*}
\sigma^{2}\{\mathrm{X}+\mathrm{Y}\}=\sigma^{2}\{\mathrm{X}\}+\sigma^{2}\{\mathrm{Y}\} \tag{L.11b}
\end{equation*}
$$

where,
c = a constant;
X and $\mathrm{Y}=$ independent random variables.
Note that the term $\check{\mathrm{D}}_{\mathrm{jk}} / \check{\mathrm{D}}$ in Equation (L.9) is the fractional contribution to total dose by pathway and radionuclide based on the point estimate calculation, i.e., $\mathrm{E}_{\mathrm{jk}}$. This quantity is a constant for each choice of j and k . The variance of the normalized total dose can be found by determining the variance of each of the contributing terms in Equation (L.10) and then using the rules in Equations (L.11) to combine them. This procedure yields:

$$
\begin{equation*}
\sigma^{2}\{F\}=\sum_{1}^{N} \sum_{1}^{M} \sigma^{2}\left\{\frac{D_{j k}}{\bar{D}_{j k}} \cdot \frac{\breve{D}_{j k}}{\check{D}}\right\}=\sum_{1}^{N} \sum_{1}^{M} \sigma^{2}\left\{\frac{D_{j k}}{\stackrel{D}{j k}^{D_{j k}}}\right\} E_{j k}^{2} \tag{L.12}
\end{equation*}
$$

In other words, the variance of the dose is equal to the variance of the normalized dose contributors by radionuclide and exposure pathway, multiplied by the square of the contribution of that radionuclide and exposure pathway to the point estimate dose. If the variance of each normalized contributor to dose, $\sigma^{2}\left\{\mathrm{D}_{\mathrm{ij}} / \check{\mathrm{D}}_{\mathrm{ij}}\right\}$, is bounded by some quantity, say $\mathrm{Q}^{2}$, then the sum can be approximated by eliminating any terms for which:

$$
\begin{equation*}
\mathrm{E}_{\mathrm{ij}}^{2} / \max \left\{\mathrm{E}_{\mathrm{ij}}^{2}\right\} \ll 1 \tag{L.13}
\end{equation*}
$$

and where max $\left\{\mathrm{x}_{\mathrm{ij}}\right\}$ is the maximum value of the indexed variable. In other words, if the variances of the normalized contributors to dose are bounded, only the terms with a large fractional contribution to the point estimate dose $\left(\mathrm{E}_{\mathrm{ij}}^{2}\right)$ will be important. Based on the results of the point estimate runs, the ratios indicated by Equation (L.13) can be calculated. For example, the results for the adult female in Rural Family 2 are given in Table L-1. Table L-2 shows the reduced renormalized set after some nuclides and pathways were eliminated because they had only a minor contribution to dose. Table L-3 shows the
values indicated in Equation (L.12) and demonstrates that only four values account for over 98\% of the total. This clearly highlights four nuclide/pathway pairs that are candidates to account for most of the variance in the dose. Table L-4 shows the equivalent of Table L-3, but for the child born in 1964. This shows more and different pathways and nuclides, primarily those involving tritium.

To reach this point, however, it was assumed the variance of each normalized contributor to dose is bounded. Consider the variance of each normalized contributor to dose, denoted in Equation (L.12) by the term:

$$
\sigma^{2}\left\{\frac{D_{j k}}{D_{j k}}\right\}
$$

To simplify the notation for the upcoming derivation, denote $\mathrm{D}_{\mathrm{jk}}$ by u . Then the normalized contributor to dose is $u / \breve{u}$. Suppose this contributor to dose is expanded in a Taylor series of the variables involved in calculating this aspect of dose; note that this set of variables may include variables unique to this nuclide and pathway, such as the uptake of iodine by milk from animal feed, but it may also include variables common to many pathways and radionuclides, such as the particle diameter used to determine deposition velocity. We can then write:

$$
\begin{equation*}
u\left(z_{1}, z_{2}, \ldots, z_{n}\right)=a_{0}+a_{1} z_{1}+a_{2} z_{2}+\ldots+a_{n} z_{n} \tag{L.14}
\end{equation*}
$$

If the variance of both sides of (L.14) are taken and the relationships in equations (L.11a) and (L.11b) are applied, then:

$$
\begin{equation*}
\sigma^{2}\{\mathrm{u}\}=\sum \mathrm{a}_{\mathrm{m}}^{2} \sigma^{2}\left\{\mathrm{z}_{\mathrm{m}}\right\} \tag{L.15}
\end{equation*}
$$

where $m$ is the index for the $n$ independent variables. Note also that the variance of a constant $\left(a_{0}\right)$ is zero.

Similar to Equation (L.5) an expression for the variance of $u / u ̆$ can be written as:

$$
\begin{equation*}
\sigma^{2}\{u / \breve{u}\}=\sigma^{2}\{u\} / \breve{u}^{2}=\sum_{1}^{n} \frac{a_{m}^{2} \sigma^{2}\left\{z_{m}\right\}}{\breve{u}^{2}}=\sum_{1}^{n} \frac{a_{m}^{2} \breve{z}_{m}^{2}}{\breve{u}^{2}} \frac{\sigma^{2}\left\{z_{m}\right\}}{\breve{z}_{m}^{2}} \tag{L.16}
\end{equation*}
$$

But, from the definition of the Taylor series,

$$
\begin{equation*}
a_{m}=\left(\frac{\partial u}{\partial z_{m}}\right)_{z_{m}=\bar{z}_{\mathrm{m}}} \tag{L.17}
\end{equation*}
$$

Furthermore, the definition of the dimensionless sensitivity coefficient, $\mathrm{S}_{\mathrm{m}}$, is given by:

$$
\begin{equation*}
\mathrm{S}_{\mathrm{m}}=\left(\frac{\check{\mathrm{z}}_{\mathrm{m}}}{\breve{\mathrm{u}}} \frac{\partial \mathrm{u}}{\partial \mathrm{z}_{\mathrm{m}}}\right)_{\mathrm{z}_{\mathrm{m}}=\bar{z}_{\mathrm{m}}} \tag{L.18}
\end{equation*}
$$

Then combining (L.16), (L.17), and (L.18) yields:

$$
\begin{equation*}
\frac{\sigma^{2}\{u\}}{\breve{u}^{2}}=\sum_{1}^{n} S_{1}^{2} \cdot \frac{\sigma^{2}\left\{z_{m}\right\}}{\breve{Z}_{m}^{2}}=\sum_{1} S_{m}^{2} \cdot V_{m}^{2} \tag{L.19}
\end{equation*}
$$

Where the term, $V_{m}$, denotes the coefficient of variation for variable $z_{m}$; i.e., the standard deviation of $z_{m}$ divided by its mean. However, this requires that the mean value of the random variable is approximately equal to the point-estimate value of the same variable. This can be achieved by "anchoring" the distribution of the random variable, by making its mean (or possibly its median for a log-normal distribution) to the point estimate value for that variable. If the means of the independent random variables are equal to their point estimate values, it would be expected that the mean of the dose to be approximately equal to its point estimate value.

Although this may seem to be a lot of mathematical manipulation, the result summarized by Equation (L.19) is quite useful. Essentially the square of the coefficient of variation of $u$, (recall that $u$ is another name for $D_{j k}$, the contribution to dose from a particular radionuclide and pathway) is equal to the sum over all variables of the square of the product of the sensitivity coefficient for the variable $z_{m}$ and the coefficient of variation for that same variable. This is an approximate relationship, because it was assumed that the mean of the random variables is approximately equal to the point estimate value of the variable (including the dose). Although this is an approximate relationship, depending on the degree of fit of the Taylor series expansion, it provides a means to determine which variables contribute substantially to the variance of the incremental dose.

Equation (L.18) can be simplified further, by noting the following. Many of the equations used to estimate dose are of the form:

$$
\begin{equation*}
\mathrm{u}=\mathrm{A} \mathrm{z}_{1} \mathrm{z}_{2} \mathrm{z}_{3} \tag{L.20}
\end{equation*}
$$

Where A is a constant and the $\mathrm{z}_{\mathrm{m}}$ 's are variables. Then,

$$
\begin{equation*}
\frac{\partial \mathrm{u}}{\partial \mathrm{z}_{1}}=\mathrm{A} \mathrm{z}_{2} \mathrm{z}_{3} \tag{L.21}
\end{equation*}
$$

and

$$
\begin{equation*}
S_{1}=\frac{\breve{z}_{1}}{\breve{u}} \frac{\partial u}{\partial z_{1}}=\frac{\breve{z}_{1}=\breve{z}_{m}}{A \breve{z}_{1} \breve{z}_{2} \breve{z}_{3}} A \breve{z}_{2} \breve{z}_{3}=1 \tag{L.22}
\end{equation*}
$$

If the equation is of the form:

$$
\begin{equation*}
\mathrm{u}=\mathrm{A} \mathrm{z}_{2} \mathrm{z}_{3} / \mathrm{z}_{1} \tag{L.23}
\end{equation*}
$$

then,

$$
\begin{equation*}
\frac{\partial u}{\partial z_{1}}=-A z_{2} z_{3} / z_{1}^{2} \tag{L.24}
\end{equation*}
$$

and

$$
\begin{equation*}
\mathrm{S}_{1}=\frac{\breve{z}_{1}}{\breve{\mathrm{u}}} \frac{\partial \mathrm{u}}{\partial \mathrm{z}_{1 z_{\mathrm{m}}=\breve{z}_{\mathrm{m} 1}}}=\frac{\breve{\mathrm{z}}_{1}}{\left(\mathrm{~A} \breve{z}_{2} \breve{z}_{3} / \breve{z}_{1}\right)}\left(-\mathrm{A} \breve{z}_{2} \breve{\mathrm{z}}_{3} / \breve{\mathrm{z}}_{1}^{2}\right)=-1 \tag{L.25}
\end{equation*}
$$

Since many of the dose equations are of the form in Equation (L.20) or (L.23) the value to be used for $\mathrm{S}_{\mathrm{m}}$ becomes either +1 or -1 in these cases. Since $\mathrm{S}_{\mathrm{m}}$ is squared in Equation (L.19), the squared value, in many cases, will be unity. On this basis Equations (L.18) and (L.11) can be combined to yield:

$$
\begin{equation*}
\frac{\sigma^{2}\{D\}}{D^{2}}=\sum_{1}^{N} \sum_{1}^{M} E_{j k}^{2} \sigma^{2}\left\{\frac{D_{j k}}{\tilde{D}_{j \mathrm{k}}}\right\}=\sum_{1}^{N} \sum_{1}^{M} E_{j \mathrm{k}}^{2} \sum_{1}^{n} S_{\mathrm{jkm}}^{2} \mathrm{~V}_{\mathrm{m}}^{2} \tag{L.26}
\end{equation*}
$$

Note that now the sensitivity coefficient depends on the independent variable, $m$, as well as the Taylor series representation for the particular contributor to dose, jk . If the m ranges over all variables, then $\mathrm{S}_{\mathrm{jkm}}$ will be approximately unity if the variable enters into the dose equation for that nuclide and pathway, otherwise it will be zero.

In evaluating this expression to determine which variables to include in the variance estimation, there are two distinct cases: (1) a variable enters into only one increment of dose, $\mathrm{E}_{\mathrm{jk}}$, and (2) a variable enters into two or more increments of dose. An example of case 1 is the uptake factor for ruthenium-106 from animal feed to milk. This variable enters only into the dose from the milk pathway for the isotope ruthenium-106. An example of case 2 is the absolute humidity variable that affects all food-chain pathways involving tritium. For case 1 the contribution to the normalized variance by variable 1 is given simply by:

$$
\begin{equation*}
V_{m}^{2} E_{j k}^{2} \tag{L.27}
\end{equation*}
$$

where the $\mathrm{E}_{\mathrm{jk}}$ is defined as before as the value associated with the pathway and nuclide in which the variable 1 plays a role. For case 2 the contribution to the normalized variance by variable 1 is given by:

$$
\begin{equation*}
\sum_{j^{\prime}} \sum_{k^{\prime}} V_{m}^{2} E_{j k}^{2} \tag{L.28}
\end{equation*}
$$

where the summation is over all nuclides, $\mathrm{j}^{\prime}$, and pathways, $\mathrm{k}^{\prime}$, involving variable m. However, because $\mathrm{V}_{\mathrm{m}}$ is frequently unity, then $\mathrm{V}_{\mathrm{m}}{ }^{2}$ will be unity, and a good approximation to Equation (L.28) will usually be:

$$
\begin{equation*}
\sum_{j^{\prime}} \sum_{k^{\prime}} E_{j k}^{2} \tag{L.29}
\end{equation*}
$$

where again the summation is over all nuclides, $\mathrm{j}^{\prime}$, and pathways, $\mathrm{k}^{\prime}$, involving variable m . The types of functional dependence that will make the sensitivity coefficient substantially different from unity are exponential, logarithmic, and power functions of the variables. This type of functional dependence in dose assessment does occur, but infrequently.

## L. 2 Summary of Implementation Approach

1. Take the entire set of about 350 input variables and eliminate from consideration any variables solely associated with the following pathways and radionuclides:
a. Air Release:
i. Radionuclides: ${ }^{241} \mathrm{Am},{ }^{137} \mathrm{Cs},{ }^{129} \mathrm{I},{ }^{89} \mathrm{Sr},{ }^{90} \mathrm{Sr}$, all isotopes of U
ii. Pathways: ground contamination, grain, and soil ingestion
b. Water Release:
i. Radionuclides: ${ }^{129} \mathrm{I},{ }^{95} \mathrm{Nb},{ }^{89} \mathrm{Sr}$, all isotopes of U.
ii. Pathways: Boating, swimming immersion, swimming inadvertent ingestion.

These radionuclides and pathways were eliminated from the dose assessment model used for the uncertainty analysis, because their cumulative contribution to the dose of all receptors was less than $5 \%$.
2. Eliminate categories of variables not considered suitable for inclusion in the uncertainty analysis. For example, dose and risk coefficients, although uncertain, were considered to be certain variables. These variables, whose nominal values are established by national and international standards organizations, have less uncertainty than many other variables in the study; furthermore, incorporating their variability would confound the results from other variables. Variables specified by scenarios, e.g. consumption values such as breathing rate and quantities of food ingested, were considered to be certain. The variability in these variables was considered to be addressed by the creation and use of the scenarios; incorporating variability for these variables would confound the effects of the various scenarios and blur their differences.
3. Develop Tables of the $E^{2} j k$ based on a couple of representative receptors for air and water (e.g., Rural Family 2, the adult female and child born in 1964).
4. Use the results of Step 3 to focus on a few key pathways and radionuclides. Identify the variables involved in those pathways and radionuclides. Then develop the distributions describing the uncertainty in those variables. Form the products indicated in Equation (L.27). If any product is noticeably small, eliminate that variable from further consideration. In the case that a variable is used for all pathways, but a single radionuclide, an appropriate sum based on Equation (L.29) can be used instead of Equation (L.27);
5. Look at the entire list (singleton variables) and the approximate contribution to variance computed for each. Order according to expected importance and draw an appropriate line for inclusion/elimination.
6. Evaluate the eliminated variables to make sure consideration of multiple radionuclides or pathways would not cause it to be included; include if appropriate.

## L. 3 Tables

Table L-1 presents the contribution to total dose by pathway and radionuclide. The contribution of some radionuclides has been eliminated $\left({ }^{241} \mathrm{Am},{ }^{137} \mathrm{Cs},{ }^{129} \mathrm{I},{ }^{89} \mathrm{Sr},{ }^{90} \mathrm{Sr}\right.$, all isotopes of U$)$, as has the contribution of some pathways (ground contamination, grain, and soil ingestion). These numbers are the $\mathrm{D}_{\mathrm{jk}}$ in the notation of this Appendix. Note that the maximum value is $3.48 \mathrm{E}-04$ for ${ }^{131} \mathrm{I}$ and beef ingestion.

Table L-2 identifies the reduced renormalized matrix. For this matrix, we have eliminated the noncontributing pathways and radionuclides and renormalized by the total dose based on those pathways and nuclides included. The highlighted cell indicates the maximum contributor to dose $\left({ }^{131}\right.$ I through beef ingestion). The entries in this table are approximately equal to $\mathrm{E}_{\mathrm{jk}} / \mathrm{E}_{\text {total }}$.

Table L-3 matrix is the square of the normalized contribution to dose ( $\mathrm{E}^{2} \mathrm{jk}$ in my notation) divided by the maximum value of these numbers. The highlighted terms contribute at least $1 \%$ of the maximum value. The sum of these four terms constitutes over $98 \%$ of the sum over all terms.

The matrix presented in Table L-4 is the square of the normalized contribution to dose ( $\mathrm{E}^{2} \mathrm{jk}$ in my notation) divided by the maximum value of these numbers. The highlighted terms contribute at least $1 \%$ of the maximum value. The sum of these four terms constitutes over $98 \%$ of the sum over all terms.

## Table L-1 Contribution to Total Dose by Pathway and Radionuclide

|  | A | 41 | C-14 |  | H-3 | I-129 | -131 | Pu-238 |  | Ru-106 | Sr-89 | Sr-90 | U-234 |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Air Immers | 0.00E | 5.39-05 | , | 0.00E+00 | 000 | 0.00E+00 | , | 1.5 | 1.00E | 3.63E-10 | 0.00E+00 | 0.00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |  |
| ound Co | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |  |
| Beef | $0.00 \mathrm{E}+0$ | 0.00E+00 | 1.74E-06 | 0.00E+00 | $1.14 \mathrm{E}-05$ | 0.00E+00 | 3.48E-04 | $1.56 \mathrm{E}-10$ | 1.31E-09 | 1.83E-06 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 3.6 |
| Eggs | 0.00E | 0.00E+00 | 7.13E-08 | 0.00E+ | 2.60E-06 | 0.00 | 2. | 4.69E | 3.3 | 7.4 | 0.00E+00 | 0. | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0 |  |
| Fruit | 0.00E+00 | 0.00E+00 | 3.72E-07 | 0.00E+00 | 9.70E-06 | 0.00E+00 | 1.28E-05 | $2.20 \mathrm{E}-08$ | 1.84E-07 | 5.12E-08 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |  |
| Grain | $0.00 \mathrm{E}+$ | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00 | 0.00E+00 | 0.00E+00 | 0.0 | 0 |  |
| aafy Veg | 0.00E+00 | 0.00E+00 | 1.10E-07 | 0.00E+00 | .10E-06 | 0.00E+00 | 3.77E-05 | 6.50E-08 | 5.42E-07 | 1.51 | 0.00 | 0.00 | 0.00E+00 | 0.00 | 0.00E+00 | 0.00E+00 |  |
| , | 0.00E+00 | 0.00E+00 | 8.80E-07 | 0.00E+00 | 1.63E-05 | 0.00E+00 | 9.45E-05 | $2.06 \mathrm{E}-$ | 1.45E-10 | 1.37E-10 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |  |
| Poultry | 0.00E+00 | 0.00E+00 | 6.66E-08 | 0.00E+00 | 1.85E-06 | 0.00E+00 | 4.26 | 1.92E | 1.60E-12 | 8.42E-10 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |  |
| Root Vege | 0.00E+00 | 0.00E+00 | 3.76E-07 | $0.00 \mathrm{E}+00$ | 9.81E-06 | 0.00E+00 | 4.21E-06 | $2.22 \mathrm{E}-0$ | 1.86E-07 | 5.05E-08 | 0.00E+00 | 0.00E+ | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 1.47E-0 |
| Soil | 0.00E+00 | 0.00E+00 | 0.00E+00 | $0.00 \mathrm{E}+00$ | 0.00E+ | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.0 |
| Air Inhalatir | 0.00E+00 | 0.00E+00 | 8.04E-10 | $0.00 \mathrm{E}+00$ | 1.30E-05 | 0.00E+00 | 2.75E-05 | 2.72E-06 | 1.89E-05 | 1.25E-07 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 |  |
| Resuspe | 0.00E+00 | 0.00E+00 | 0.00E+00 | $0.00 \mathrm{E}+00$ | 0.00E+00 | 0.00E+00 | 3.98E-07 | 1.17E-06 | 8.13E-06 | 4.30E-08 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 9.74E-06 |
| Total Ove | 0.00E+00 | 5.39E-05 | 3.62E-06 | 0.00E+00 | 6.78E-05 | 0.00E+00 | 5.26E-04 | 4.00E-06 | 2.79E-05 | 2.25E-06 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | 0.00E+00 | $0.00 \mathrm{E}+00$ | 6.85E-04 |
| otal Dos | E-04 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Table L-2 Reduced Renormalized Matrix

| REDUCED RENORMALIZED MATRIX |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Rural Family \#2 |  |  |  |  |  |  |  |  |
| Adult Female |  |  |  |  |  |  |  |  |
|  | Ar-41 | C-14 | H-3 | I-131 | Pu-238 | Pu-239 | Ru-106 | Sum Over Pathway |
| Air Immersion | 7.86E-02 | 3.73E-09 | 0.00E+00 | $3.36 \mathrm{E}-04$ | $2.31 \mathrm{E}-12$ | 1.46E-11 | 5.30E-07 | 7.90E-02 |
| Beef | $0.00 \mathrm{E}+00$ | $2.54 \mathrm{E}-03$ | $1.66 \mathrm{E}-02$ | 5.08E-01 | $2.27 \mathrm{E}-07$ | 1.91E-06 | $2.67 \mathrm{E}-03$ | 5.30E-01 |
| Eggs | $0.00 \mathrm{E}+00$ | $1.04 \mathrm{E}-04$ | 3.79E-03 | 3.56E-11 | $6.85 \mathrm{E}-11$ | 4.82E-10 | 1.09E-09 | $3.89 \mathrm{E}-03$ |
| Fruit | $0.00 \mathrm{E}+00$ | $5.44 \mathrm{E}-04$ | $1.42 \mathrm{E}-02$ | 1.87E-02 | 3.21E-05 | $2.68 \mathrm{E}-04$ | $7.47 \mathrm{E}-05$ | 3.37E-02 |
| Leafy Vegetables | $0.00 \mathrm{E}+00$ | $1.60 \mathrm{E}-04$ | $4.52 \mathrm{E}-03$ | 5.51E-02 | $9.48 \mathrm{E}-05$ | 7.91E-04 | $2.20 \mathrm{E}-04$ | 6.09E-02 |
| Milk | $0.00 \mathrm{E}+00$ | $1.28 \mathrm{E}-03$ | $2.38 \mathrm{E}-02$ | 1.38E-01 | $3.00 \mathrm{E}-08$ | 2.11E-07 | $2.01 \mathrm{E}-07$ | 1.63E-01 |
| Poultry | $0.00 \mathrm{E}+00$ | $9.71 \mathrm{E}-05$ | $2.70 \mathrm{E}-03$ | 6.22E-14 | $2.80 \mathrm{E}-10$ | $2.34 \mathrm{E}-09$ | $1.23 \mathrm{E}-06$ | 2.79E-03 |
| Root Vegetables | $0.00 \mathrm{E}+00$ | 5.49E-04 | 1.43E-02 | 6.14E-03 | 3.23E-05 | $2.71 \mathrm{E}-04$ | $7.36 \mathrm{E}-05$ | $2.14 \mathrm{E}-02$ |
| Air Inhalation | $0.00 \mathrm{E}+00$ | 1.17E-06 | $1.90 \mathrm{E}-02$ | 4.02E-02 | 3.98E-03 | 2.76E-02 | 1.83E-04 | 9.09E-02 |
| Resuspended Soil | $0.00 \mathrm{E}+00$ | 0.00E+00 | 0.00E+00 | 5.81E-04 | $1.70 \mathrm{E}-03$ | 1.19E-02 | $6.27 \mathrm{E}-05$ | 1.42E-02 |
| Total Over Isotope | 7.86E-02 | $5.28 \mathrm{E}-03$ | $9.89 \mathrm{E}-02$ | 7.67E-01 | $5.84 \mathrm{E}-03$ | 4.08E-02 | $3.28 \mathrm{E}-03$ | $1.00 \mathrm{E}+00$ |

Table L-3 Eij^2/max Eij^2

| Eij^2 / max Eij^2 Rural Family \#2 Adult Female |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Ar-41 | C-14 | H-3 | I-131 | Pu-238 | Pu-239 | Ru-106 | Sum Over Pathway |
| Air Immersion | $2.39 \mathrm{E}-02$ | 5.38E-17 | 0.00E+00 | 4.38E-07 | 2.06E-23 | 8.29E-22 | 1.09E-12 | 2.39E-02 |
| Beef | 0.00E+00 | 2.50E-05 | $1.06 \mathrm{E}-03$ | 1.00E+00 | 2.00E-13 | $1.41 \mathrm{E}-11$ | 2.75E-05 | $1.00 \mathrm{E}+00$ |
| Eggs | 0.00E+00 | 4.19E-08 | 5.56E-05 | 4.92E-21 | 1.82E-20 | 8.99E-19 | 4.56E-18 | 5.56E-05 |
| Fruit | 0.00E+00 | 1.14E-06 | 7.75E-04 | 1.35E-03 | 3.99E-09 | 2.78E-07 | 2.16E-08 | 2.12E-03 |
| Leafy Vegetables | 0.00E+00 | 9.96E-08 | 7.91E-05 | 1.17E-02 | 3.48E-08 | 2.42E-06 | 1.87E-07 | 1.18E-02 |
| Milk | 0.00E+00 | 6.38E-06 | $2.20 \mathrm{E}-03$ | 7.36E-02 | 3.49E-15 | $1.73 \mathrm{E}-13$ | 1.56E-13 | 7.58E-02 |
| Poultry | 0.00E+00 | 3.65E-08 | 2.81E-05 | 1.49E-26 | 3.03E-19 | 2.11E-17 | 5.85E-12 | 2.82E-05 |
| Root Vegetables | 0.00E+00 | 1.17E-06 | 7.94E-04 | $1.46 \mathrm{E}-04$ | 4.04E-09 | 2.84E-07 | 2.10E-08 | 9.41E-04 |
| Air Inhalation | 0.00E+00 | 5.33E-12 | $1.40 \mathrm{E}-03$ | 6.24E-03 | 6.11E-05 | $2.94 \mathrm{E}-03$ | 1.29E-07 | 1.06E-02 |
| Resuspended Soil | 0.00E+00 | 0.00E+00 | 0.00E+00 | $1.30 \mathrm{E}-06$ | 1.12E-05 | 5.45E-04 | 1.52E-08 | 5.57E-04 |
| Total Over Isotope | 2.39E-02 | $3.38 \mathrm{E}-05$ | $6.39 \mathrm{E}-03$ | $1.09 \mathrm{E}+00$ | 7.24E-05 | 3.49E-03 | 2.79E-05 | $1.13 \mathrm{E}+00$ |

Table L-4 Child Born in 1964


