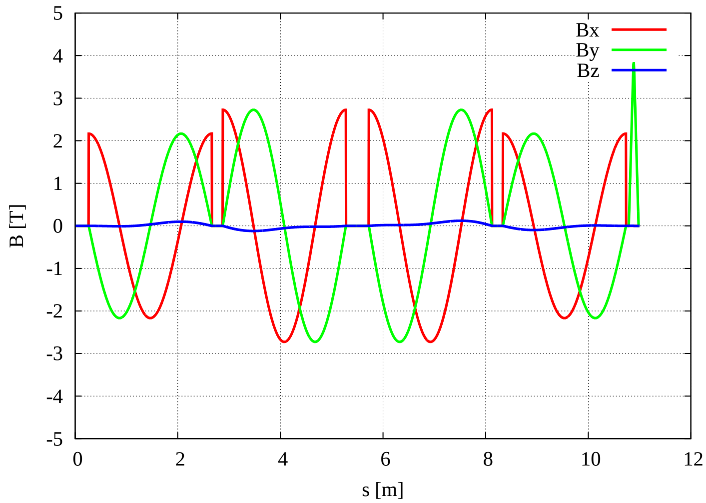
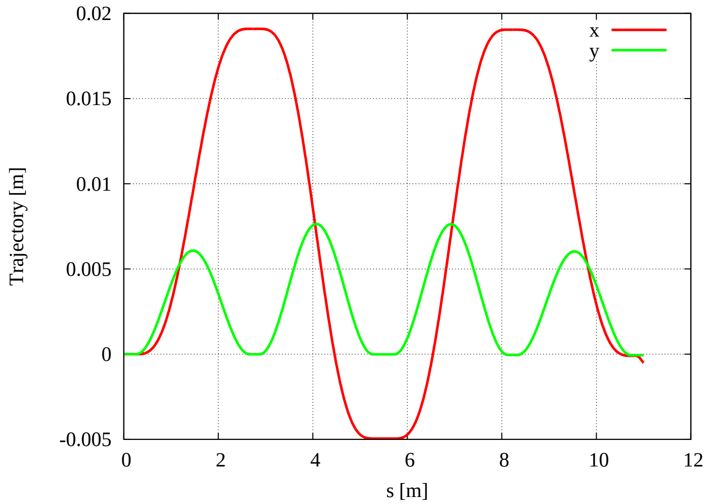
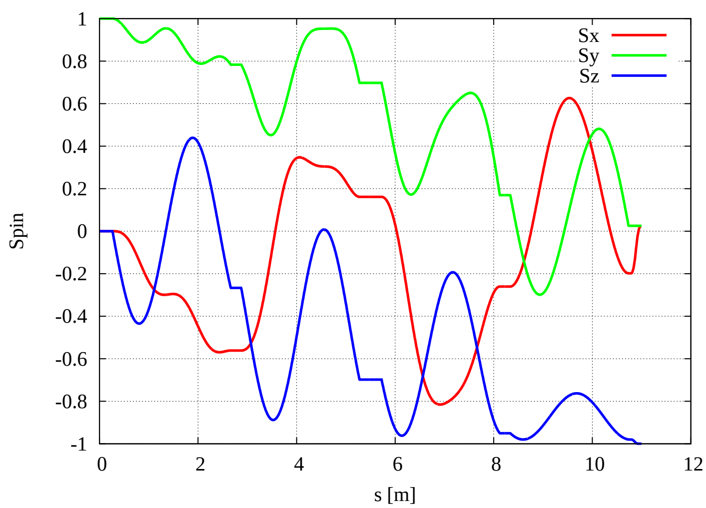
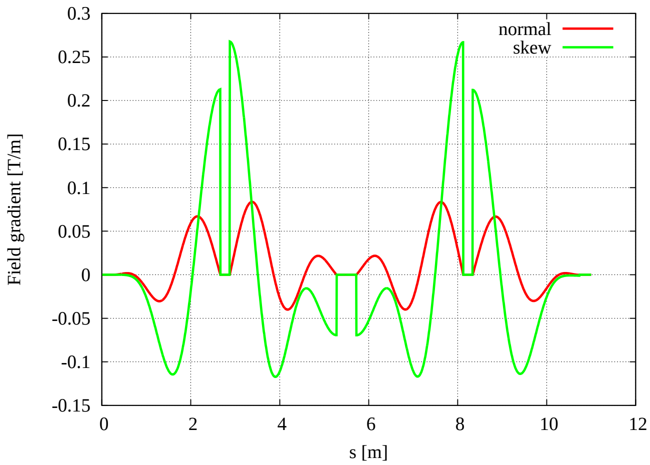
syrk4track: yo8-rot3 31.200 GeV proton with Iout= 162.0 A Iin= 205.0 A







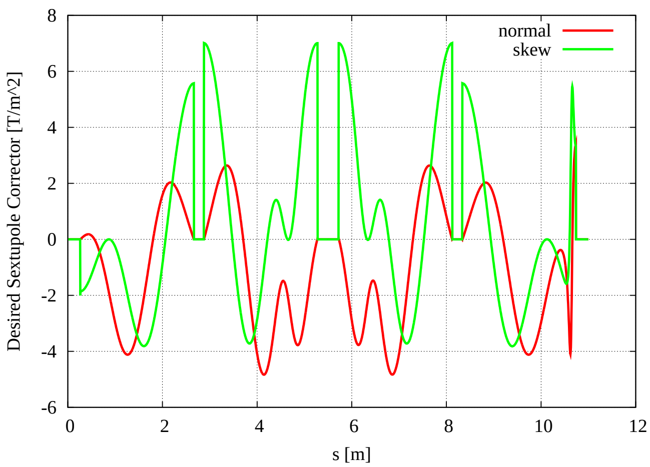


Particle Species:	proton $Z = 1$
Rest Mass	938.272 MeV/c^2
G	1.79284740
γ	33.252607
β	0.999548
Energy	31.200 GeV
Momentum	31.186 GeV/c
p/q	104.0249 Tm
γG	59.617
Pipe radius	4.5 cm

syrk4track: Rotator simulation with total length= 11.000 m

$$I_{\text{out}} = 162.000 \text{ A}$$
$$I_{\text{in}} = 205.000 \text{ A}$$

Type	z_{up} [m]	z_{dn} [m]	θ_{up}	θ_{dn}	B_0 [T]	$\tilde{b}_1$ [m^{-1}]	$\tilde{a}_1$ [m^{-1}]	$\tilde{b}_2$ [m^{-2}]	$\tilde{a}_2$ [m^{-2}]
helix	0.264	2.664	90.000°	-270.000°	-2.1671	0.00	0.00	0.00	0.00
helix	2.876	5.276	90.000°	450.000°	-2.7261	0.00	0.00	0.00	0.00
helix	5.724	8.124	90.000°	-270.000°	-2.7261	0.00	0.00	0.00	0.00
helix	8.336	10.736	90.000°	450.000°	-2.1671	0.00	0.00	0.00	0.00
dipole	10.836	10.936	90.000°	90.000°	3.8224				



The above gradients along the trajectory may be mostly correctable with distributed sextupole components as shown to the left.

For the trajectory starting at $\begin{pmatrix} x \\ w_x \\ y \\ w_y \\ z \\ w_z \end{pmatrix}_0 = \begin{pmatrix} 0.000000 \\ 0.000000 \\ 0.000000 \\ 0.000000 \\ 0.000000 \\ 1.000000 \end{pmatrix}$ the linearized transformation is given by:

$$\begin{pmatrix} x \\ w_x \\ y \\ w_y \\ z \\ w_z \end{pmatrix}_1 = \mathbf{M} \begin{pmatrix} x \\ w_x \\ y \\ w_y \\ z \\ w_z \end{pmatrix}_0 + \begin{pmatrix} -0.000518 \\ -0.003691 \\ -0.000065 \\ -0.000012 \\ 0.000015 \\ -0.000007 \end{pmatrix}.$$

$$\mathbf{M} = \begin{pmatrix} \mathbf{A} & \mathbf{B} & \mathbf{E} \\ \mathbf{C} & \mathbf{D} & \mathbf{F} \\ \mathbf{G} & \mathbf{H} & \mathbf{J} \end{pmatrix} = \begin{pmatrix} 0.948655 & 10.785314 & -0.000006 & 0.000799 & -0.003715 & 0.001063 \\ -0.009528 & 0.945798 & -0.000000 & -0.000007 & 0.000000 & 0.000106 \\ -0.000004 & 0.000820 & 0.953557 & 10.807386 & -0.000071 & 0.000348 \\ -0.000000 & -0.000005 & -0.008395 & 0.953554 & 0.000000 & 0.000051 \\ -0.000111 & -0.000096 & -0.000052 & -0.000220 & 0.999993 & 0.011517 \\ -0.000035 & 0.003514 & -0.000000 & 0.000070 & -0.000004 & 1.000007 \end{pmatrix}.$$

The determinant of the Jacobian matrix is $\det(\mathbf{M}) = 1.000000$. The integrated path length is 11.000800 m, giving an increase in path length of 0.000785 m beyond a simple drift.

Checking the symplecticity and coupling of $\mathbf{M}$ we have:

$$|\mathbf{M_{13}}| + |\mathbf{M_{14}}| + |\mathbf{M_{23}}| + |\mathbf{M_{24}}| = 0.000812$$
$$|\mathbf{M_{31}}| + |\mathbf{M_{32}}| + |\mathbf{M_{41}}| + |\mathbf{M_{42}}| = 0.000829.$$

$$\begin{aligned} \det(\mathbf{A}) &= 1.000000 \\ \det(\mathbf{B}) &= 0.000000 \\ \det(\mathbf{C}) &= 0.000000 \\ \det(\mathbf{D}) &= 1.000000 \\ \det(\mathbf{A}) + \det(\mathbf{B}) &= 1.000000 \\ \det(\mathbf{C}) + \det(\mathbf{D}) &= 1.000000 \\ \det(\mathbf{A}) + \det(\mathbf{C}) &= 1.000000 \\ \det(\mathbf{B}) + \det(\mathbf{D}) &= 1.000000. \end{aligned}$$

$$\begin{aligned} \mathbf{T} &= \mathbf{MSM}^{\mathrm{T}} - \mathbf{S} \\ &= \begin{pmatrix} -0.000000 & -0.000000 & -0.000000 & -0.000000 & 0.000000 & -0.000000 \\ 0.000000 & 0.000000 & -0.000000 & -0.000000 & -0.000000 & -0.000000 \\ 0.000000 & 0.000000 & -0.000000 & 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 0.000000 & -0.000000 & -0.000000 & 0.000000 & 0.000000 \\ -0.000000 & 0.000000 & -0.000000 & -0.000000 & 0.000000 & -0.000000 \\ 0.000000 & 0.000000 & -0.000000 & -0.000000 & 0.000000 & -0.000000 \end{pmatrix} \end{aligned}$$

with

$$\sqrt{\sum_{i<j} T_{ij}^2} = 0.998058 \times 10^{-12}.$$

Applying the Teng-Edwards uncoupling procedure gives:

$$\mathbf{D} = \begin{pmatrix} 1.046828 & -1.894062 \\ 0.001706 & 0.943264 \end{pmatrix},$$

with $\phi = 3.141593$ and $\det(\mathbf{D}) = 0.990666$.

$$\mathbf{A} = \begin{pmatrix} 0.948655 & 10.785314 \\ -0.009528 & 0.945798 \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} 0.953557 & 10.807386 \\ -0.008395 & 0.953554 \end{pmatrix},$$

$$\mathbf{RUR}^{-1} = \begin{pmatrix} 0.948655 & 10.785314 & -0.000000 & 0.000000 \\ -0.009528 & 0.945798 & -0.000000 & -0.000000 \\ -0.000000 & 0.000000 & 0.953557 & 10.807386 \\ -0.000000 & -0.000000 & -0.008395 & 0.953554 \end{pmatrix},$$

Thin lens transfer matrix:

$$\mathbf{N} = \mathbf{ML}^{-1} = \begin{pmatrix} 0.948655 & 0.350104 & -0.000006 & 0.000860 \\ -0.009528 & 1.050608 & -0.000000 & -0.000004 \\ -0.000004 & 0.000866 & 0.953557 & 0.318259 \\ -0.000000 & -0.000001 & -0.008395 & 1.045903 \end{pmatrix}.$$

$|\mathbf{N_{13}}| + |\mathbf{N_{14}}| + |\mathbf{N_{23}}| + |\mathbf{N_{24}}| = 0.000870$. $|\mathbf{N_{31}}| + |\mathbf{N_{32}}| + |\mathbf{N_{41}}| + |\mathbf{N_{42}}| = 0.000872$.

The spin rotation matrix along the trajectory was found to be

$$\mathbf{W} = \begin{pmatrix} 0.980049 & 0.023400 & 0.197375 \\ -0.197946 & 0.025275 & 0.979887 \\ 0.017941 & -0.999407 & 0.029403 \end{pmatrix},$$

with determinant $-\ 1 = 0.541789 \times 10^{-12}$. Checking the orthogonality we get:

$$\mathbf{W}\mathbf{W}^{\mathrm{T}} = \begin{pmatrix} 1.000000 & 0.000000 & -0.000000 \\ 0.000000 & 1.000000 & 0.000000 \\ -0.000000 & 0.000000 & 1.000000 \end{pmatrix},$$

The spin is rotated by the angle $\mu = 89.01°$ (49.45% snake) about the axis:

$$\hat{n} = \begin{pmatrix} 0.989796 \\ -0.089730 \\ 0.110690 \end{pmatrix} \quad \text{with, } \phi = 83.62° \text{ and } \theta = -5.15°.$$

Recheck:

$$\mathbf{W}\hat{n} = \begin{pmatrix} 0.989796 \\ -0.089730 \\ 0.110690 \end{pmatrix}$$