CP and $\pi\nu\bar{\nu}$

$$K_L = \frac{K_2+\epsilon K_1}{\sqrt{(1+|\epsilon|^2)}} \quad CP \approx -1$$
$$K_S = \frac{K_1+\epsilon K_2}{\sqrt{(1+|\epsilon|^2)}} \quad CP \approx +1$$

$\pi^0\nu\bar{\nu}$ is CP even state

$$\begin{array}{lll}
A(K^0 \to \pi^0\nu\bar{\nu}) & = (1/\sqrt{2})A(K^+ \to \pi^+\nu\bar{\nu}) & \\
A(K_1 \to \pi^0\nu\bar{\nu}) & = Re(A(K^+ \to \pi^+\nu\bar{\nu})) & = R \\
A(K_2 \to \pi^0\nu\bar{\nu}) & = iIm(A(K^+ \to \pi^+\nu\bar{\nu})) & = iI \\
B(K_L \to \pi^0\nu\bar{\nu}) & = I^2 + 2Re(-i\epsilon RI) + |\epsilon|^2R^2 & \simeq I^2
\end{array}$$

$$R/I \simeq Re(V_{ts}^* V_{td})/Im(V_{ts}^* V_{td}) \simeq \frac{3.0\times10^{-4}}{1.4\times10^{-4}}$$

But the smallness of $|\epsilon| \sim 0.0022$ makes $K_L \to \pi^0\nu\bar{\nu}$ almost entirely through direct CP violation ($K_2 \to \pi^0\nu\bar{\nu}$).
Littenberg, 1989

Measurements of $K \to \pi\nu\bar{\nu}$ provide CKM parameters with small theoretical uncertainties, but the experiments are hard.

Extend analysis to get limit on branching ratio:

$$B(K_L \to \pi^0\nu\bar{\nu}) < 4.4 \times B(K^+ \to \pi^+\nu\bar{\nu})$$

Grossman and Nir, 1997

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