Chiral symmetry restoration and in-medium hadron properties

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thanks to:
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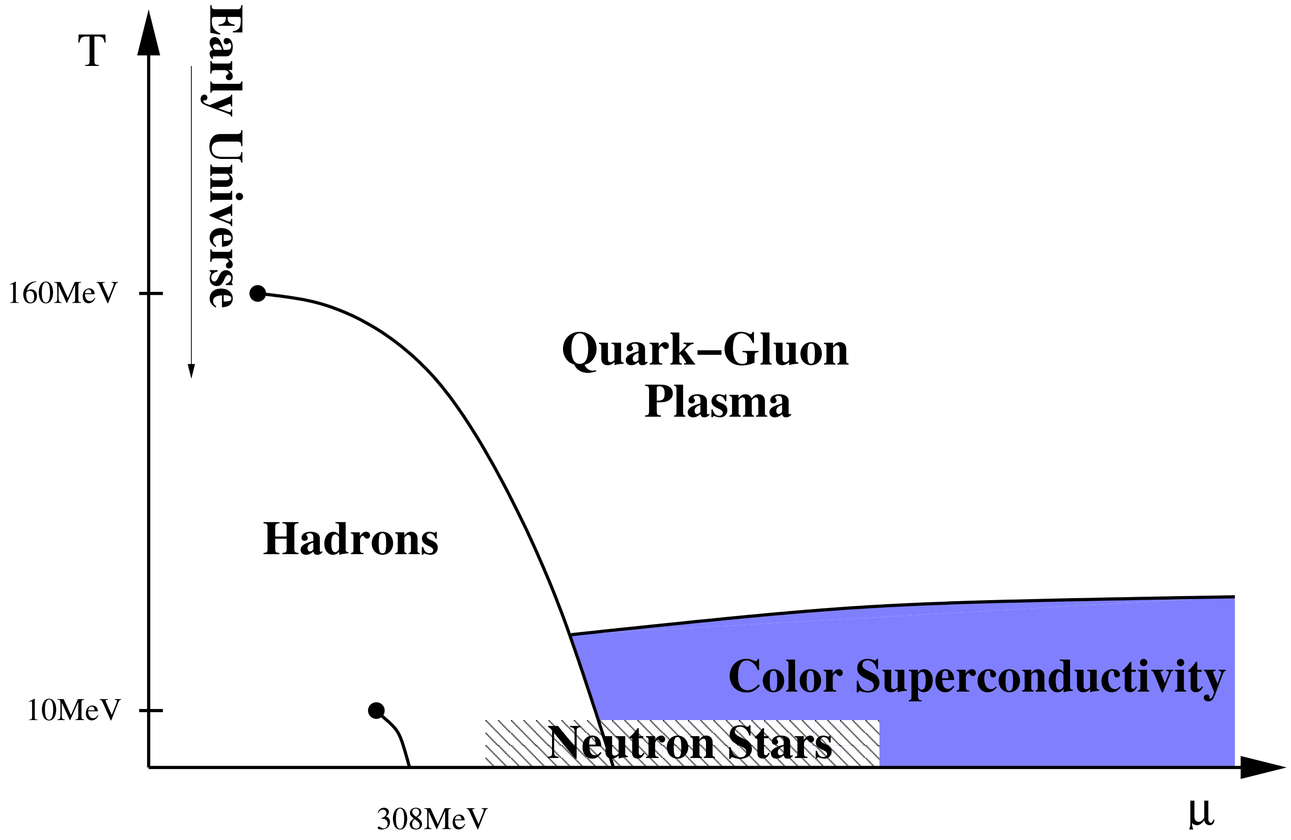
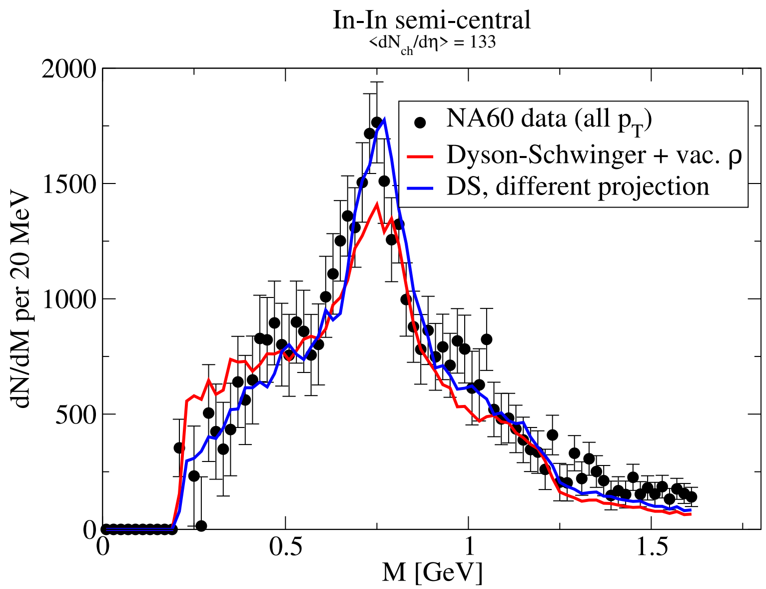
Jürgen Schaffner-Bielich

QCD phase diagram



Motivation

Dileptons carry information from hot and dense stages of hic's:



NA60 collaboration

(fig. courtesy of Thorsten Renk)

$\Longrightarrow$ **Effective theory of QCD:**

- **Hadronic d.o.f.'s**
- **Same global symmetries as QCD**

$\Longrightarrow$ **Linear sigma models!**

- **Analytic treatment possible**
- **Generalizable to $\mu \neq 0$**

Linear Sigma Models (I)

Why linear sigma models?

- **Chiral partners become degenerate above chiral restoration temperature T_c**
- **Linear sigma models treat chiral partners on same footing**

  (nonlinear sigma models: condensate $\phi = const.$)

1. **$O(N)$ model:** $\vec{\phi} = (\phi_1, \phi_2, \ldots, \phi_N) \equiv (\sigma, \vec{\pi})$

$$\boxed{\mathcal{L} = \frac{1}{2}\left(\partial_\mu \vec{\phi} \cdot \partial^\mu \vec{\phi} - m^2\, \vec{\phi} \cdot \vec{\phi}\right) - \frac{\lambda}{N}\left(\vec{\phi} \cdot \vec{\phi}\right)^2 + H\sigma}$$

$H = 0$, $m^2 > 0$: $O(N)$ **invariant**

$H = 0$, $m^2 < 0$: **vacuum expectation value (v.e.v.)** $\langle \vec{\phi} \rangle = (\phi, \vec{0})$, $\phi \equiv \langle \sigma \rangle > 0$

spontaneous symmetry breaking (SSB): $O(N) \longrightarrow O(N-1)$

$\dim[O(N)/O(N-1)] = \frac{1}{2}\left[N(N-1) - (N-1)(N-2)\right] = N - 1$

$\Longrightarrow$ $N-1$ **Goldstone bosons**

$H > 0$: **explicit symmetry breaking (ESB)**

$m^2 < 0$: $\Longrightarrow$ $N-1$ **pseudo - Goldstone bosons**

$N = 4$: σ **meson, 3 pions**

Linear Sigma Models (II)

2. $U(N_f)_r \times U(N_f)_\ell$ model (global symmetry of $\mathcal{L}_{\mathrm{QCD}}$):

$\Phi \in (N_f^*, N_f) \quad \Longrightarrow \quad \Phi \equiv \phi_a T_a, \quad T_a$ generators of $U(N_f)$, $\quad \phi_a \equiv \sigma_a + i\pi_a$

$$\boxed{\begin{aligned}\mathcal{L} = \mathrm{Tr}\left(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2\, \Phi^\dagger \Phi\right) - \lambda_1 \left[\mathrm{Tr}\left(\Phi^\dagger \Phi\right)\right]^2 - \lambda_2 \mathrm{Tr}\left(\Phi^\dagger \Phi\right)^2 \\ + c\left(\det\Phi + \det\Phi^\dagger\right) + \mathrm{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right]\end{aligned}}$$

ESB term: $H \equiv h_a T_a$

$h_a = c = 0,\ m^2 > 0$: $U(N_f)_r \times U(N_f)_\ell$ invariant

$h_a = c = 0,\ m^2 < 0$: v.e.v. $\langle \Phi \rangle = \phi\, N_f\, T_0,\ \phi \equiv \langle \sigma_0 \rangle > 0$

SSB: $U(N_f)_r \times U(N_f)_\ell \longrightarrow U(N_f)_V$ $(V \equiv \ell + r)$

$\dim[U(N_f)_r \times U(N_f)_\ell / U(N_f)_V] = 2\,N_f^2 - N_f^2 = N_f^2$

$\Longrightarrow$ N_f^2 Goldstone bosons

$h_a = 0,\ c \neq 0$: $U(1)_A$ anomaly $(A \equiv \ell - r)$

ESB: $U(N_f)_r \times U(N_f)_\ell \longrightarrow SU(N_f)_r \times SU(N_f)_\ell \times U(1)_V$

$m^2 < 0$: SSB: $SU(N_f)_r \times SU(N_f)_\ell \longrightarrow SU(N_f)_V$

$\dim[SU(N_f)_r \times SU(N_f)_\ell / SU(N_f)_V] = 2\,(N_f^2 - 1) - (N_f^2 - 1) = N_f^2 - 1$

$\Longrightarrow$ $N_f^2 - 1$ Goldstone bosons

$h_a,\ c \neq 0,\ m^2 < 0$: ESB $\Longrightarrow$ $N_f^2 - 1$ pseudo - Goldstone bosons

Linear Sigma Models (III)

Particle content:

1. $O(N)$ **model:**

N	scalars σ	pseudoscalars $\vec{\pi}$
4	σ	(π^+, π^0, π^-)

2. $U(N_f)_r \times U(N_f)_\ell$ **model:**

N_f	scalars σ_a	pseudoscalars π_a
2	σ, (a_0^+, a_0^0, a_0^-)	η, (π^+, π^0, π^-)
3	σ, (a_0^+, a_0^0, a_0^-) f_0, (K_0^{*+}, K_0^{*0}), $(\bar{K}_0^{*0}, K_0^{*-})$	η, (π^+, π^0, π^-) η', (K^+, K^0), $(\bar{K}^0, K^-)$
4	σ, (a_0^+, a_0^0, a_0^-) f_0, (K_0^{*+}, K_0^{*0}), $(\bar{K}_0^{*0}, K_0^{*-})$ χ_{c0}, (D_0^+, D_0^0, D_{s0}^+), $(\bar{D}_0^0, D_0^-, D_{s0}^-)$	η, (π^+, π^0, π^-) η', (K^+, K^0), $(\bar{K}^0, K^-)$ η_c, (D^+, D^0, D_s^+), $(\bar{D}^0, D^-, D_s^-)$

Note: $O(4) \cong SU(2) \times SU(2)$

$\Longrightarrow$ $O(4)$ **model is** $U(2)_r \times U(2)_\ell$ **model with maximal** $U(1)_A$ **anomaly,**

$c \to \infty \Longrightarrow m_\eta, m_{a_0} \to \infty$

Cornwall-Jackiw-Tomboulis formalism (I)

G. Baym, PR 127 (1962) 127
J.M. Cornwall, R. Jackiw, E. Tomboulis, PRD 10 (1974) 2428

Effective action for 1- and 2-point functions φ, G:

$$\boxed{\Gamma[\varphi, G] = I[\varphi] - \tfrac{1}{2}\,\mathrm{Tr}\ln G^{-1} - \tfrac{1}{2}\,\mathrm{Tr}\left(D^{-1}G - 1\right) + \Gamma_2[\varphi, G]}$$

$I[\varphi] \equiv \int_X \mathcal{L}(\varphi)$: tree-level action

$D^{-1}(X,Y) \equiv -\dfrac{\delta^2 I[\varphi]}{\delta\varphi(X)\delta\varphi(Y)}$: inverse tree-level propagator

Γ_2: sum of all 2PI vacuum diagrams, internal lines: G

v.e.v. ϕ and full propagator $\mathcal{G}$ follow from stationarity conditions:

$$\boxed{\left.\frac{\delta\Gamma}{\delta\varphi}\right|_{\varphi=\phi, G=\mathcal{G}} = \left.\frac{\delta\Gamma}{\delta G}\right|_{\varphi=\phi, G=\mathcal{G}} = 0}$$

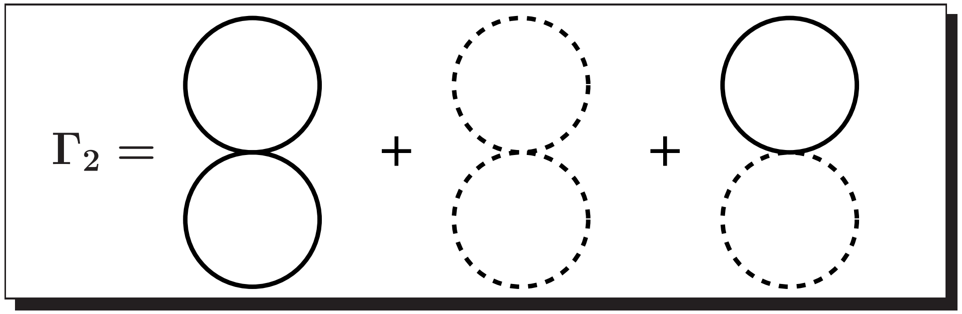
$\Longrightarrow$ **Dyson-Schwinger equation (DSE):** $\boxed{\mathcal{G}^{-1} = D^{-1} + \Pi}$

Self-energy: $\boxed{\Pi \equiv -2\left.\dfrac{\delta\Gamma_2}{\delta G}\right|_{\varphi=\phi, G=\mathcal{G}}}$

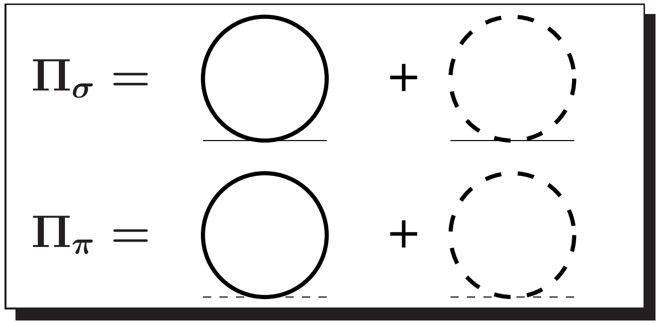
Cornwall-Jackiw-Tomboulis formalism (II)

Truncation of Γ_2 $\Longleftrightarrow$ Many-body approximation

Hartree-Fock approximation:

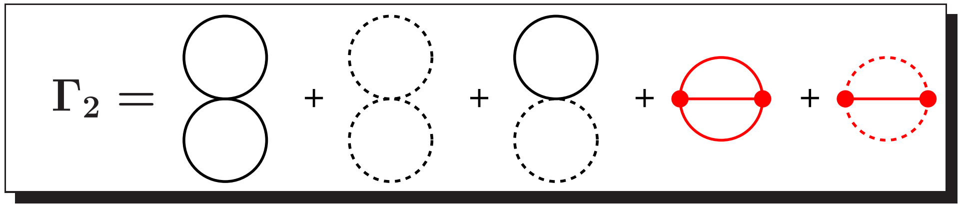


$\Longrightarrow$

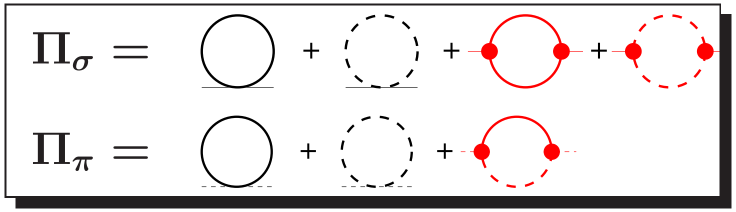


s.-e.'s independent of energy, momentum $\Longrightarrow$ DSE's: gap eqs. for masses

Full 2-loop approximation:



$\Longrightarrow$

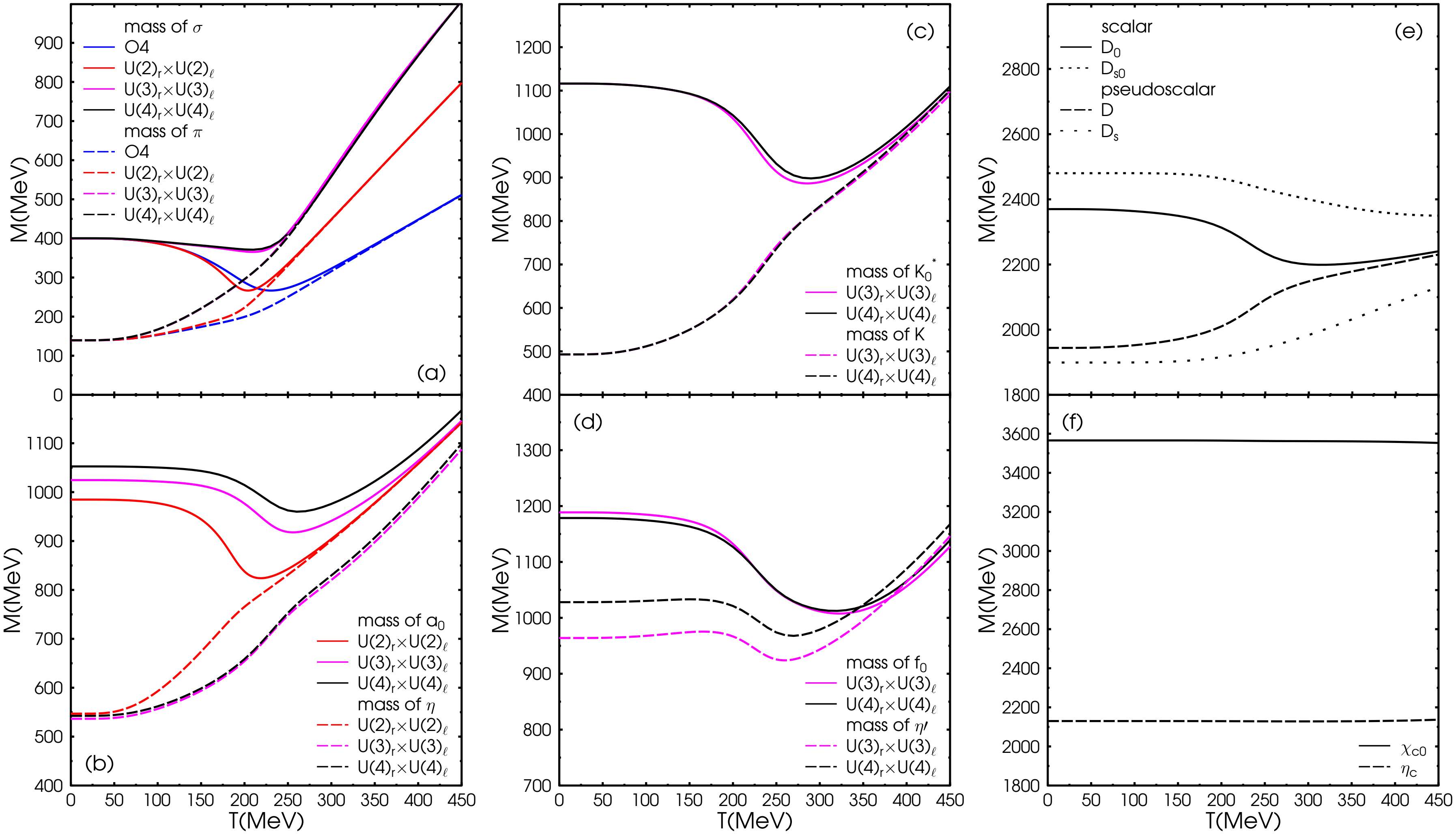


s.-e.'s depend on energy, momentum $\Longrightarrow$ DSE's: integral eqs. for spectral densities

$$\rho(\omega, k) \equiv 2 \, \mathrm{Im} \, \mathcal{G}(\omega + i\eta, k)$$

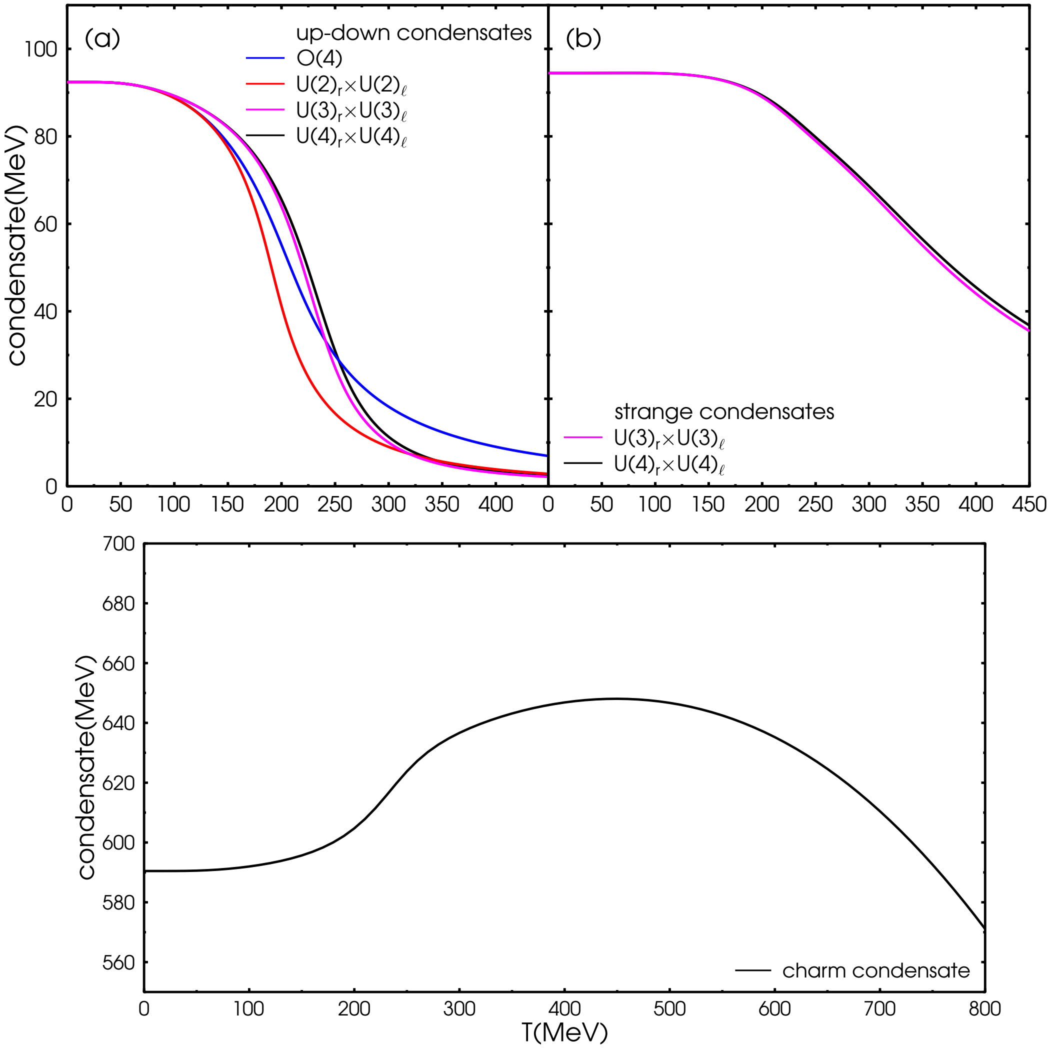
Linear Sigma Models (IV)

Masses in HF approximation: Dirk Röder, Jörg Ruppert, DHR, PRD 68 (2003) 016003



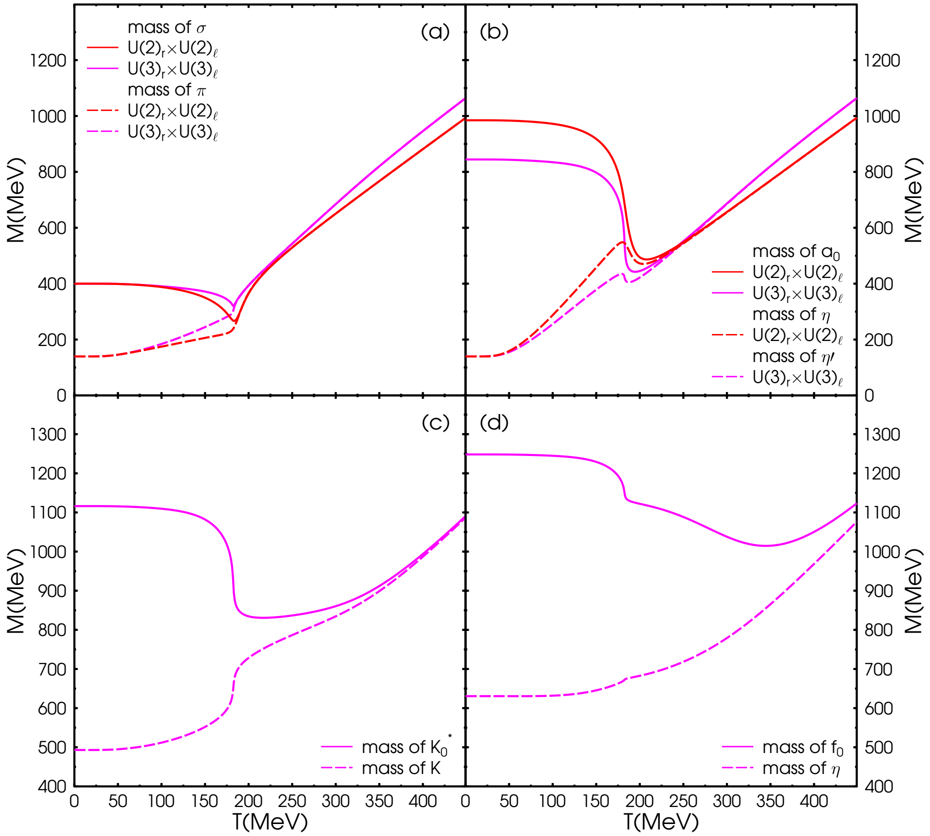
Linear Sigma Models (V)

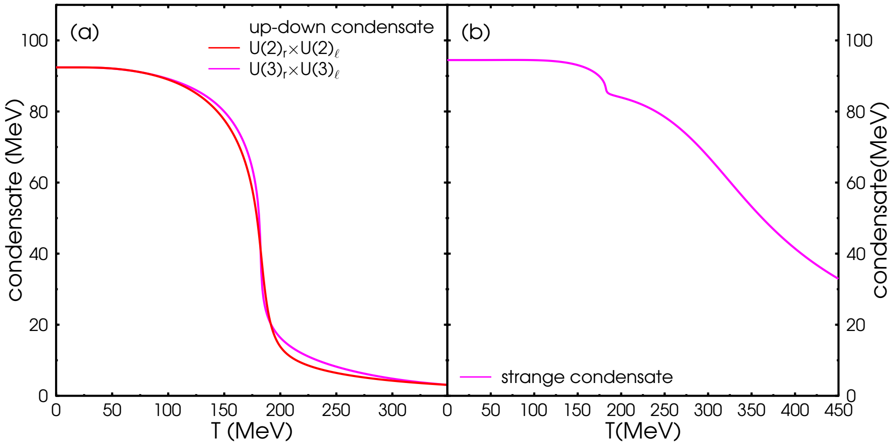
Condensates in HF approximation: Dirk Röder, Jörg Ruppert, DHR, PRD 68 (2003) 016003



Linear Sigma Models (VI)

Absence of $U(1)_A$ anomaly, HF approximation:

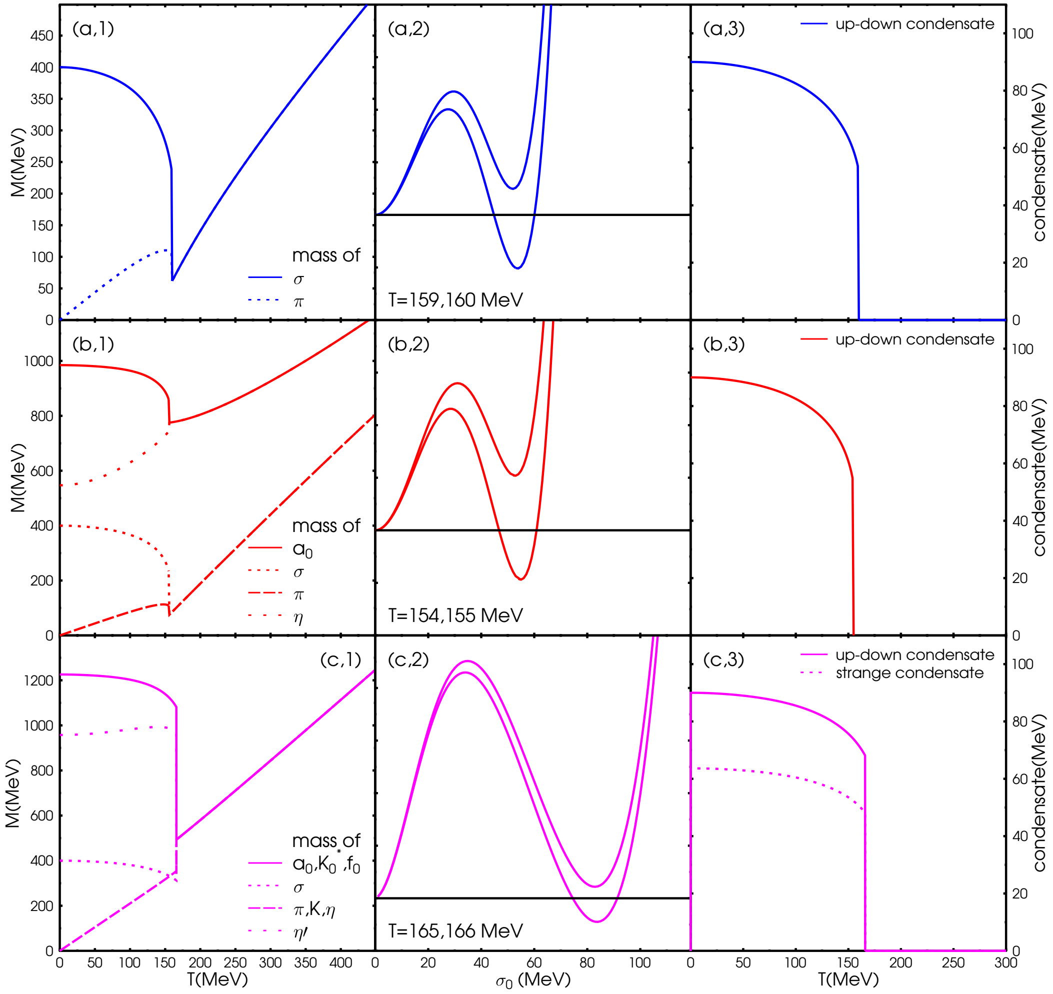




Dirk Röder, Jörg Ruppert, DHR, PRD 68 (2003) 016003

Linear Sigma Models (VII)

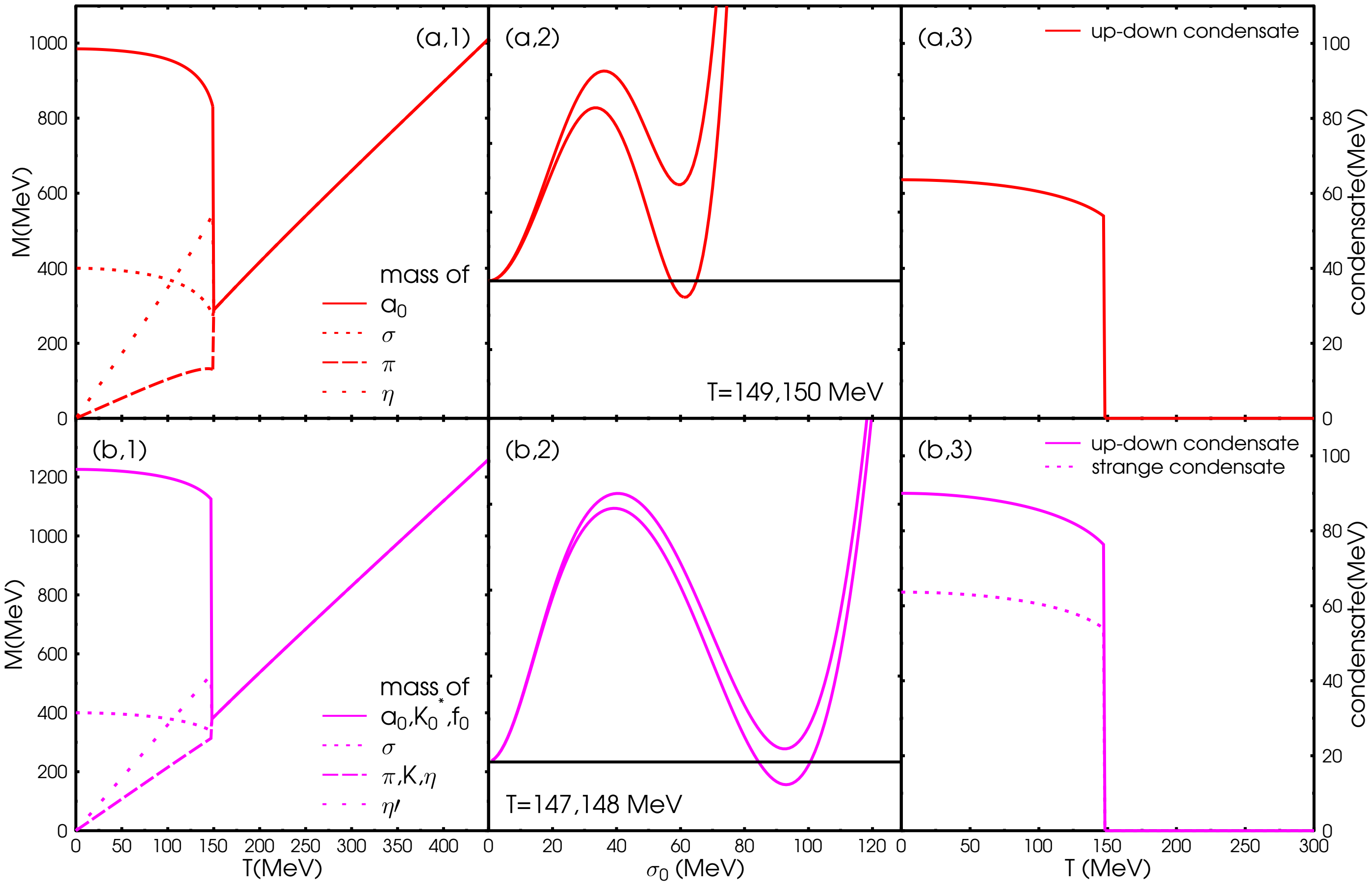
Chiral limit, HF approximation: **Dirk Röder, Jörg Ruppert, DHR, PRD 68 (2003) 016003**



Linear Sigma Models (VIII)

Chiral limit, no $U(1)_A$ anomaly, HF approximation:

Dirk Röder, Jörg Ruppert, DHR, PRD 68 (2003) 016003



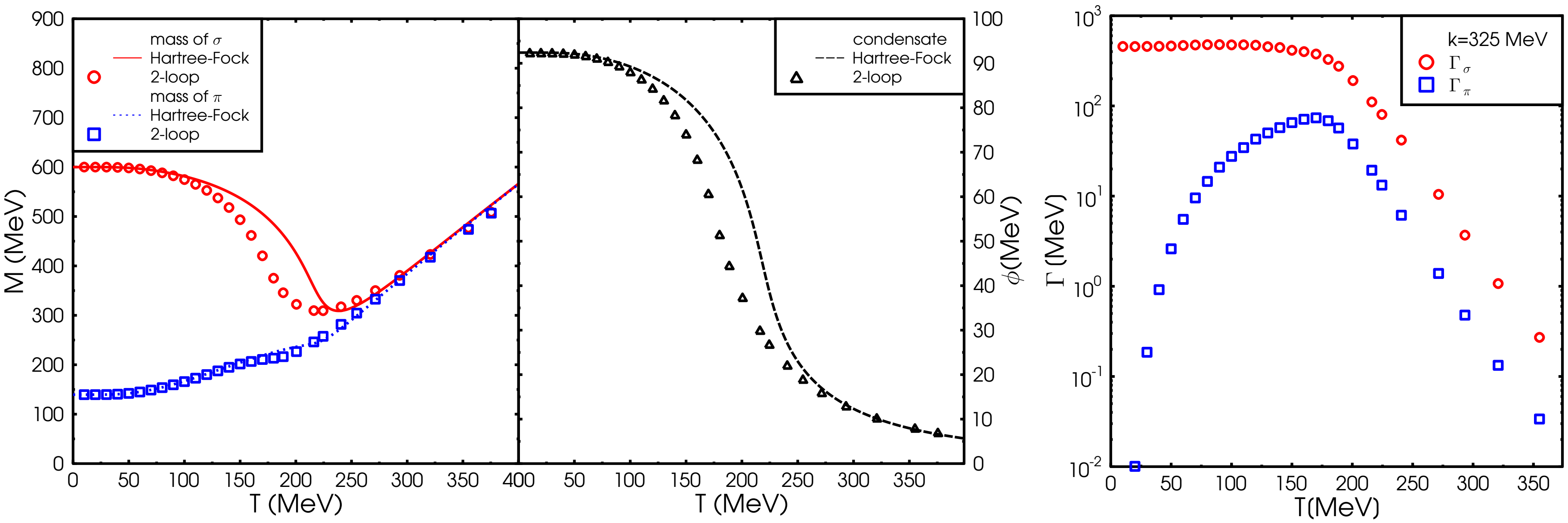
Linear Sigma Models (IX)

2-loop approximation for $O(4)$ model ($\mathrm{Re}\,\Pi \equiv \Pi_{\mathrm{tadpole}}$):

$\Longrightarrow$ **mass** $M^2 = M^2_{\mathrm{tree-level}} + \Pi_{\mathrm{tadpole}}\,,\quad \Pi_{\mathrm{tadpole}} \sim \int d\omega\, d^3k\, \rho(\omega, k)\, \dfrac{1}{e^{\omega/T} - 1}$

$\Longrightarrow$ **on-shell energy** $\omega(k) = \sqrt{k^2 + M^2}$

$\Longrightarrow$ **decay width** $\Gamma(k) \equiv \dfrac{\mathrm{Im}\,\Pi(\omega(k), k)}{\omega(k)}$

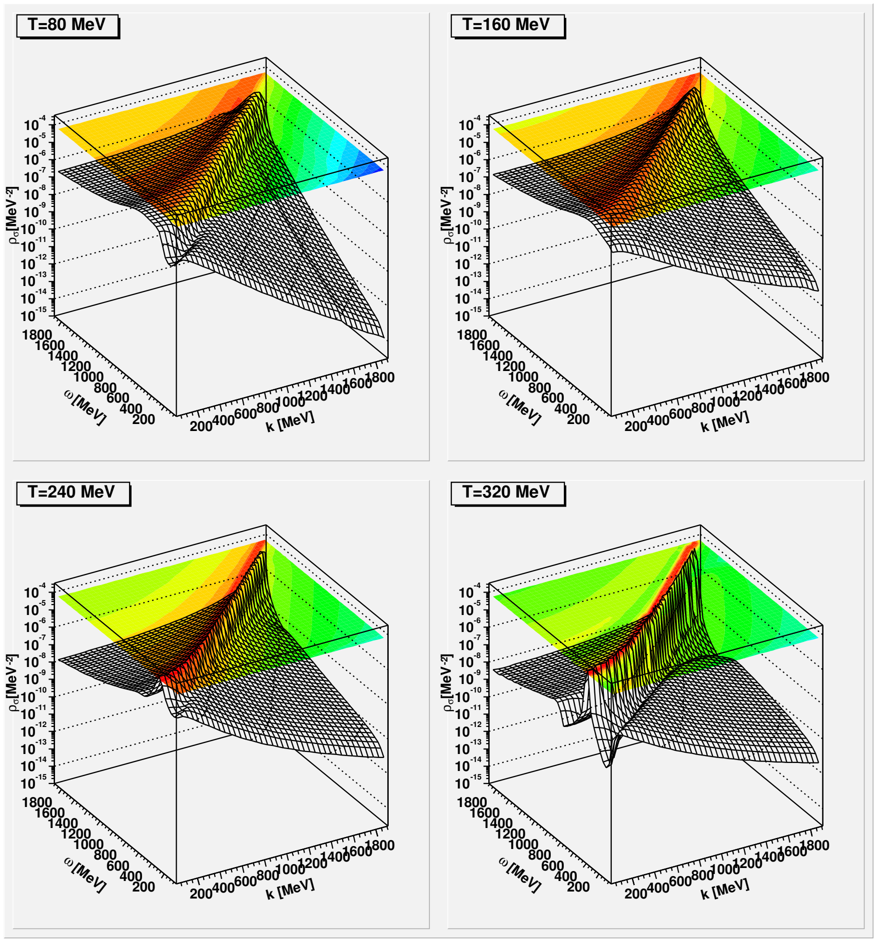


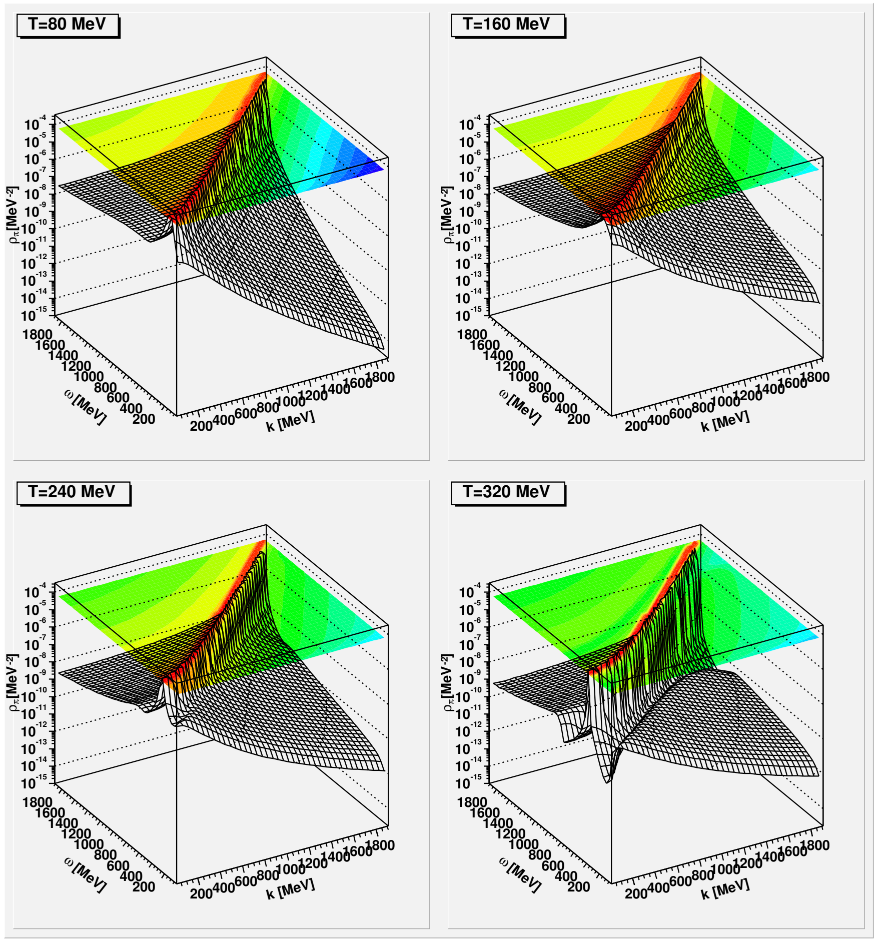
Dirk Röder, Jörg Ruppert, DHR, hep-ph/0503042 (NPA, to be published)

Linear Sigma Models (X)

Spectral densities in 2-loop approximation for $O(4)$ model:

σ meson **pion**

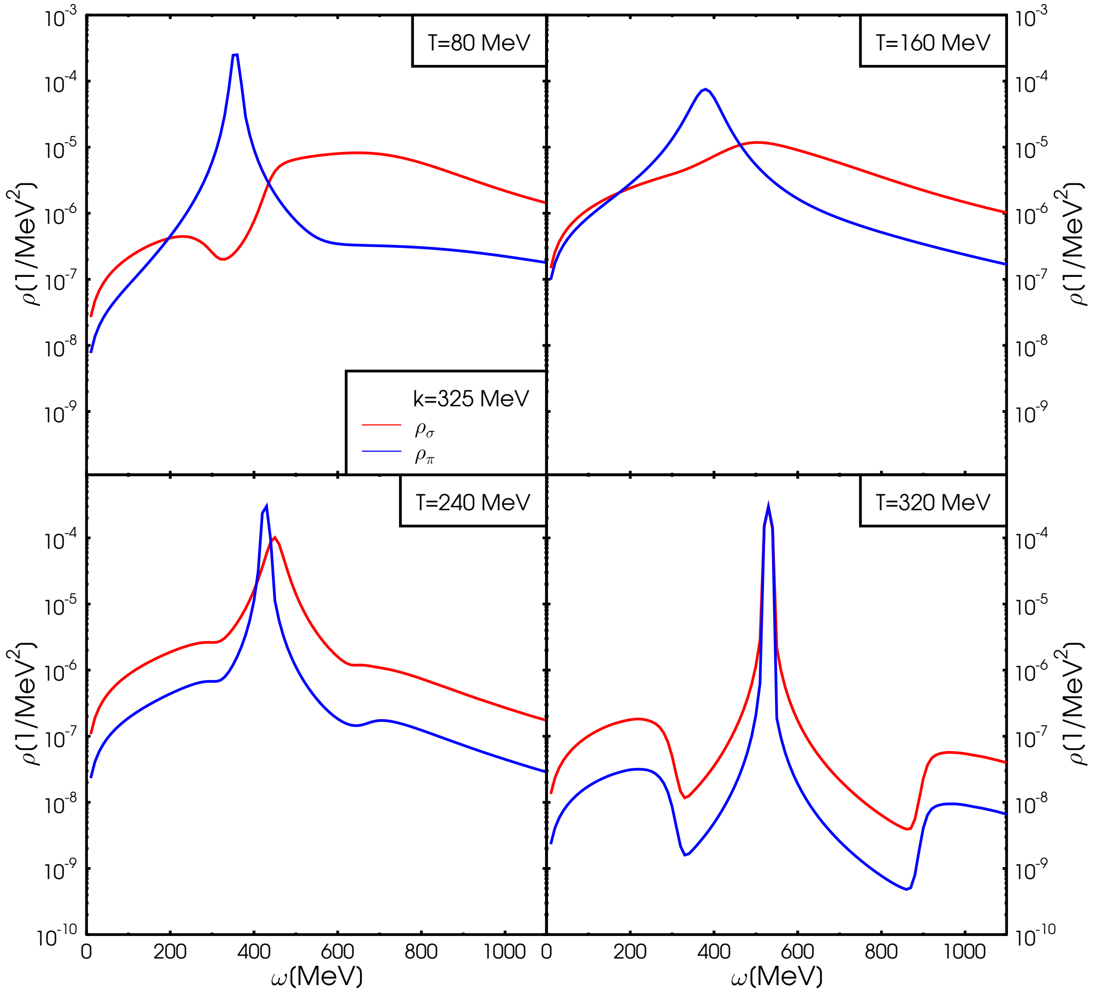




Dirk Röder, Jörg Ruppert, DHR, hep-ph/0503042 (NPA, to be published)

Linear Sigma Models (XI)

Spectral densities in 2-loop approximation for $O(4)$ model:



Dirk Röder, Jörg Ruppert, DHR, hep-ph/0503042 (NPA, to be published)

Vector Mesons (I)

Local gauge symmetry:

$\Longrightarrow$ **covariant derivative:** $\boxed{\partial_\mu \Phi \longrightarrow D_\mu \Phi = \partial_\mu \Phi - ig\mathcal{L}_\mu \Phi + ig\Phi \mathcal{R}_\mu}$

$\mathcal{L}_\mu \equiv L_\mu^a T_a$, $\mathcal{R}_\mu \equiv R_\mu^a T_a$

vector mesons: $V_\mu^a \equiv \frac{1}{2}\left(L_a^\mu + R_\mu^a\right)$

axialvector mesons: $A_\mu^a \equiv \frac{1}{2}\left(L_a^\mu - R_\mu^a\right)$

$$\Longrightarrow \boxed{\begin{aligned} \mathcal{L} = \mathrm{Tr}\left[(D_\mu\Phi)^\dagger D^\mu\Phi - m^2\,\Phi^\dagger\Phi\right) - \lambda_1\left[\mathrm{Tr}\left(\Phi^\dagger\Phi\right)\right]^2 - \lambda_2\mathrm{Tr}\left(\Phi^\dagger\Phi\right)^2 \\ + c\left(\det\Phi + \det\Phi^\dagger\right) + \mathrm{Tr}\left[H\left(\Phi + \Phi^\dagger\right)\right] \\ - \frac{1}{4}\mathrm{Tr}(\mathcal{L}_{\mu\nu}\mathcal{L}^{\mu\nu} + \mathcal{R}_{\mu\nu}\mathcal{R}^{\mu\nu}) + \frac{m_1^2}{2}\mathrm{Tr}(\mathcal{L}_\mu\mathcal{L}^\mu + \mathcal{R}_\mu\mathcal{R}^\mu) \end{aligned}}$$

$\mathcal{L}_{\mu\nu} \equiv \partial_\mu\mathcal{L}_\nu - \partial_\nu\mathcal{L}_\mu - ig[\mathcal{L}_\mu, \mathcal{L}_\nu]$, $\mathcal{R}_{\mu\nu} \equiv \partial_\mu\mathcal{R}_\nu - \partial_\nu\mathcal{R}_\mu - ig[\mathcal{R}_\mu, \mathcal{R}_\nu]$

$h_a = c = 0$, $m_1^2 = 0$: **Local** $U(N_f)_r \times U(N_f)_\ell$ **symmetry**

$m_1^2 \neq 0$: **Global** $U(N_f)_r \times U(N_f)_\ell$ **symmetry**

Current-field proportionality: $\mathcal{J}_{V\mu}^a \equiv \frac{m_1^2}{g} V_\mu^a$, $\mathcal{J}_{A\mu}^a \equiv \frac{m_1^2}{g} A_\mu^a$

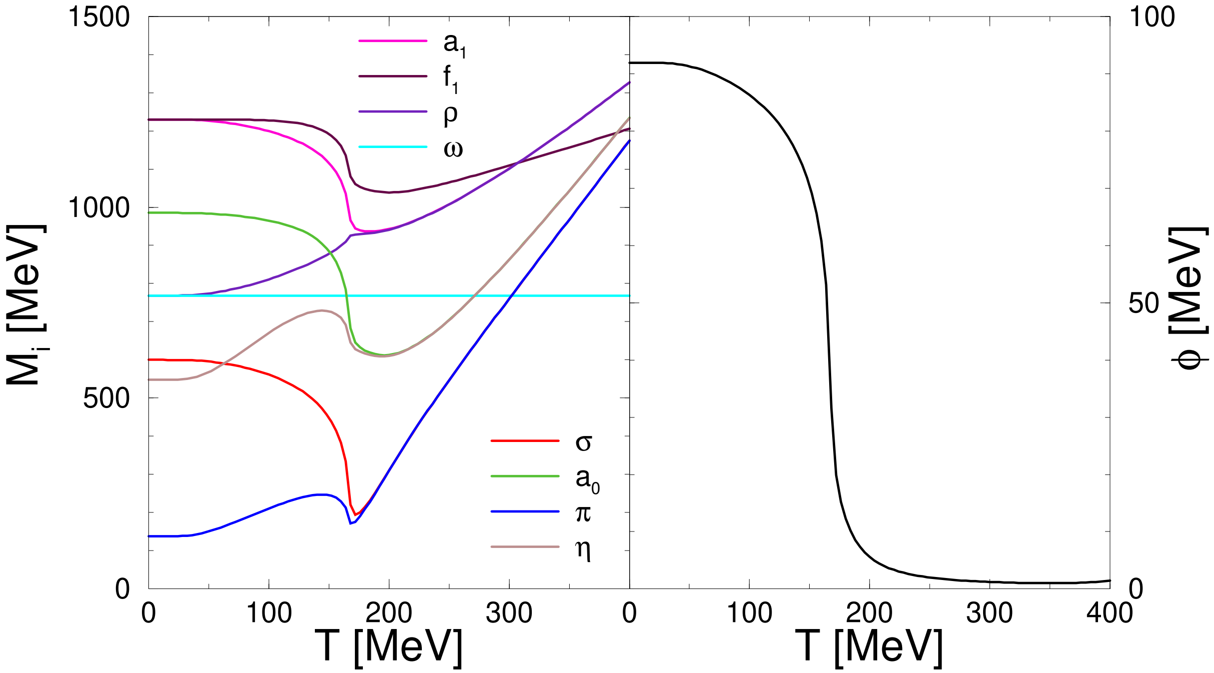
Conservation laws: $\partial_\mu \mathcal{J}_V^{a\mu} = f_{abc}\sigma_b h_c$, $\partial_\mu \mathcal{J}_A^{a\mu} = d_{abc}\pi_b h_c + c\,\delta_{a0}\,\mathrm{Im}\det\Phi$

Vector Mesons (II)

Particle content of $U(2)_r \times U(2)_\ell$ model:

scalars σ_a	σ, (a_0^+, a_0^0, a_0^-)
pseudoscalars π_a	η, (π^+, π^0, π^-)
vectors V_μ^a	ω, (ρ^+, ρ^0, ρ^-)
axial vectors A_μ^a	f_1, (a_1^+, a_1^0, a_1^-)

Hartree-Fock approximation:



Stefan Strüber, DHR, in preparation

Baryons

Inclusion of baryons:

$$\boxed{\mathcal{L}_B = \bar{\Psi}_\ell \, i \not{D}_\ell \, \Psi_\ell + \bar{\Psi}_r \, i \not{D}_r \, \Psi_r + \bar{\Psi}_\ell \, \hat{g} \Phi \, \Psi_r + \bar{\Psi}_r \, \hat{g} \Phi^\dagger \, \Psi_\ell}$$

$$D_\ell^\mu \equiv \partial^\mu + i g L_a^\mu T_a \, , \quad D_r^\mu \equiv \partial^\mu + i g R_a^\mu T_a \, , \quad L_a^\mu \equiv V_a^\mu + A_a^\mu \, , \quad R_a^\mu \equiv V_a^\mu - A_a^\mu$$

$N_f = 2$:

$$\boxed{\mathcal{L}_B = \bar{\Psi} \left[i \not{\partial} - \frac{g}{2} \left(\not{\omega} + \vec{\tau} \cdot \vec{\not{\rho}} + \gamma_5 \not{f}_1 + \gamma_5 \, \vec{\tau} \cdot \vec{\not{a}}_1 \right) + \frac{\hat{g}}{2} \left(\sigma + \vec{\tau} \cdot \vec{a}_0 + i \gamma_5 \, \eta + i \gamma_5 \, \vec{\tau} \cdot \vec{\pi} \right) \right] \Psi}$$

Chiral symmetry breaking induces term $\sim g \phi \, \vec{a}_1^{\,\mu} \cdot \partial_\mu \vec{\pi}$

$\Longrightarrow$ **eliminate by shift** $\vec{a}_1^{\,\mu} \longrightarrow \vec{a}_1^{\,\mu} - w(\phi) \, \partial_\mu \vec{\pi} \, , \quad w(\phi) \equiv \frac{g\phi}{m_1^2 + g^2 \phi^2}$

$\Longrightarrow$ **induces p-wave πN coupling** $\sim g_{\text{eff}} \, \bar{\Psi} \, \gamma_5 \, \vec{\tau} \cdot \not{\partial} \vec{\pi} \, \Psi \, , \quad g_{\text{eff}} \equiv \frac{g}{2} \, w(\phi)$

G. Carter, P.J. Ellis, S. Rudaz, NPA 603 (1996) 367

$\Longrightarrow$ **πN scattering length $a_0^{(+)} \equiv 0$ (as dictated by chiral symmetry)**

Conclusions and open problems

Chiral symmetry restoration in linear sigma models:

1. **Scalar and pseudoscalar mesons:**
   - $O(4)$ and $U(N_f)_r \times U(N_f)_\ell$ models $(N_f = 2, 3, 4)$ in Hartree-Fock approximation
   - $O(4)$ model in 2-loop approximation $(\mathrm{Re}\,\Pi \equiv \Pi_{\mathrm{tadpole}})$
   - Inclusion of energy-momentum dependent part of $\mathrm{Re}\,\Pi$
   - $U(N_f)_r \times U(N_f)_\ell$ models in 2-loop approximation
2. **Vector and axialvector mesons:**
   - $U(2)_r \times U(2)_\ell$ model in HF approximation
   - Full 2-loop approximation
   - Extension to $N_f = 3, 4$
3. **Baryons**
4. **Coupling to photon**
5. **Dilepton rate, spectrum**