AMR on Structured Meshes

PARAMESH

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Today

- Why AMR ?
- Different AMR approaches.
- Story of block−based AMR
- Examples

Tomorrow

- Introduction to PARAMESH
- Functionality
- Design choices for parallel environment.
- Weaknesses

- Tutorial

Motivation

show images from SOHO and TRACE websites

http://sohowww.nascom.nasa.gov/

http://vestige.lmsal.com/TRACE/

AMR approaches

Fixed number of mesh cells

- Lagrangian
- Rezoning

Variable number of mesh cells

- Unstructured finite element
- Structured cell–by–cell
- Structured sub–grid blocks

Observation :

For schemes which use a fixed number of mesh cells, adding cells in one location takes cells from somewhere else. This makes refinement control more difficult when many features compete for refinement.

Composite Meshes

LAGRANGIAN METHODS

Grid moves with the fluid flow.

Advantages

- Lagrangian schemes are often less dissipative than eulerian schemes for hydro
- Fixed memory size
- Conservation laws are easy to enforce

Disadvantages

- Refinement control is somewhat clumsy. Needs modification if anything other than mass flow is to control refinement.
- Mesh can get hopelessly tangled in 2 and 3D.

  Variant called the 'free lagrange method' designed to solve this problem.

  Good for front tracking but has problems with multiple interacting fronts.

Rezoning (penalty methods)

Adjust a fixed number of grid cells using some weighting function to specify relative mesh cell sizes.

eg Brackbill and Saltzmann

Variational approach – have to solve a set of Euler equations which define the coordinate transformation.

Advantages

- **fixed memory size**
- **good for fitting grid to complex boundary shapes**
- **good cache line reuse**
- **Conservation laws are easy to enforce**

Disadvantages

- **difficult to maintain smooth grids (eg rapidly varying cell sizes, anisotropic cell shapes)**
- **may need to solve an additional elliptic equation(s) to determine the new grid**
- **accuracy concerns associated with shock–capturing schemes for strong shocks where mesh cell sizes change too quickly.**
- **time variation of mesh is non–trivial**

Brackbill and Saltzmann figure

TeX pages for Brackbill and Saltzmann method

Unstructured Finite Element

Advantages

- Most flexible refinement
- Best approach for complex boundary geometries
- Conservation laws are easy to enforce

Disadvantages

- Relatively slow in execution because of high levels of indirect data referencing
- Requires significant memory for element and node locations and interconnectivity arrays
- Generating ‘good’ grids is a difficult art

Finite element example

Tsunami model – Myers, Baptista and Priest

‘Structured’ cell–by–cell

Equivalent to the unstructured FEM approach except that grid cells are rectangular in coordinate space, and have hanging nodes to deal with.

eg Cart3D or ALLA

http://www.nas.nasa.gov/~aftosmis/cart3d
http://tycho.as.utexas.edu/~ajk/PROJECTS/ALLA/alla.html

Advantages

- **Flexible and efficient refinement patterns**
- **Low memory overhead**

Disadvantages

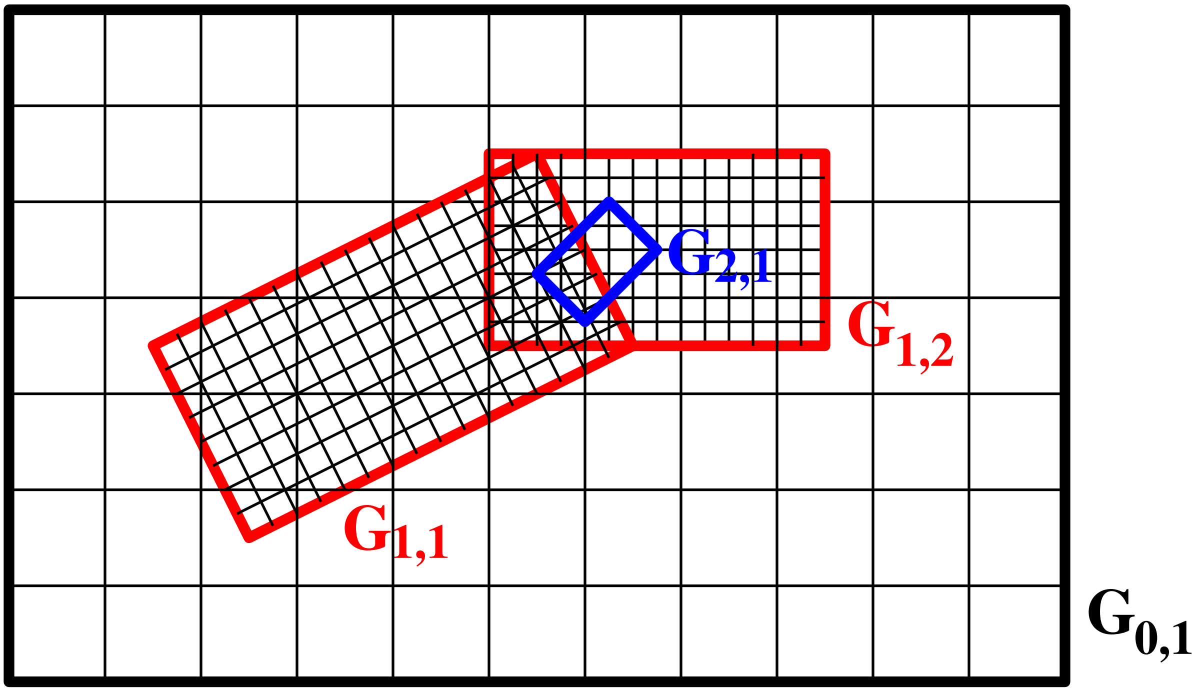
- **Expensive general interpolation formulae or use of expensive non–uniform stencils for derivatives of required order of accuracy.**
- **Large tree data–structure to keep track of cells and their neighbors**
- **Conservation laws require additional work at refinement jumps**
- **Hard to deal with complex boundary shapes.**

Show Khokhlov example

Structured Sub–Grids

Grid is a combination of 'logically cartesian' sub–grids at various refinement levels, overlaid so as to cover the computational domain with the desired resolution.

Berger and Oliger – finite difference hydrodynamics



Uses a tree data–structure to manage the grids

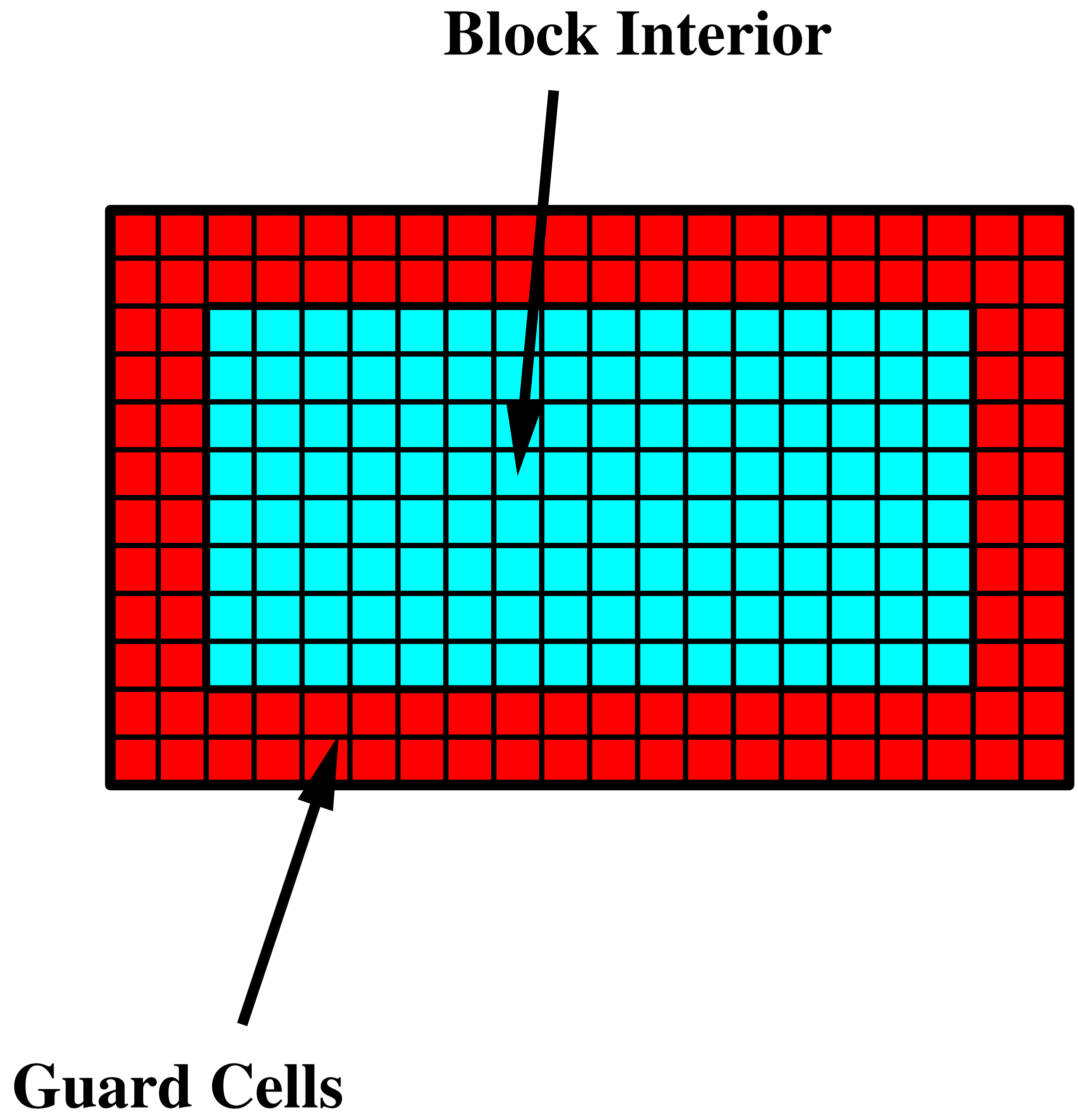
Refinements must be properly nested

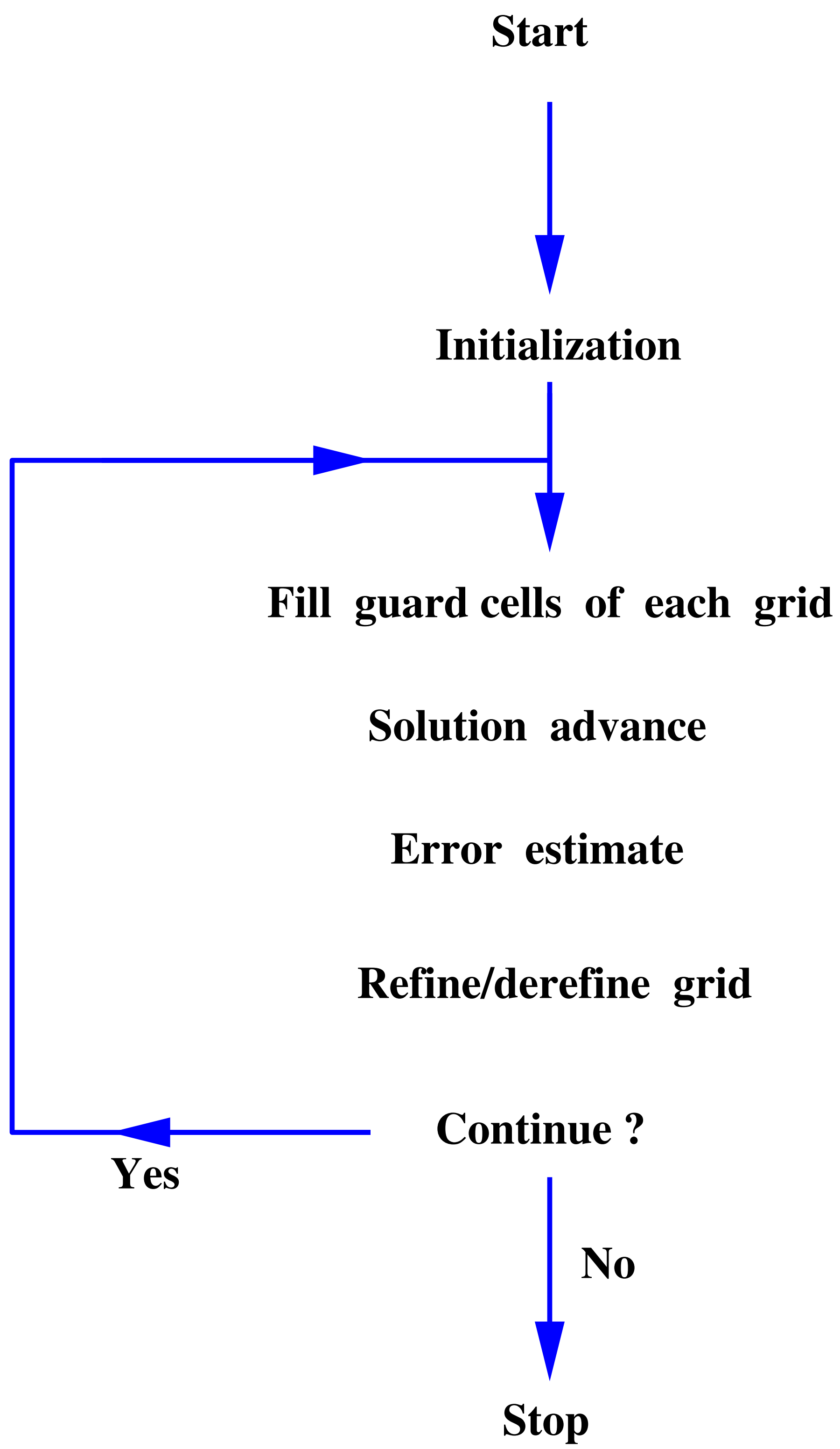
Advantages

- **regular sub−grids produce efficient code**
- **convergence properties better understood on uniform grids.**
- **shock capturing schemes work better on uniform meshes**

Disadvantages

- **Hard to deal with complex boundary shapes.**
- **Memory overhead associated with guard cells**
- **Conservation laws require additional work at refinement jumps**


Block Interior
Guard Cells


Start
Initialization
Fill guard cells of each grid
Solution advance
Error estimate
Refine/derefine grid
Continue ?
Yes
No
Stop

Berger and Oliger Bells and whistles

- **Supports different size sub–grids**
- **Allows sub–grids to be rotated with respect to the coordinate axes**
- **Allows user to choose different integer refinement ratios (usually choose 4)**
- **Refines integration timestep along with grid spacing.**
- **Can combine sub–grids at the same refinement level if it would be more efficient.**
- **Advances solution on all grids.**
- **Does convergence test to estimate error.**

Complex code. Very difficult to parallelize efficiently.

http://cs.nyu.edu/cs/faculty/berger/amrsoftware.html

http://seesar.lbl.gov/anag/chombo/

PARAMESH is a simplified version of this basic strategy.

Analysis of Algorithms

Accuracy

Is order of scheme maintained at resolution jumps?

If you use a 4th order spatial difference, then you need a 4th order interpolation scheme to fill guardcells at a block boundary with a coarser region.

Stability

Convergence

AMR adds an additional complication to convergence properties.

Stability analyses for solvers on AMR grids

Few and far between !

Stability proofs exist for leap–frog and Lax–Wendroff algorithms for the Euler equations with certain interpolation functions at refinement jumps.

Bergers Ph.D. thesis – Lax–Wendroff
Trefethen, 1985, Math. Comp., vol 45, p.279 – leapfrog
Wilds Ph.D. thesis – wave equation

Trefethen :

Considers block–structured AMR as a multi–interface model.

" Any multi–interface model potentially admits trapped numerical waves that reflect back and forth repeatedly from one interface to another. If the reflections cause amplification, this can lead to unbounded growth of numerical solutions. The factors that control this are: magnitude of the reflection coefficients, which is the source of growth; dissipation of waves as they travel between interfaces, which is a source of attenuation; and travel time between interfaces, which determines how frequently any reflection circuit that causes growth is repeated."

Reflection coefficients are influenced by :

1. **the size of the refinement jump**

2. **the interpolation schemes used to compute data in guardcells at refinement discontinuities.**

Other algorithmic properties affected by internal boundaries :

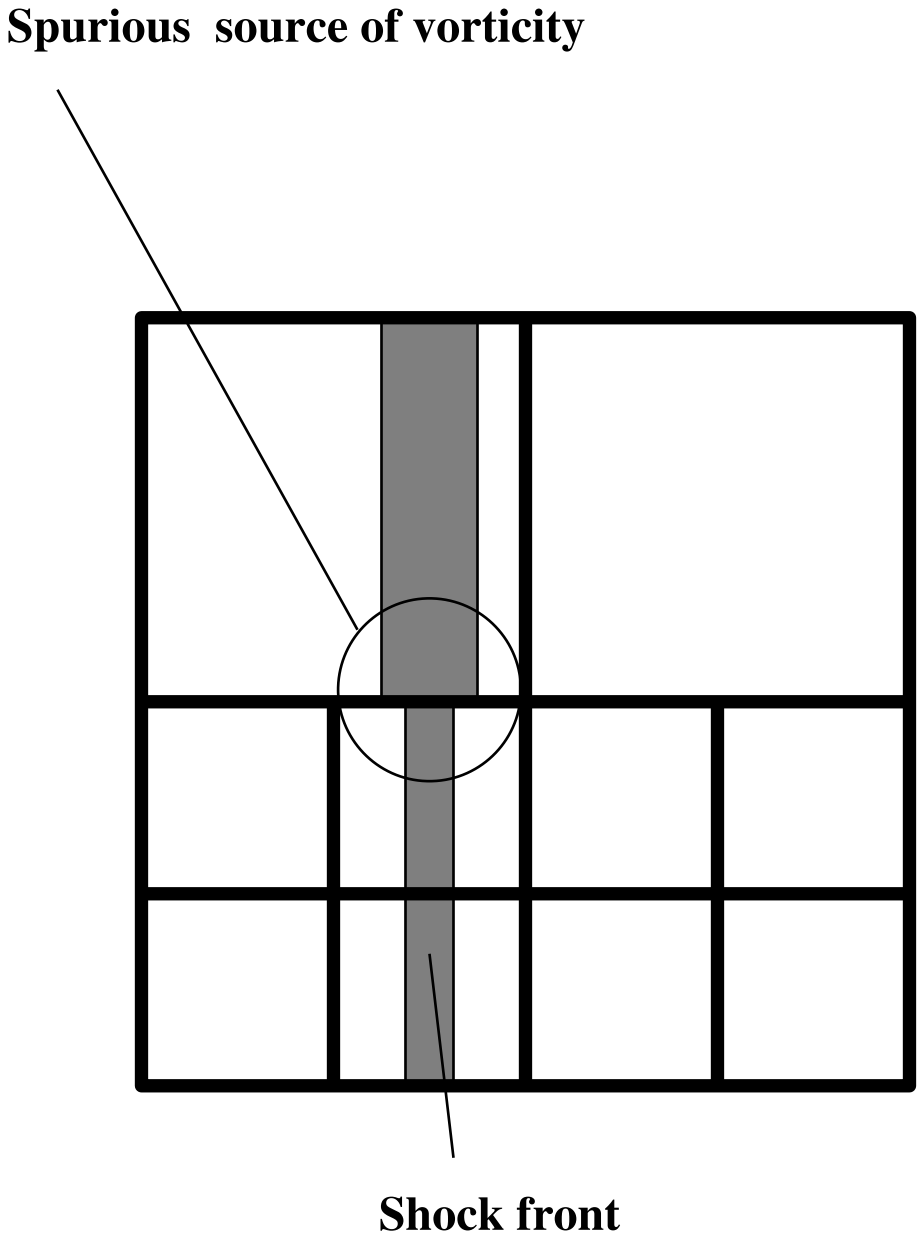
- **Conservation**

- **Monotonicity – Quirk. Monotonicity constraint forces Courant number to be reduced by about 20%**

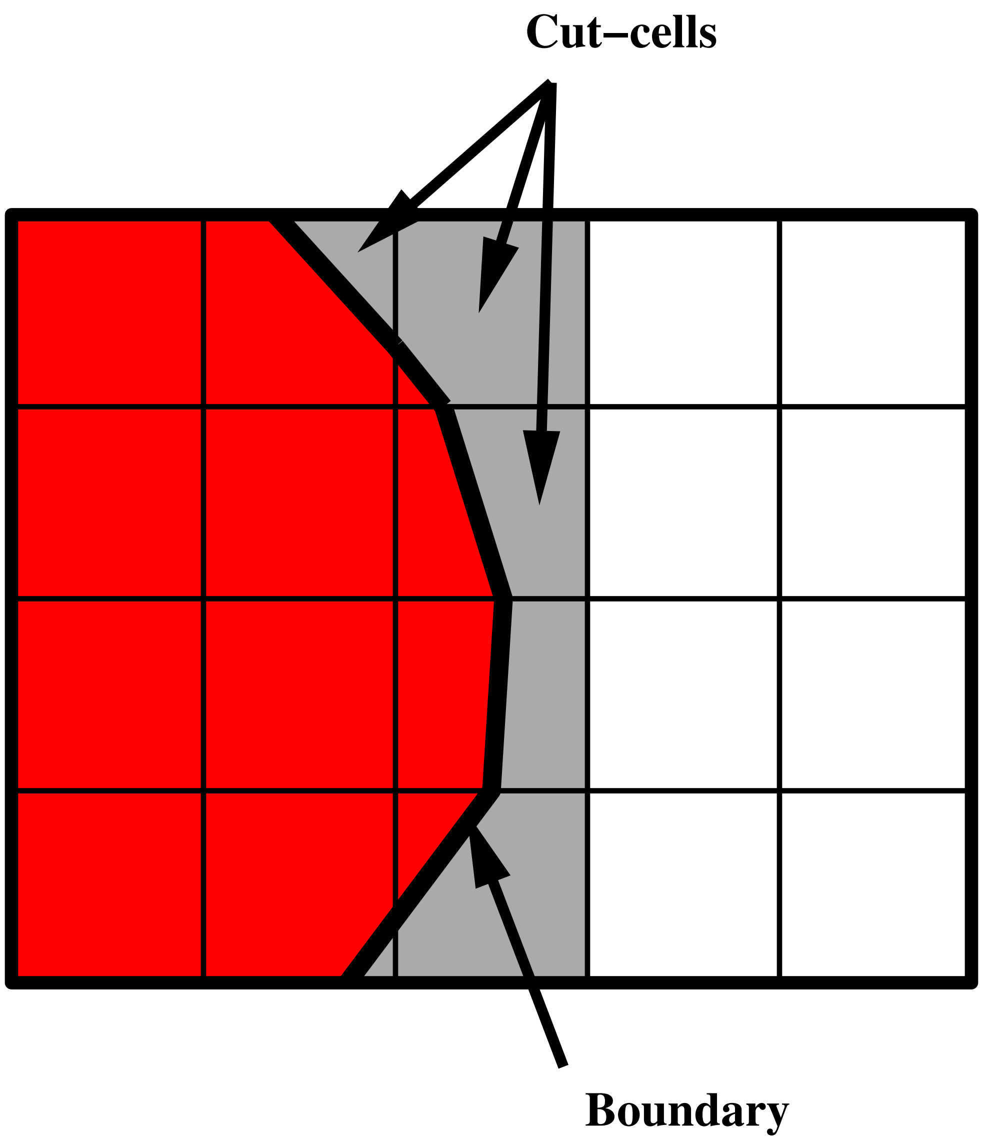
Never let a strong shock cross a refinement discontinuity!

Shock capturing problem on mesh with variable spacing for strong shocks – figure from Berger and Colella

show multiple reflections at block interfaces from Wild's thesis


Spurious source of vorticity
Shock front

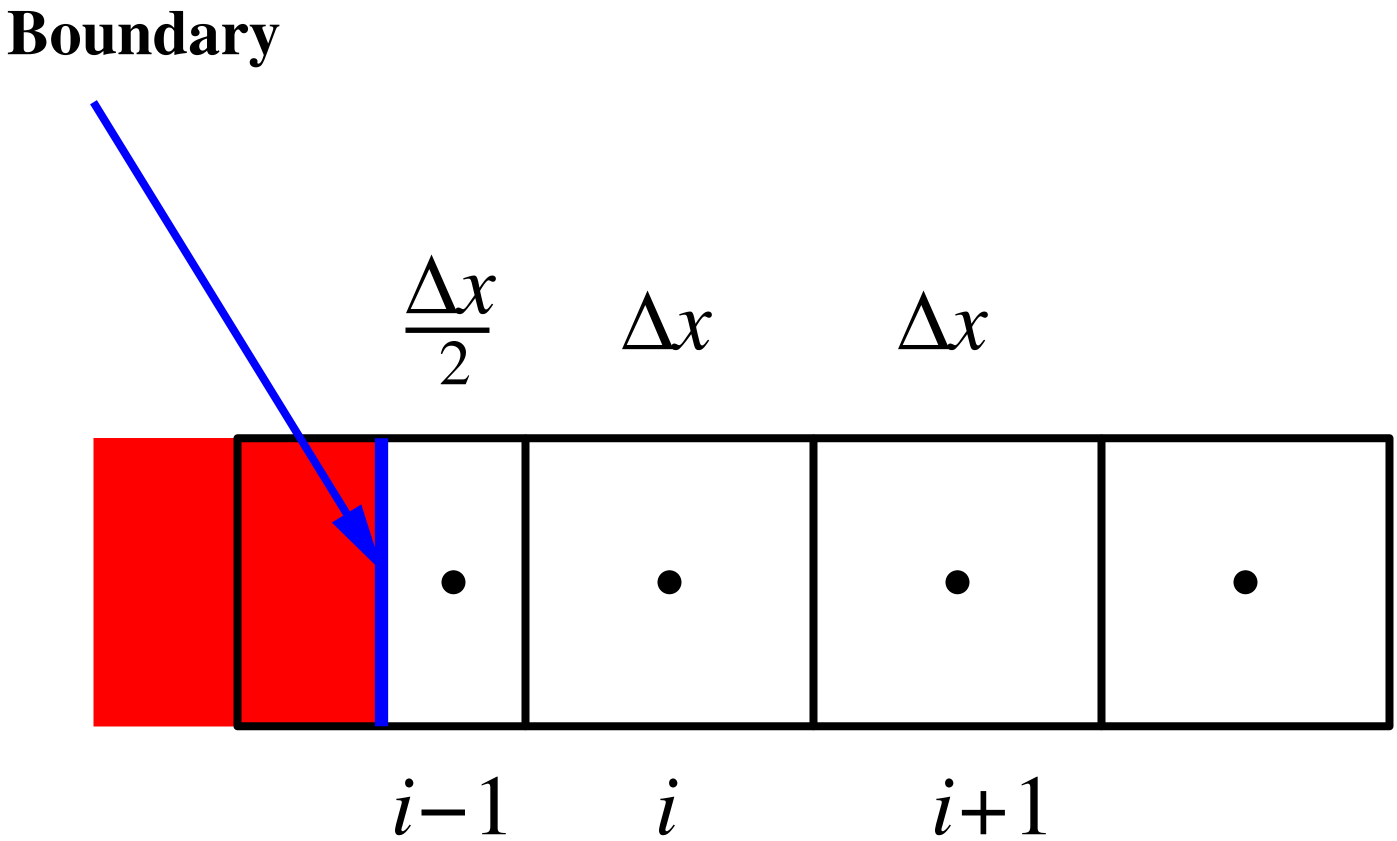
Complex boundary geometries



Cut−cells have accuracy and stability problems.

Two basic strategies :

- Cell merging
- Wave propagation based

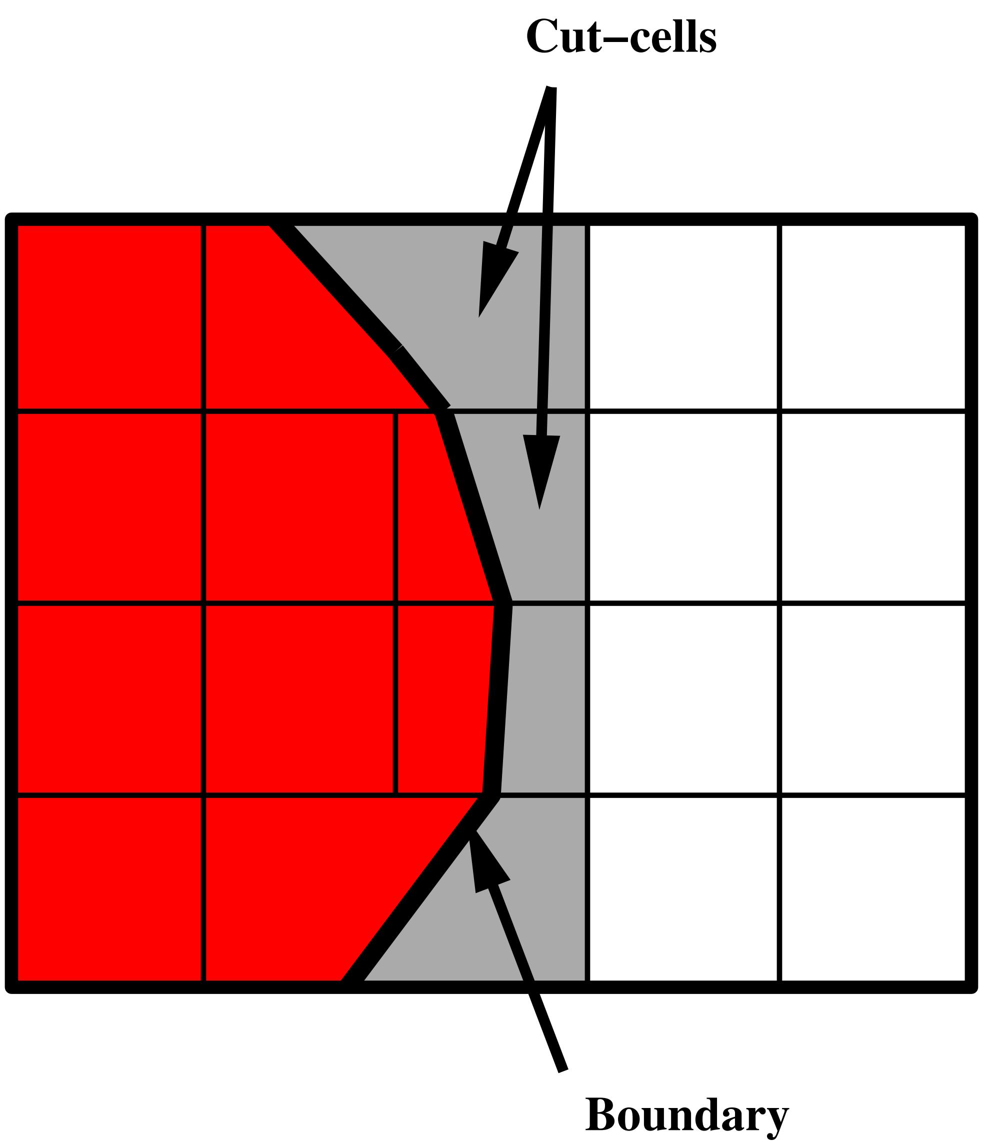


Boundary
$\frac{\Delta x}{2}$
Δx
Δx
$i-1$
i
$i+1$

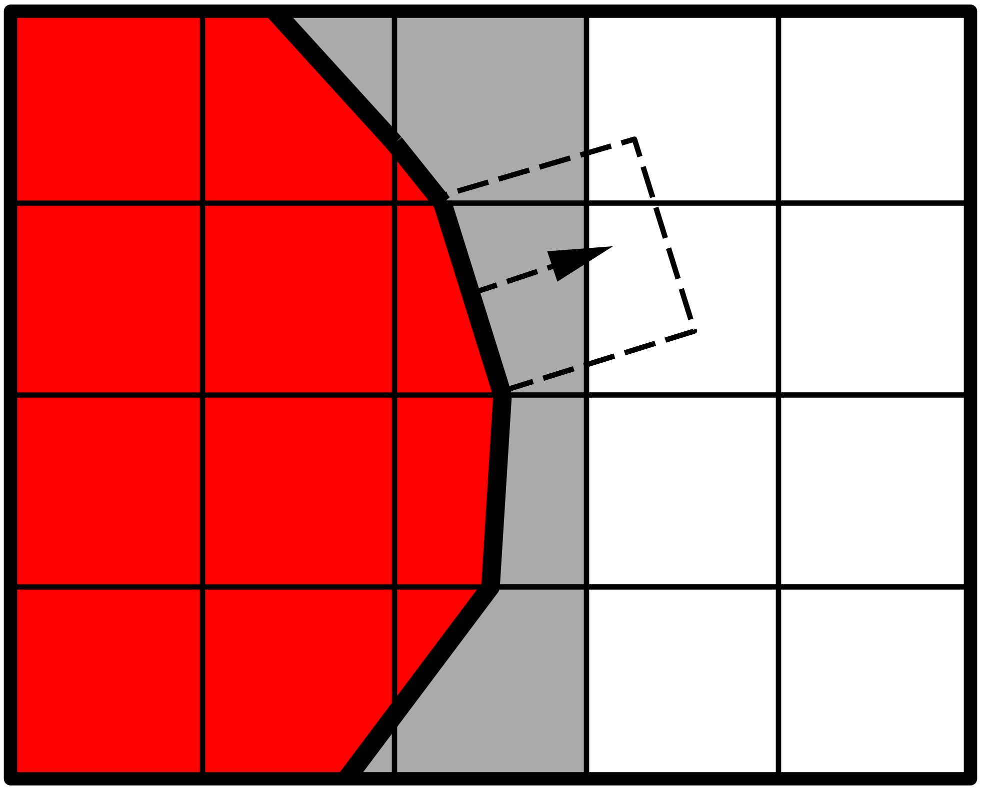
Accuracy

Stability

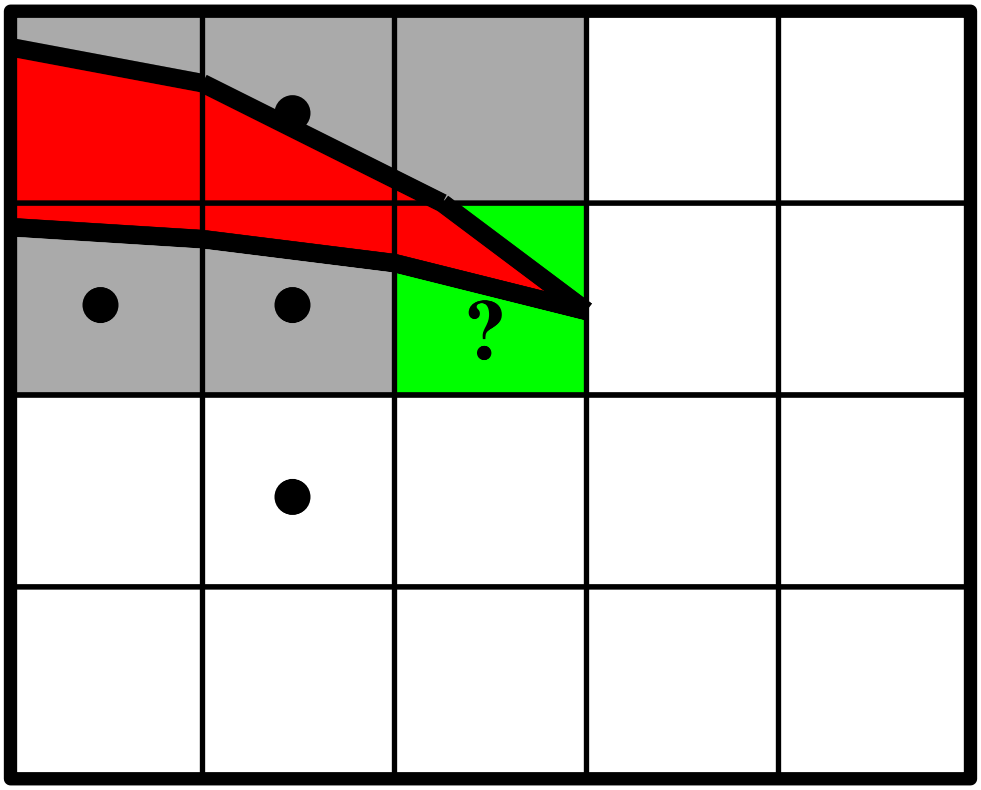
Cell Merging



Wave propagation based approach



Double Valued Cells



Components of Block Structured AMR codes.

Data structures:

- solution
- mesh

Computational Tasks:

- guard cell filling
- prolongation (coarse to fine)
- restriction (fine to coarse)
- error estimation
- mesh rebuilding
- supporting conservation laws

Parallel computing :

- load balancing

Communication Requirements

Operations involving communication

- guardcell filling
- prolongation
- restriction
- conservation support
- modifying grid
- I/O

Setting up interprocessor communication

Is communication network fast?

Yes – eg T3E or SGI Origin

Fetch data as it is required. Easiest programming model.

No – eg PC clusters

Analyse remote data required and pre–fetch.
Much more difficult programming model. Requires additional memory to buffer messages.

Things that make life much easier

one sided communication calls

blocking gets

Bibliography

Berger, M. , *Adaptive Mesh refinement for hyperbolic Partial Differential Equations,* Ph.D thesis.

Quirk, J.J., *An Adaptive Grid Algorithm for Computational Shock Hydrodynamics,* Ph.D thesis

Berger, M.J. and Oliger,J. , 1984, JCP, **53,** 484–512

Software Packages

PARAMESH – F90. Available

CHOMBO – C++ and F77. Available

GrACE – C++ . Available

AMRCLAW – AMR + Godunov teaching codes

OVERTURE/AMR++ – C++ . Available

HAMR ?

SAMRAI – C++. Available