

# EXPLORING COLLECTIVE DYNAMICS IN LARGE-SCALE NETWORKS

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# ***Exploring Collective Dynamics in Large-Scale Networks***

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***What?*** We show some fundamental ideas behind our modeling approaches to study collective dynamics in large-scale networks, and here we focus on the close relationship of network scale and time scale in modeling and simulation.

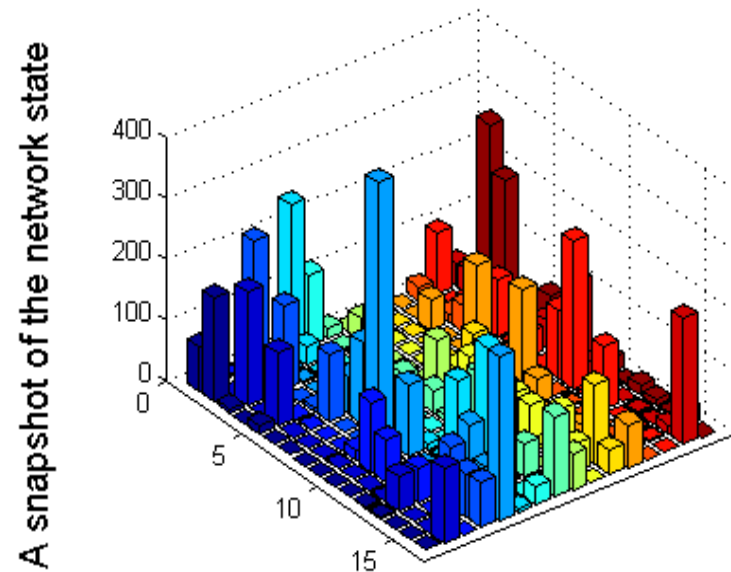
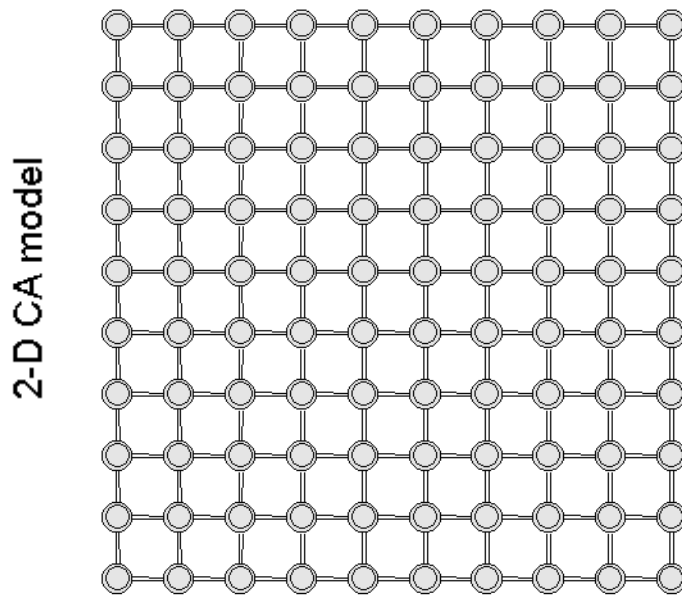
***Why?*** We seek models that can help network researchers and designers to understand the global implications of proposed control mechanisms for large-scale networks.

***Our Twist?*** When viewed as a whole, networks might exhibit emergent properties, which cannot be deduced from properties of individual components. For example, we find that spatial-temporal evolution of network congestion mainly results from complex interactions among many traffic flows exchanged between various source-destination pairs, and routed among many shared nodes. Collective phenomena, which may emerge at large scale, can turn a collection of interactions into an individual, coherent whole. We explore some approaches to understand such global emergence. We expect that identifying differences between small systems and large systems, and even subsystems, in network modeling and simulation will have important engineering values.

# Our Cellular Automata Approach

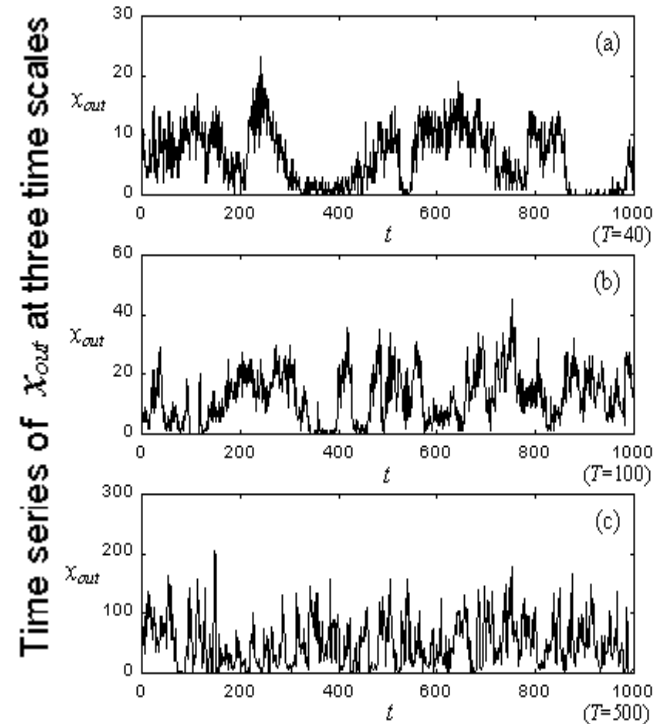
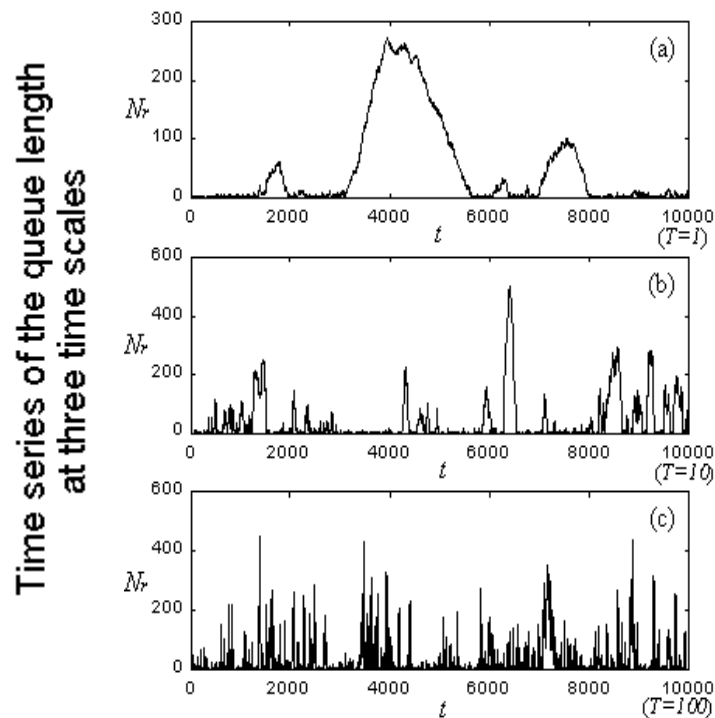
We use a two-dimensional cellular automata (CA) grid to reveal collective behavior from abstract mathematical models of individual network nodes. In the square grid with  $L \times L$  nodes, each cell corresponds to a node with four neighbors. The queue length of each node represents the state of the cell. Such CA models can provide a discrete representation of the continuous behavior of a large-scale network. This provides a natural mechanism to investigate behavior at multiple time scales.

- To mitigate any long-memory in each node due to the input process, we generate traffic by “on/off” sources exponentially distributed with respective parameters  $\lambda_{on}$  and  $\lambda_{off}$ .
- At each time step, every node forwards in parallel a packet to one of its nearest neighbors. Each packet is routed along the shortest path. The “ripple effect” occurs on the packet level because the finer-scale description contains the coarser-scale description.
- TCP is used as a feedback control mechanism, which plays an important role in collective dynamics at a fine time scale.



## Multiple time scales from small to large

To give an intuition about measures of interest at multiple time scales, in the left-hand column below we show three time series of the queue length,  $N_r$ , at three time scales,  $T = 1, 10,$  and  $100$  (time steps) with the same system size  $L = 16$ . In the right-hand column, we show three time series of the aggregate arrival traffic,  $X_{out}$ , which denotes the number of packets consumed by a destination node during  $T$  at three time scales,  $T = 40, 100,$  and  $500$ . We find that as  $T$  increases, the queue length and the consumed traffic change more violently. This suggests that dynamic behaviors at different time scales show different characteristics, which cannot be substituted for each other in simulations.

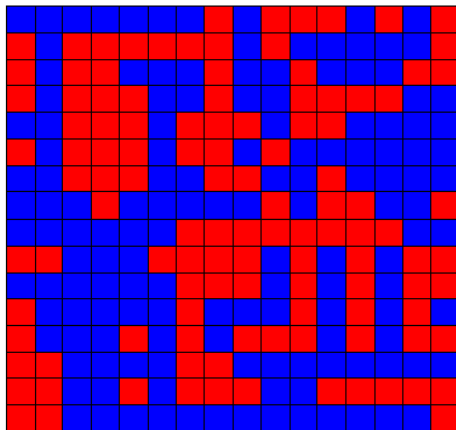


# Understanding the emergence of collective dynamics

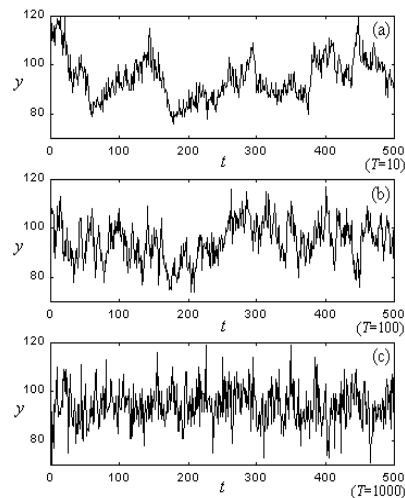
Collective dynamics denotes behavior that parts of a system exhibit together that they would not exhibit individually. We seek to understand the emergence of collective dynamics arising from detailed structure, behavior, and relationships among individual nodes.

- **We start with a disordered system** so that any propagating influence will quickly disperse and eventually be destroyed by random perturbations; thus, distant parts of the network are basically independent. **Correlation gradually increases** over time from feedback control mechanisms that adapt to changes in TCP state in all directions. Eventually, **correlation expands into a global order** that arises as the complex system evolves over time.
- The **emergent phenomena occurs at a global or macro level**, in contrast to the micro-level of the components. Observation of emergence, therefore, relates to behavior on this macro level. We have to ignore some details to “see the forest” conveniently when we describe the correlative pattern in evolving networks.
- To study collective dynamics of the model in both space and time, we develop a method: **map the three dimensional structure of the network state to a binary pattern** where the state of each node is congested or not congested, based on a threshold  $Y$  ( $Y = 5$  in the checkerboard figure below). The middle column below shows the time series of the number of congested nodes ( $y$ ) at three time scales  $T = 10, 100, \text{ and } 1000$ . In the right-hand column appear two time series of  $y$  (with  $T = 100$  for  $Y = 5$  and  $30$ ), and the corresponding power spectra,  $S_y(f)$ , of  $y$ . These power spectra are  $1/f$ -like, with similar slopes, near  $-1.2$ . So **we find that our method is robust w.r.t. the congestion threshold**.

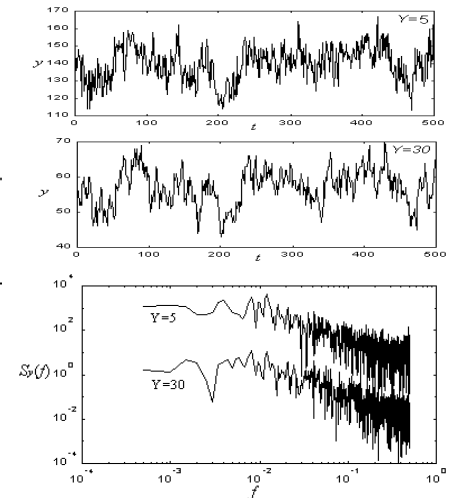
A binary network pattern



Time series of the number of congestion nodes at three time scales



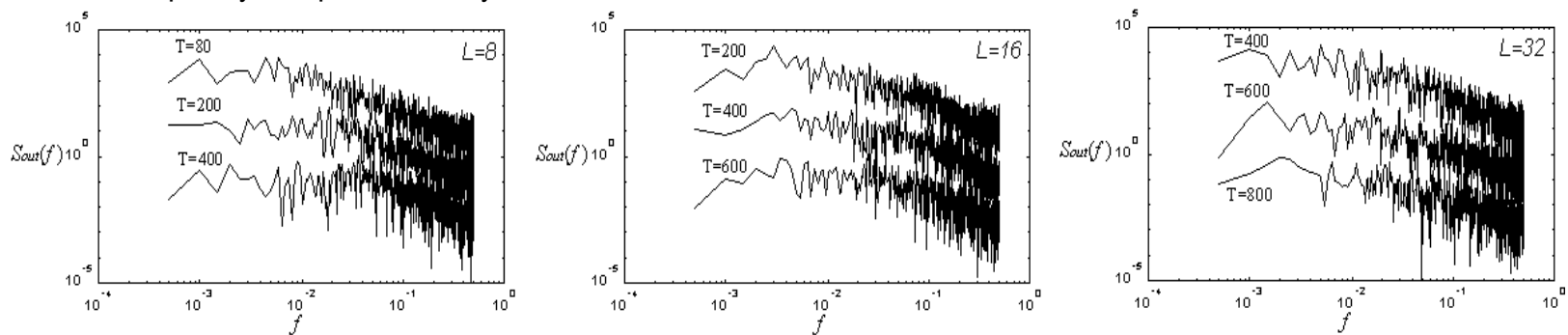
Time series of  $y$  with  $T=100, Y=5, 30$  and their power spectra



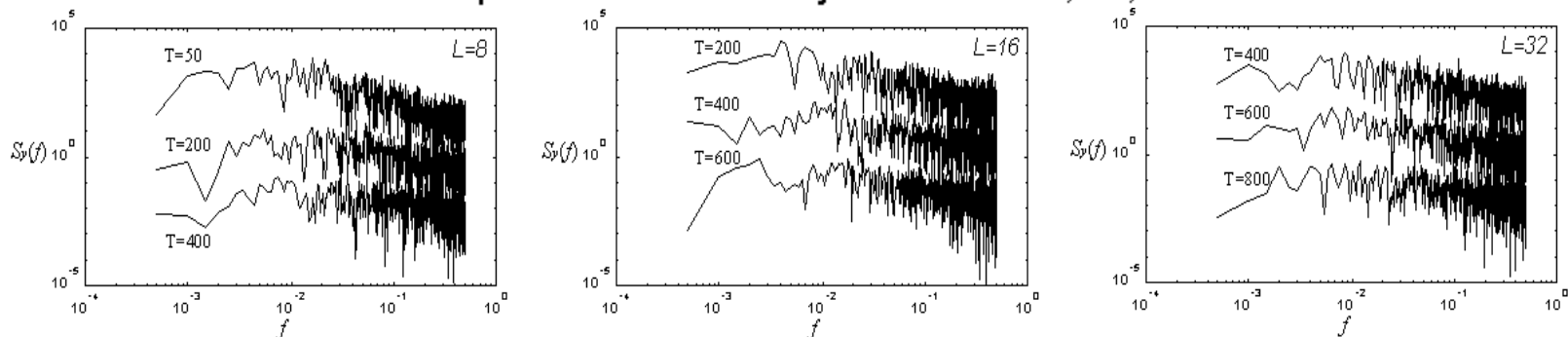
# Multiple network scales from small to large

How do collective dynamics differ between small and large networks? Correlation provides a useful measure to study transition a disordered to ordered state. The top row of three figures below shows the power spectra  $S_{out}(f)$  of  $x_{out}$  with three system sizes,  $L = 8, 16$  and  $32$ . These curves appear as  $1/f$  noise, and have slopes near  $-1.4$ . The bottom row of three figures shows the power spectra of  $y$  with three system sizes,  $L = 8, 16$  and  $32$ . These curves appear as  $1/f$  noise, and have slopes about  $-1.0 \sim -1.2$ .  $1/f$  noise suggests that correlation extends over a wide range of time scales, and provides indication of cooperative effect. Here, such long-range dependence may arise from complex collective interactions in the model.

- Long-range dependence decays as  $T$  increases, but should emerge at larger time scale as system size  $L$  increases.
- This suggests that the multiple timescale characteristics of the traffic patterns might be related to complex interactions.
- The low frequency component decays faster in a small network as  $T$  increases.



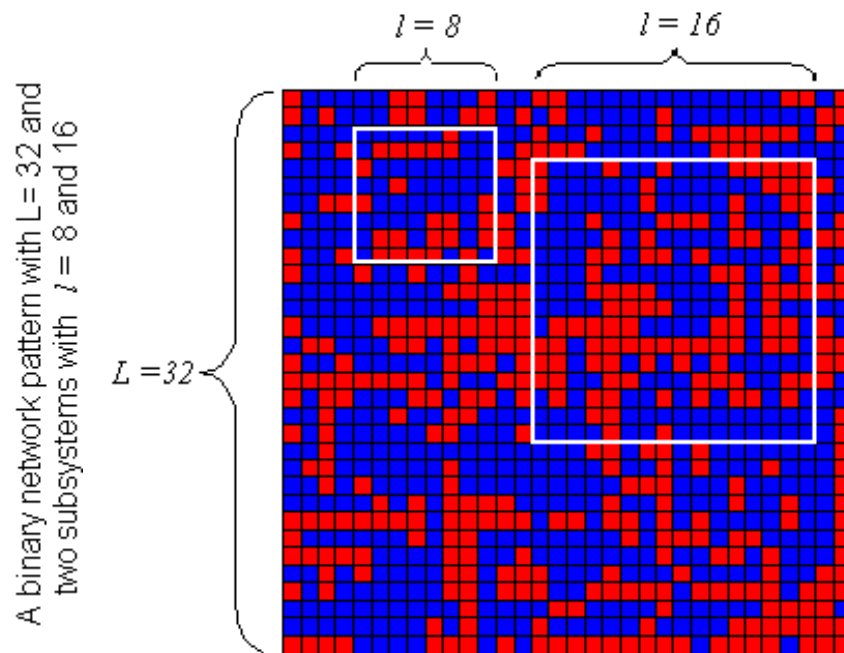
Power spectra of  $x_{out}$  with system size  $L=8, 16,$  and  $32$



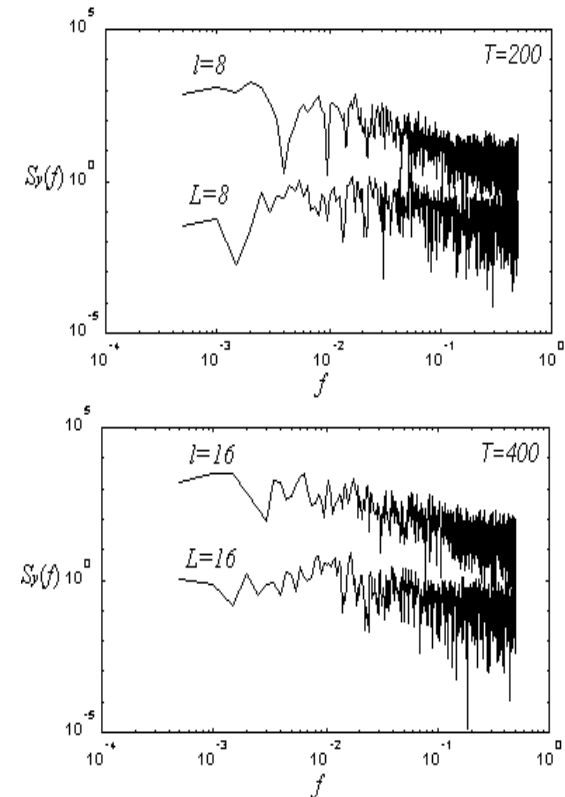
Power spectra of  $y$  with system size  $L=8, 16,$  and  $32$

## Sub-network scales vs. small networks

A sub-area within a larger network may have different features as compared with a small network of the same size. In the checkerboard below, we illustrate a binary network pattern with  $L = 32$  and indicate two sub-areas with system sizes  $l = 8$  and  $16$ . The two graphs below compare power spectra of  $y$ , the number of congested nodes. In the top graph, we compare a network of size  $L = 8$  with a sub-area of size  $l = 8$  in a network of size  $L = 32$ . In the bottom graph, we show similar results for a network of  $L = 16$  compared to a sub-area of size  $l = 16$  in a network of size  $L = 32$ . These results show that sub-areas may more likely keep the  $1/f$  noise feature for the same system size and time scale. Isolating a sub-area completely from the original system when simulating a network, ignores relationships and interdependencies, and may yield inaccurate analyses in some aspects of dynamic behavior in a large-scale network. This means that network scale and time scale are two closely related facets in the collective dynamics of a network.



Power spectra of  $y$  with  $L = 8$  and the subsystem  $l = 8$  of the system  $L = 32$  (up), and  $L = 16$  and  $l = 16$  (down)



## *Expected impact of our work*

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- Improved understanding of global emergence in large-scale networks, which should help to guide the analysis of network measurements
- Help network researchers and designers to understand the global implications of proposed control mechanisms for large-scale networks without ignoring the close relationship between network scale and time scale in modeling and simulation