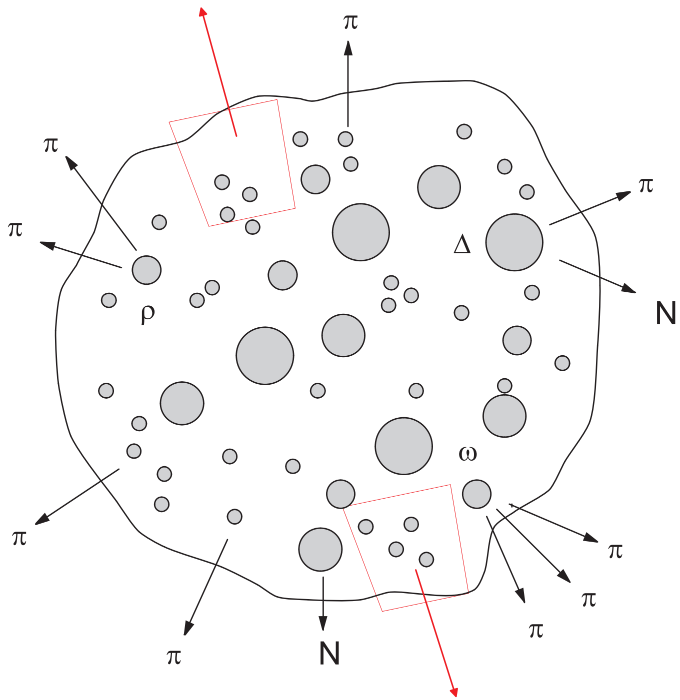
Thermal Model of Particle Production at RHIC

Wojciech Broniowski
H. Niewodniczański Institute of Nuclear Physics
Polish Academy of Sciences, Cracow, Poland

Twenty Years of Collective Expansion
Lawrence Berkeley National Laboratory, 19-27 May 2005

Wojciech Florkowski

Piotr Bożek
Brigitte Hiller
Mariusz Michalec
Anna Baran
Mikołaj Chojnacki



Thermal approach

Koppe (1948), Fermi (1950), Landau, Hagedorn, Rafelski, Letessier, Torrieri, Bjorken, Gorenstein, Gaździcki, Bugaev, Sinyukov, Heinz, Sollfrank, Braun-Munzinger, Stachel, Magestro, Andronic, Turko, Redlich, Prorok, Xu, Kaneta, Csörgő, Csanad, Lörstad, Becattini, Cleymans, Wheaton, ...

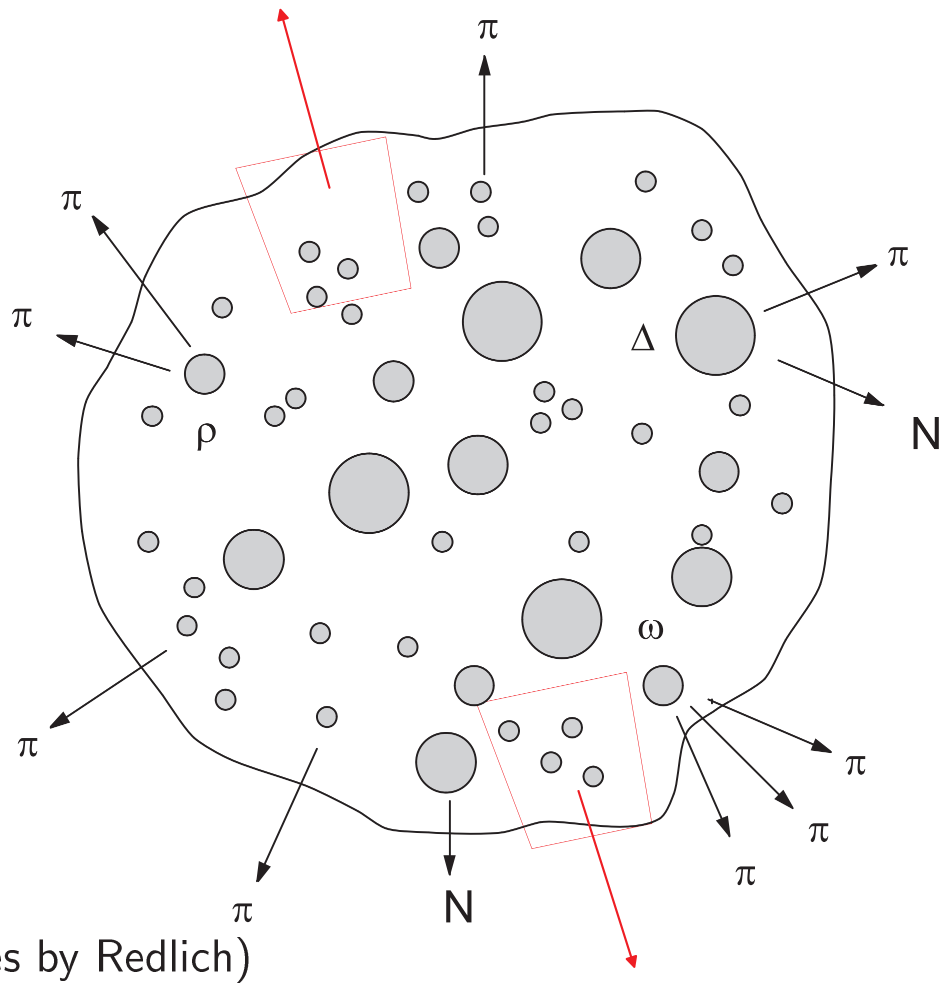
Lectures based on

WB+WF, Phys. Lett. B490 (2000) 223 (Hagedorn spectra)
WB+WF, PRL 87 (2001) 272302 (p_T-spectra of pions, kaons, and protons)
WB+WF, PRC 65 (2002) 064905 (p_T-spectra of strange particles)
WB+WF+ Anna Baran, AIP Conf. Proc. **660** (2003) 185 [nucl-th/0212053] (v_2)
AB+WB+WF, Acta Phys. Polon. B35 (2004) 779 (p_T-spectra at various centralities)
WB+WF+ Brigitte Hiller, PRC 68 (2003) 034911 (pion invariant-mass distributions)
Piotr Bożek +WB+WF, Heavy-Ion Physics (2004) (pion balance functions)
PB, Phys. Lett. B609 (2005) 247 (balance functions in the azimuthal angle)
WF+WB, Acta Phys. Polon. B35 (2004) 2895 (review of expansion models) [references to other people's work inside]

Outline

• Hagedorn hypothesis • Freeze-out and flow • Two new computer packages: `SHARE` and `THERMINATOR` • p_T-spectra at RHIC • Departing from mid-rapidity • Elliptic flow, v_2 • $\pi^+\pi^-$ invariant mass spectra • HBT radii • Balance functions in rapidity • Balance functions in the azimuthal angle • Event-by-event physics

Basics



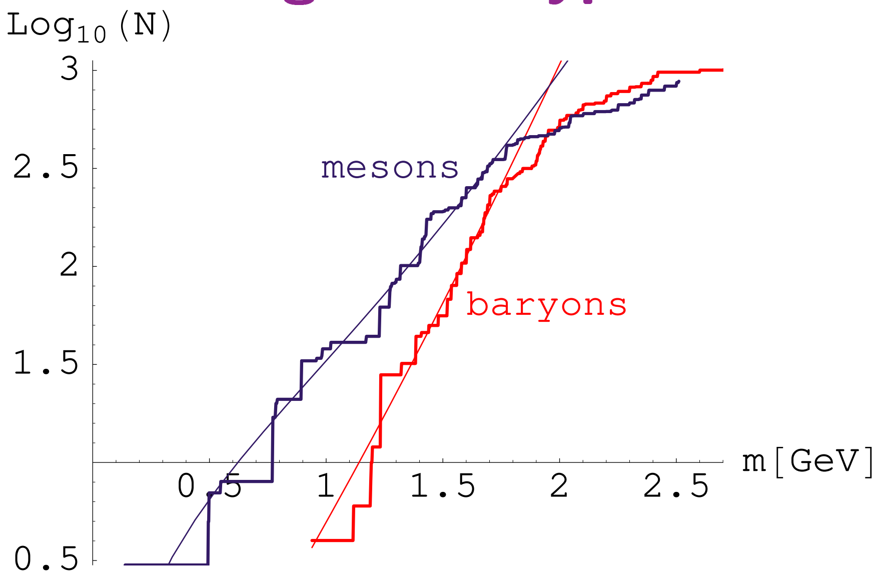
- $\sim e^{-(E-\mu)/T}$
  - freezeout
  - expansion
  - resonances

(see also the lectures by Redlich)

Various models differ in assumptions: statistical factors (γ's), model of freeze-out, model of collective expansion, finite volume corrections, Van der Waals corrections, rescattering

Include resonances!

The Hagedorn hypothesis



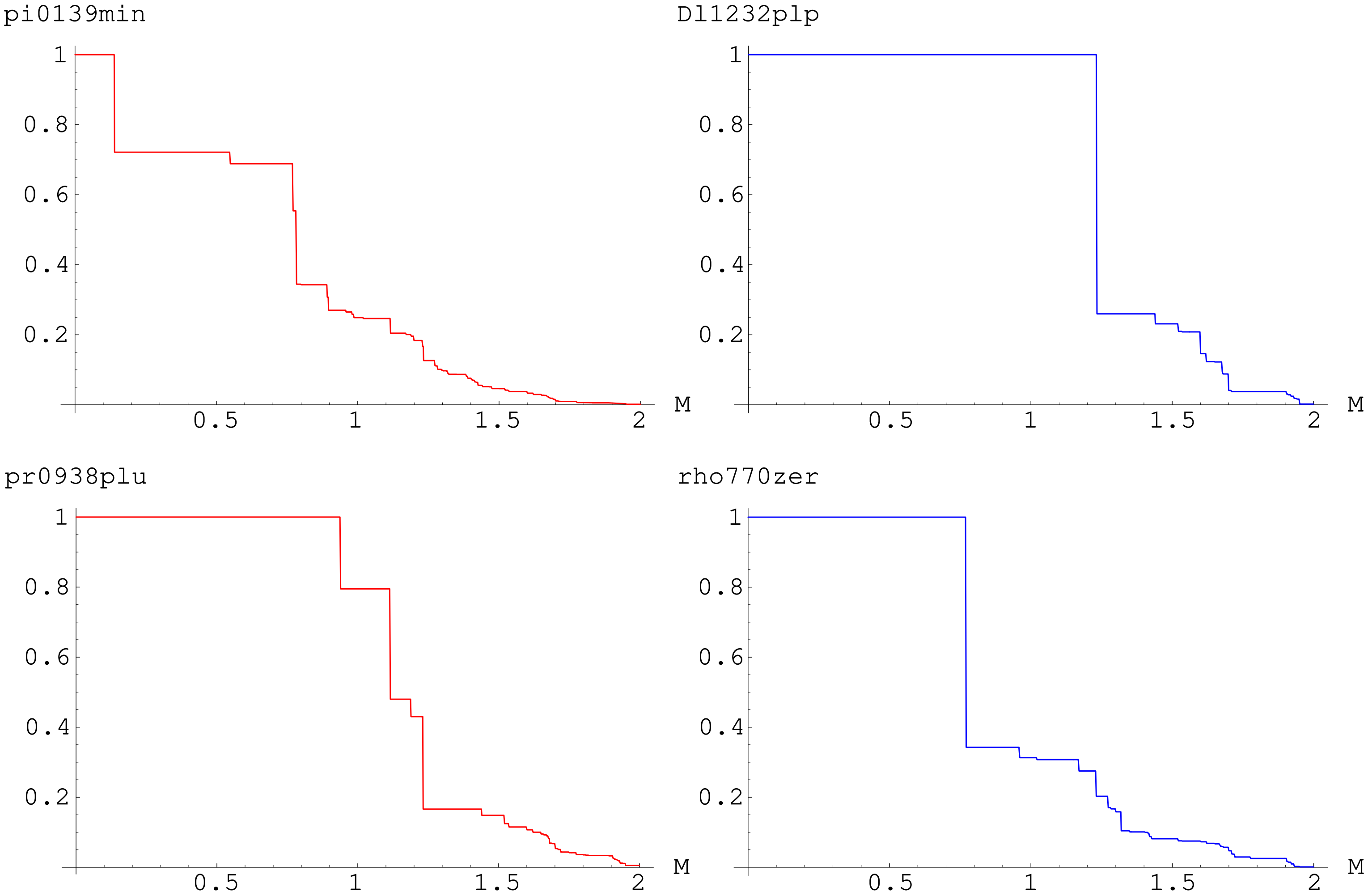
[from WB+WF, PLB 490 (2000) 223,
update in WB+WF+ L. Glozman, PRD70 (2004) 117503]

372 light-flavor (u, d, s) particles, ~1500 DOF, ~1800 decay channels!
Complete treatment of resonances important due to the exponential growth of N

Remarks

- Different growth rate for mesons and baryons
- Independence of flavor
- T_H depends on the form of f the formula $dN/dM \sim f(M) \exp(M/T_H)$. One cannot quote a single number for T_H
- Accuracy of a few % requires inclusion of states at least up to 1.7 GeV
- Inclusion of resonances is a way of describing interactions between stable hadrons
- More resonances $\rightarrow$ lower temperature in thermal fits

Resonance feeding of π^-,Δ^{++}, p, and ρ_0



Plots made for $T = 165$ MeV, $\mu_B = 28$ MeV with equilibrium distributions [MathSHARE]

Approximately 75% of pions come from decays of higher states, 80% of protons and Λ's, 60% of Ξ's, 30% of Δ's and ρ_0's, . . . , 3% of ϕ, 0% of Ω's

Particle properties and decay channels

```
# Name       mass   width    spin     I       I3     q      s      aq     as       c       ac       MC#
...
Ns2600plu    2.60   0.65     5.5     0.5     0.5    3.     0.     0.     0.       0.      0.       9401
Ns2600zer    2.60   0.65     5.5     0.5    -0.5    3.     0.     0.     0.       0.      0.       9400
Ns2600plb    2.60   0.65     5.5     0.5    -0.5    0.     0.     3.     0.       0.      0.      -9401
Ns2600zrb    2.60   0.65     5.5     0.5     0.5    0.     0.     3.     0.       0.      0.      -9400
Dl2420plp    2.42   0.40     5.5     1.5     1.5    3.     0.     0.     0.       0.      0.       9297
...
Lm2100zer   Ka0492zrb   ne0939zer   0.2    1
Lm2100zer   Ka0492min   pr0938plu   0.2    1
Lm2100zer   Ka0892zrb   ne0939zer   0.5    1
Lm2100zer   Ka0892min   pr0938plu   0.5    1
Lm2100zer   pi0139plu   Sg1189min   0.3    1
Lm2100zer   pi0139min   Sg1189plu   0.3    1
Lm2100zer   pi0135zer   Sg1192zer   0.3    1
Lm2100zrb   Ka0492zer   ne0939zrb   0.2    1
Lm2100zrb   Ka0492plu   pr0938plb   0.2    1
Lm2100zrb   Ka0892zer   ne0939zrb   0.5    1
Lm2100zrb   Ka0892plu   pr0938plb   0.5    1
Lm2100zrb   pi0139min   Sg1189mnb   0.3    1
Lm2100zrb   pi0139plu   Sg1189plb   0.3    1
Lm2100zrb   pi0135zer   Sg1192zrb   0.3    1
a42040plu   Ka0492plu   Ka0492zrb   0.3333   0
a42040plu   pi0139plu   pi0139plu   pi0139min   0.3333   0
a42040plu   pi0139plu   eta547zer   0.33     0
a42040min   Ka0492min   Ka0492zer   0.3333   0
a42040min   pi0139min   pi0139plu   pi0139min   0.3333   0
a42040min   pi0139min   eta547zer   0.33     0
a42040zer   Ka0492plu   Ka0492min   0.1667   0
a42040zer   Ka0492zer   Ka0492zrb   0.1667   0
a42040zer   pi0135zer   pi0139plu   pi0139min   0.3333   0
a42040zer   pi0135zer   eta547zer   0.3333   0
...
```

Appeal to the Particle Data Group

Please, provide the data electronically!

SHARE

We have done it for you! The Fortran and Mathematica MathSHARE packages for the analyses of the ratios of particle abundances in thermal models

G. Torrieri, S. Steinke, W. Broniowski, W. Florkowski, J. Letessier and J. Rafelski, SHARE: Statistical HAdronization with REsonances, nucl-th/0404083.

http://www.ifj.edu.pl/Dept4/share.html
or
http://www.physics.arizona.edu/~torrieri/SHARE/share.html

A similar effort by S. Wheaton and J. Cleymans, THERMUS: *A thermal model package for ROOT*, hep-ph/0407174.

See the Thermal Web Calculator by Steve Steinke!

- Convenient and fast
- Watch out for the weak decay policies!

Ratios

[see the very detailed presentation by Andronic]

Comment:
For a boost-invariant model

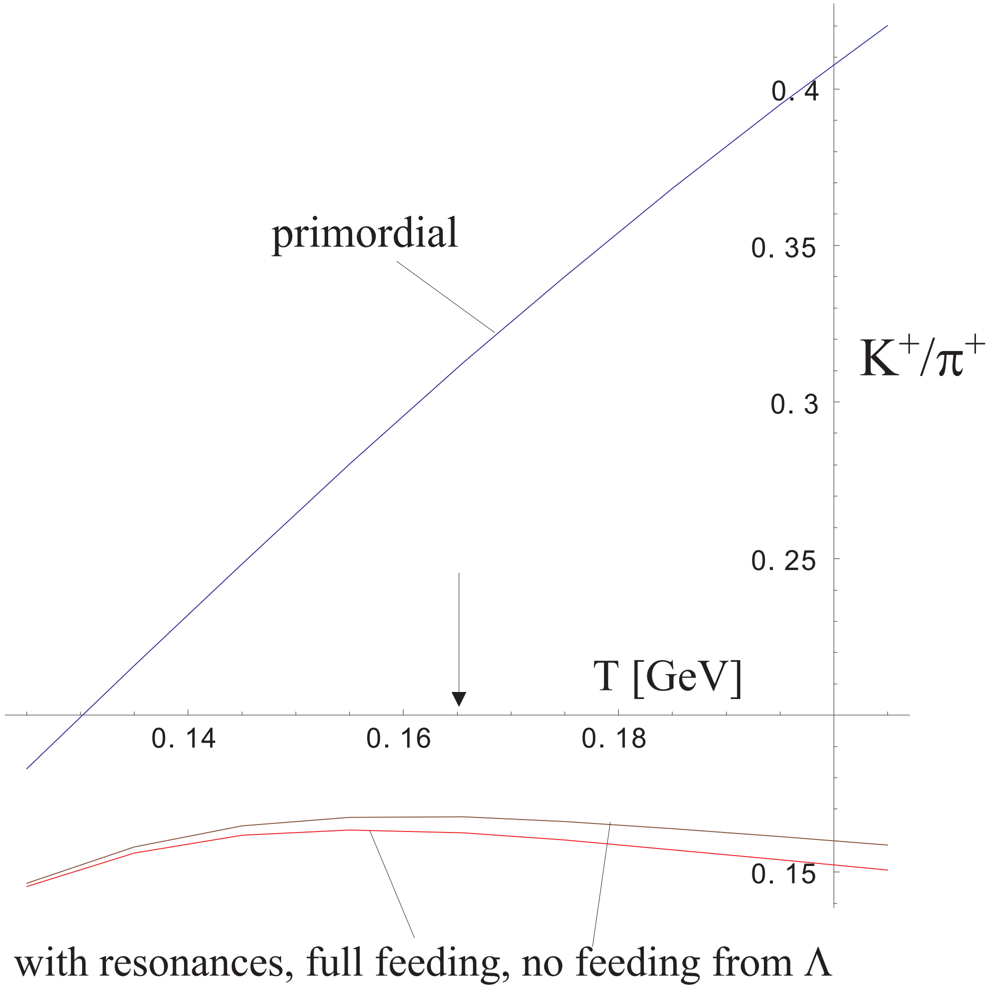
$$\frac{dN_i/dy}{dN_j/dy} = \frac{N_i}{N_j}$$

and the ratios do not depend on geometry/flow. This is a good approximation for RHIC, where the rapidity spectra are flat (up to a few %) for $-1 < y < 1$. **This is not a very good approximation** for lower collision energies, where a more detailed modelling, including the dependence of parameters on rapidity, would be appropriate. In that case the ratios become dependent of freeze-out hypersurface and expansion!

The values of thermal parameters used in these lectures:

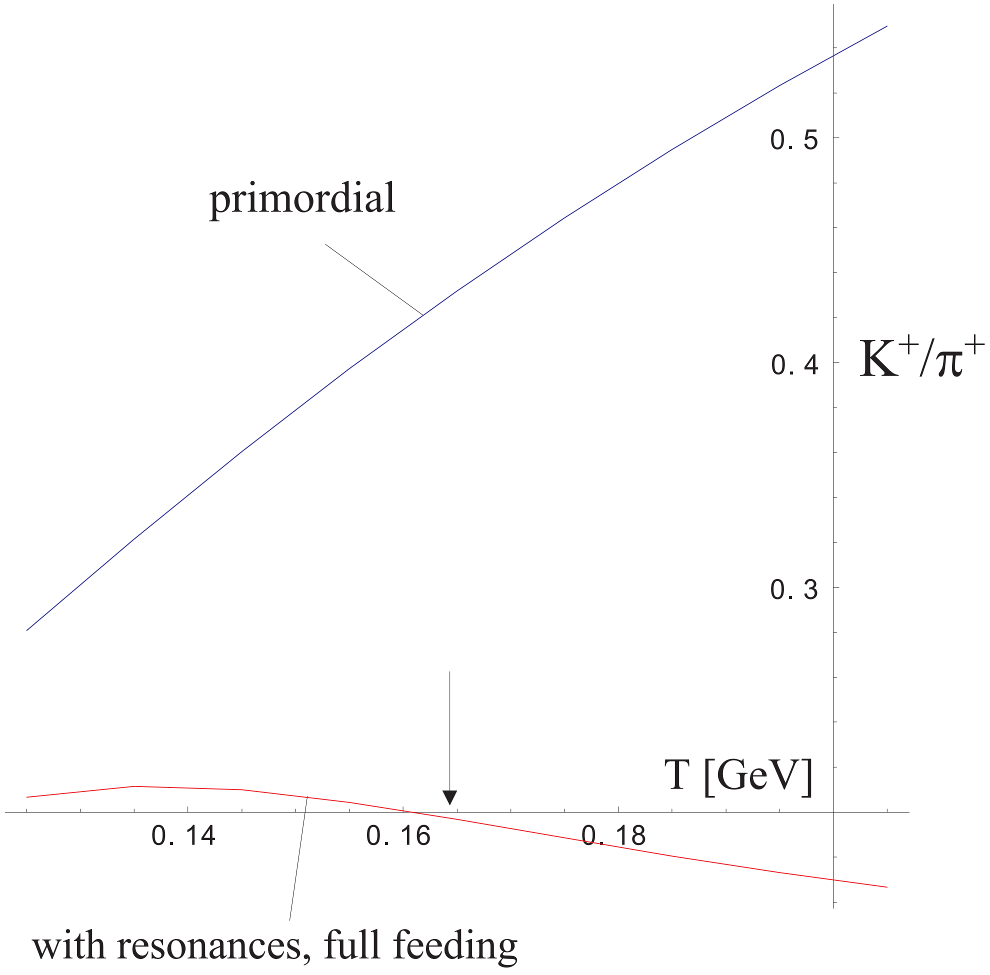
$\sqrt{s_{NN}}$ [GeV]	Pb+Pb @ 17	Au+Au @ 130	Au+Au @ 200
T [MeV]	164 ± 3	165 ± 7	165.6 ± 4.5
μ_B [MeV]	234 ± 7	41 ± 5	28.5 ± 3.7
μ_S [MeV]	56	9	7
μ_I [MeV]	-8	-1	-1
χ^2/DOF	0.9	1.0	0.2

K^+/π^+ at mid-rapidity for RHIC @ 200



The ratio of primordial abundances is large and growing with T, while the ratio of K^+/π^+ remains remarkably flat, as the feeding to π^+ grows faster than the feeding to K^+. The experimental number is 0.156 ± 0.020 at $c = 0 - 5\%$. The calculation includes the feeding of K_S to pions.

K^+/π^+ at mid-rapidity for SPS @ 17



The ratios at RHIC are remarkably well reproduced by the thermal model

Fine print:

- Watch out how the feeding from the weak decays is included
- "dominant", "seen", mass of σ
- In order to obtain the yield, the experimental results typically include extrapolations
- At lower energies the boost-invariance near $y = 0$ is not a good approximation and a more elaborate modelling, including the rapidity dependence of the thermal parameters, is necessary

One should not demand a too high accuracy!

Final comments:

- Effects of widths are completely negligible
- Dropping masses lead to lowering of T

see the PhD thesis of M. Michalec, nucl-th/0112044

Expansion

Only the ratios of dN/dy are independent of the freeze-out geometry and expansion. If one wants to go further, modelling of the geometry and flow is necessary.

Impossible to heat up without subsequent expansion!

The system expands and at some point it freezes (this is a far reaching simplification: may occur at different times for different processes, such as chemical and thermal freeze-outs, may be washed-out – will come back)

The following part follows WF+WB, Acta. Phys. Polon. B35 (2004) 2895

- Basics
- The blast model of Siemens and Rasmussen
- The model of Schnedermann, Sollfrank, and Heinz
- Our choice

The Cooper-Frye formalism

$$E\,\frac{dN}{d^3p} = \frac{dN}{dy\,d^2p_\perp} = \int d^3\Sigma_\mu(x) p^\mu f(x,p)$$

$$d^3\Sigma_\mu = \varepsilon_{\mu\alpha\beta\gamma}\frac{dx^\alpha}{d\alpha}\frac{dx^\beta}{d\beta}\frac{dx^\gamma}{d\gamma}d\alpha d\beta d\gamma,$$

where α, β, γ are the three independent coordinates introduced to parameterize the hypersurface. For systems in local thermodynamic equilibrium we have

$$E\,\frac{dN}{d^3p} = \int d^3\Sigma_\mu(x)\, p^\mu f\,(u_\mu(x)\, p^\mu)\,,$$

where f is the equilibrium distribution function. For a static fireball

$$d^3\Sigma_\mu = (dV, 0, 0, 0), \quad u_\mu = (1, 0, 0, 0),$$

and

$$\frac{dN}{d^3p} = V f(E)$$

Spherically symmetric freeze-out

For a spherically symmetric case

$$x^\mu = (t, x, y, z) = (t(\zeta), r(\zeta)\sin\theta\cos\phi, r(\zeta)\sin\theta\sin\phi, r(\zeta)\cos\theta)$$

and the freeze-out hypersurface is completely defined by the mapping $\zeta \longrightarrow (t(\zeta), r(\zeta))$ in the $t - r$ space. The range of ζ may be restricted to $0 \le \zeta \le 1$. Then

$$d^3\Sigma^\mu = \big(r'(\zeta), t'(\zeta)\sin\theta\cos\phi, t'(\zeta)\sin\theta\sin\phi, t'(\zeta)\cos\theta\big)\, r^2(\zeta)\sin\theta\, d\theta\, d\phi\, d\zeta$$

where $'$ denotes $d/d\zeta$. We also introduce the spherically symmetric flow

$$u^\mu = (1 - v^2(\zeta))^{-1/2}\,(1, v(\zeta)\sin\theta\cos\phi, v(\zeta)\sin\theta\sin\phi, v(\zeta)\cos\theta)$$

Since the four-momentum of a hadron is parameterized as

$$p^\mu = [E, p\sin\theta_p\cos\phi_p, p\sin\theta_p\sin\phi_p, p\cos\theta_p]\,,$$

we find (the sphericall symetry allows to choose $\theta_p = 0$)

$$p\cdot u = (E - pv(\zeta)\cos\theta)\;\gamma(\zeta),$$

$$d^3\Sigma\cdot p = \big(Er'(\zeta) - pt'(\zeta)\cos\theta\big)\, r^2(\zeta)\sin\theta\, d\theta\, d\phi\, d\zeta.$$

In the case of the Boltzmann statistics we carry the angular integration and obtain

$$E\frac{dN}{d^3p} = \int_0^1 \frac{e^{-(E\gamma-\mu)/T}}{2\pi^2}\left[E\,\frac{\sinh a}{a}\,\frac{dr}{d\zeta} + T\,\frac{(\sinh a - a\cosh a)}{a\gamma v}\frac{dt}{d\zeta}\right] r^2(\zeta)d\zeta$$

Here v, $\gamma = (1-v^2)^{-1/2}$, r and t are functions of ζ, and $a = \frac{\gamma v p}{T}$. The thermodynamic parameters T and μ may also depend on ζ. To proceed further we need to make certain assumptions about the ζ-dependence of these quantities.

Motivated by the work of Bondorf, Garpman, Zimanyi (1978), Siemens and Rasmussen (1979) proposed the blast-wave model. They assumed that the thermodynamic parameters as well as the transverse flow velocity are constant

$$T = \text{const}, \quad \mu = \text{const}, \quad v = \text{const} \quad (\gamma = \text{const}, \quad a = \text{const}).$$

Moreover, they assumed that the freeze-out curve in the $t-r$ space satisfies the condition

$$dt = v\,dr, \quad t = t_0 + vr$$

In this case we obtain the formula

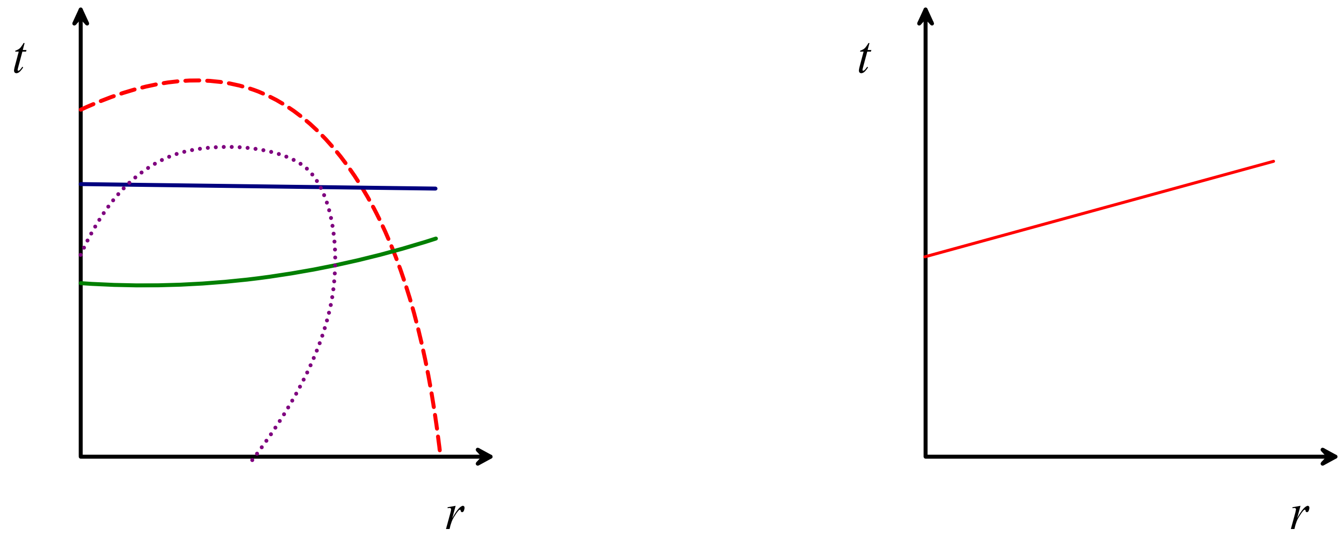
$$\frac{dN}{d^3p} = \frac{e^{-(E\gamma-\mu)/T}}{2\pi^2}\left[\left(1+\frac{T}{\gamma E}\right)\frac{\sinh a}{a} - \frac{T}{\gamma E}\cosh a\right]\int_0^1 r^2(\zeta)\frac{dr}{d\zeta}d\zeta.$$

This coincides with the original Siemens-Rasmussen formula

$$\frac{dN}{d^3p} = Z \exp\left(-\frac{\gamma E}{T}\right) \left[\left(1 + \frac{T}{\gamma E}\right) \frac{\sinh a}{a} - \frac{T}{\gamma E} \cosh a\right]$$

if we identify

$$Z = \frac{V}{(2\pi)^3} \exp\left(\frac{\mu}{T}\right), \quad \int r^2(\zeta) \frac{dr}{d\zeta} d\zeta = \frac{r_{\max}^3}{3}, \quad V = \frac{4}{3}\pi r_{\max}^3$$



Left: A priori possible different freeze-out curves in the $t - r$ space. The dotted and dashed lines describe the cases where both the space-like and time-like parts are present. The solid lines describe the cases where only the time-like part is present. Right: The (time-like) freeze-out curve assumed in the blast-wave model of Siemens and Rasmussen

Boost-inv. BW of Schnedermann, Sollfrank and Heinz

$$x^\mu = (t, x, y, z) = \left(\tilde{\tau}(\zeta)\text{cosh } \alpha_\parallel, \rho(\zeta)\cos\phi, \rho(\zeta)\sin\phi, \tilde{\tau}(\zeta)\text{sinh } \alpha_\parallel\right) .$$

The surface is defined with $\zeta \to (\tilde{\tau}(\zeta), \rho(\zeta))$, which determines the freeze-out times of the cylindrical shells with the radius ρ. Because of boost-invariance it is enough to define this curve at $z = 0$, since for finite values of z the freeze-out points may be obtained by the Lorentz transformation. The boost-invariant four-velocity field can be parameterized as[1]

$$u^\mu = \left(ch\alpha_\perp(\zeta)ch\alpha_\parallel, sh\alpha_\perp(\zeta)\cos\phi, sh\alpha_\perp(\zeta)\sin\phi, ch\alpha_\perp(\zeta)ch\alpha_\parallel\right) .$$

We note that the longitudinal flow is simply $v_z = \text{tanh}\,\alpha_\parallel = z/t$ (as in the one-dim. Bjorken model), whereas the transverse flow is $v_r = \text{tanh}\,\alpha_\perp(\zeta)$.

For the Boltzmann statistics ($\beta = 1/T$) one gets

$$\begin{aligned}\frac{dN}{dyd^2p_\perp} &= \frac{e^{\beta\mu}}{(2\pi)^3}\int_0^{2\pi} d\phi \int_{-\infty}^{\infty} d\alpha_\parallel \int_0^1 d\zeta\ \rho(\zeta)\tilde{\tau}(\zeta)\left[m_\perp\text{cosh}(\alpha_\parallel - y)\frac{d\rho}{d\zeta} - p_\perp\cos(\phi-\varphi)\frac{d\tilde{\tau}}{d\zeta}\right] \\ &\times \quad \exp\left[-\beta m_\perp\text{cosh}(\alpha_\perp)\text{cosh}(\alpha_\parallel - y) + \beta p_\perp\text{sinh}(\alpha_\perp)\cos(\phi-\varphi)\right]\end{aligned}$$

The distribution is independent of y and φ in accordance with the boost-invariance and cylindrical symmetry.

[1] Watch out for a typo in Eq. (24) of our paper

The integrals over $\alpha_{\|}$ and ϕ are analytic and lead to the Bessel functions K and I,

$$
\begin{aligned}
\frac{dN}{dy d^2 p_\perp} = \; & \frac{e^{\beta\mu}}{2\pi^2} m_\perp K_1 \left[\beta m_\perp \cosh(\alpha_\perp)\right] I_0 \left[\beta p_\perp \sinh(\alpha_\perp)\right] \int_0^1 d\zeta \;\; \rho(\zeta)\tilde{\tau}(\zeta) \frac{d\rho}{d\zeta} \\
& - \frac{e^{\beta\mu}}{2\pi^2} p_\perp K_0 \left[\beta m_\perp \cosh(\alpha_\perp)\right] I_1 \left[\beta p_\perp \sinh(\alpha_\perp)\right] \int_0^1 d\zeta \;\; \rho(\zeta)\tilde{\tau}(\zeta) \frac{d\tilde{\tau}}{d\zeta}.
\end{aligned}
$$

In the spirit of the BW model of Siemens and Rasmussen it is assumed here that the radial velocity is constant, $v_r = \tanh \alpha_\perp(\zeta) = \text{const}$. In order to achieve a simpler form, the common practice is to neglect the second line. This means that one assumes the condition $d\tilde{\tau}/d\zeta = 0$. Then

$$
\frac{dN}{dy d^2 p_\perp} = \text{const}\, m_\perp K_1 \left[\beta m_\perp \cosh(\alpha_\perp)\right] I_0 \left[\beta p_\perp \sinh(\alpha_\perp)\right]
$$

This equation forms the basis of numerous phenomenological analyses of the transverse-momentum spectra measured at the SPS and RHIC energies.

Lessons

• 26 years after: It is profitable to use a suitable parameterization of the freeze-out hypersurface and the flow velocity. Parameters are chosen to reproduce the data
• Eventually, the choice should be supported by the underlying theory (hydrodynamics)

Our freeze-out

[similar to the Buda-Lund parametrization]

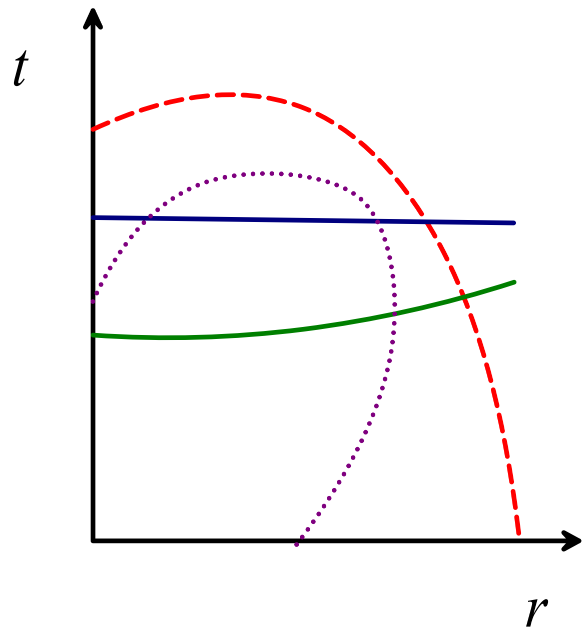
$$t = \tau \cosh \alpha_{\parallel} \cosh \alpha_{\perp}, \quad z = \tau \sinh \alpha_{\parallel} \cosh \alpha_{\perp},$$
$$x = \tau \sinh \alpha_{\perp} \cos \phi, \quad y = \tau \sinh \alpha_{\perp} \sin \phi$$

$$\tau = \sqrt{t^2 - r_x^2 - r_y^2 - r_z^2} = \text{const.}$$

The transverse size of the system is defined by $\rho_{\max}$
$\rho = \sqrt{r_x^2 + r_y^2}, \quad \rho < \rho_{\max}$
and the velocity field at freeze-out has the Hubble form

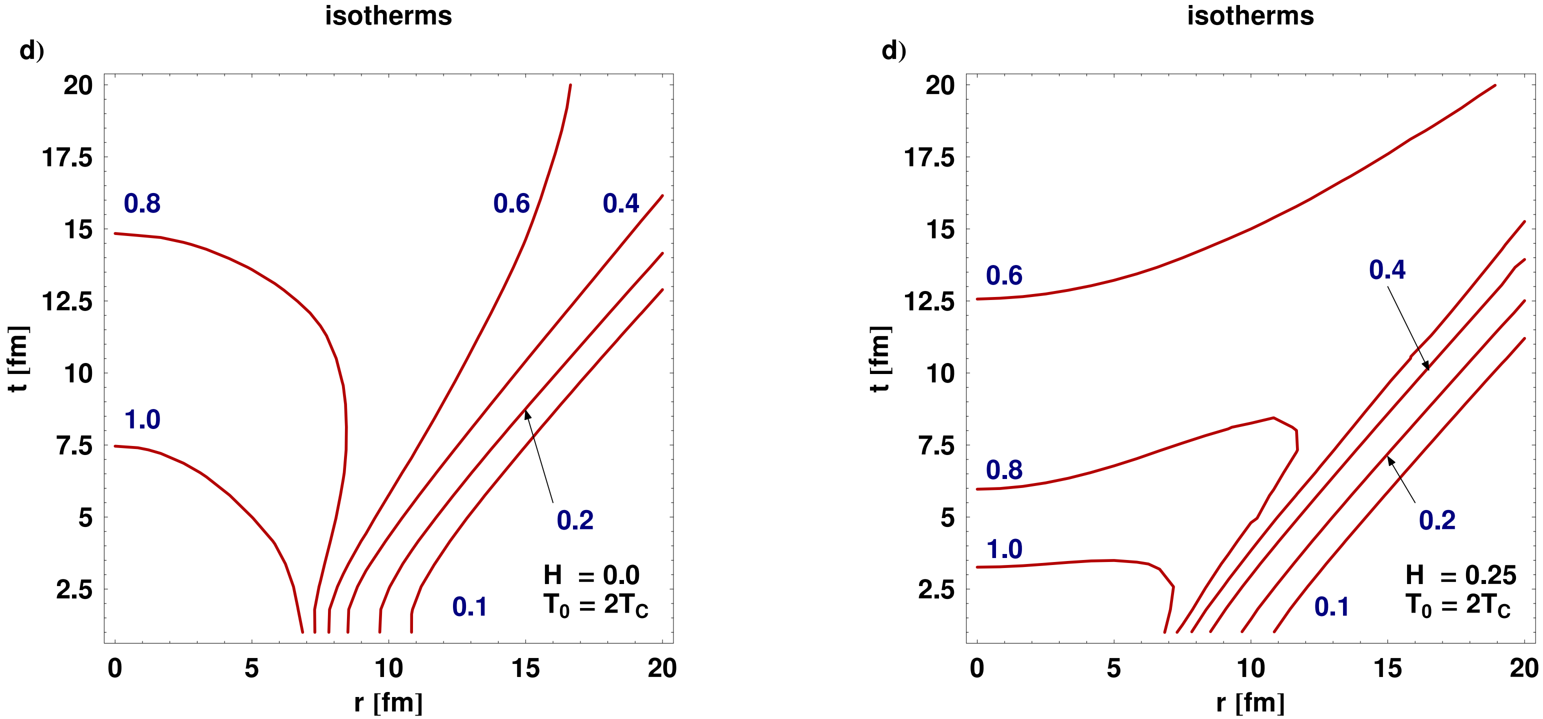
$$u^\mu = \frac{x^\mu}{\tau} = \frac{t}{\tau}\left(1, \frac{x}{t}, \frac{y}{t}, \frac{z}{t}\right)$$

blue - boost-inv. blast wave , green - our model



Hints from hydrodynamics

[M. Chojnacki +WF+ T. Csörgő, ...]
Cylindrically symmetric hydro with initial flow of the form $v(r) = Hr/\sqrt{1+H^2r^2}$

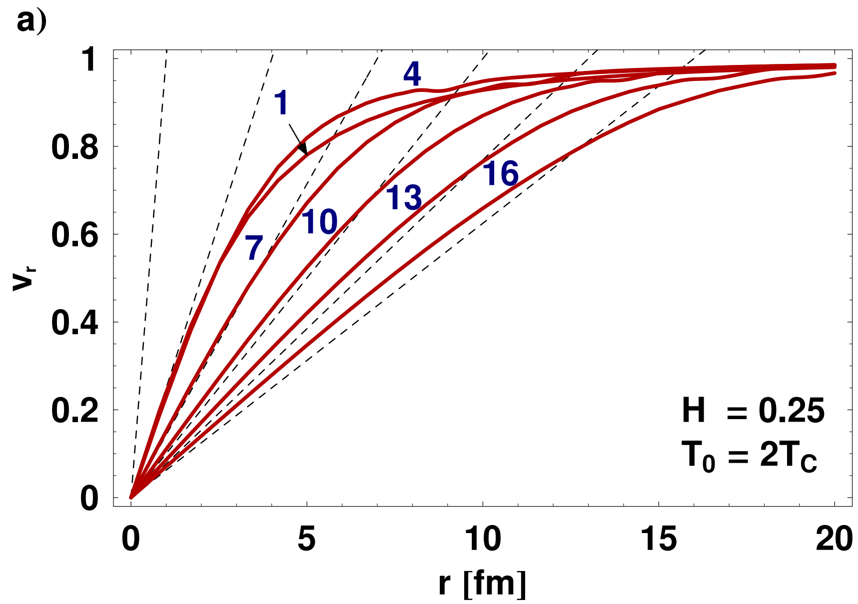


Left: no flow, right: $H = 0.25$ fm^{-1}. Blue labels indicate T/T_c

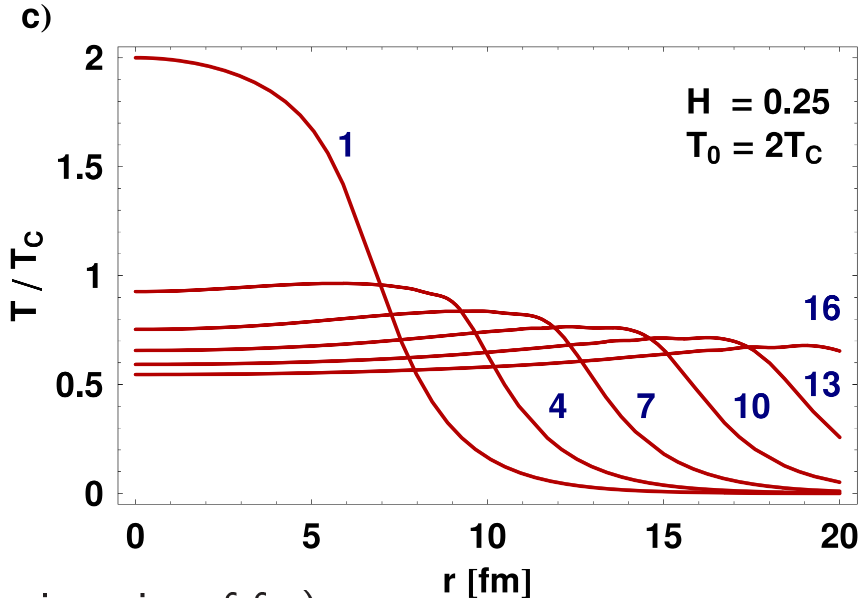
A large chunk of a *scaling solution* is found at quite large T. The scaling solution has precisely the features implemented in our expansion model: τ =const. and the Hubble flow.

Some more details from this work:

The Hubble flow



The dependence of T of the radius r.



(The blue labels indicate time in units of fm)

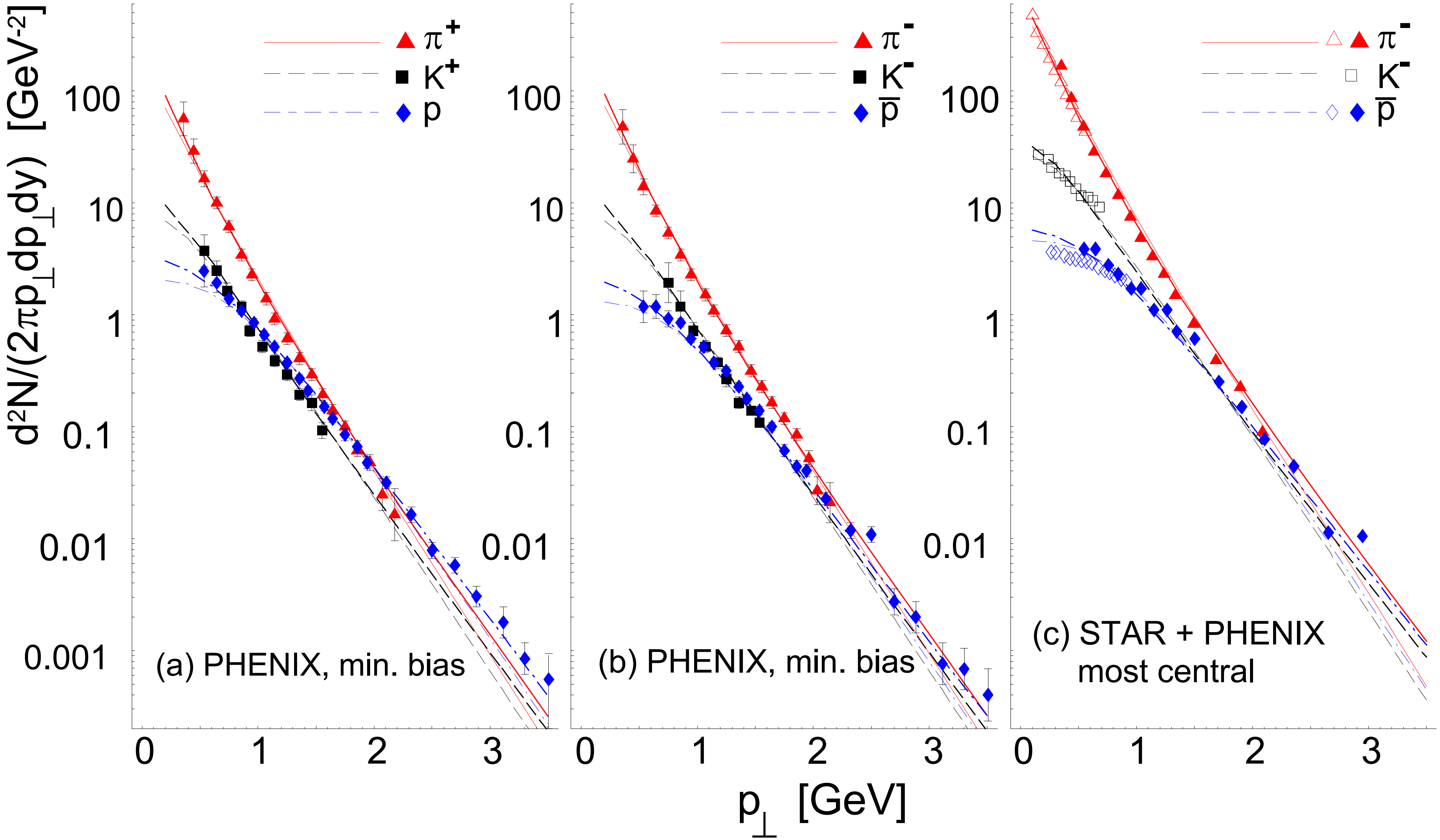
Single freeze-out model

1. Grand canonical ensamble, equlibrium distrubutions
2. Single freeze-out approximation: $T_{\rm chem} = T_{\rm kin} \equiv T$
3. Complete treatment of resonances in *all* analyses
4. The freeze-out hypersurface assumed in the form $\tau = \sqrt{t^2 - x^2 - y^2 - z^2} =$ const, with $x^2 + y^2 < \rho^2_{\rm max}$.
5. Hubble-like flow, $u^\mu = \frac{x^\mu}{\tau} = \frac{t}{\tau}(1, \frac{x}{t}, \frac{y}{t}, \frac{z}{t})$
6. Only **4** parameters: T,μ_B (fixed earlier by the ratios of the particle abundances), invariant time at freeze-out τ (controls the overall normalization), and the transverse size $\rho_{\rm max}$ ($\rho_{\rm max}/\tau$ controls the slopes of the $p_\perp$ spectra). The 2 geometry/flow parameters are fitted **globally** to the spectra of $\pi^\pm$,$K^\pm$, p, and $\bar{p}$ and then are kept constant for a given centrality (and experimental group!)
7. $\rho_{\rm max}$ also controls the flow, with the typical value $\langle \beta_\perp \rangle \sim 0.5$

Discussion of **2.**

This is an approximation. With the present software we can count the average number of collisions experienced by particles. For the pion from mid-rapidity this number is ~ 2, which is not negligible, but not devastating! The collision rate is inhibited by expansion and surface effects. Furthermore, the system may be diluted by the excluded-volume corrections, which does not change the ratios nor p_T-spectra [WB+WF, Hirschegg 2002, p.146]. With this dilution the number of collisions per particle becomes ~ 1.

The first shot

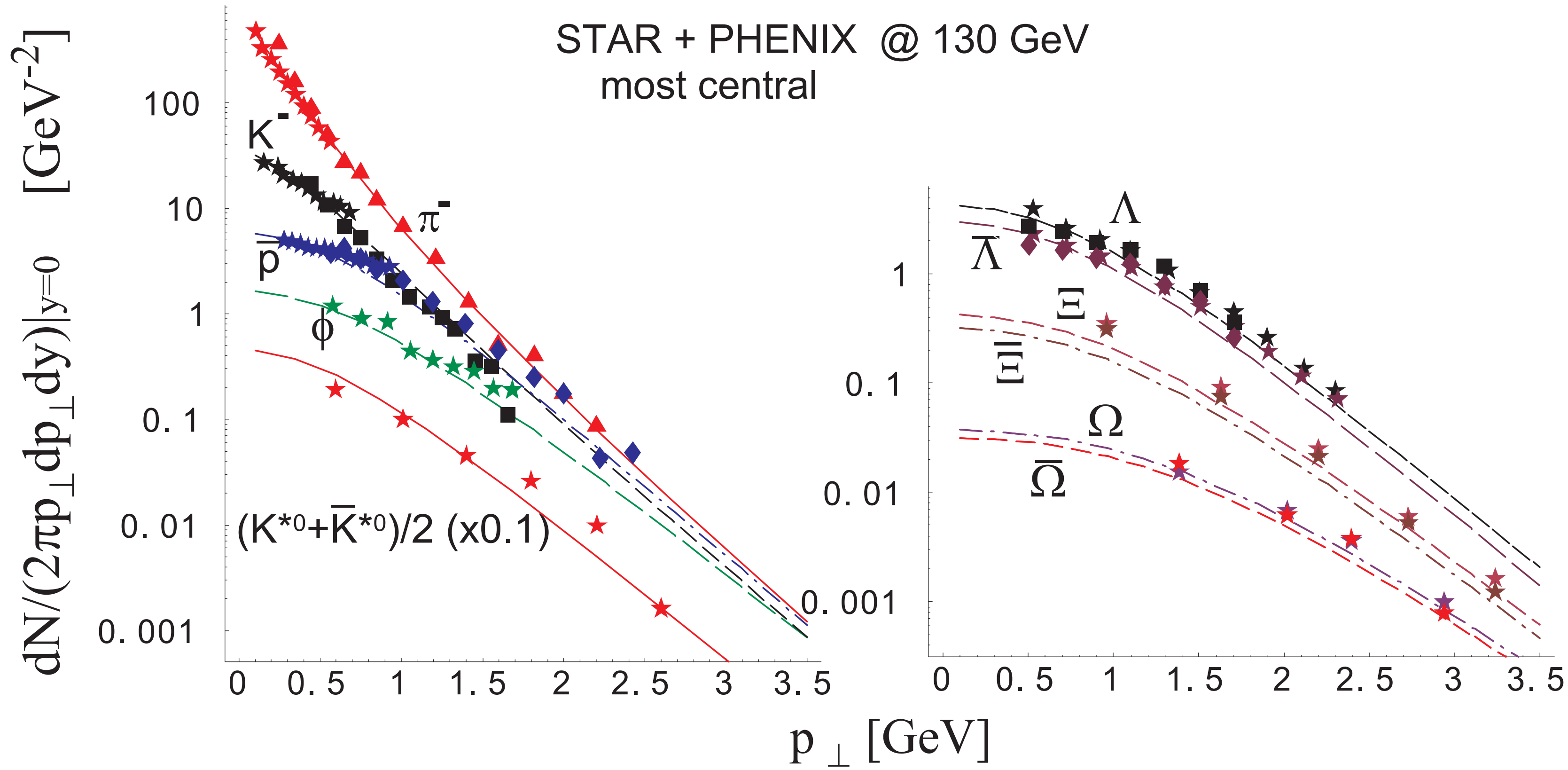


[Data: J. Velkovska, PHENIX, NPA **698** (2002) 507c, J. Harris, STAR, ibid. 64c]

Thicker lines: our model, thin lines: blast-wave + resonances

The geometry/flow model "bends" the curves

Strange sector



Prediction for Ξ and Ω, PRC 65 (2002) 064905

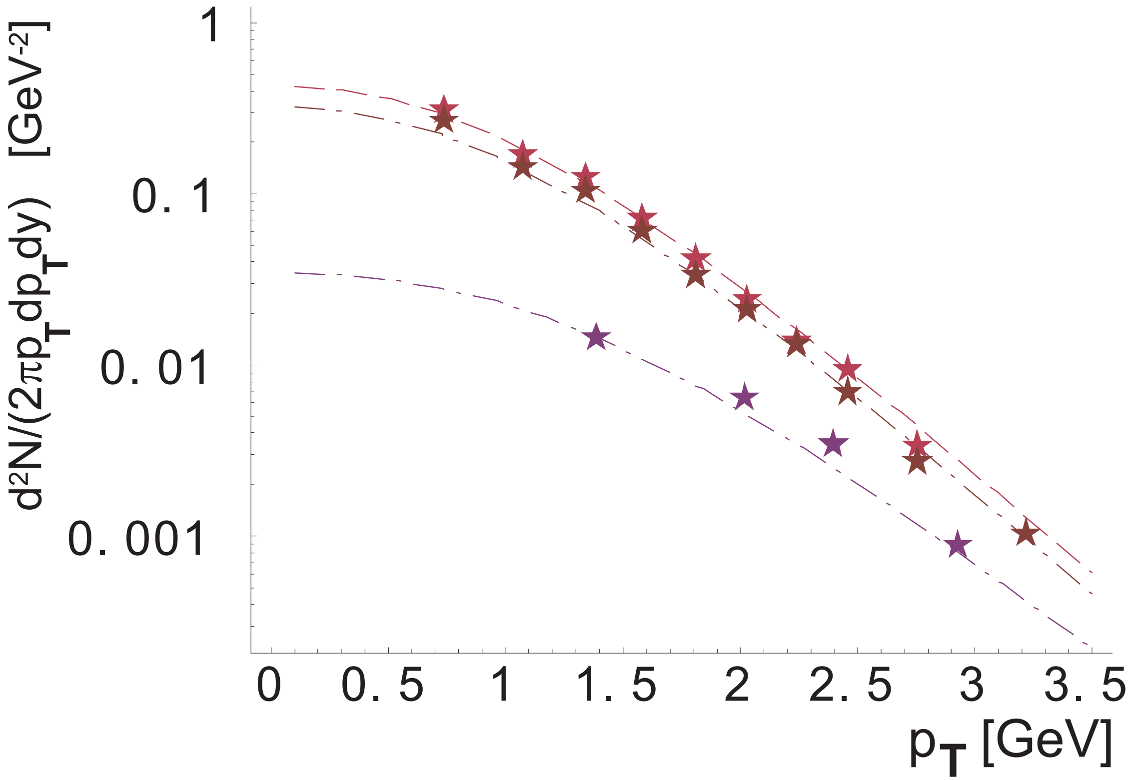
Just the two geometric parameters, $\tau = 7.7$ fm, $\rho_{\max} = 6.7$ fm

ϕ – very weak interactions, no rescattering, serves as a good thermometer

K^* – resonance, lower T would lead to much less K^*'s

No special treatment of Ω's

The Ξ went down



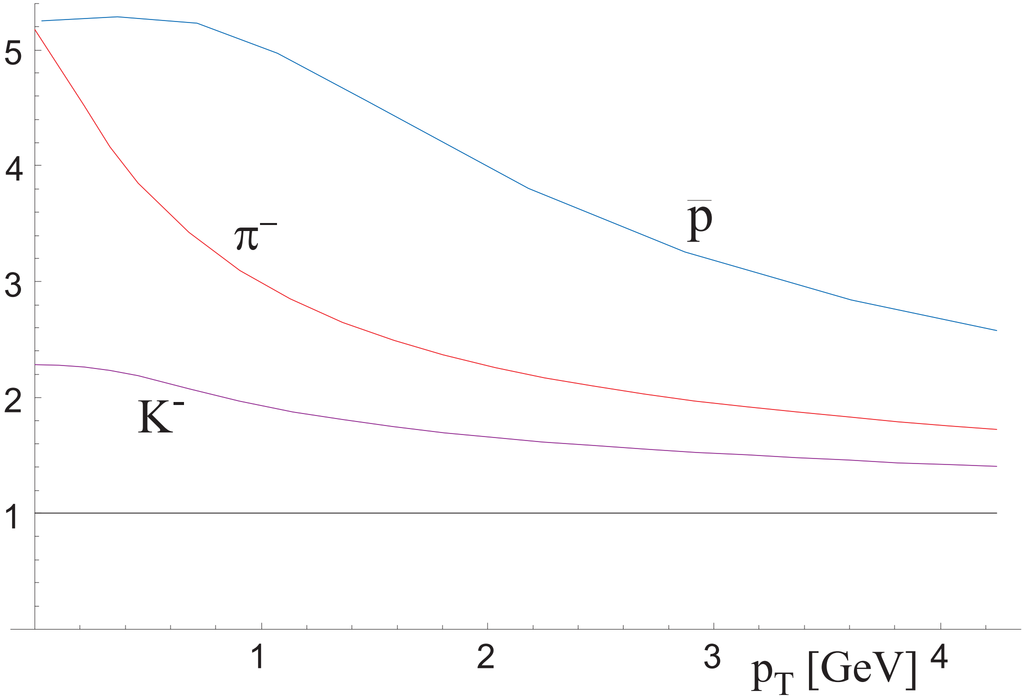
Ξ and $\overline{\Xi}$ – upper curves, $(\Omega + \overline{\Omega})/2$ – lower curve

This is a surprisingly good agreement for such a simple model

Fine print:

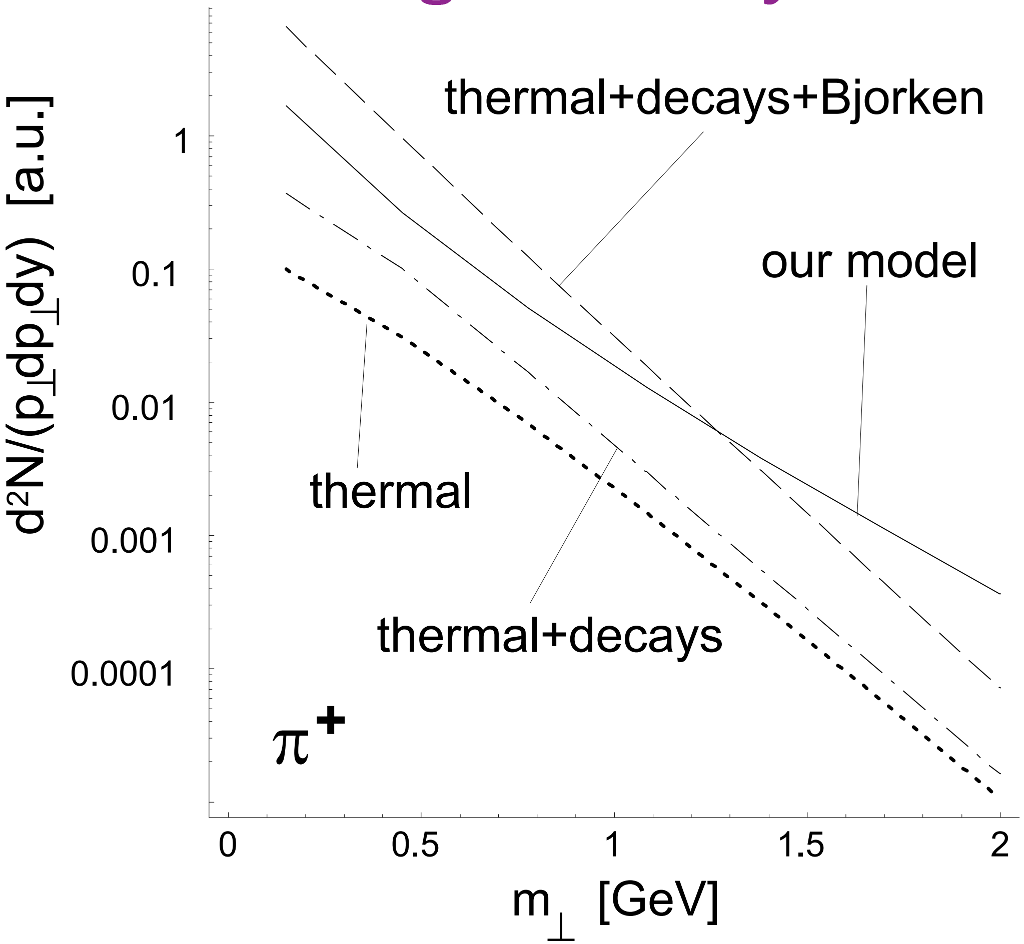
All comparisons are made on the log plot. The agreement of course is not perfect and works with the expected, say, 20-30%, accuracy. Nevertheless, for some particles one goes down a few decades and the agreement holds.

Resonance contribution to p_T-spectra



Ratio of spectra with full feeding from resonances to spectra with no feeding
Note that the feeding is important even at large p_T ! This is general statement, applying to all approaches which use resonances

"Cooling" via decays



(with flow the scale on the plot is arbitrary)

Resonance decays lower the inverse slope by about 30 MeV

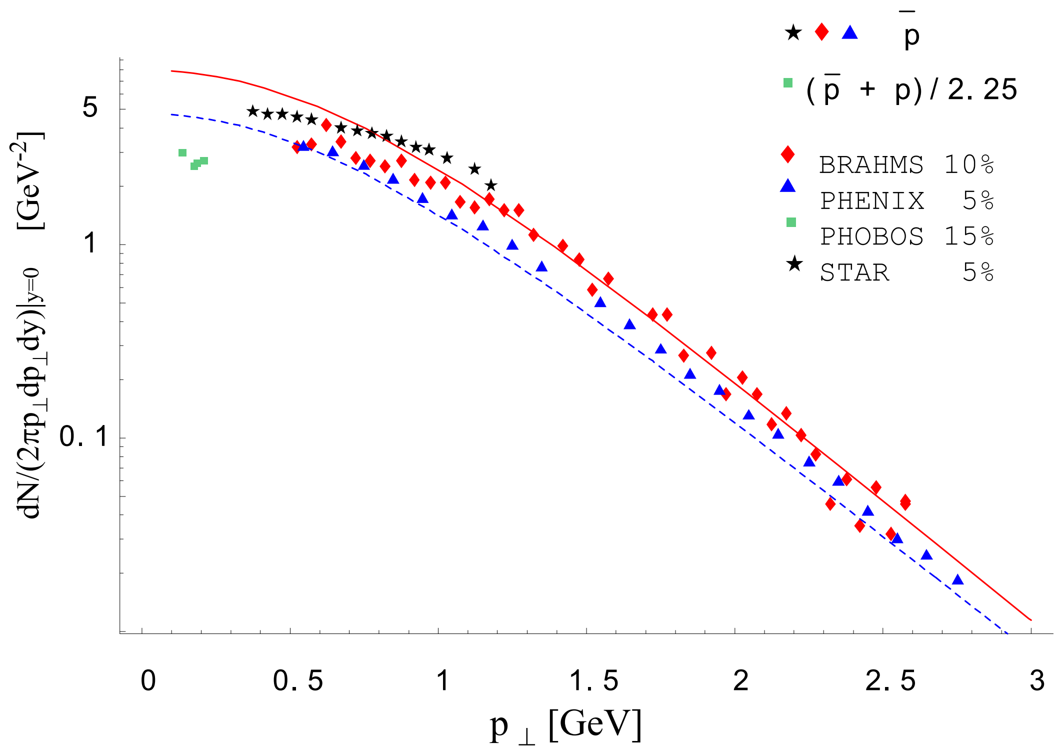
This "protects" the pion spectra against rescattering

Bending reflects the freeze-out surface/flow: blue-shift and red-shift

Correlation of T, flow velocity, and the chosen shape of the freeze-out!

The constant $\tau = \sqrt{t^2 - x^2 - y^2 - z^2}$ works

Antiprotons @ 200 GeV



solid line: our model with full feeding, dashed line: no feeding from weak decays
STAR somewhat more flat and higher, the model has problems for antiprotons at low p_T

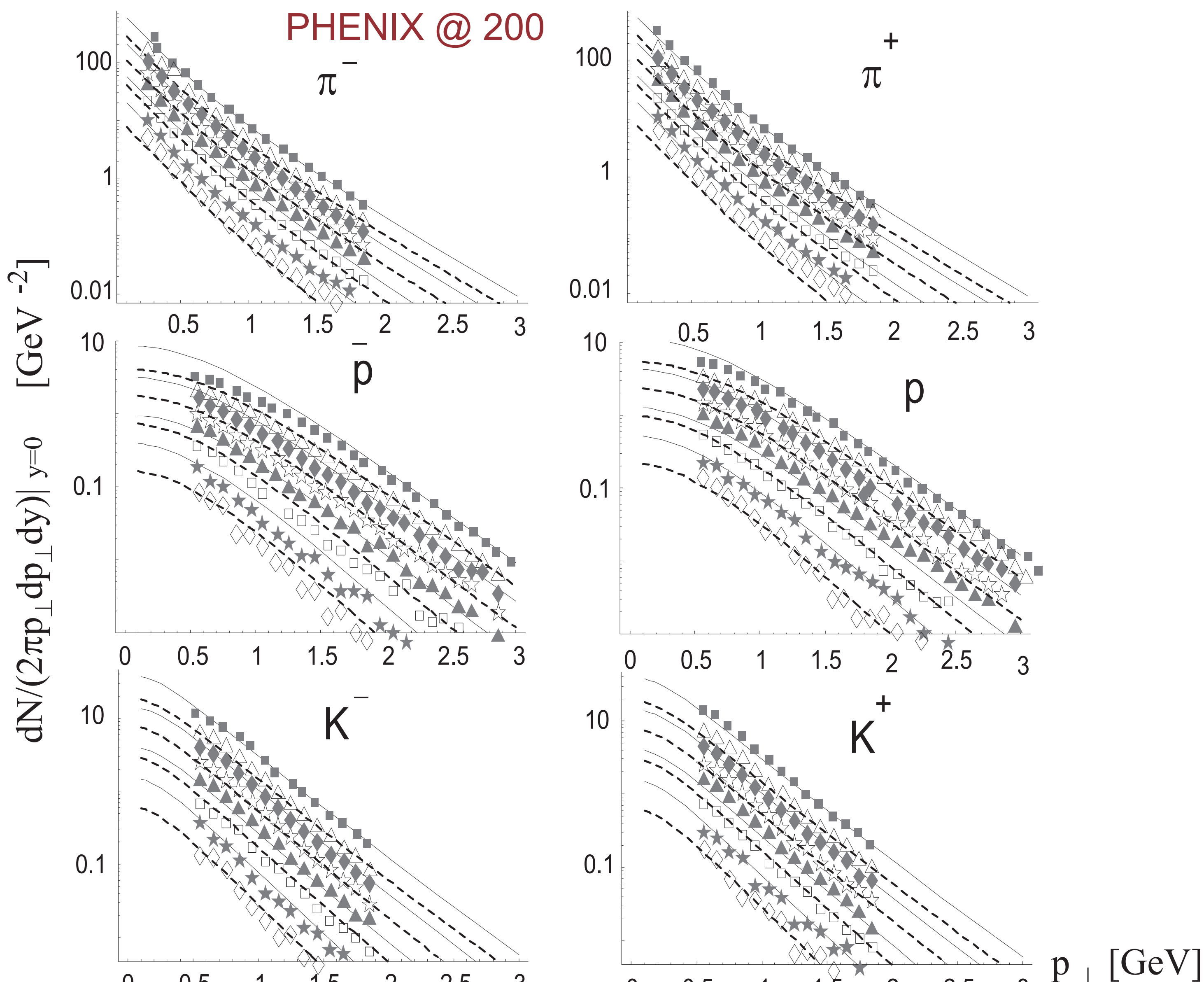
Extension for non-central collisions

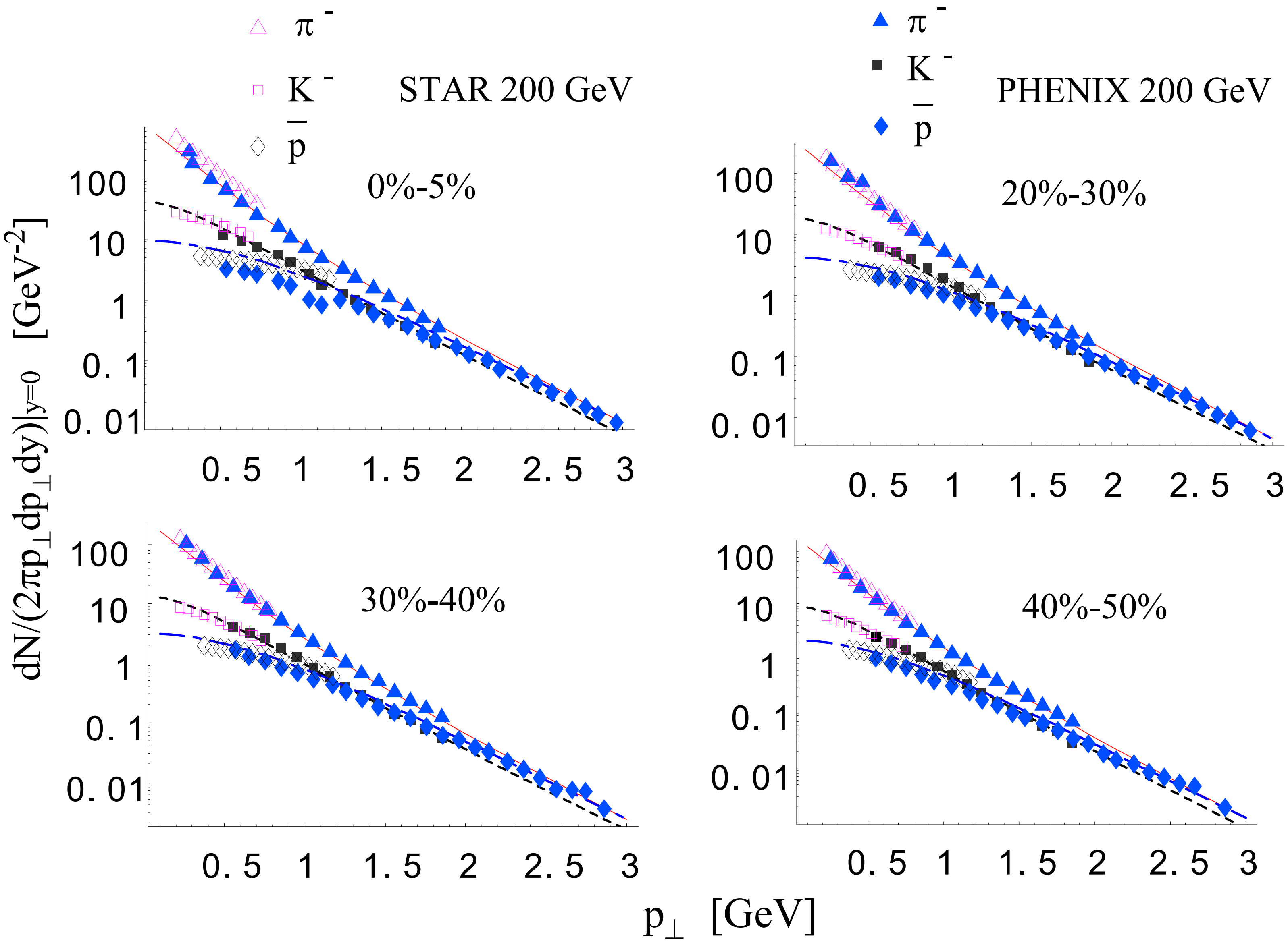
To a very good accuracy

$$c \simeq \frac{\pi b^2}{\sigma_{\mathrm{inel}}^{\mathrm{tot}}} \simeq \frac{b^2}{4R^2}$$

(WB+WF, PRC 65 (2002) 024905)

- The thermal parameters kept independent of centrality
- The geometry parameters, obviously, do depend on centrality
- In the calculation of the p_T-spectra we can neglect, as A. Baran has found in an explicit numerical calculation, the departure from the cylindrical symmetry. The effects for the p_T-spectra are at the level of 1%


PHENIX @ 200
π^-
π^+
$\bar{p}$
p
K^-
K^+
$dN/(2\pi p_\perp dp_\perp dy)|_{y=0}$ [GeV^{-2}]
$p_\perp$ [GeV]

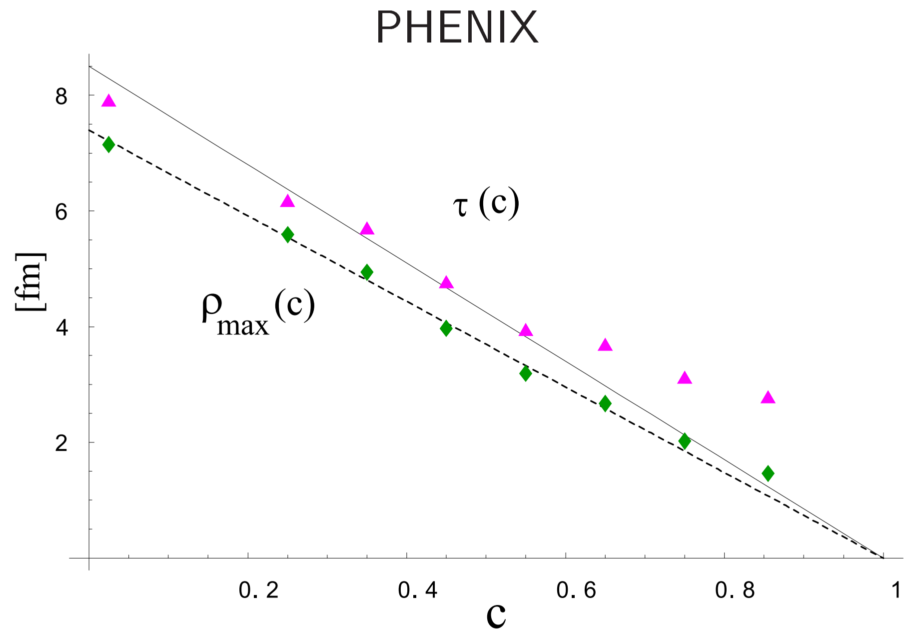


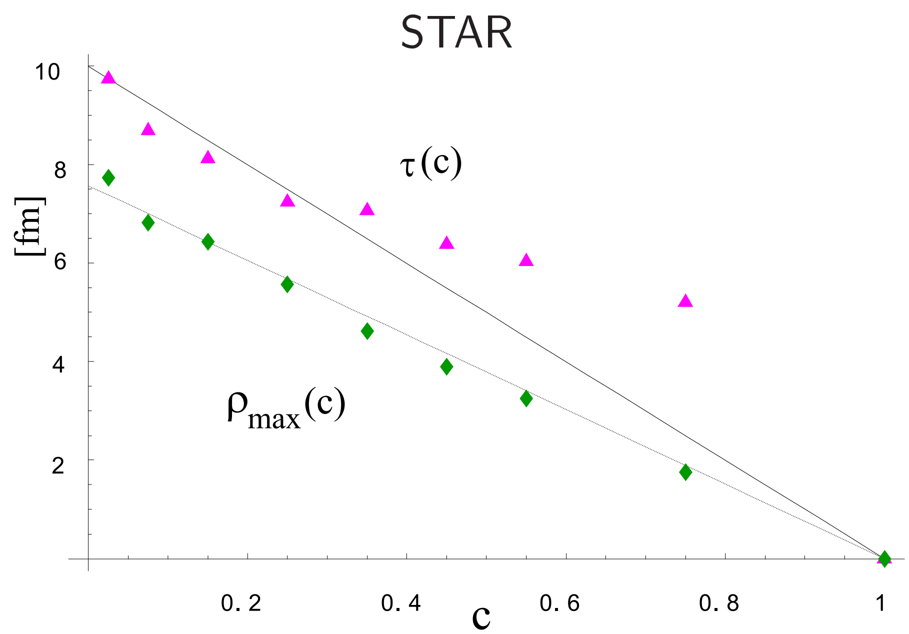
π -
K -
p̄
STAR 200 GeV
π -
K -
p̄
PHENIX 200 GeV
0%-5%
20%-30%
30%-40%
40%-50%
100
10
1
0. 1
0. 01
0. 5
1
1. 5
2
2. 5
3
dN/(2πp⊥dp⊥dy)|y=0 [GeV-2]
p⊥ [GeV]

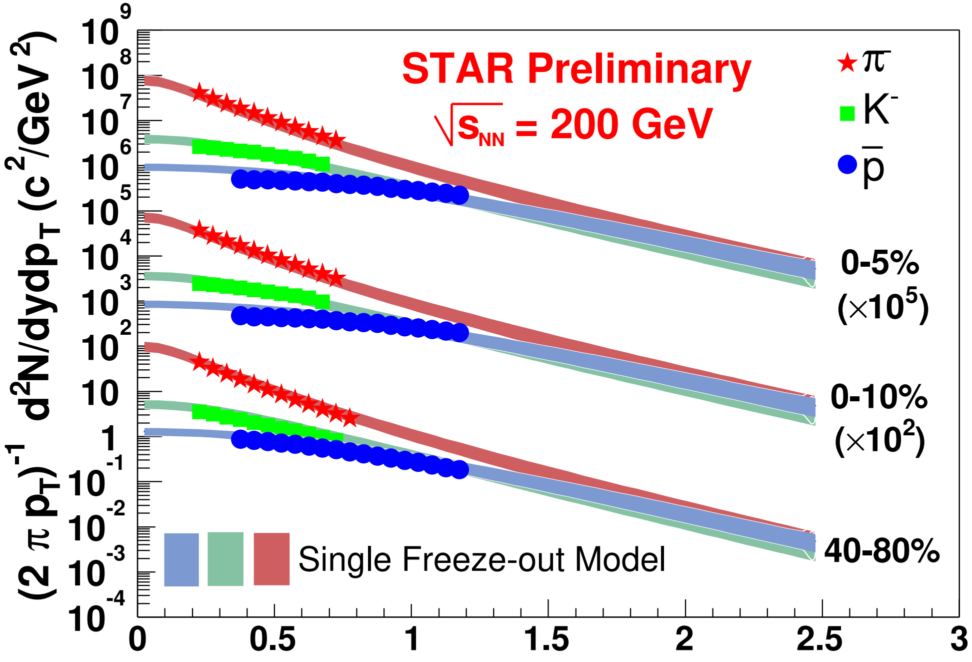
Compilation of geometric parameters (A. Baran)

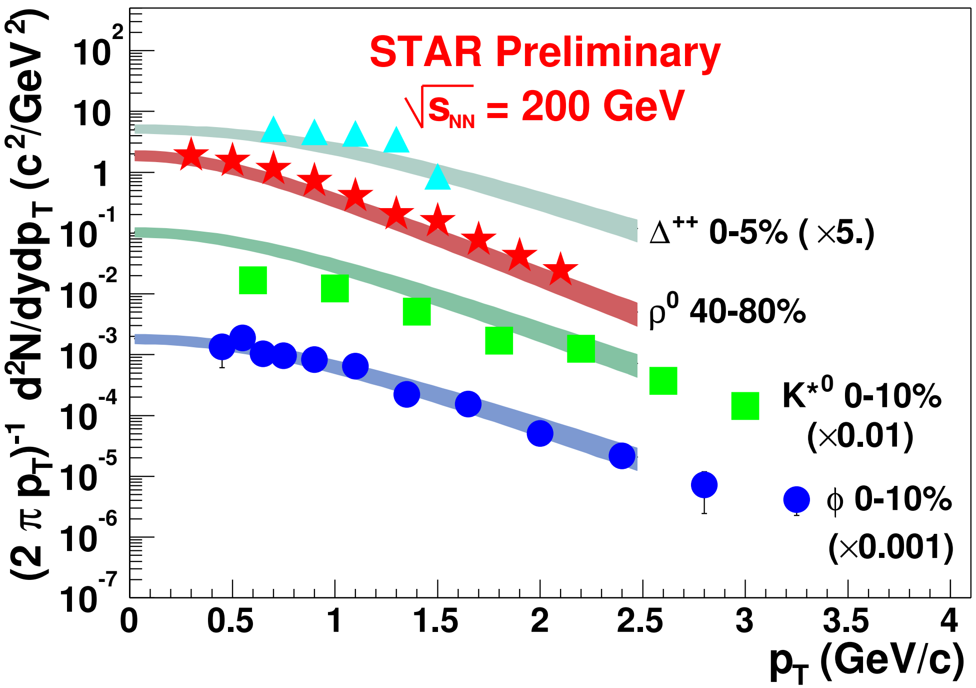
	c [%]	τ [fm] (norm)	$\rho_{\max}$ [fm] (slope)	$\langle\beta_\perp\rangle = \langle\rho/\sqrt{\tau^2+\rho^2}\rangle$
ALL	$0-5/10$	7.58 ± 0.32	7.27 ± 0.12	0.51 ± 0.02
BRAHMS	10	7.68 ± 0.19	7.46 ± 0.05	0.52 ± 0.01
STAR	$0-5$	9.74 ± 1.57	7.74 ± 0.68	0.45 ± 0.08
	$5-10$	8.69 ± 1.39	7.18 ± 0.64	0.47 ± 0.08
	$10-20$	8.12 ± 1.31	6.44 ± 0.57	0.45 ± 0.08
	$20-30$	7.24 ± 1.18	5.57 ± 0.50	0.44 ± 0.08
	$30-40$	7.07 ± 1.17	4.63 ± 0.39	0.39 ± 0.08
	$40-50$	6.38 ± 1.02	3.91 ± 0.33	0.37 ± 0.07
	$50-60$	6.19 ± 1.09	3.25 ± 0.28	0.32 ± 0.07
	$70-80$	5.48 ± 0.81	4.03 ± 0.10	0.43 ± 0.06
PHENIX	$0-5$	7.86 ± 0.38	7.15 ± 0.13	0.50 ± 0.02
	$20-30$	6.14 ± 0.32	5.62 ± 0.11	0.50 ± 0.02
	$30-40$	5.73 ± 0.16	4.95 ± 0.05	0.48 ± 0.01
	$40-50$	4.75 ± 0.28	3.96 ± 0.09	0.47 ± 0.03
	$50-60$	3.91 ± 0.23	3.12 ± 0.07	0.45 ± 0.03
	$60-70$	3.67 ± 0.12	2.67 ± 0.03	0.42 ± 0.01
	$70-80$	3.09 ± 0.11	2.02 ± 0.02	0.39 ± 0.01
	$80-91$	2.76 ± 0.20	1.43 ± 0.03	0.32 ± 0.03

For periferic collisions the transverse flow is weaker


PHENIX
8
6
4
2
[fm]
τ (c)
ρmax (c)
0.2
0.4
0.6
0.8
1
c


STAR
10
8
6
4
2
[fm]
τ (c)
ρmax (c)
0.2
0.4
0.6
0.8
1
c





(plotted by P. Fachini, STAR, QM'04)

THERMINATOR

THERMINATOR

THERMal heavy-IoN generATOR

Adam Kisiel, Tomasz Tałuć, Wojciech Broniowski, Wojciech Florkowski

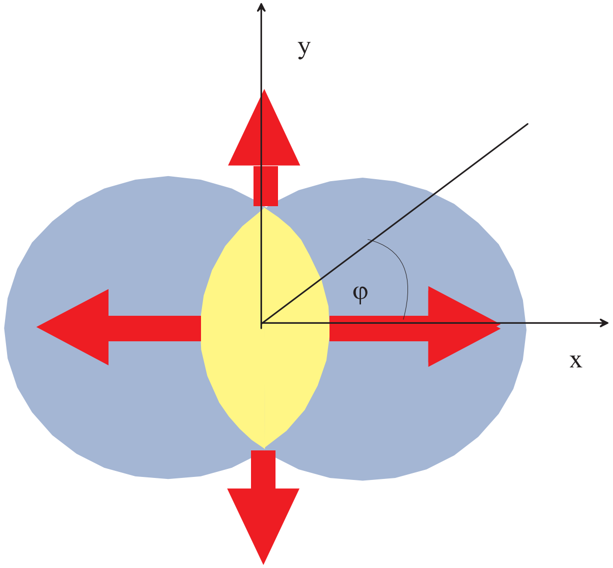
THERMINATOR is a Monte Carlo event generator designed for studies of particle production in relativistic heavy-ion collisions performed at such experimental facilities as the SPS, RHIC, or LHC. The program implements the thermal model of particle production with single freeze-out, performing the following tasks: 1) generation of stable particles and unstable resonances at the chosen freeze-out hypersurface with the local phase-space density of particles given by statistical distribution functions, 2) subsequent space-time evolution and decays of hadronic resonances, 3) calculation of the transverse-momentum spectra or other observables. The code is written in c++ and uses the ROOT environment. It uses the same universal input for particle properties as SHARE.

See the talk by Adam Kisiel

Elliptic flow

Anna Baran, PhD thesis, 2004

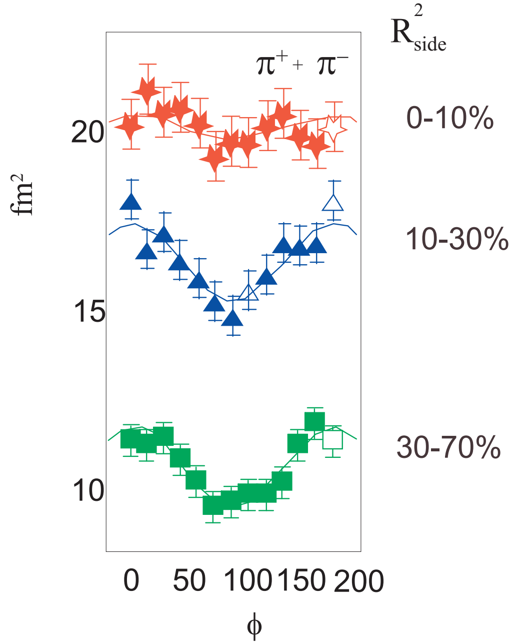
WB+WF+AB, AIP 660 (2003) 185 [nucl-th/0212053]

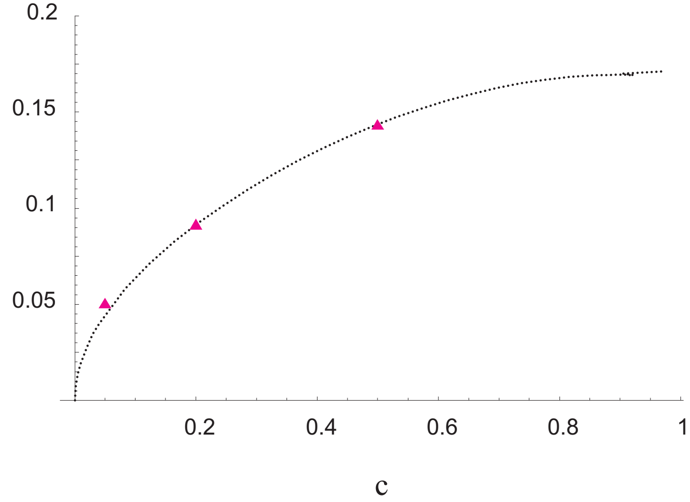


When the nuclei collide at non-zero impact parameter, $b \neq 0$, the momentum distribution of the produced particles carries azimuthal asymmetry. At mid-rapidity for same nuclei

$$\left.\frac{dN}{d^2p_\perp dy}\right|_{y=0} = \left.\frac{dN}{2\pi p_\perp dp_\perp dy}\right|_{y=0} (1 + 2\, v_2\, \cos 2\phi + 2\, v_4\, \cos 4\phi + \ldots)$$

We estimate the shape excentricity parameter, ϵ, from the measured values of $R_{\rm side}(\phi)$ [STAR, nucl-ex/0301005]





$$\epsilon = \frac{\langle y \rangle^2 - \langle x \rangle^2}{\langle y \rangle^2 + \langle x \rangle^2}$$

Modification of the freeze-out hypersurface (out-of-plane elongation)

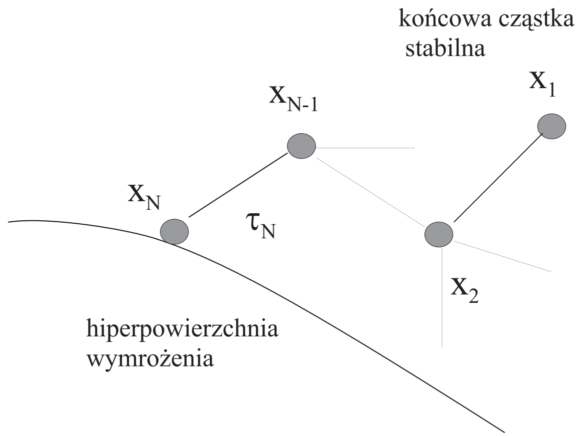
$$r_x = \rho_{\rm max} \sqrt{1-\epsilon}\, \cos\varphi, \quad r_y = \rho_{\rm max} \sqrt{1+\epsilon}\, \sin\varphi$$

Modification of the flow profile (stronger in the reaction plane)

$$u_x = \frac{r_x}{N}\sqrt{1+\delta}\,\cos\varphi, \quad u_y = \frac{r_y}{N}\sqrt{1-\delta}\,\sin\varphi, \quad u_z = \frac{r_z}{N}, \quad u_t = \frac{t}{N}$$

N obtained from the normalization condition $u^\mu u_\mu = 1$

Resonance decays



$$n(x_1,p_1) \quad = \int \frac{d^3p_2}{E_{p_2}} B(p_2,p_1) \int d\tau_2 \Gamma_2 e^{-\Gamma_2\tau_2} \int d^4x_2 \delta^{(4)}(x_2 + \frac{p_2\tau_2}{m_2} - x_1) \times$$

$$\cdots \times \int \frac{d^3p_N}{E_{p_N}} B(p_N,p_{N-1}) \int d\tau_N \Gamma_N e^{-\Gamma_N\tau_N}$$

$$\int d\Sigma_\mu(x_N) p_N^\mu \delta^{(4)}(x_N + \frac{p_N\tau_N}{m_N} - x_{N-1}) f_N(p_N \cdot u(x_N))$$

$$B(q,k) = \frac{b}{4\pi p^*}\delta(\frac{k\cdot q}{m_R} - E^*)$$

$$E_{p_1}\frac{dN_1}{d^3p_1} = \int \frac{d^3p_2}{E_{p_2}} B(p_2,p_1) \times \ldots \times \int \frac{d^3p_N}{E_{p_N}} B(p_N,p_{N-1}) \int d\Sigma_\mu(x_N) p_N^\mu f_N[p_N \cdot u_N]$$

Independence of Fourier components

Each step of the cascade involves

$$g(q_\perp, \varphi_q) = \int k_\perp dk_\perp \int_0^{2\pi} d\varphi_k J(k_\perp, q_\perp, \varphi_q - \varphi_k) f(k_\perp, \varphi_k)$$

We introduce the Fourier decomposition

$$g(q_\perp, \varphi_q) = \sum_n \cos(n\varphi_q) g_n(q_\perp), \, f(k_\perp, \varphi_k) = \sum_m \cos(m\varphi_k) f_m(k_\perp)$$

and immediately find that the evolution is diagonal in the Fourier index n:

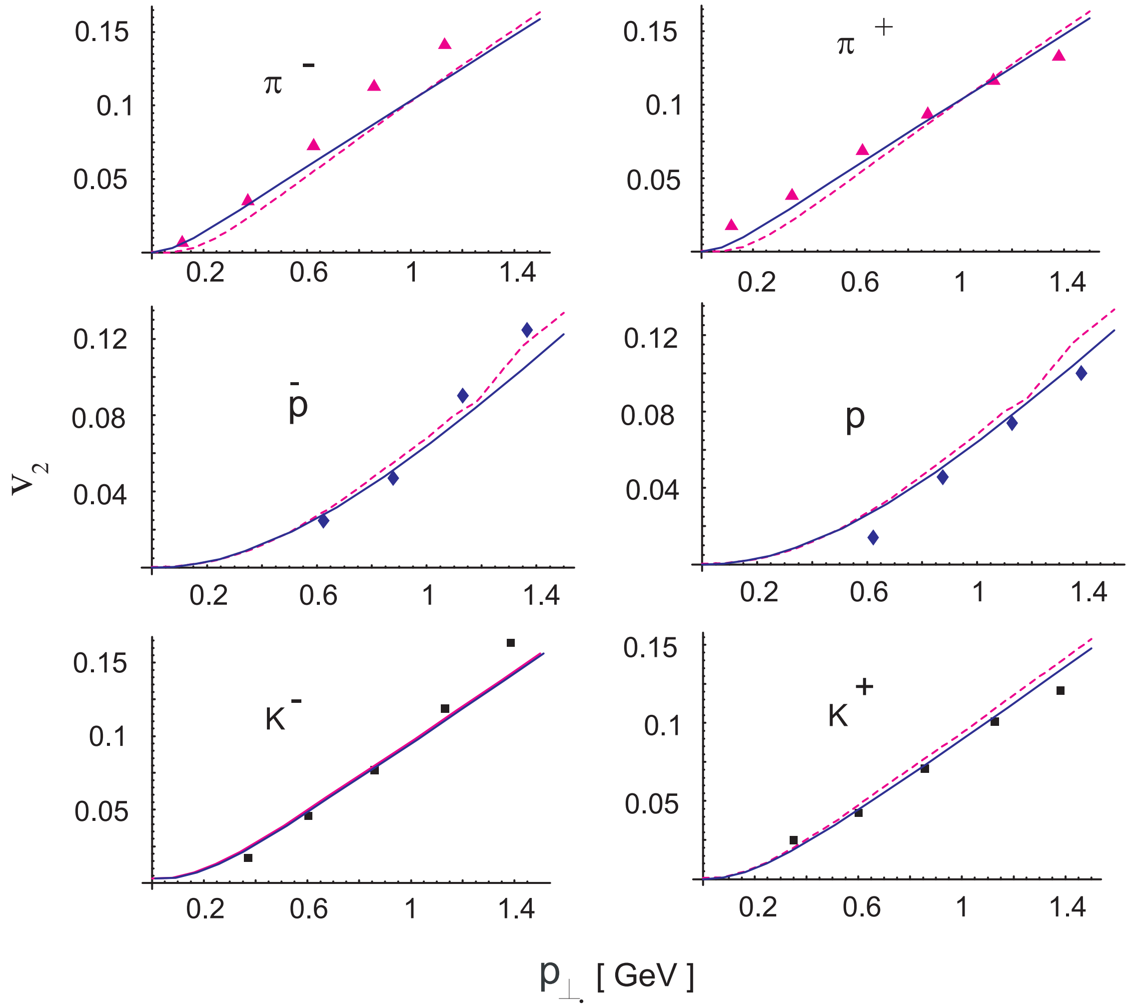
$$g_n(q_\perp) = \int k_\perp dk_\perp \int_0^{2\pi} d\varphi_k J(k_\perp, q_\perp, \varphi) \cos(n\varphi) f_n(k_\perp)$$

The whole cascade proceeds independently for each Fourier component of the spectrum

Now we apply the same method as for the φ-integrated spectra: fit the new parameter δ (asymmetry of the flow) to "good" particles and make predictions for other particles.

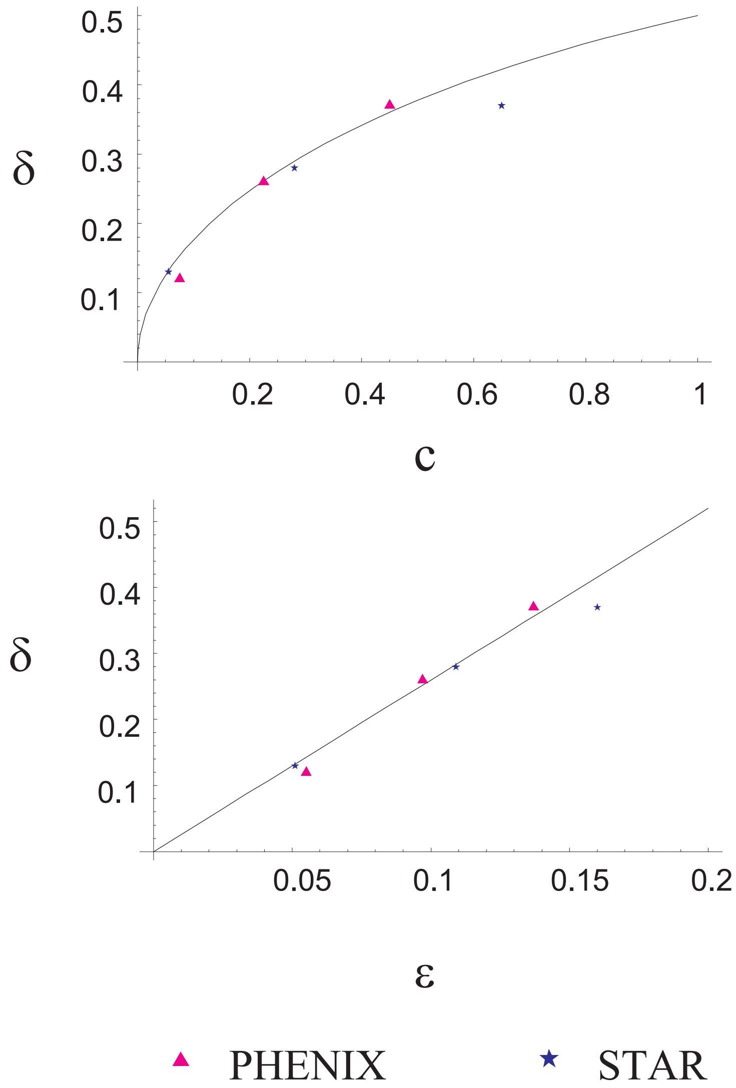
$$v_2 = \frac{g_2}{g_0}$$

data from PHENIX @ 200 GeV, S. A. Voloshin, NP **A715** (2003) 379c



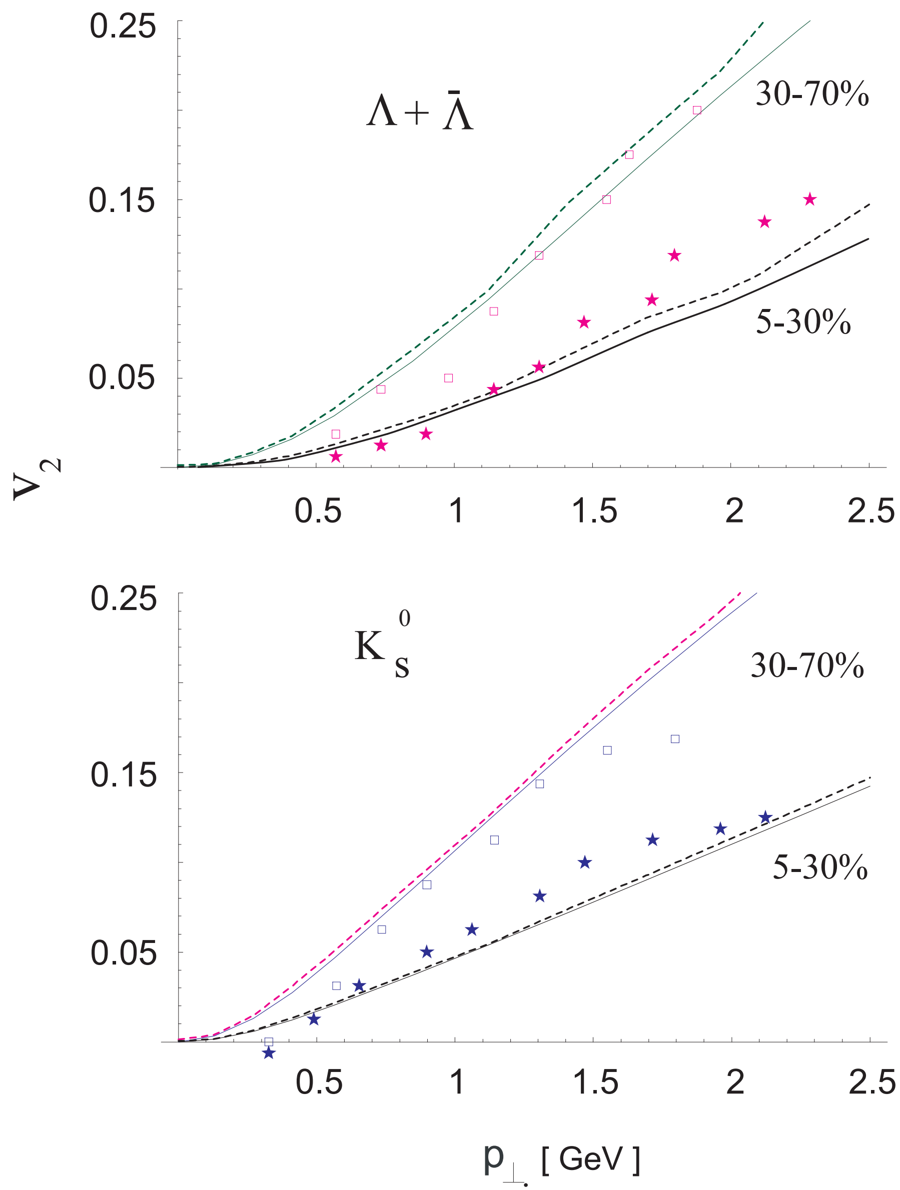
model fit: T and μ_B from the ratios, $\tau = 4.04$ fm, $\rho_{\max} = 3.70$ fm from the p_T-spectra, $\epsilon = 0.13$ (from $R_{\rm side}$), $\delta = 0.25$ (from v_2)

solid – no resonanses, dashed – with resonances

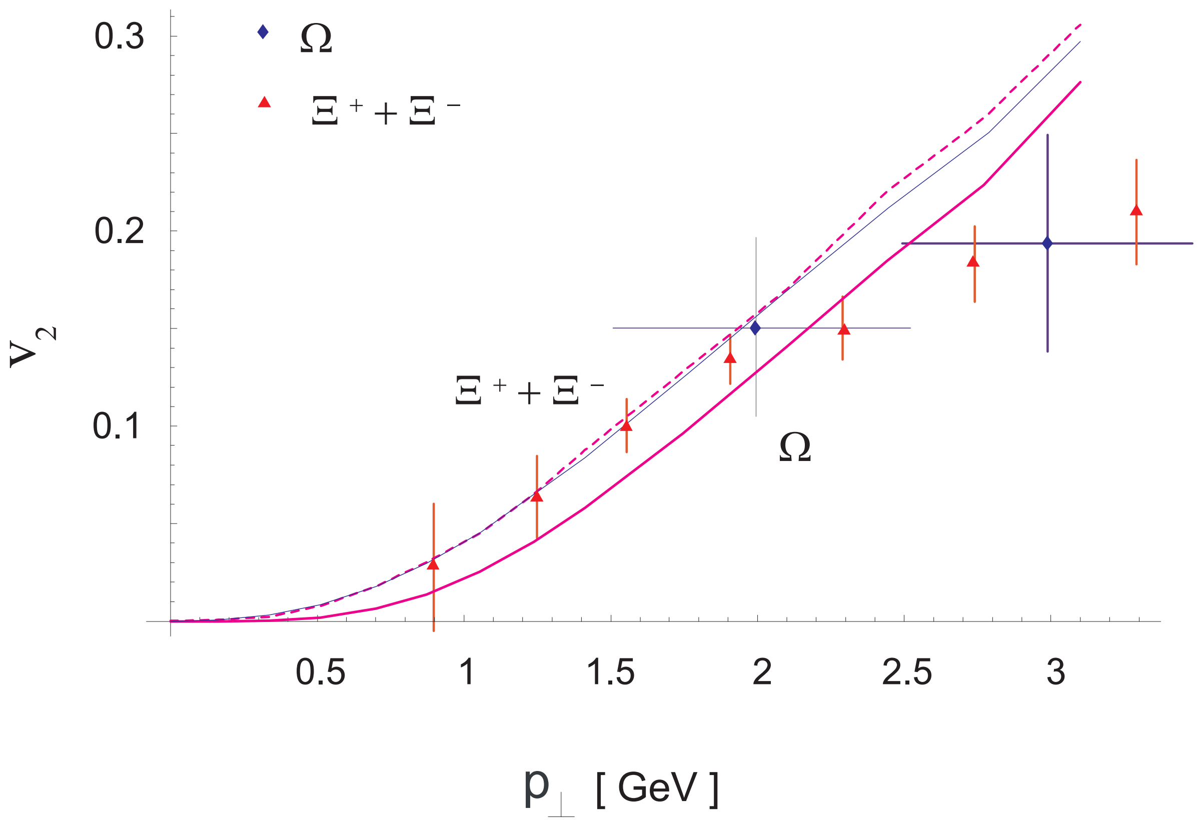

δ
0.5
0.4
0.3
0.2
0.1
0.2
0.4
0.6
0.8
1
c
δ
0.5
0.4
0.3
0.2
0.1
0.05
0.1
0.15
0.2
ε
PHENIX
STAR

Verification / predictions

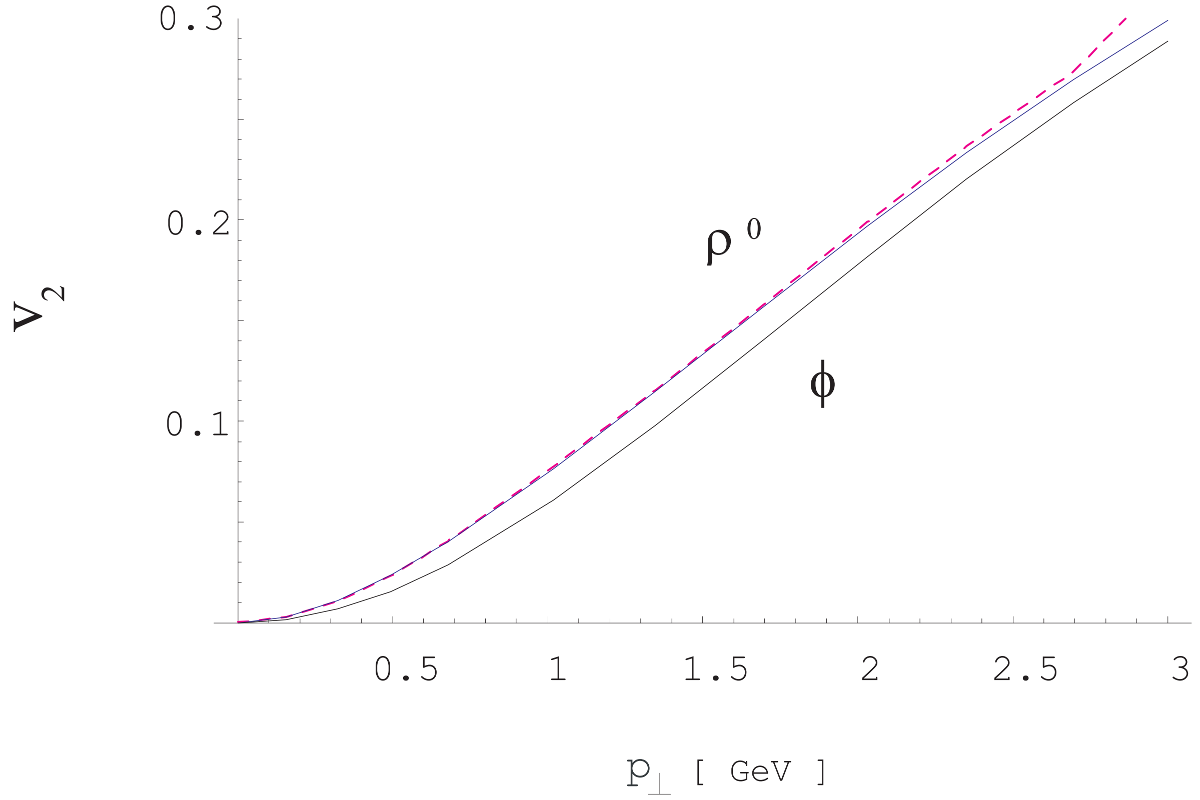
[data from STAR @ 200 GeV, PRL 92 (2004) 052302]



[data from J. Castillo, QM04, J. Phys. G30 (2004) S1207, min. bias]



Predictions for ρ and ϕ @ 200 GeV, min. bias
(see the talk by Sarah Blyth)



Summary of v_2: model works for not-too-large $p_\perp$ (no saturation at large momenta), results similar to hydro, works well for hyperons, supports strongly the flow explanation of azimuthal asymmetry. Contributions of resonances “accidently” cancel out!
PANTA REI scenario

Departure from the boost invariance

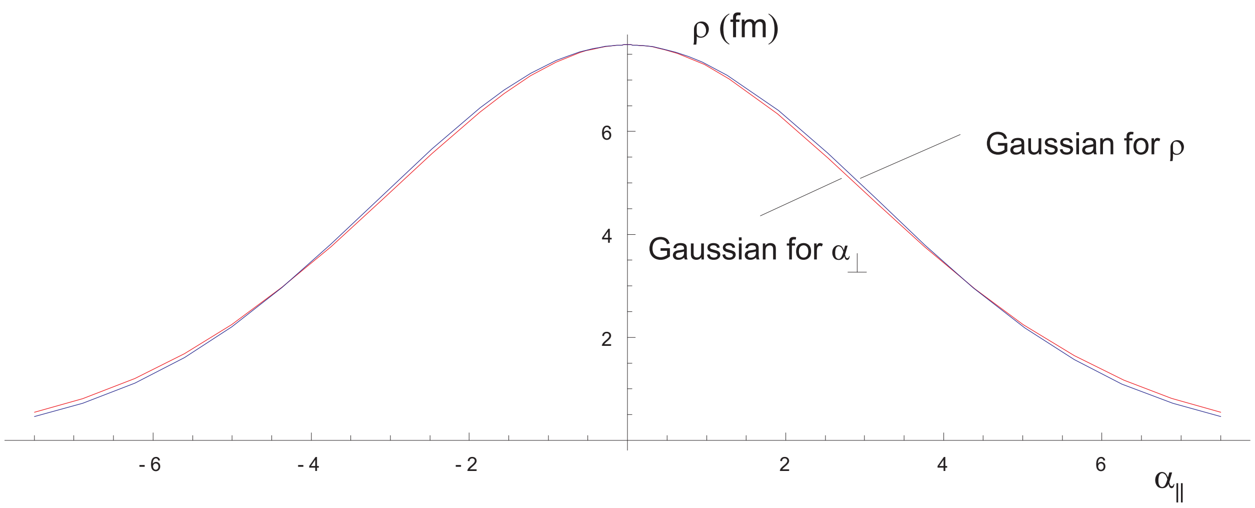
Recall our geometry:

$$x = \tau \sinh \alpha_\perp \cos\phi, \quad y = \tau \sinh \alpha_\perp \sin\phi$$
$$z = \tau \sinh \alpha_\parallel \cosh \alpha_\perp, \quad t = \tau \cosh \alpha_\parallel \cosh \alpha_\perp,$$

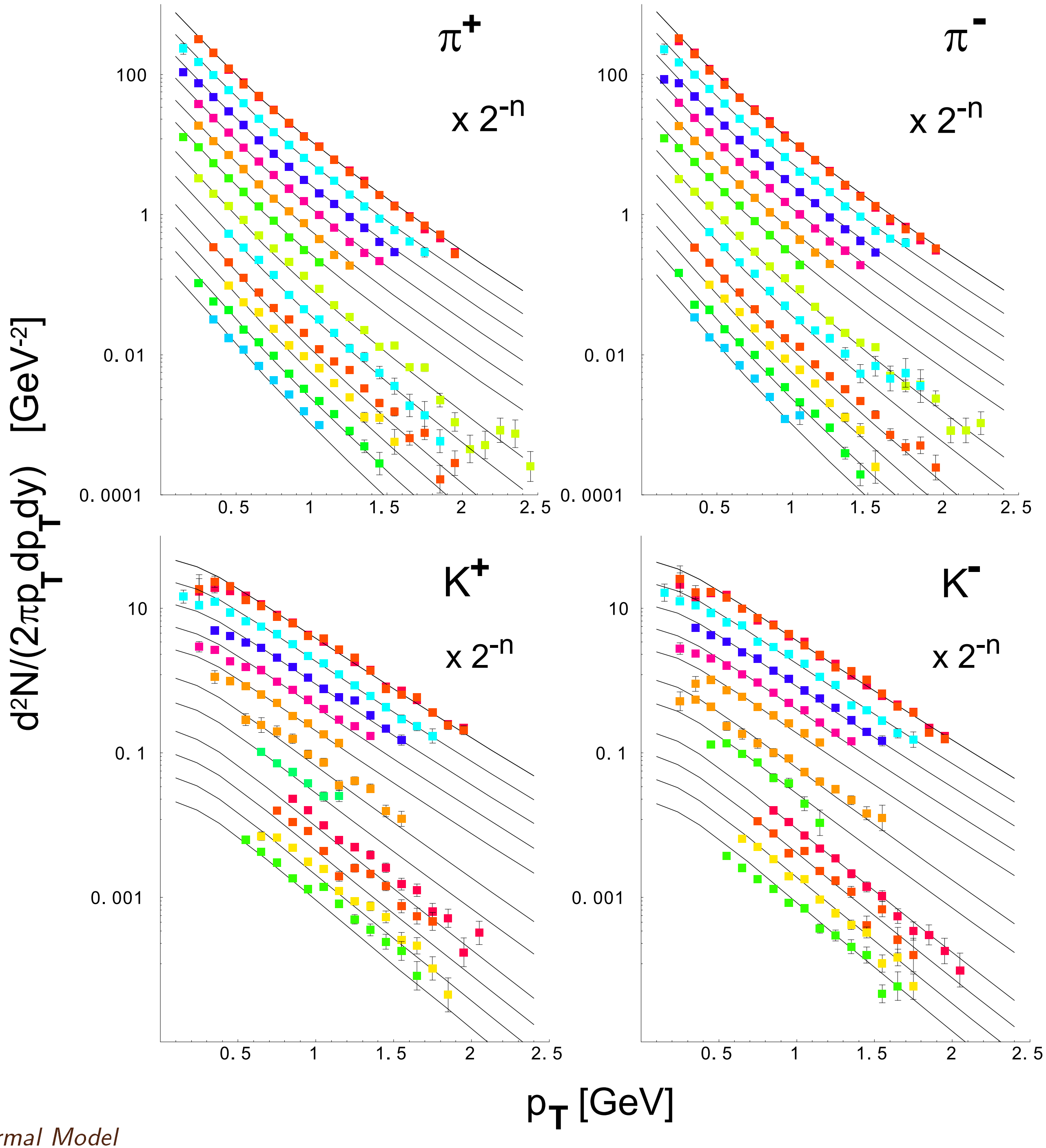
Now we take $\alpha_\perp \in [0, \alpha_2 \exp[-\alpha_\parallel^2/(2\Delta^2)]$ (larger z – narrower transverse size)
The optimum values of parameters from fits to the BRAHMS spectra are

$$\tau = 8.33 \text{ fm}, \quad \alpha_2 = 0.825, \quad \Delta = 3.33$$

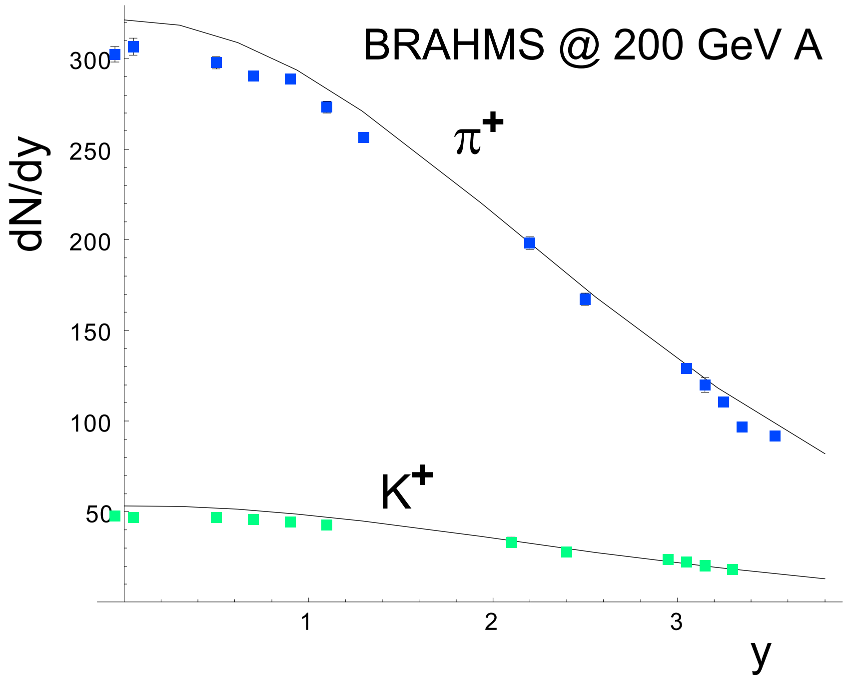
ρ vs. $\alpha_\parallel$



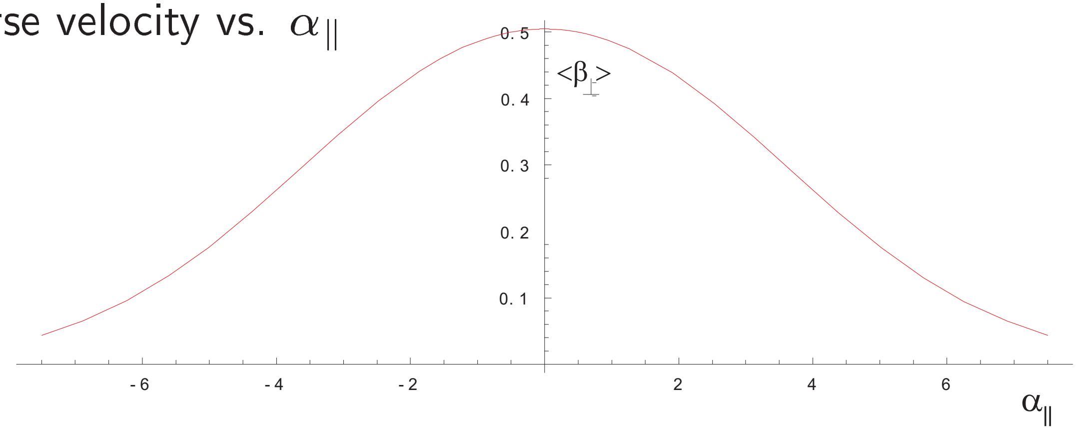
BRAHMS @ 200 GeV A



dN/dy from BRAHMS vs. the model



Average transverse velocity vs. $\alpha_{||}$



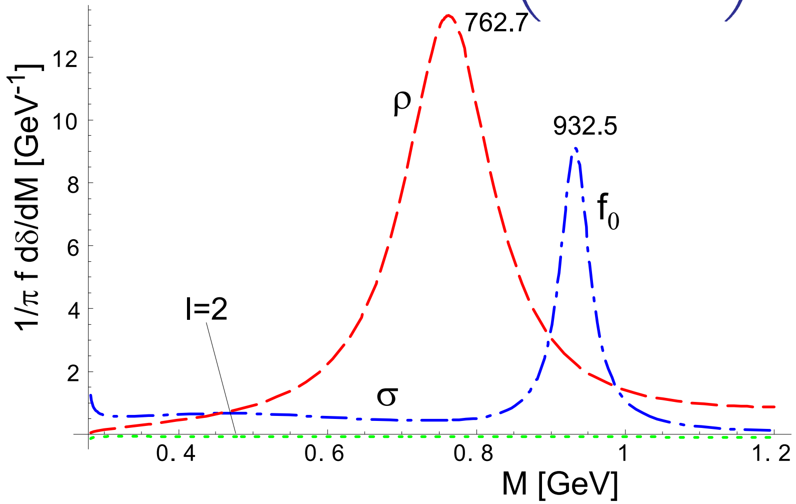
Future: More accurate model requires dependence of chemical potentials on y

$\pi^+\pi^-$ invariant-mass correlations

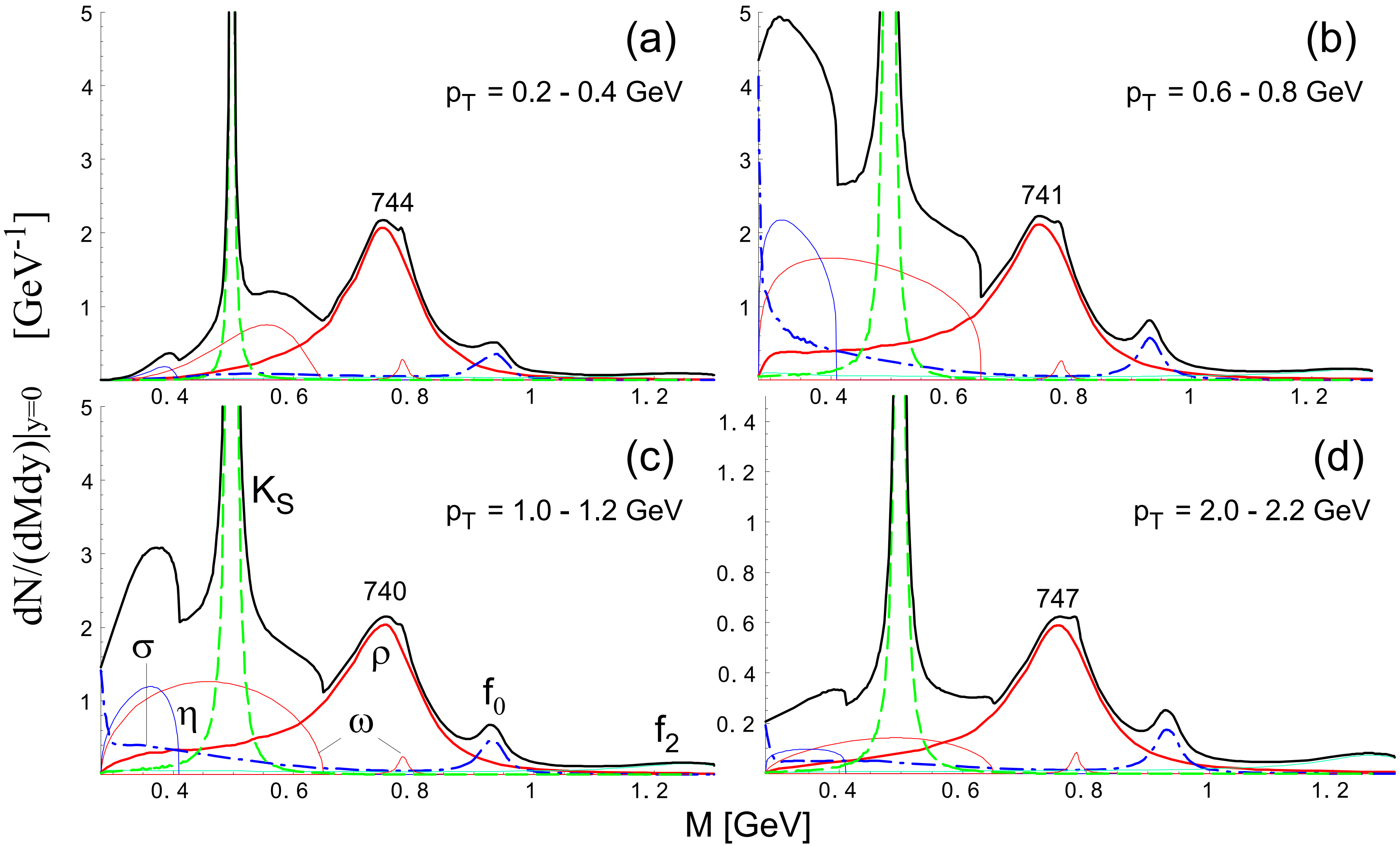
The phase-shift formula for the density of resonances

Beth,Uhlenbeck (1937); Dashen, Ma, Bernstein, Rajaraman (1974); **Weinhold (1998)**, Friman, Nörenberg; **WB,WF,B. Hiller, nucl-th/0306034**; Pratt, Bauer, nucl-th/0308087

$$\frac{dn}{dM} = f \int \frac{d^3p}{(2\pi)^3} \frac{d\delta_{\pi\pi}(M)}{\pi dM} \frac{1}{\exp\left(\frac{\sqrt{M^2+\boldsymbol{p}^2}}{T}\right) \pm 1}$$



Small contribution from σ, negative and tiny contribution from $I = 2$, ρ-peak slightly shifted to lower M, $1/\sqrt{M - 4m_\pi^2}$ behavior for the σ
(data from J. Adams et al., nucl-ex/0307023; P. Fachini, nucl-ex/0305034)
(Theoretical papers by Brown+Shuryak, Kolb-Prakash, Rapp, Pratt+Bauer)

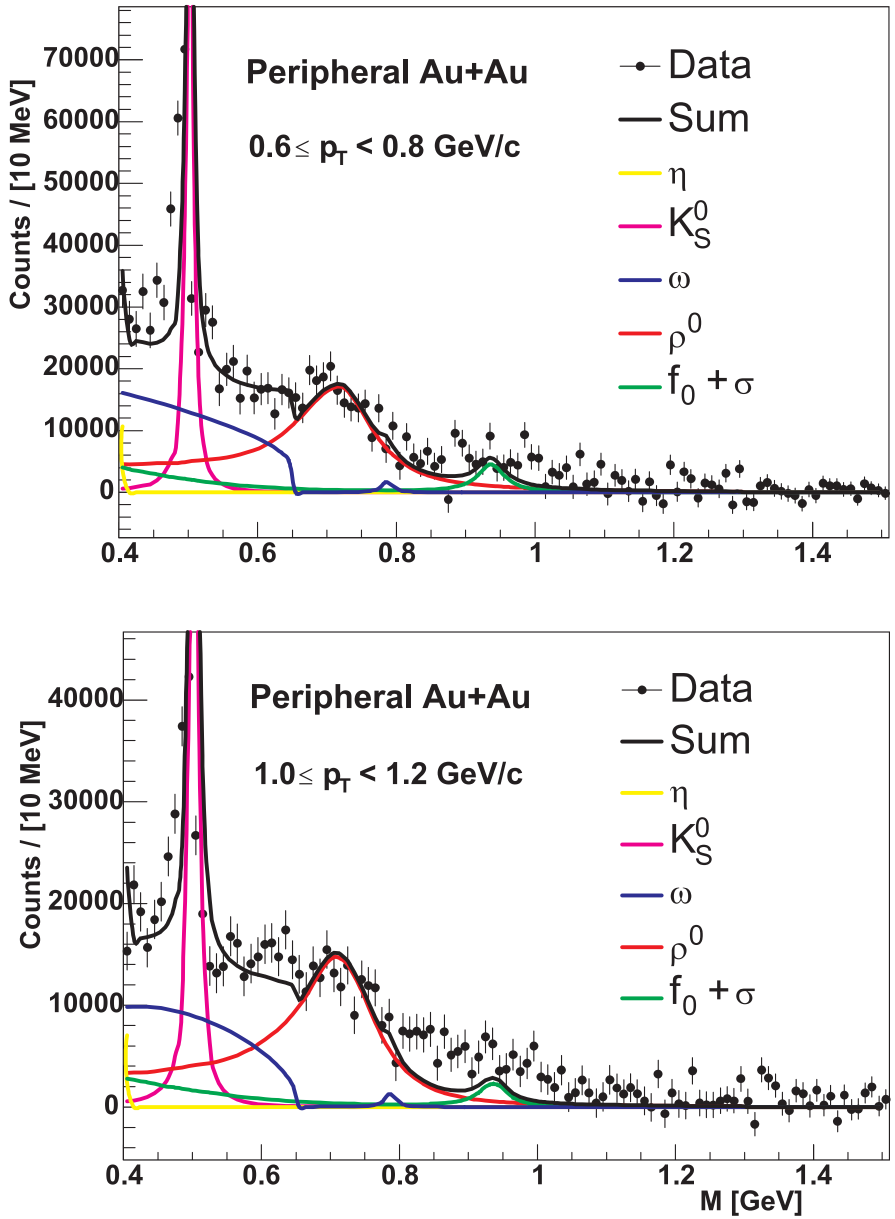


The invariant $\pi^+\pi^-$ mass spectra in the single-freeze-out model for four sample bins in the transverse momentum of the pair, p_T, plotted as a function of M. η indicates $\eta + \eta'$. The kinematic cuts of the STAR experiment are incorporated

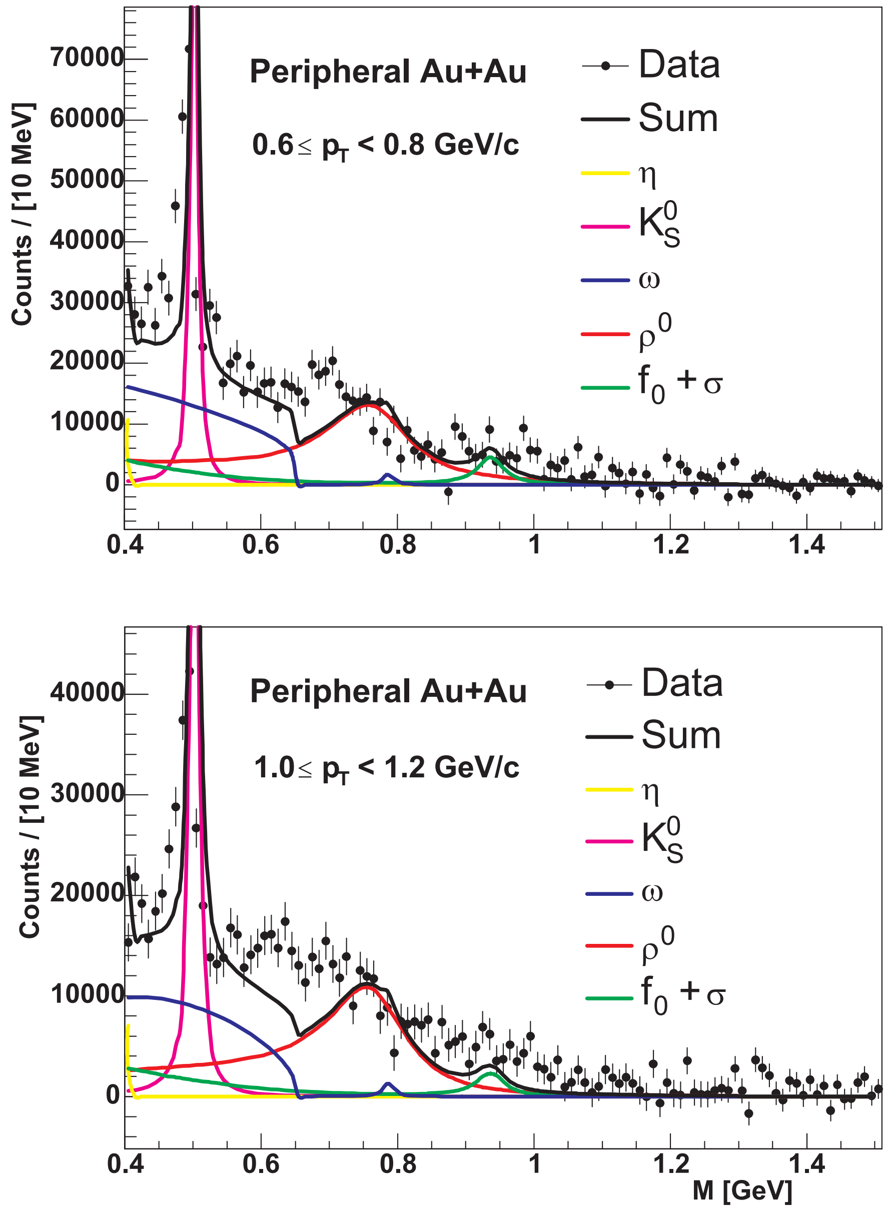
Would look different at different T

ρ shifted down in M by 10%

lines – our model



(prepared by P. Fachini)



(worse agreement)

Balance functions

S. Bass, P. Danielewicz, and S. Pratt, PRL 85 (2000) 2689
S. Jeon and V. Koch, hep-ph/0304012
A. Białas and V. Koch, Phys. Lett. B456 (1999) 1
Asakawa, Heinz, and Müller, PRL 85 (2000) 2072
Jeon and Koch, PRL 85 (2000) 2076
P. Bożek +WB+WF, nucl-th/0310062, Heavy-Ion Phys.

$$B(\delta, Y) = \frac{1}{2}\left\{\frac{\langle N_{+-}(\delta)\rangle - \langle N_{++}(\delta)\rangle}{\langle N_{+}\rangle} + \frac{\langle N_{-+}(\delta)\rangle - \langle N_{--}(\delta)\rangle}{\langle N_{-}\rangle}\right\}$$

N_{+-} and N_{-+} – number of the unlike-sign pairs
N_{++} and N_{--} – number of the like-sign pairs The two members of the pair fall into the rapidity window Y, with relative rapidity

$$\delta = \Delta y = |y_2 - y_1|$$

N_{+} (N_{-}) – number of positive (negative) particles in the interval Y

Relation to charge fluctuations

After integration over δ

$$\int_0^Y d\delta\, B(\delta, Y) = \frac{1}{2}\left\{\frac{\langle N_+ N_- \rangle - \langle N_+(N_+ - 1)\rangle}{\langle N_+\rangle} + (+ \to -)\right\}$$

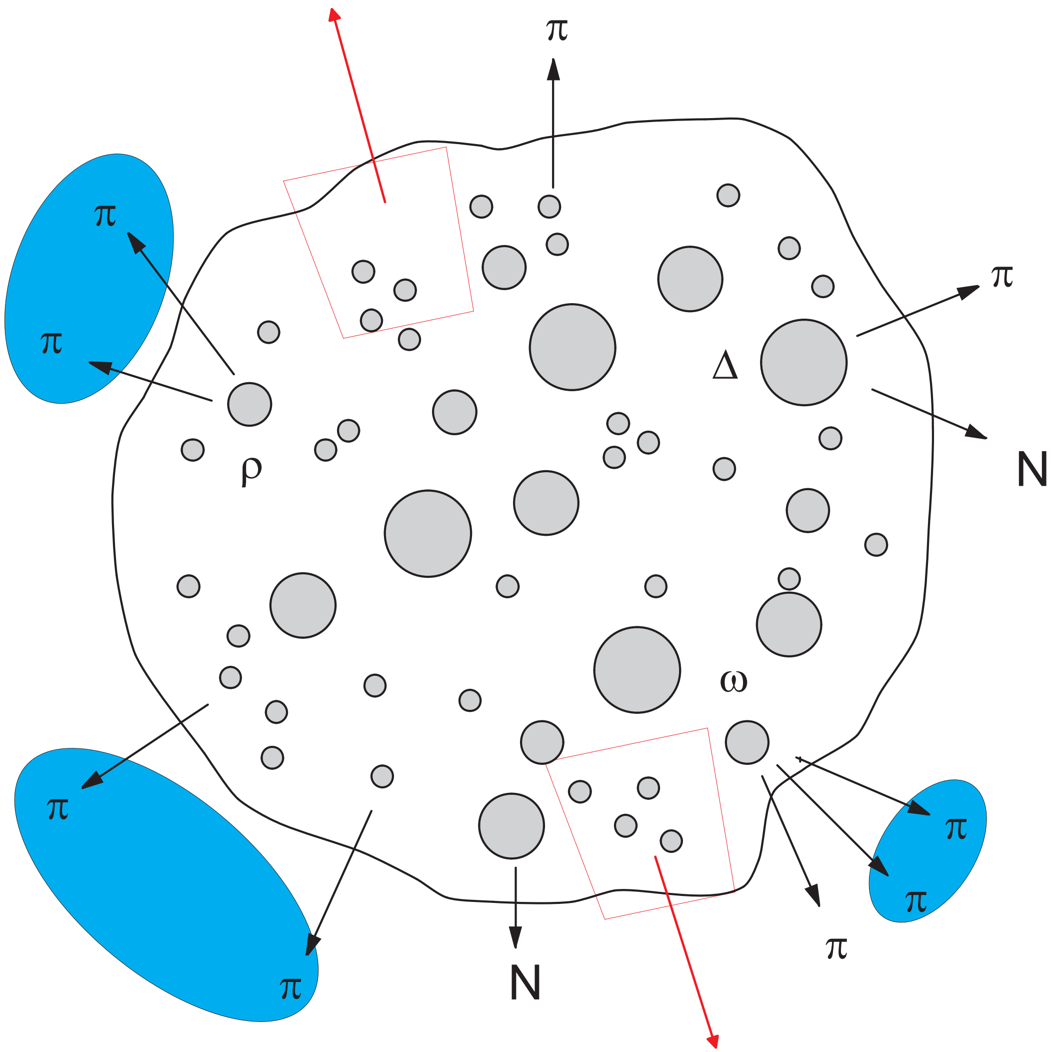
charge: $Q = N_+ - N_-$, multiplicity of charged particles: $N_{\rm ch} = N_+ + N_-$

$$\frac{\langle (Q - \langle Q \rangle)^2 \rangle}{\langle N_{\rm ch}\rangle} = 1 - \int_0^Y d\delta\, B(\delta, Y)$$

For sufficiently large Y we have $\int_0^Y d\delta\, B(\delta, Y) = 1$

The width of B in δ gives info about the hadronization time
small width $\equiv$ late-stage hadronization
large width $\equiv$ production of hadrons at early stage

Subtraction of $++$ and $--$ pairs effectively removes the uncorrelated $+-$ pairs from the distribution


π
π
π
π
Δ
N
ρ
ω
π
π
π
π
π
N

Two contributions for the $\pi^+\pi^-$ balance function

1) RESONANCE CONTRIBUTION (R) is determined by the decays of neutral hadronic resonances which have a $\pi^+\pi^-$ pair in the final state

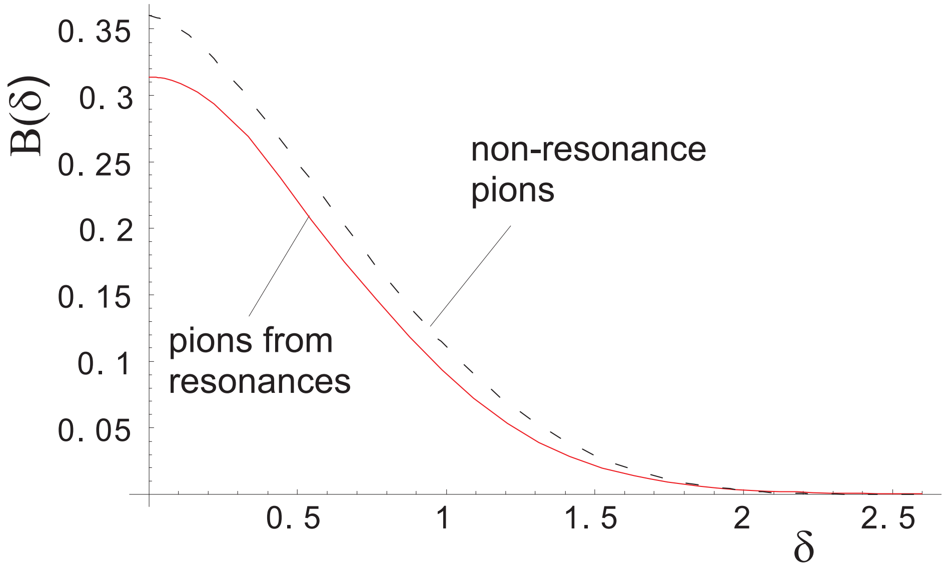
$$K_S,\ \eta,\ \eta',\ \rho^0,\ \omega,\ \sigma,\ f_0$$

2) NON-RESONANCE CONTRIBUTION (NR) other possible correlations among the charged pions

The pion balance function is constructed as a sum of the two terms

$$B(\delta, Y) = B_{\mathrm{R}}(\delta, Y) + B_{\mathrm{NR}}(\delta, Y)$$

Results

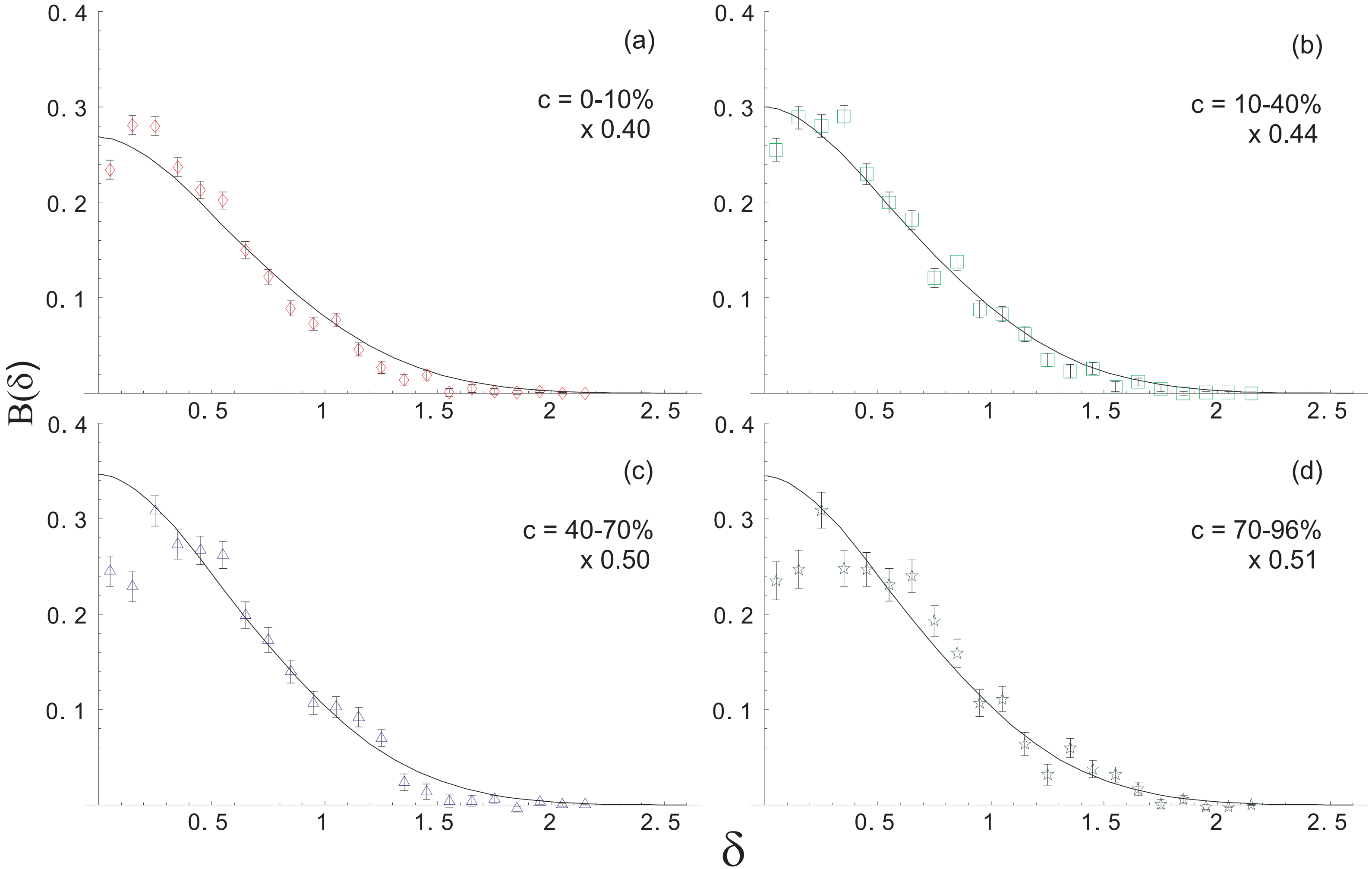


$\rho_{\text{max}}/\tau = 0.9 \rightarrow \langle \beta_\perp \rangle = 0.5$

$\langle \delta \rangle \equiv \int_{0.2}^{2.4} \delta\, B(\delta)\, d\delta / \int_{0.2}^{2.4} B(\delta)\, d\delta$

$\langle \delta \rangle_{NR} = 0.67,\ \langle \delta \rangle_{R} = 0.65,\ \langle \delta \rangle_{R+NR} = 0.66,\ (\text{exp} : 0.59 - 0.66)$

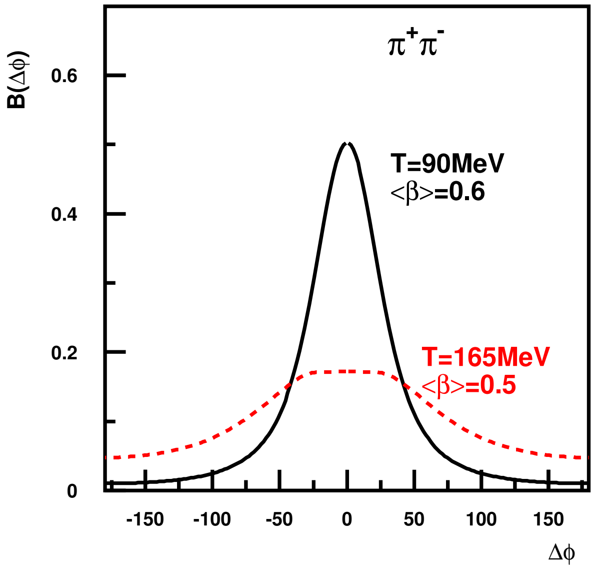
Comparison to the STAR data

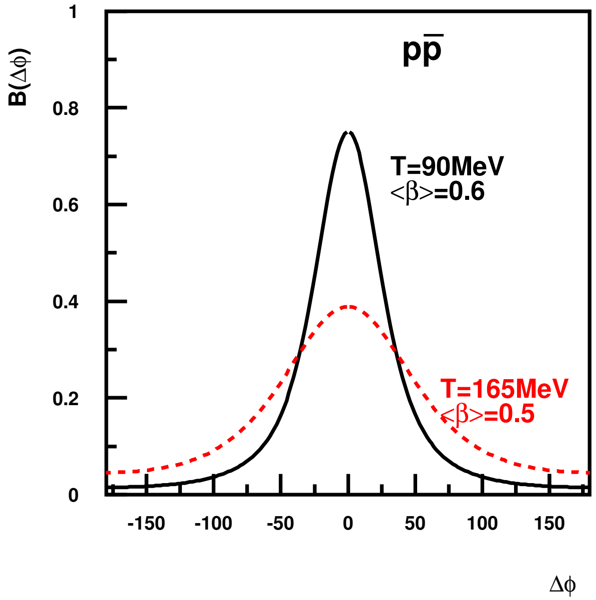


Rescaling factors (from χ^2 fits) are poor man's way of taking into account the detector acceptance and efficiency

Balancing in the azimuthal angle

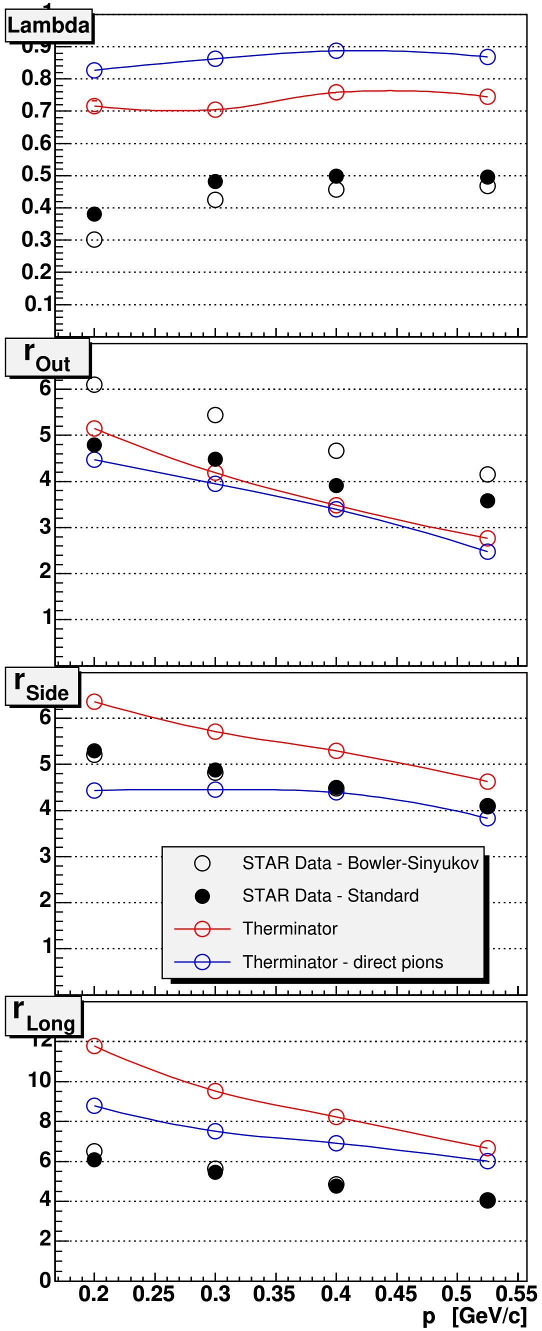
[Piotr Bożek, PLB 609 (2005) 247]
Instead of rapidity use the azimuthal angle



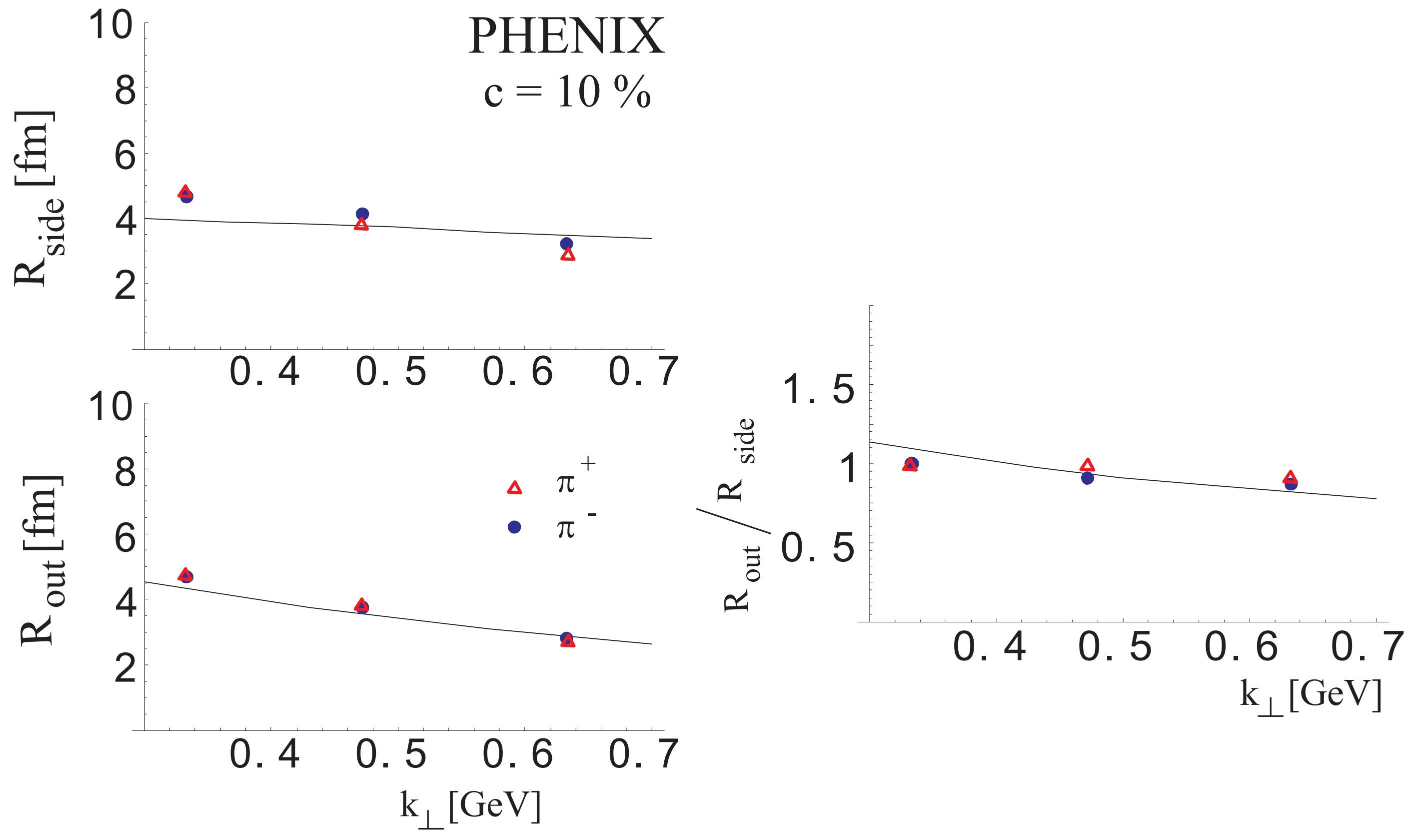


Strong dependence of the shape on the temperature!

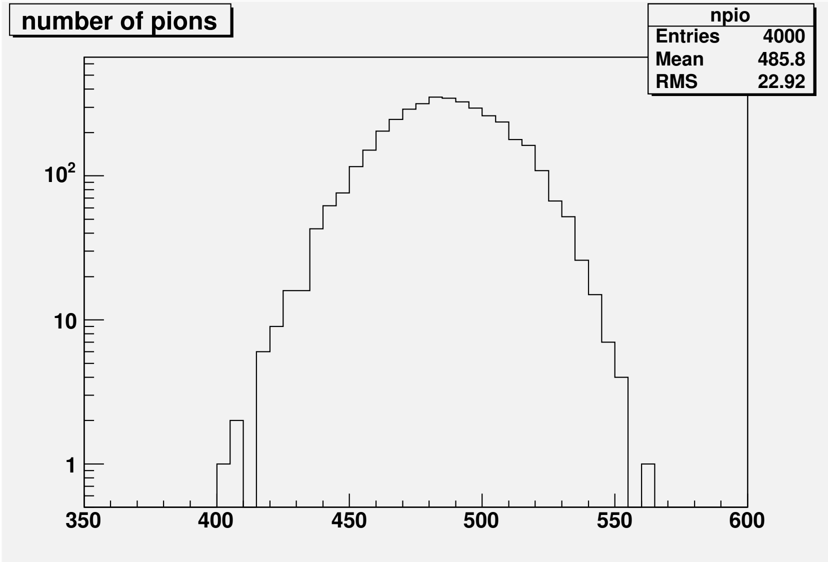
HBT from Adam Kisiel

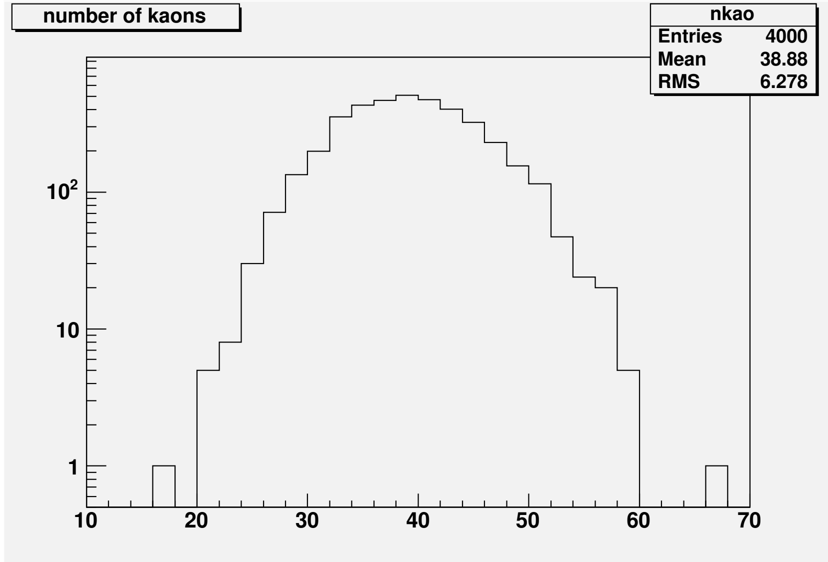


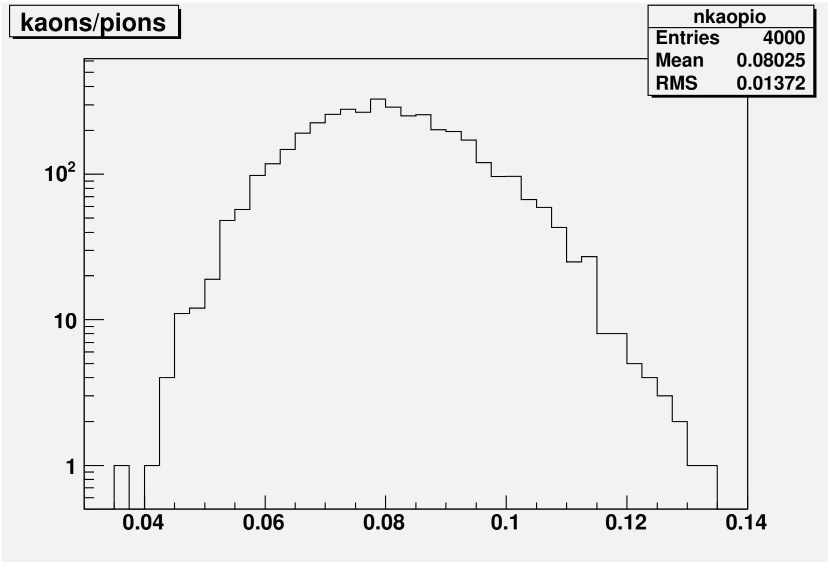
Our earlier calculation

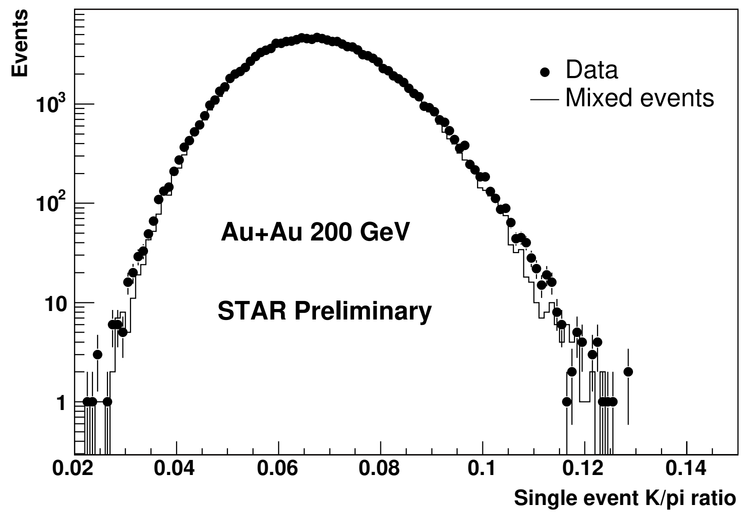


Event-by-event physics with THERMINATOR









[data from Supriya Das, STAR, Kolkata talk]

The generator gives the statistical features + correlations from the resonance decays

Summary

- SHARE,web calculator
- THERMINATOR
- Big role of resonances, a few is not enough
- Single freee-out approximation with $T \sim 165$ MeV works to the expected accuracy for spectra, also of strange particles, spectra at various centralities, spectra in rapidity, elliptic flow, pion invariant mass spectra, balance functions in rapidity, and HBT
- Failure: $R_{\rm long}$
- Difference to the BW fits: presence of resonances, no adjustment of the norm!
- Balance functions in ϕ
- Possibility to investigate event-by-event physics
- Flow velocity weakly drops from central to periferic
- Flow velocity decreases with y
- All particle flow in the same way: the PANTA REI scenario

Back-up slides

Ratios used for the thermal fit for RHIC@200

200 GeV A	Model	Experiment
π^-/π^+	1.009 ± 0.003	$1.025 \pm 0.006 \pm 0.018$ $1.02 \pm 0.02 \pm 0.10$
K^-/K^+	0.939 ± 0.008	$0.95 \pm 0.03 \pm 0.03$ $0.92 \pm 0.03 \pm 0.10$
$\overline{p}/p$	0.74 ± 0.04	$0.73 \pm 0.02 \pm 0.03$ $0.70 \pm 0.04 \pm 0.10$ 0.78 ± 0.05
$\overline{p}/\pi^-$	0.104 ± 0.010	0.083 ± 0.015
K^-/π^-	0.174 ± 0.001	0.156 ± 0.020
$\Omega/h^- \quad \times 10^3$	0.990 ± 0.120	$0.887 \pm 0.111 \pm 0.133$
$\overline{\Omega}/h^- \quad \times 10^3$	0.900 ± 0.124	$0.935 \pm 0.105 \pm 0.140$

Fugacity factors

[Rafelski, Letessier, Torrieri, Cleymans, Kampfer, Kaneta, Wheaton, Xu, ...]

The saturation factors γ are defined through

$$n(m_i, g_i; T, \Upsilon_i) = g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\Upsilon_i^{-1} \exp(\sqrt{p^2 + m_i^2}/T) \pm 1}$$

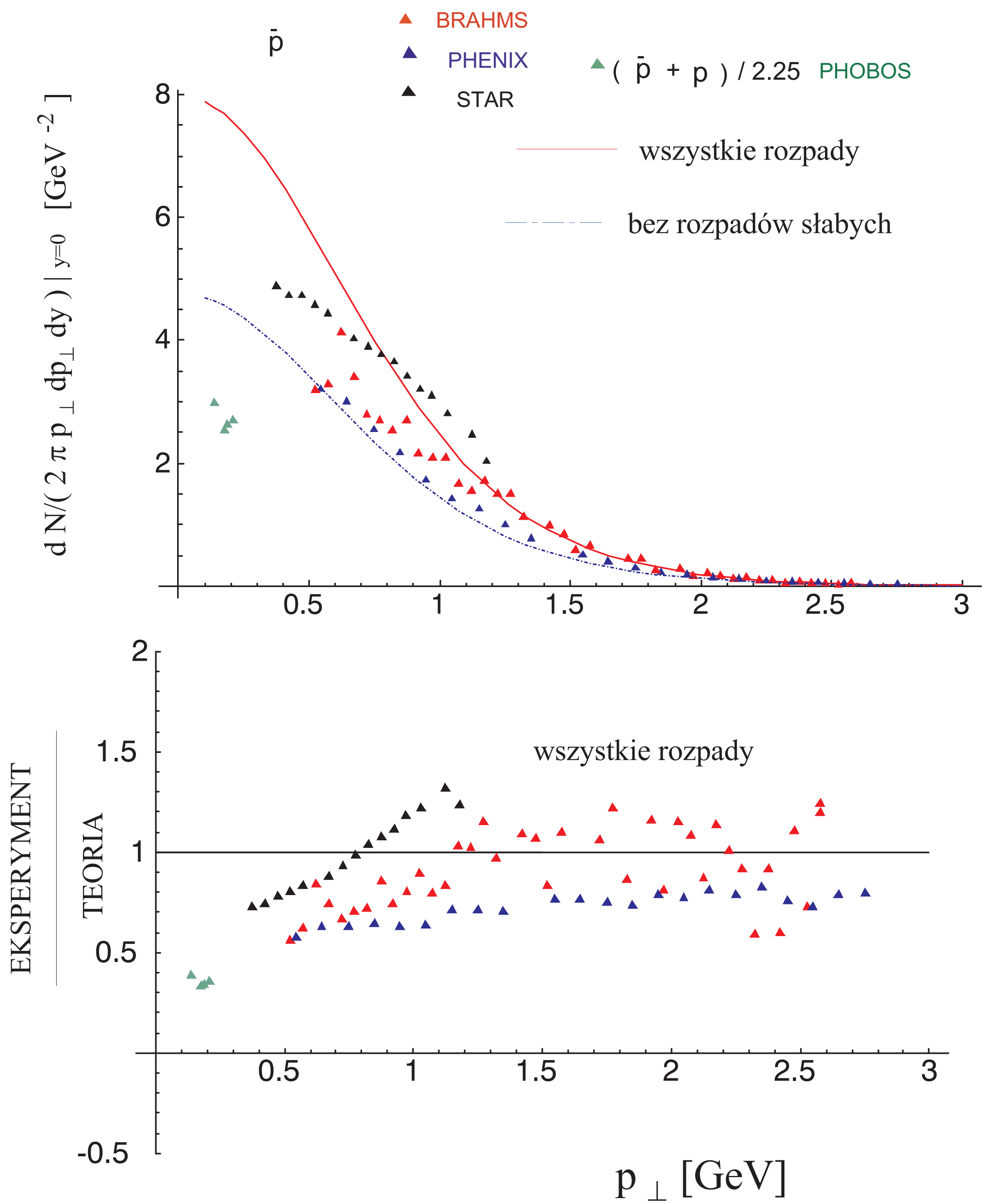
The fugacity Υ_i is defined as

$$\Upsilon_i = \lambda_{I_3^i} \left(\lambda_q \gamma_q\right)^{N_q^i} \left(\lambda_s \gamma_s\right)^{N_s^i} \left(\lambda_c \gamma_c\right)^{N_c^i} \left(\lambda_{\bar{q}} \gamma_{\bar{q}}\right)^{N_{\bar{q}}^i} \left(\lambda_{\bar{s}} \gamma_{\bar{s}}\right)^{N_{\bar{s}}^i} \left(\lambda_{\bar{c}} \gamma_{\bar{c}}\right)^{N_{\bar{c}}^i},$$

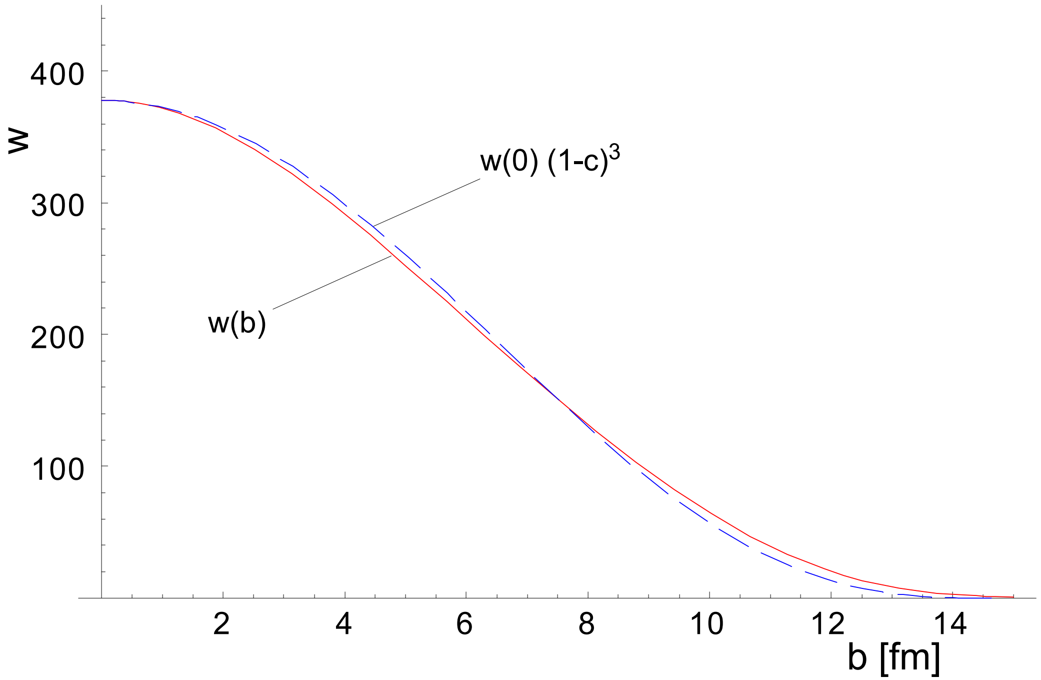
$$\lambda_q = \lambda_{\bar{q}}^{-1}, \qquad \lambda_s = \lambda_{\bar{s}}^{-1}, \qquad \lambda_c = \lambda_{\bar{c}}^{-1},$$
$$\gamma_q = \gamma_{\bar{q}}, \qquad \gamma_s = \gamma_{\bar{s}}, \qquad \gamma_c = \gamma_{\bar{c}}.$$

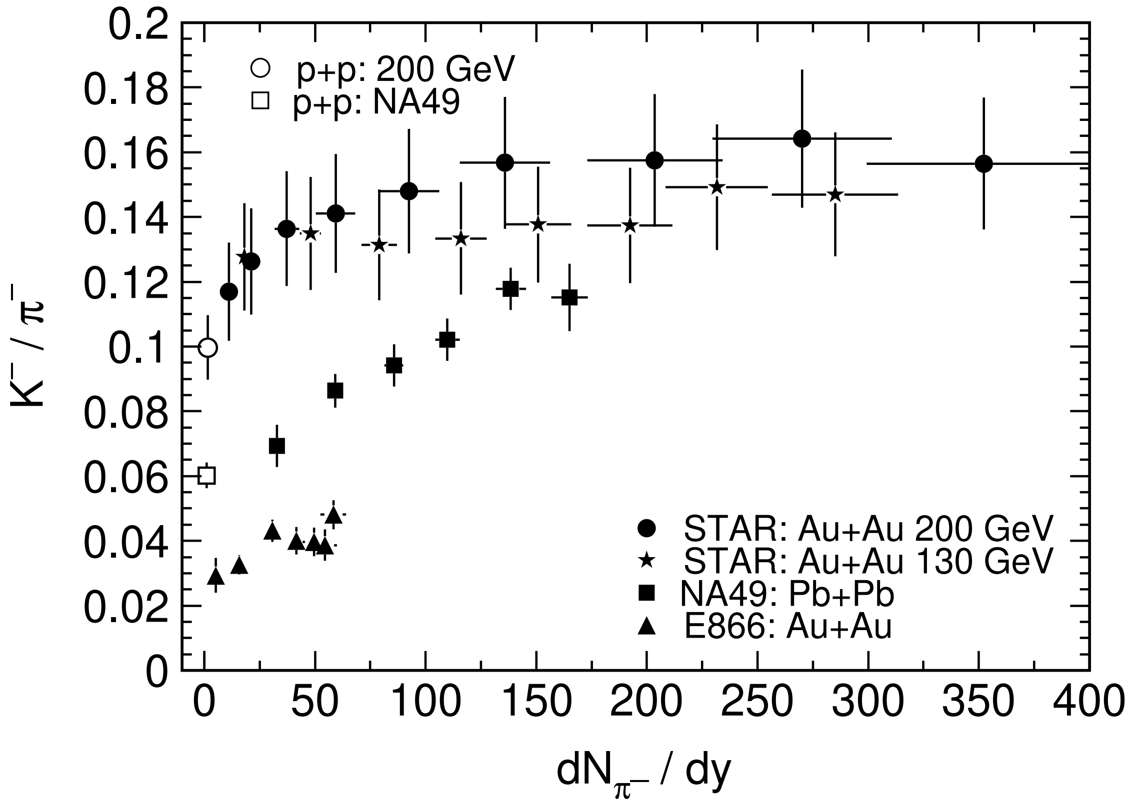
N_q^i, N_s^i and N_c^i are the numbers of light (u, d), strange (s) and charm (c) quarks in the ith hadron, and $N_{\bar{q}}^i$, $N_{\bar{s}}^i$ and $N_{\bar{c}}^i$ are the numbers of the corresponding antiquarks

We do not include γ's in our analyses


p̄
BRAHMS
PHENIX
STAR
(p̄ + p) / 2.25 PHOBOS
wszystkie rozpady
bez rozpadów słabych
d N/(2 π p ⊥ dp⊥ dy) | y=0 [GeV -2]
8
6
4
2
0.5
1
1.5
2
2.5
3
wszystkie rozpady
EKSPERYMENT
TEORIA
2
1.5
1
0.5
-0.5
0.5
1
1.5
2
2.5
3
p ⊥ [GeV]

observation: at low and moderate c the model complies to the wounded nucleon scaling

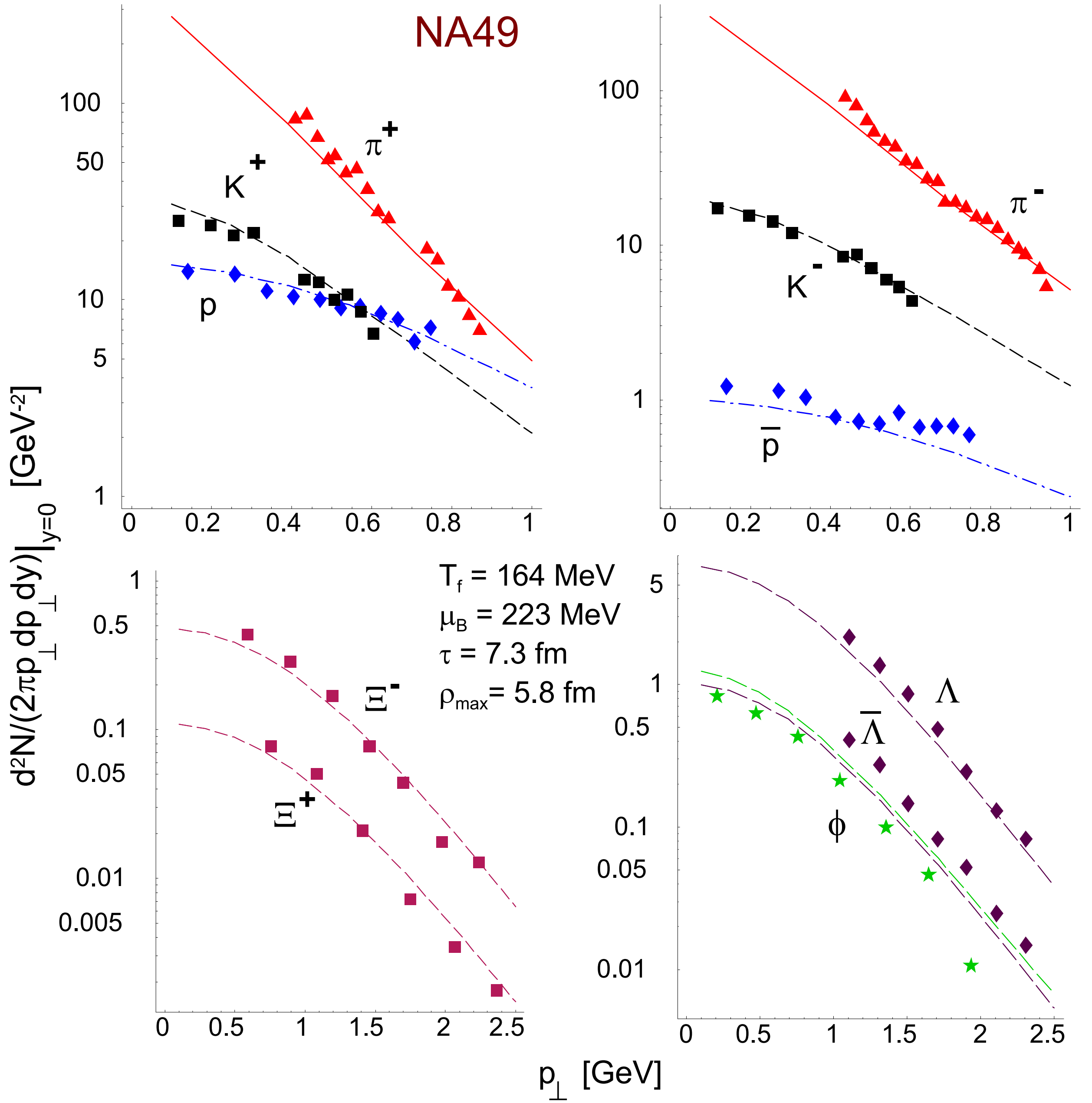




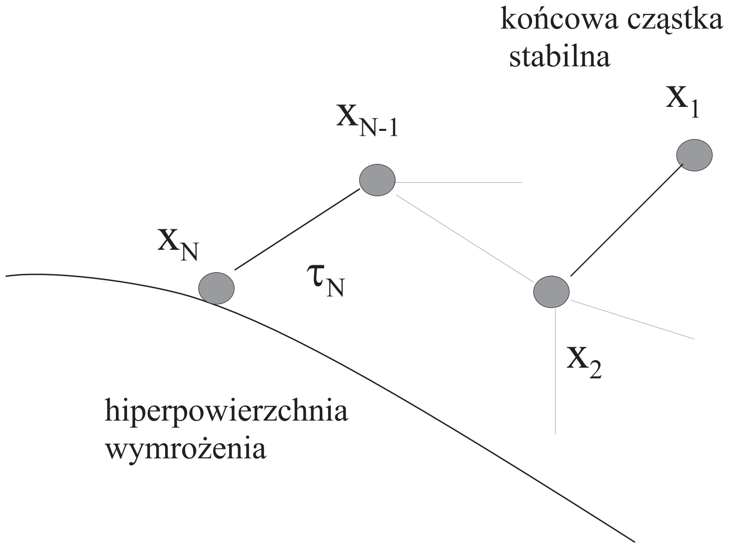
p+p: 200 GeV
p+p: NA49
STAR: Au+Au 200 GeV
STAR: Au+Au 130 GeV
NA49: Pb+Pb
E866: Au+Au
K^- / π^-
dN_{π^-} / dy

net baryon density $= 0.03$ fm^{-3}
density of baryons $= 0.12$ fm^{-3}
density of antibaryons $= 0.09$ fm^{-3}
density of $\pi^+ = 0.23$ fm^{-3}
density of all particles $= 1.3$ fm^{-3}

SPS


NA49
100
50
10
5
1
0
0.2
0.4
0.6
0.8
1
π^+
K^+
p
π^-
K^-
$\bar{p}$
$d^2N/(2\pi p_\perp dp_\perp dy)|_{y=0}$ [GeV^{-2}]
T_f = 164 MeV
μ_B = 223 MeV
τ = 7.3 fm
ρ_{max}= 5.8 fm
0.5
0.1
0.05
0.01
0.005
1.5
2
2.5
Ξ^-
Ξ^+
Λ
$\bar{\Lambda}$
ϕ
$p_\perp$ [GeV]

Semi-analytic formalism of resonance decays



$$n(x_1,p_1) = \int \frac{d^3p_2}{E_{p_2}} B(p_2,p_1) \int d\tau_2 \Gamma_2 e^{-\Gamma_2 \tau_2} \int d^4x_2 \delta^{(4)}(x_2 + \frac{p_2\tau_2}{m_2} - x_1) \times$$

$$\cdots \times \int \frac{d^3p_N}{E_{p_N}} B(p_N,p_{N-1}) \int d\tau_N \Gamma_N e^{-\Gamma_N \tau_N}$$

$$\int d\Sigma_\mu(x_N) p_N^\mu \delta^{(4)}(x_N + \frac{p_N\tau_N}{m_N} - x_{N-1}) f_N(p_N \cdot u(x_N))$$

$$B(q,k) = \frac{b}{4\pi p^*} \delta(\frac{k \cdot q}{m_R} - E^*)$$

$$E_{p_1} \frac{dN_1}{d^3p_1} = \int \frac{d^3p_2}{E_{p_2}} B(p_2,p_1) \times \ldots \times \int \frac{d^3p_N}{E_{p_N}} B(p_N,p_{N-1}) \int d\Sigma_\mu(x_N) p_N^\mu f_N[p_N \cdot u_N]$$

A technically important feature of our expansion model is that $d\Sigma^\mu \sim u^\mu$. This feature holds also in the model of Siemens and Rasmussen. In this case the treatment of the resonance is very much facilitated, since

$$E_{p_1}\frac{dN_1}{d^3p_1} = \int d\Sigma\left(x_N\right)\int\frac{d^3p_2}{E_{p_2}}B\left(p_2,p_1\right)$$

$$\dots\int\frac{d^3p_N}{E_{p_N}}B\left(p_N,p_{N-1}\right)p_N\cdot u\left(x_N\right)\ f_N\left[p_N\cdot u\left(x_N\right)\right]$$

$$= \int d\Sigma\left(x_N\right)p_1\cdot u\left(x_N\right)\ f_1\left[p_1\cdot u\left(x_N\right)\right]$$

where we have introduced the notation

$$p_{i-1}\cdot u\left(x_N\right)\ f_{i-1}\left[p_{i-1}\cdot u\left(x_N\right)\right] = \int\frac{d^3p_i}{E_{p_i}}B\left(p_i,p_{i-1}\right)p_i\cdot u\left(x_N\right)\ f_i\left[p_i\cdot u\left(x_N\right)\right]$$

In the local rest frame, the iterative procedure becomes a simple one-dimensional integral transform

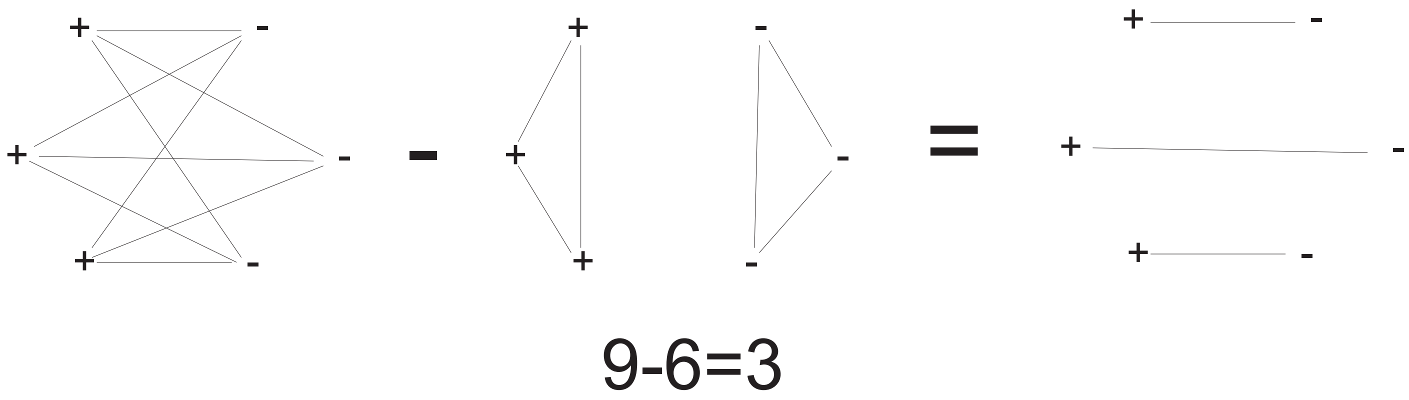
$$f_{i-1}\left(q\right) = \frac{bm_R}{2E_qp^*q}\int_{k_-(q)}^{k_+(q)}dk\,k\,f_i\left(k\right)$$

where $k_\pm(q) = m_R|E^*q \pm p^*E_q|/m_1^2$. Now the cascade decays can be done very efficiently, similarly to the calculation of the hadron abundances.

foilhead[-.5in]The Białas clusters

Because of the charge conservation, in the late-hadronization scenario, the opposite-charge particles may be treated as created from neutral clusters. In the calculation of the two-particle distributions one has to take into account that particles of the same charge must originate from different clusters, whereas the particles of opposite charge may come either from different cluster or from the same cluster (A. Białas, hep-ph/0308245)

In this case the difference of the two-particle distributions, $\rho_{+-}(p_1, p_2) - \rho_{++}(p_1, p_2)$, may be reduced to a two-particle distribution in a single cluster



What are the clusters in the thermal model?

Resonance contribution

$$\frac{dN_R^{+-}}{dy_1 dy_2} = \int dy d^2 p_\perp \int d^2 p_1^\perp \, d^2 p_2^\perp C_\pi \frac{dN_R}{dy d^2 p_\perp} \rho_{R\to\pi^+\pi^-}(p, p_1, p_2)$$

C_π indicates the kinematic cuts for the pions $(|\eta| < 1.3,\ p_\perp > 100\mathrm{MeV})$ The momentum distribution of the resonance R is obtained from the Cooper-Frye formula

$$\frac{dN_R}{dy d^2 p_\perp} = \int d\Sigma(x) \cdot p \, f_R \left(p \cdot u(x)\right)$$

where f_R is the phase-space distribution function of the resonance

The two-particle pion momentum distribution in a two-body $(\pi^+\pi^-)$ resonance decay is

$$\rho_{R\to\pi^+\pi^-} = \frac{b_{\pi\pi}}{N_2} \delta^{(4)}\left(p - p_1 - p_2\right)$$

$b_{\pi\pi}$ – the branching ratio, $N_2 = \int \frac{d^3 p_1}{E_1} \frac{d^3 p_2}{E_2} \delta^{(4)}\left(p - p_1 - p_2\right)$ – normalization

Finally,

$$B_{\mathrm{R}}(\delta) = \frac{1}{N_\pi} \sum_R \int dy_1 dy_2 C_\pi \frac{dN_R^{+-}}{dy_1 dy_2} \delta(|y_2 - y_1| - \delta)$$

Non-resonance contribution

$$\frac{dN^{+-}_{NR}}{dy_1 dy_2} = A \int d^2p_1^\perp d^2p_2^\perp C_\pi \int d\Sigma(x) p_1 \cdot u(x) f^\pi_{NR}(p_1 \cdot u(x))\, p_2 \cdot u(x) f^\pi_{NR}(p_2 \cdot u(x))$$

f^π_{NR} – phase-space distribution function of non-resonance pions

normalization constant A obtained from the condition $\int dy_2 \left(\frac{dN^{+-}_{NR}}{dy_1 dy_2} \right) = \frac{dN^\pi_{NR}}{dy_1}$

$$\tilde{B}_{\rm NR}(\delta) = \frac{1}{N_\pi} \int dy_1 dy_2 C_\pi \frac{dN^{+-}_{NR}}{dy_1 dy_2} \delta(|y_2 - y_1| - \delta)$$

R + NR contributions

$\int_0^Y d\delta B_R(\delta) = N_{\mathrm{R}}^{\pi}/N_{\pi}, \quad \int_0^Y d\delta \tilde{B}_{NR}(\delta) = N_{NR}^{\pi}/N_{\pi}$

Since some of the non-resonance pions are balanced by other charged hadrons, the final expression for the pion balance function is

$$B(\delta) = B_{\mathrm{R}}(\delta) + \frac{N_{\mathrm{NR}}^{\pi}}{N_{\mathrm{charged}} - N_{\mathrm{R}}^{\pi}} \tilde{B}_{\mathrm{NR}}(\delta)$$

From the thermal model

$N_{\mathrm{charged}} = N_{\mathrm{R}}^{\pi} + N_{\mathrm{NR}}^{\pi} + \Delta N \;\rightarrow\; N_{\mathrm{NR}}^{\pi} + \Delta N = N_{\mathrm{charged}} - N_{\mathrm{R}}^{\pi}$

$N_{NR}^{\pi}/(N_{\mathrm{charged}} - N_{\mathrm{R}}^{\pi}) = 0.68$