Exploiting Algebraic Dependences Between Local Partial Derivatives in Jacobian Accumulation [*]

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We introduce a novel undirected graph model for the optimal Jacobian accumulation problem. This new graph addresses the need illustrated in [1] for accumulation techniques capable of exploiting algebraic dependences between the local partial derivatives.

Let $\mathbf{G}$ be a linearized computational graph (LCG). The corresponding *path-clique graph* (CPG) $\mathbf{G}^p$ has vertex set $V^p = E \in \mathbf{G}$; two vertices $i, j \in V^p$ are adjacent iff their corresponding edges in $\mathbf{G}$ lie on a common path. We allow two vertices to be *merged* if their respective labels are identified as equivalent.

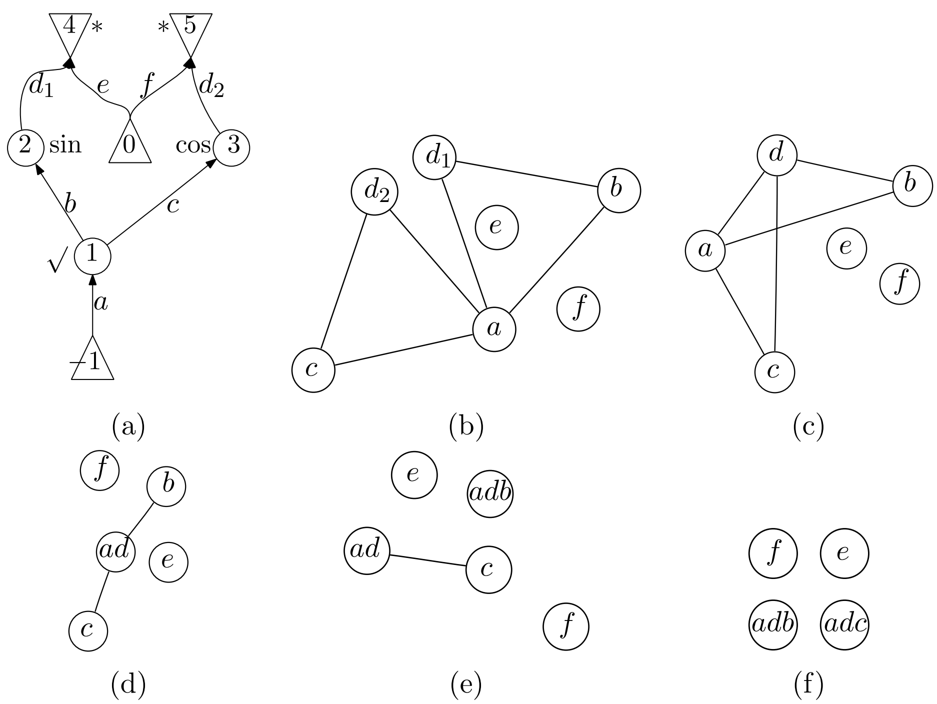

Figure 1: An example LCG $\mathbf{G}$ (a), the corresponding CPG $\mathbf{G}^p$ (b), $\mathbf{G}^p$ after vertices d_1 and d_2 have been merged (c), $\mathbf{G}^p$ after the contraction of edges $\{a, d\}$ (d), $\{ad, b\}$ (e), and $\{ad, c\}$ (f).

A sequence of *edge contraction* operations reduces a CPG to an edgeless graph whose vertices yield the values of Jacobian entries. Each edge contraction implies a single *fused multiply-add* operation in the generated derivative code.

References

[1] Uwe Naumann. Optimal Jacobian accumulation is np-complete. *Math. Prog.* Online First.

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