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RHIC PROJECT

Brookhaven National Laboratory

**Transmission Line Analysis of
Dielectric-Loaded Ferrite Kicker**

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TRANSMISSION LINE ANALYSIS OF DIELECTRIC-LOADED FERRITE KICKER (Anisotropic Approximation)

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Introduction

The RHIC injection kicker is constructed as a C-shaped ferrite kicker in which the ferrite blocks are spaced apart and the interstice is filled with high-permittivity ceramic.¹ Due to this ferrite-dielectric composite structure, the kicker behaves like a transmission line rather than a lumped inductor.²

The RHIC kicker cross section is shown in Fig. 1. In the beam direction, the 15 ferrite layers are each $w^F = 5$ cm and the 14 ceramic layers each $w^D = 2.5$ cm thick, for a total length of 1.1 m. The kicker aperture is $2a = 48.4$ mm and ferrite/dielectric thickness $t = 13.9$ mm. The conductors are shown as heavy solid lines.

The electric properties of the kicker, notably characteristic impedance and propagation velocity, have been analyzed by Forsyth and his colleagues, Claus and Zhang, as a low pass filter with lumped L and C elements. Claus has also made a transmission line analysis assuming uniform isotropic properties of the ferrite/dielectric layers.³ In the present paper, the kicker will be considered a transmission line with anisotropic medium in order to correctly describe the layered ferrite/ceramic structure. This treatment is, of course, valid only at low frequencies where the wavelength is much larger than a ferrite/dielectric cell length but the skin depth is smaller than the conductor thickness. The anisotropic approximation will be compared with expressions obtained by treating the kicker as a periodically loaded transmission line. The results will indicate that, in practice, the anisotropic approximation in the long-wavelength limit represents an adequate kicker description.

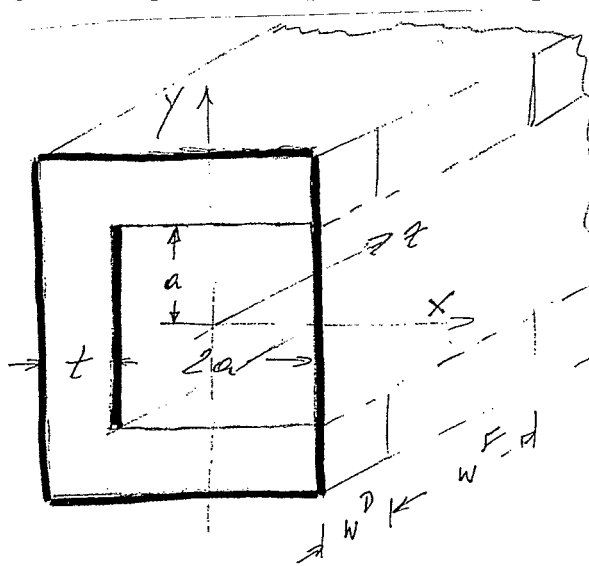


Fig. 1

The Field Expressions

A full waveguide analysis of the kicker, even in the anisotropic approximation, is impractical, and the simplified model shown in Fig. 2 will be used for the present study. The model neglects details of the top and bottom regions, and thus assumes that the inductance is primarily determined by the kicker aperture and the capacitance by the straight return path. Essentially, the same model has been used by Forsyth to define the basic kicker parameters and to choose the dielectric constant of the ceramic to achieve the desired $\sim 25 \Omega$ characteristic impedance.

The expressions for the time harmonic electric and magnetic fields propagating in the (opposite) direction of the beam are given by (natural units $c = \mu_0 = 1$, time-dependence $e^{j\omega t}$ suppressed)

- in the aperture

$$\begin{aligned} E_x &= B_0 \frac{\omega}{\kappa} \cosh \Upsilon y e^{-j\kappa z} \\ H_y &= B_0 \cosh \Upsilon y e^{-j\kappa z} \\ H_z &= -j B_0 \frac{\Upsilon}{\kappa} \sinh \Upsilon y e^{-j\kappa z} \end{aligned}$$

with $\Upsilon^2 = \kappa^2 - \omega^2$

- and in the ferrite/dielectric composite

$$\begin{aligned} E_x^C &= q^C \frac{\omega}{\kappa} \cos \eta^C (y - h) e^{-j\kappa z} \\ H_y^C &= q^C \frac{1}{\mu_T} \cos \eta^C (y - h) e^{-j\kappa z} \\ H_z^C &= j q^C \frac{\eta^F}{\mu_z \kappa} \sin \eta^C (y - h) e^{-j\kappa z} \end{aligned}$$

with q^C a constant still to be determined and

$$\eta^{C2} = \epsilon_T \mu_z \omega^2 - \frac{\mu_z}{\mu_T} \kappa^2$$

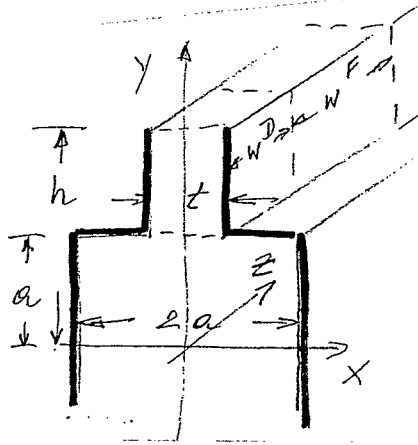


Fig. 2

The ferrite/dielectric composite has ferrite layers, w^F thick with μ^F and ϵ^F , and dielectric layers, w^D thick with ϵ^D and $\mu^D = 1$. In the long-wavelength limit, i.e. $\kappa w^F, \kappa w^D \ll 1$, the composite is quasi-uniform with anisotropic properties, given by ⁴

$$\begin{aligned}\epsilon_T &= \frac{\epsilon^D w^D + \epsilon^F w^F}{w^D + w^F} \\ \mu_T &= \frac{\mu^F w^F + w^D}{w^D + w^F} \\ \mu_z &= \frac{\mu^F (w^D + w^F)}{\mu^F w^D + w^F}\end{aligned}$$

For the RHIC kicker one has

$$w^F = 5 \text{ cm}, \mu^F \approx 1500, \epsilon^F \approx 10, w^D = 2.5 \text{ cm}, \epsilon^D \approx 100$$

resulting in

$$\epsilon_T \approx 40, \mu_T \approx 1000, \mu_z \approx 3.$$

The amplitude constant q^C is determined by the voltage condition at $y = a$

$$\int_{-a}^a E_x dx = \int_{-t/2}^{t/2} E_x^C dx$$

leading to

$$q^C = \frac{2a}{t} \frac{\cosh \Upsilon a}{\cos \eta^C (h - a)} B_0$$

The dispersion relation $\omega = \omega(\kappa)$ is obtained from continuity of the transverse power flow at $y = a$

$$\int_{-a}^a E_x H_z^* dx = \int_{-t/2}^{t/2} E_x^C H_z^{C*} dx$$

leading to

$$\Upsilon \tanh \Upsilon a = \frac{2a}{\mu_z t} \eta^C \operatorname{tg} \eta^C (h - a)$$

In the low frequency limit, where $\tanh x \approx \operatorname{tg} x \approx x$, one obtains

$$\kappa^2 = \omega^2 \frac{\epsilon_T \frac{2a}{t} \left(\frac{h}{a} - 1\right) + 1}{1 + \frac{1}{\mu_T} \frac{2a}{t} \left(\frac{h}{a} - 1\right)} \approx \omega^2 \epsilon_T \frac{2a}{t} \left(\frac{h}{a} - 1\right)$$

and

$$q^C = \frac{2a}{t} B_0$$

with the wave propagation velocity

$$v/c = \frac{\omega}{\kappa} \approx \sqrt{\frac{t}{2a\epsilon_T \left(\frac{h}{a} - 1\right)}}$$

The Transmission Line Expressions

The transmission line representation of waveguide propagation is very convenient, but the definitions of voltage, current and impedance somewhat arbitrary provided only that the power flow is given by the product voltage \times current.⁵ In order to establish a correlation with measurable quantities, the current is here defined by the integral

$$\begin{aligned} I &= 2 \int_0^a H_y dy + 2 \int_a^h H_y^C dy \\ &= 2 \left\{ \frac{\sinh \Upsilon a}{\Upsilon} + \frac{q^C}{\mu_T B_0} \frac{\sin \eta^C (h-a)}{\eta^C} \right\} B_0 \end{aligned}$$

In the low frequency limit

$$I \approx 2a \left\{ 1 + \frac{2a}{\mu_T t} \left(\frac{h}{a} - 1 \right) \right\} B_0$$

The characteristic impedance now follows from the definition involving power flow and current

$$Z_c = \frac{P_z}{I^* I} = \frac{\int E_x H_y^* dS}{I^* I}$$

The power flow is given by

$$P_z = 4a^2 \frac{\omega}{\kappa} B_0^2 \left\{ \left[\frac{1}{2} + \frac{\sinh 2\Upsilon a}{4\Upsilon a} \right] + \frac{t}{2a\mu_T} \left(\frac{h}{a} - 1 \right) \frac{q^{C2}}{B_0^2} \left[\frac{1}{2} + \frac{\sin 2\eta^C (h-a)}{4\eta^C (h-a)} \right] \right\}$$

In the low frequency limit

$$P_z = 4a^2 \frac{\omega}{\kappa} B_0^2 \left\{ 1 + \frac{2a}{\mu_T t} \left(\frac{h}{a} - 1 \right) \right\}$$

and the characteristic impedance in the low frequency limit

$$Z_c = \sqrt{\frac{1}{\{\epsilon_T \frac{2a}{t} (\frac{h}{a} - 1) + 1\} \{1 + \frac{2a}{\mu_T t} (\frac{h}{a} - 1)\}}} \approx \sqrt{\frac{t}{2a\epsilon_T (h/a - 1)}}$$

The geometrical quantities determining the characteristic impedance are well defined with the exception of h , which represents the effective length of the return path and thus includes the details of the top and bottom region. Since Z_c depends primarily on the transverse dielectric constant, a choice of h which best renders the low frequency capacitance is indicated. A plausible value is $h \approx 2a + t$.

For the RHIC kicker one finds in SI units ($Z_0 = c\mu_0 \approx 377 \Omega$)

$$\frac{v}{c} \approx \frac{1}{14.8}$$

and

$$Z_c \approx \frac{377}{14.8} \Omega = 25.5 \Omega$$

in reasonable agreement with the measured results⁶ of 24.5 Ω .

The full expressions for the dispersion relation and the characteristic impedance were programmed, including the frequency dependence of μ ,

$$\mu \sim \frac{1500}{\sqrt{1 + (f/2 \text{ GHz})^2}}$$

The frequency dependence of Z_c was found to be negligible in the range over which the low frequency approximation can be expected to remain valid (i.e. up to ~ 10 MHz).

Equivalent LC Transmission Line

A transmission line with uniform inductance L and capacitance C per unit length has a wave impedance

$$Z_c = \sqrt{\frac{L}{C}}$$

and a wave propagation velocity

$$v = \frac{1}{\sqrt{LC}}$$

It is therefore possible to interpret the above anisotropic waveguide results for the kicker in terms of an equivalent uniform LC transmission line. In order to allow a comparison with static estimates, the following expressions for the inductance and capacitance of one cell (one ferrite plus one dielectric layer) have been obtained in the low frequency limit,

$$L_{cell} = (w^F + w^D) \frac{1 + \frac{2a}{\mu_T t}}{\left\{1 + \frac{2a}{\mu_T t} \left(\frac{h}{a} - 1\right)\right\}^2} \approx w^F + w^D$$

and

$$C_{cell} = (w^F + w^D) \left\{ \frac{2a\epsilon_T}{t} \left(\frac{h}{a} - 1\right) + 1 \right\} \frac{1 + \frac{2a}{\mu_T t} \left(\frac{h}{a} - 1\right)}{1 + \frac{2a}{\mu_T t}} \approx (w^F + w^D) \frac{2a\epsilon_T}{t} \left(\frac{h}{a} - 1\right)$$

with the numerical results $L \approx 94$ nH and $C \approx 146$ pF where again $h = 2a + t$ was taken.

Periodically Loaded Transmission Line

An alternate description of the kicker is obtained by treating it as a periodically loaded transmission line, where one cell consisting of $\frac{1}{2}$ ferrite layer + 1 dielectric layer + $\frac{1}{2}$ ferrite layer represents the basic period. The cascade matrix of one cell has the general form

$$\begin{pmatrix} u_{in} \\ i_{in} \end{pmatrix} = \begin{pmatrix} \cos \Phi & jZ_c \sin \phi \\ jZ_c^{-1} \sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix} u_{out} \\ i_{out} \end{pmatrix}$$

where the phase shift per cell

$$\kappa w = \phi = \phi(\omega)$$

with the propagation constant κ as defined above. By treating each layer as a section of a transmission line, one can derive the expressions for the phase shift per cell

$$\cos \phi = \cos \kappa^D w^D \cos \kappa^F w^F - \frac{1}{2} \left(\frac{Z^F}{Z^D} + \frac{Z^D}{Z^F} \right) \sin \kappa^D w^D \sin \kappa^F w^F$$

and for the characteristic impedance

$$Z_c^2 = Z^D Z^F \frac{Z^D \sin \frac{1}{2} \kappa^D w^D \cos \frac{1}{2} \kappa^F w^F + Z^F \sin \frac{1}{2} \kappa^F w^F \cos \frac{1}{2} \kappa^D w^D}{Z^D \sin \frac{1}{2} \kappa^F w^F \cos \frac{1}{2} \kappa^D w^D + Z^F \sin \frac{1}{2} \kappa^D w^D \cos \frac{1}{2} \kappa^F w^F}$$

Applying the above anisotropic results to uniform transmission line sections, one finds $\kappa^\% = \kappa^\%(\omega)$ from the condition

$$\Upsilon^\% \tanh \Upsilon^\% a = \frac{2a}{\mu^\% t} \eta^\% \operatorname{tg} \eta^\% (h - a)$$

with

$$\begin{aligned} \Upsilon^\%{}^2 &= \kappa^\%{}^2 - \omega^2 \\ \eta^\%{}^2 &= \epsilon^\% \mu^\% \omega^2 - \kappa^\%{}^2 \end{aligned}$$

and $\%$ representing either a dielectric D or ferrite F section. The characteristic impedance $Z^\%$ is given by

$$Z^\% = \frac{P^\%}{I^\%{}^2}$$

with

$$\begin{aligned} P^\% &= 4a^2 \frac{\omega}{\kappa^\%} \left\{ \left[\frac{1}{2} + \frac{\sinh 2\Upsilon^\% a}{4\Upsilon^\% a} \right] + \frac{t}{2a\mu^\%} \left(\frac{h}{a} - 1 \right) q^\%{}^2 \left[\frac{1}{2} + \frac{\sin 2\eta^\% (h - a)}{4\eta^\% (h - a)} \right] \right\} \\ I^\% &= 2 \left\{ \frac{\sinh \Upsilon^\% a}{\Upsilon^\%} + \frac{q^\% \sin \eta^\% (h - a)}{\mu^\% \eta^\%} \right\} \end{aligned}$$

and

$$q^\% = \frac{2a}{t} \frac{\cosh \Upsilon^\% a}{\cos \eta^\% (h - a)}$$

The above expressions were programmed and numerical results were obtained for the RHIC kicker parameters, assuming the frequency dependence of μ given above. At low frequencies (< 10 MHz) the results obtained are

- for the uniform ferrite section

$$Z^F \approx 50.3 \, \Omega \text{ and } v/c \approx 1/7.5$$

- for the uniform dielectric section

$$Z^D \approx 6.3 \, \Omega \text{ and } v/c \approx 1/9.2$$

- and for the ferrite-dielectric composite structure

$$Z_c \approx 21.5 \Omega \text{ and } v/c \approx 1/12.6$$

in reasonable agreement with the anisotropic results. At higher frequencies the results start to differ, with the present solution showing a low-pass cutoff frequency of ~ 107 MHz and a much lower cutoff frequency of ~ 17 MHz if $\mu \approx \text{const}$ is assumed.

References

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