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NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
NATIONAL WEATHER SERVICE
NATIONAL METEOROLOGICAL CENTER

OFFICE NOTE 188

Real Data Experiments with a Fourth Order Version of the Operational Seven-Layer Model

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1. Introduction

There are several techniques available which reduce space truncation error in grid point atmospheric models. Two of them have been used in the seven-layer primitive equation model (7L PE) at NMC. The first one uses a finer grid mesh to obtain more accurate resolution of meteorological waves--the smaller grid spacing also makes the finite differences closer approximations to the continuous equations. By changing grid distance from 1 bedient (381 km at 60°N) to ½ bedient, the operational hemispheric 7L PE model has successfully reduced the effects of spatial truncation error--translational speeds of meteorological waves have been improved and both locked-in error and cross contour flow problems have been reduced.

An alternative to the fine-mesh technique is use of higher order, more accurate, finite difference approximations to the continuous equations, while keeping the same grid size. Application of fourth-order finite differences to the advective terms in a semi-implicit version of the "old" 6L PE (1 bedient grid) has been quite successful in reducing spatial truncation error (Campana, 1977). If the fourth-order scheme can duplicate the success of the fine-mesh technique, then its computational efficiency becomes attractive for NMC's operational environment. Of course the 'saved' time might be used for better (more complex) physical parameterizations.

The 7L PE has been changed to a fourth-order model and five real data experiments have been made with it on a 1-bedient grid. The purpose of this note is to discuss the form of the fourth-order scheme and then show comparisons of the experimental model with the operational 7L PE (½ bedient). Both statistical verifications and Varian maps for 48-hour forecasts are presented.

2. Finite Differencing

The 7L PE equations are

$$\frac{\partial u}{\partial t} = v[f + m^2(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})] - \frac{\partial E}{\partial x} - \widetilde{[\frac{\partial \phi}{\partial x} + c_p\theta\frac{\partial \pi}{\partial x}]} - \dot{\sigma}\frac{\partial u}{\partial \sigma} + F \qquad (2.1)$$

$$\frac{\partial v}{\partial t} = - u[f + m^2(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y})] - \frac{\partial E}{\partial y} - \widetilde{[\frac{\partial \phi}{\partial y} + c_p\theta\frac{\partial \pi}{\partial y}]} - \dot{\sigma}\frac{\partial v}{\partial \sigma} + F \qquad (2.2)$$

$$\frac{\partial \theta}{\partial t} = - m^2(u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}) - \dot{\sigma}\frac{\partial \theta}{\partial \sigma} + Q \qquad (2.3)$$

$$\frac{\partial w}{\partial t} = - m^2[u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})] - \dot{\sigma}\frac{\partial w}{\partial \sigma} + C \qquad (2.4)$$

$$\frac{\partial p_\sigma}{\partial t} = - m^2[\overline{u}\frac{\partial p_\sigma}{\partial x} + \overline{v}\frac{\partial p_\sigma}{\partial y} + p_\sigma\overline{(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})}] \qquad (2.5)$$

$$E \equiv \tfrac{1}{2}(u^2 + v^2),\ u \equiv \frac{u}{m},\ v \equiv \frac{v}{m} \qquad (2.6)$$

where u and v are the horizontal wind components; θ is potential temperature, w is precipitable water; p_σ is pressure thickness in one of two σ domains; f is the Coriolis parameter; m is the map factor; and π is the Exner function $(\frac{p}{1000})^{R/c_p}$. Additionally, F denotes frictional effects, Q is diabatic heating, and C is condensation and evaporation. The overriding bar, $(\overline{\ })$, in (2.5) represents a σ domain mean value, whereas the $\widetilde{(\ )}$ in (2.1) and (2.2) signifies pressure gradient averaging (Brown and Campana, 1978).

Neglecting F, Q, and C, the finite-difference forms of (2.1) through (2.5), using Shuman's (1968) notation, are:

$$\frac{\partial u}{\partial t} = \overline{\overline{v}^{xy}[\overline{f}^{xy} + \overline{m^2}^{xy}(\overline{v}^y_x - \overline{u}^x_y)] - \overline{E}^y_x - \widetilde{[\overline{\phi}^y_x + c_p\overline{\theta}^{xy}\overline{\pi}^y_x]} - \overline{(\dot{\sigma}\ \overline{u_\sigma}^{xy})}^{\sigma}}^{xy} \qquad (2.7)$$

$$\frac{\partial v}{\partial t} = \overline{- \overline{u}^{xy}[\overline{f}^{xy} + \overline{m^2}^{xy}(\overline{v}^y_x - \overline{u}^x_y)] - \overline{E}^x_y - \widetilde{[\overline{\phi}^x_y + c_p\overline{\theta}^{xy}\overline{\pi}^x_y]} - \overline{(\dot{\sigma}\ \overline{v_\sigma}^{xy})}^{\sigma}}^{xy} \qquad (2.8)$$

$$\frac{\partial \theta}{\partial t} = - \overline{\overline{m^2}^{xy} \left(\overline{u}^{xy}\,\overline{\theta_x}^{y} + \overline{v}^{xy}\,\overline{\theta_y}^{x}\right) - \overline{\left(\dot{\sigma}\;\overline{\theta_\sigma}^{xy}\right)}^{\sigma}}^{xy} \tag{2.9}$$

$$\frac{\partial w}{\partial t} = - \overline{\overline{m^2}^{xy} \left[\overline{u}^{xy}\,\overline{w_x}^{y} + \overline{v}^{xy}\,\overline{w_y}^{x} + \overline{w}^{xy}\left(\overline{u_x}^{y} + \overline{v_y}^{x}\right)\right] - \overline{\left(\dot{\sigma}\;\overline{w_\sigma}^{xy}\right)}^{\sigma}}^{xy} \tag{2.10}$$

$$\frac{\partial p_\sigma}{\partial t} = - \overline{\overline{m^2}^{xy} \left[\overline{u}^{xy}\,\overline{p_{\sigma_x}}^{y} + \overline{v}^{xy}\,\overline{p_{\sigma_y}}^{x} + \overline{p_\sigma}^{xy}\left(\overline{u_x}^{y} + \overline{v_y}^{x}\right)\right]}^{xy} \tag{2.11}$$

Eqns. (2.7)-(2.11) represent the second-order form of the 7L PE.

The fourth-order forms developed by Gerrity, McPherson, and Polger (1972) have been incorporated into the 7L PE in several ways. The fourth-order equivalent of $\overline{(\ )}^{xy}$, $\overline{(\ )_x}^{y}$, and $\overline{(\ )_y}^{x}$ in (2.7)-(2.11) are shown in detail in Office Note 163, and they are denoted by $\overline{(\ )}^{x_h y_h}$, $\overline{(\ )_{x_h}}^{y_h}$, and $\overline{(\ )_{y_h}}^{x_h}$ in the rest of this paper. Three versions of the fourth-order 7L PE which were tested are described below:

1. Version #1 - Fourth order on the tendencies only (i.e., the overriding $\overline{(\ )}^{xy}$ in (2.7)-(2.11)). Second order on all other finite differences.

2. Version #2 - Fourth order on the horizontal advection terms as well as the overriding $\overline{(\ )}^{xy}$ from version #1. Noting that the advection terms for the u and v equations are imbedded in the vorticity and E terms, (2.7)-(2.11) become:

$$\frac{\partial u}{\partial t} = \overline{\overline{v}^{xy}\left[\overline{f}^{xy} + \overline{m^2}^{xy}\left(\overline{v_{x_h}}^{y_h} - \overline{u_{y_h}}^{x_h}\right)\right] - \overline{E_{x_h}}^{y_h} - \left[\overline{\phi_x}^{y} + c_p\overline{\theta}^{xy}\,\overline{\pi_x}^{y}\right] - \overline{\left(\dot{\sigma}\;\overline{u_\sigma}^{xy}\right)}^{\sigma}}^{x_h y_h} \tag{2.12}$$

$$\frac{\partial v}{\partial t} = \overline{-\overline{u}^{xy}[\overline{f}^{xy} + \overline{m^2}^{xy}(\overline{v}^{y_h}_{x_h} - \overline{u}^{x_h}_{y_h})] - \overline{E}^{x_h}_{y_h} - [\overline{\phi}^{x}_{y} + c_p \overline{\theta}^{xy}\overline{\pi}^{x}_{y}] - \overline{(\dot{\sigma}\, \overline{v}^{xy}_{\sigma})}^{\sigma}}^{x_h y_h} \tag{2.13}$$

$$\frac{\partial \theta}{\partial t} = -\overline{\overline{m^2}^{xy}(\overline{u}^{xy}\overline{\theta}^{y_h}_{x_h} + \overline{v}^{xy}\overline{\theta}^{x_h}_{y_h}) - \overline{(\dot{\sigma}\, \overline{\theta}^{xy}_{\sigma})}^{\sigma}}^{x_h y_h} \tag{2.14}$$

$$\frac{\partial w}{\partial t} = -\overline{\overline{m^2}^{xy}[\overline{u}^{xy}\overline{w}^{y_h}_{x_h} + \overline{v}^{xy}\overline{w}^{x_h}_{y_h} + \overline{w}^{xy}(\overline{u}^{y}_{x} + \overline{v}^{x}_{y})] - \overline{(\dot{\sigma}\, \overline{w}^{xy}_{\sigma})}^{\sigma}}^{x_h y_h} \tag{2.15}$$

$$\frac{\partial p_\sigma}{\partial t} = -\overline{\overline{m^2}^{xy}[\overline{u}^{xy}\overline{p_\sigma}^{y_h}_{x_h} + \overline{v}^{xy}\overline{p_\sigma}^{x_h}_{y_h} + \overline{p_\sigma}^{xy}(\overline{u}^{y}_{x} + \overline{v}^{x}_{y})]}^{x_h y_h} \tag{2.16}$$

This version is the closest to the form used successfully in the semi-implicit 6L PE tests (Office Note 163).

3. Version #3 - Fourth order on all horizontal differences and averages, except the map factor squared (m^2). Equations (2.12) through (2.16) become:

$$\frac{\partial u}{\partial t} = \overline{\overline{v}^{x_h y_h}[\overline{f}^{x_h y_h} + \overline{m^2}^{xy}(\overline{v}^{y_h}_{x_h} - \overline{u}^{x_h}_{y_h})] - \overline{E}^{y_h}_{x_h} - [\overline{\phi}^{y_h}_{x_h} + c_p \overline{\theta}^{x_h y_h}\overline{\pi}^{y_h}_{x_h}] - \overline{(\dot{\sigma}\, \overline{u}^{x_h y_h}_{\sigma})}^{\sigma}}^{x_h y_h} \tag{2.17}$$

$$\frac{\partial v}{\partial t} = -\overline{\overline{u}^{x_h y_h}[\overline{f}^{x_h y_h} + \overline{m^2}^{xy}(\overline{v}^{y_h}_{x_h} - \overline{u}^{x_h}_{y_h})] - \overline{E}^{x_h}_{y_h} - [\overline{\phi}^{x_h}_{y_h} + c_p \overline{\theta}^{x_h y_h}\overline{\pi}^{x_h}_{y_h}] - \overline{(\dot{\sigma}\, \overline{v}^{x_h y_h}_{\sigma})}^{\sigma}}^{x_h y_h} \tag{2.18}$$

$$\frac{\partial \theta}{\partial t} = -\overline{\overline{m^2}^{xy}(\overline{u}^{x_h y_h}\overline{\theta}^{y_h}_{x_h} + \overline{v}^{x_h y_h}\overline{\theta}^{x_h}_{y_h}) - \overline{(\dot{\sigma}\, \overline{\theta}^{x_h y_h}_{\sigma})}^{\sigma}}^{x_h y_h} \tag{2.19}$$

$$\frac{\partial w}{\partial t} = -\overline{\overline{m^2}^{xy}[\overline{u}^{x_h y_h}\overline{w}^{y_h}_{x_h} + \overline{v}^{x_h y_h}\overline{w}^{x_h}_{y_h} + \overline{w}^{x_h y_h}(\overline{u}^{y_h}_{x_h} + \overline{v}^{x_h}_{y_h})] - \overline{(\dot{\sigma}\, \overline{w}^{x_h y_h}_{\sigma})}^{\sigma}}^{x_h y_h} \tag{2.20}$$

$$\frac{\partial p_\sigma}{\partial t} = -\overline{\overline{m^2}^{xy}[\overline{u}^{x_h y_h}\overline{p_\sigma}^{y_h}_{x_h} + \overline{v}^{x_h y_h}\overline{p_\sigma}^{x_h}_{y_h} + \overline{p_\sigma}^{x_h y_h}(\overline{u}^{y_h}_{x_h} + \overline{v}^{x_h}_{y_h})]}^{x_h y_h} \tag{2.21}$$

All three versions of the fourth-order 7L PE were tested on one real data case. The time step had to be reduced in the experiments because instabilities developed otherwise. The table below contains computer running times for 48-hour forecasts using the various models--all are on a 1-bedient grid:

Model	Δt(minutes)	Core	CPU(seconds)
2nd order	20	480K	486
Version #1	15	656K	669
Version #2	15	670K	790
Version #3	12	682K	1275

The required computer space must be increased for the extra data rows needed by the fourth-order scheme. Seven data rows have to be physically present in the machine because of the complexities caused by pressure gradient averaging. Note that second-order differencing will be used in regions that are too close to the lateral boundaries to allow use of the higher-order scheme.

S1 scores[1] for the North American and European verification areas are presented below for the various models. The scores are for 48-hour forecasts made from 12Z 9 January 1975. The "old" 6L PE and the operational 7L PE (½ bedient) verifications are included for reference (recall: all models, except OPNL 7L PE, use 1-bedient grid).

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[1]forecasts verified against the analyses.

Area 1--NORTH AMERICA

mb	2nd Order 7L PE	4th Order Version #1	4th Order Version #2	4th Order Version #3	6L PE	OPNL 7L PE
1000	62.2	55.2	47.8	49.4	64.9	50.0
500	47.5	43.9	40.8	41.2	50.5	40.9
300	47.3	42.9	39.9	39.1	49.8	40.0
100	58.1	57.1	62.2	60.1	57.5	58.2

Area 3--EUROPE

mb	2nd Order 7L PE	4th Order Version #1	4th Order Version #2	4th Order Version #3	6L PE	OPNL 7L PE
1000	58.9	56.1	53.5	55.8	64.6	55.7
500	41.9	39.3	38.6	38.8	46.2	39.1
300	43.6	41.5	39.6	41.0	48.1	41.0
100	42.5	42.9	42.2	41.0	44.9	43.0

For this case the statistics show that the fourth-order 7L PE on a 1-bedient grid is competitive with the operational 7L PE on a ½-bedient grid. It appears that using fourth order on all the terms in the equations (Version #3) adds no improvement to using fourth order on only the horizontal advection terms (Version #2). Note that a good part of the improvement in the fourth-order scheme seems to come from simply using the higher order interpolation on the tendencies (Version #1). Also note that Versions #2 and #3 seem to degrade the forecast at 100 mb over North America--although no major problems are discernible on the actual maps (not shown).

The 48-hour sea level, 500 mb, 500 mb height error (forecast-observed) and precipitation maps for North America are shown in figures 1-4. The second order 7L PE on a 1-bedient grid, the operational second order 7L PE on a ½-bedient grid, and the fourth order 7L PE versions #2 and #3 are presented. It is evident that the fourth order 7L PE produces a forecast very similar to the operational 7L PE (although the 500-mb vorticity in the base of the central United States trough has moved a little faster in the ½ bedient 7L PE--Fig. 2b). Note that there is very little difference between the two fourth order tests.

3. Smoother Experiment

The 7L PE uses diffusion terms in the equations to control computational noise (Shuman, 1977). Since the smoothing (diffusion) is applied every time step, reduction of wave amplitude during a specified forecast interval will depend on the length of time step (i.e., the number of times the smoother is applied). The smoothing coefficient μ = .99007998 is used for the second order 7L PE (1 bedient) with a 20-minute time step. The fourth-order version #3 model uses a 12-minute step, so the smoothing coefficient should be changed accordingly. However, it was not changed in the experiments discussed in the previous section.

A smoothing coefficient for the version #3 model which produces a wave amplitude reduction equivalent to the second order model is μ = .99405391 (μ = 1 means no smoothing). Results of a 48-hour forecast by the fourth order model with the new μ are shown in fig. 5. Comparison with the corresponding version #3 maps in figs. 1, 2, 4 show little difference other than a slight increase in computational noise. Figure 6 is a plot of the model's total kinetic energy during 48-hour forecasts for the 9 January 1975 case. The new smoother produces a version #3 model result that is equivalent to the second order model for at least the first 24 hours. Because the new smoothing coefficient should mean more consistent model comparisons, it is used in the real data tests described in the next section.[1]

4. Real Data Tests

Five real data cases were obtained from the original ten case comparative study of the 'old' 6L PE and the current operational 7L PE. Several are 'locked-in' cases and one is a summer convective situation, but the reasons for choosing them for the original study should still be valid. Testing of the fourth order 7L PE is done with the version #3 model and $\mu = .99405391$ (see preceding section). Version #3, rather than the apparently equivalent (in terms of forecast results) version #2, is chosen to alleviate fear of losing some accuracy in terms governing gravity wave motion. Tests were made for the following cases:

9 January 1975 12Z[2]

11 January 1975 00Z

21 February 1975 00Z

5 December 1976 12Z

1 July 1977 00Z

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[1]In retrospect, the μ used in the second order 1-bedient model itself is wrong. It is identical to the μ used by the ½ bedient 7L PE (10 minute time step), and as such is too weak a smoother for the 20-minute step model. This inconsistency should not cloud the test results and it can be corrected in future studies.

[2]Unlike section 2, this run is with the new smoothing coefficient.

The model results are for 48-hour forecasts, although the 21 February case has run successfully to 84 hours (not shown here). It is evident from the following table of statistical verifications over North America and Europe that the fourth order 7L PE (1 bedient) results are very similar to those of the operational second order 7L PE (½ bedient). The 6L PE is shown for reference only.

Mean (48-hr) S1 Scores	6L PE	OPNL 2nd order 7L PE (½-bedient)	4th order 7L PE (1-bedient)
1000 mb	61.7	54.0	54.7
500 mb	43.6	38.0	38.5
300 mb	44.3	38.2	38.9
100 mb	51.2	51.7	51.8
Mean (48-hr) RMS Vector Wind Error (m/sec)			
1000 mb	9.53	7.67	7.82
500 mb	11.13	8.66	8.89
300 mb	16.26	12.77	13.04
100 mb	--	8.13	8.27
Mean (48-hr) RMS Temperature Error (^{o}C)			
850 mb	3.78	3.13	3.16
500 mb	3.06	2.51	2.56
300 mb	3.08	2.57	2.59
100 mb	--	3.48	3.53

The verification statistics show that both 7L PE models produce the same improvements over the 6L PE (except the S1 at 100 mb where both show a 'negative improvement'). The similarity of S1 scores for both 7L PE models is shown graphically in fig. 7--note that points lying on the 45° diagonal imply equivalency.

Without displaying a whole multitude of Varian maps for the tests, 48-hour surface forecasts are shown in figs. 8-11 to further point out the similarity in the two 7L PE models.

5. Concluding Remarks

The 7L PE has been successfully restructured for fourth-order finite differencing and five real data tests have been made using a 1-bedient grid. The near equivalency of the fourth-order 1-bedient model and the second-order ½-bedient model shows the higher order technique to be an efficient alternative to finer meshes for reducing spatial truncation error.

Experimentation with the fourth-order scheme on the fine-mesh 7L PE (½-bedient) or the LFM should be done. Although further reduction in truncation error will be smaller than reported in this note, a benefit might show up in precipitation forecasts (Campana, 1978).

Applying fourth-order differencing only to the advective terms (version #2) seems to produce forecast results similar to those from a model using the higher-order scheme on all terms (version #3). Although only tested on one case, version #2 produces the desired results at a cheaper computation cost--saving approximately 8 minutes for a 48-hour forecast compared with version #3.

REFERENCES

Brown, J. A., Jr., and K. A. Campana, "An economical time-differencing system for numerical weather prediction," Mon. Wea. Rev., 106, 1978, pp. 1125-1136.

Campana, K. A., "Real data experimentation with higher order finite differencing in the semi-implicit version of the Shuman-Hovermale model," NMC Office Note 163, 1977, 9 pp.

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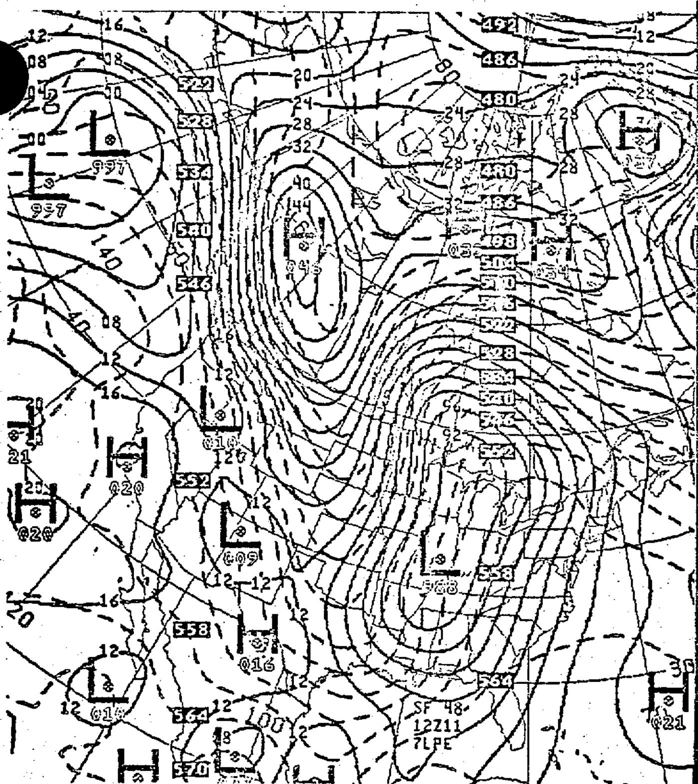
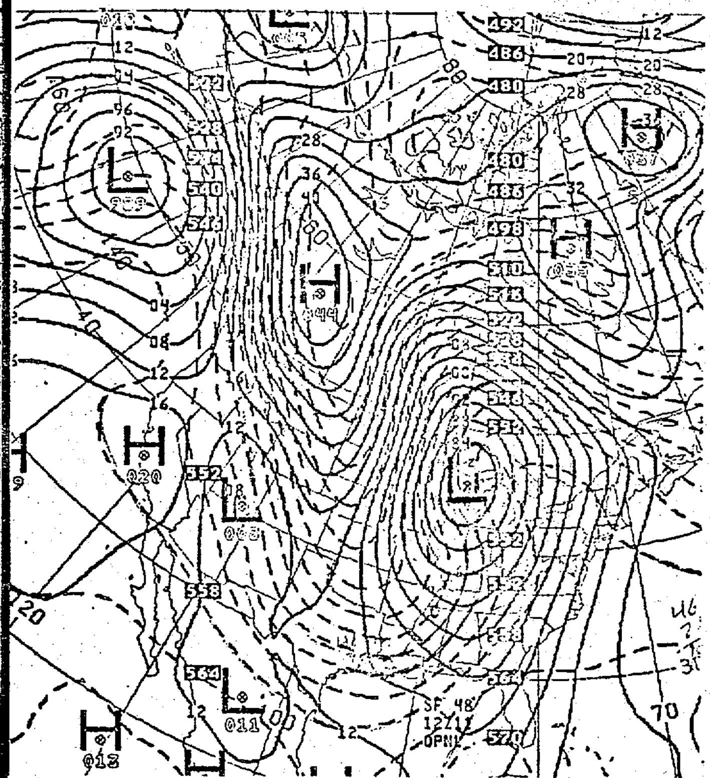
Gerrity, J. P., R. D. McPherson, and P. D. Polger, "On the efficient reduction of truncation error in numerical weather prediction models," Mon. Wea. Rev., 100, 1972, pp. 637-643.

Shuman, F. G., and J. B. Hovermale, "An operational six-layer primitive equation model," Jour. of App. Meteor., 7, 1968, pp. 525-547.

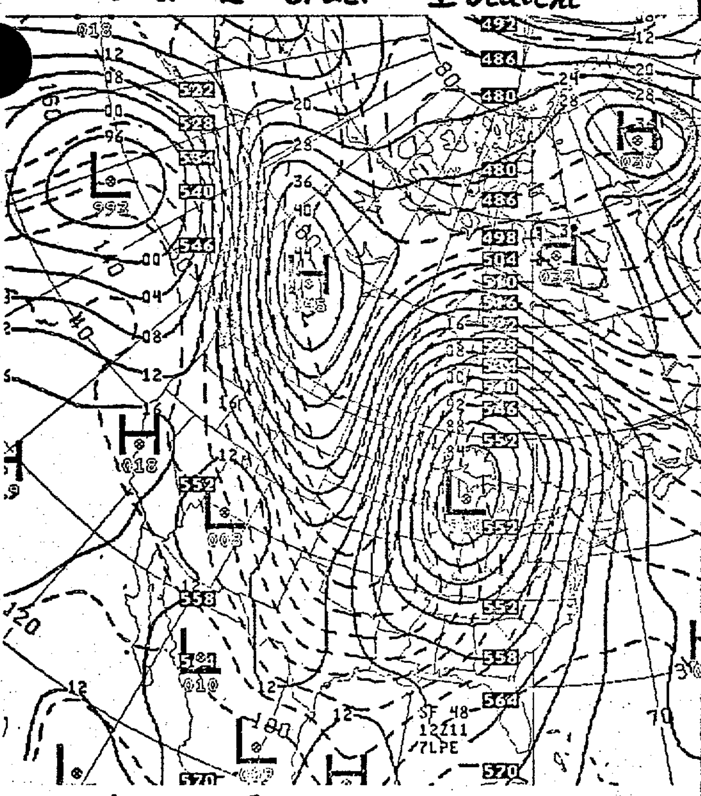
____________, "Some remarks on the use of smoothing and linear diffusion terms to control noise," NMC Office Note 162, 1977, 6 pp.



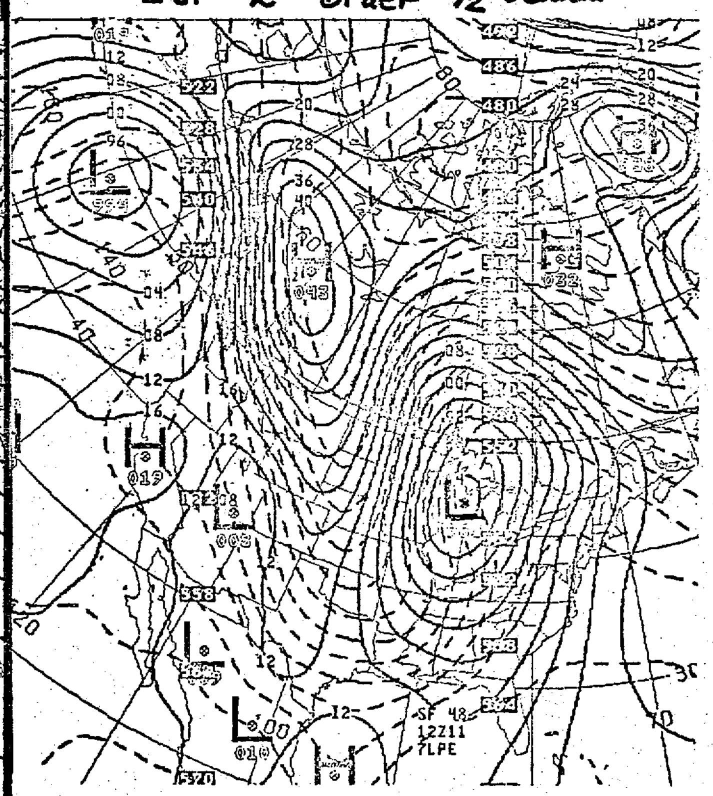
1a. 2ND order – 1 bedient



1b. 2ND order – ½ bedient

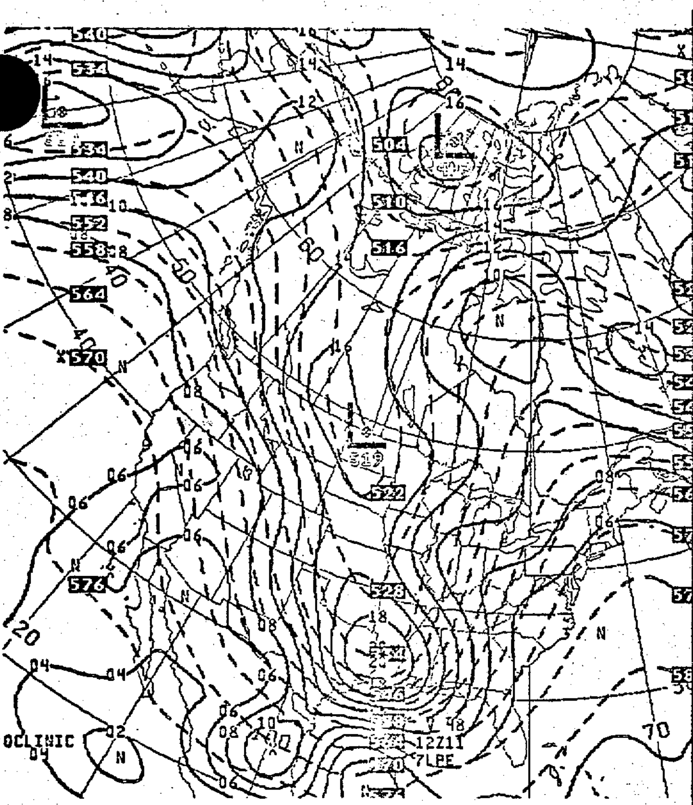


1c. 4TH order – version #2

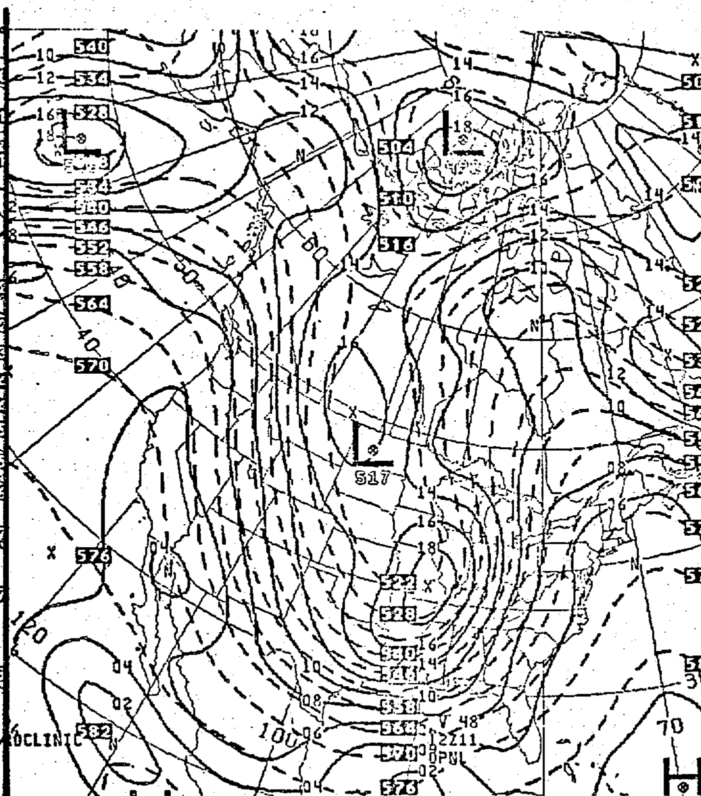


1d. 4TH order – version #3

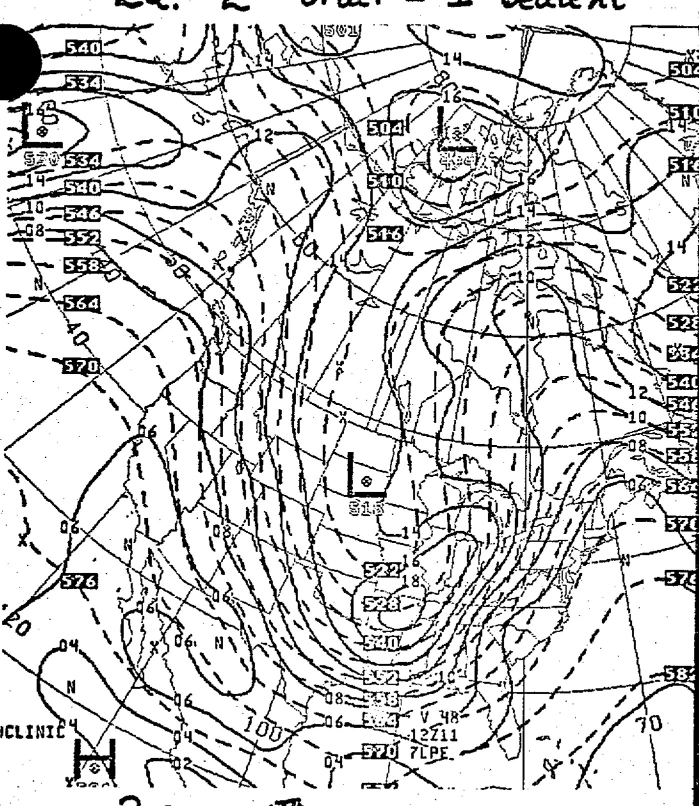
48HR FCST SFC/1000-500 THICKNESS VALID 12Z SAT 11 JAN 1975



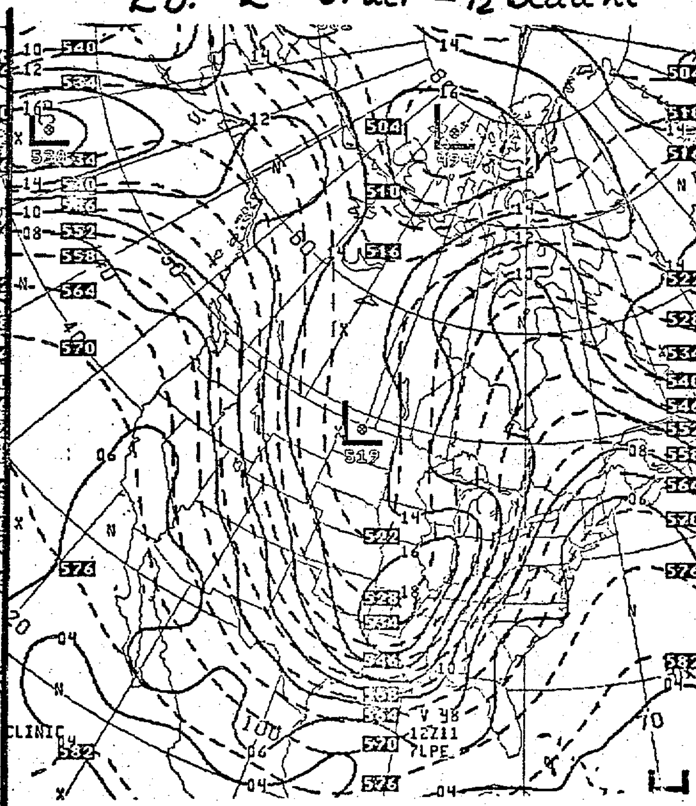
2a. 2ND order – 1 bedient



2b. 2ND order – ½ bedient

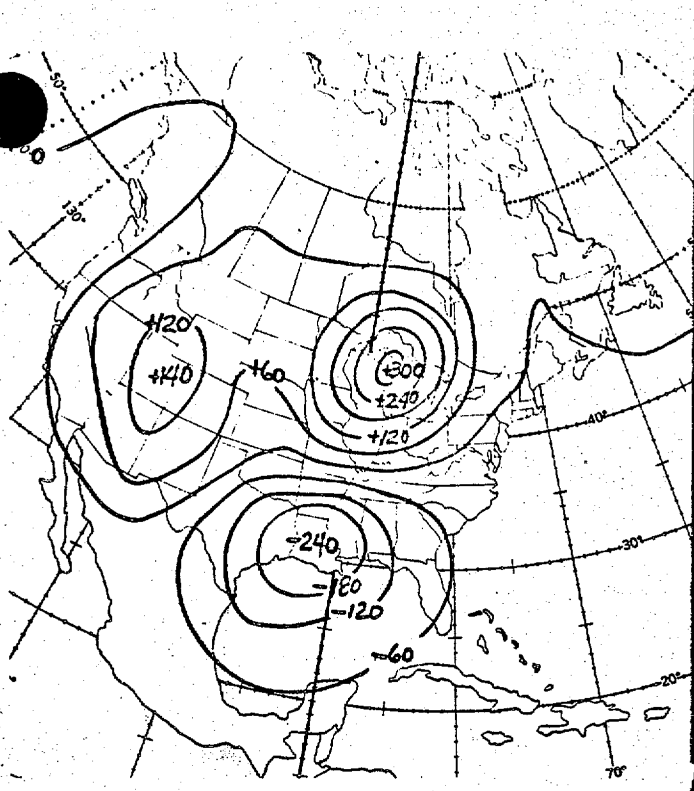


2c. 4TH order – version #2

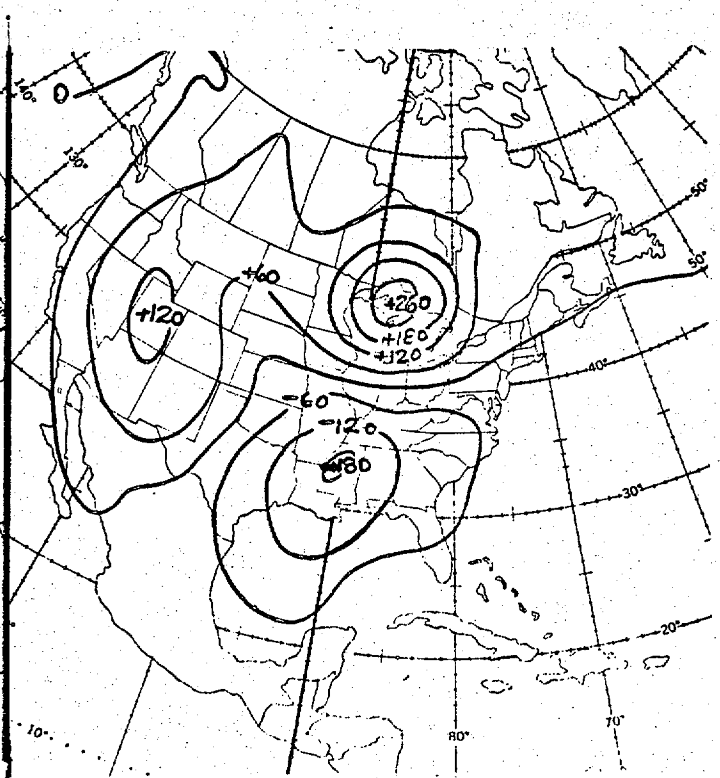


2d. 4TH order – v. rsion #3

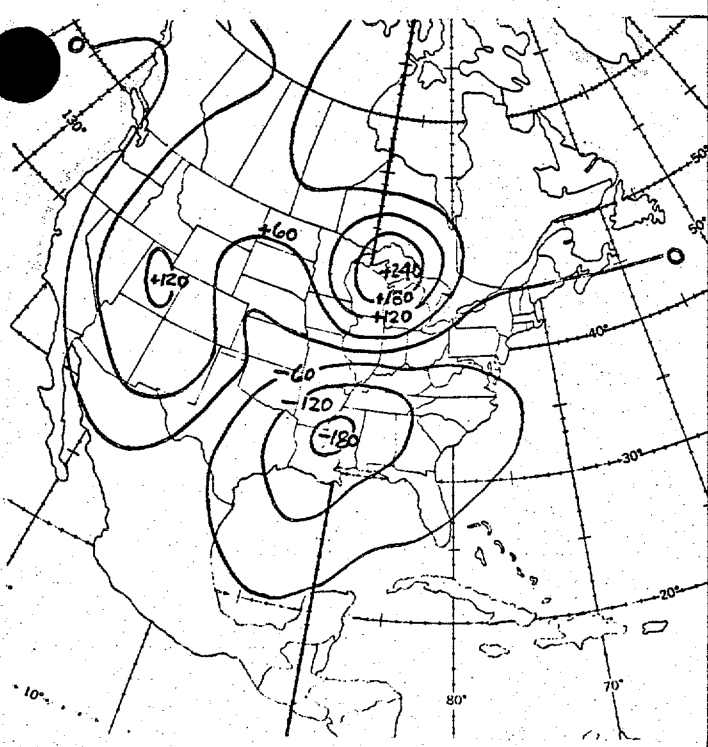
48HR FCST 500MB HEIGHTS/VORTICITY VALID 12Z SAT 11 JAN 1975



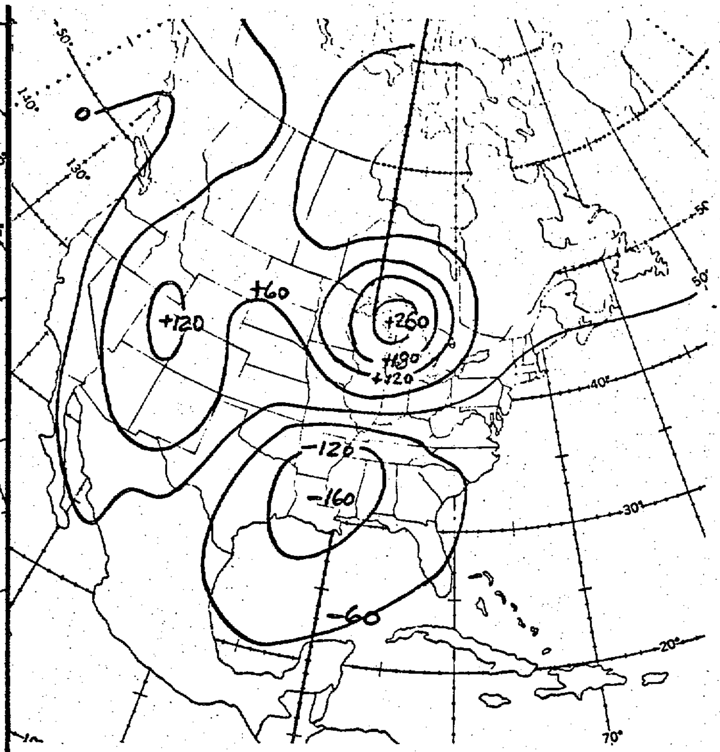
3a. 2ND order — 1 bedient



3b. 2ND order — ½ bedient

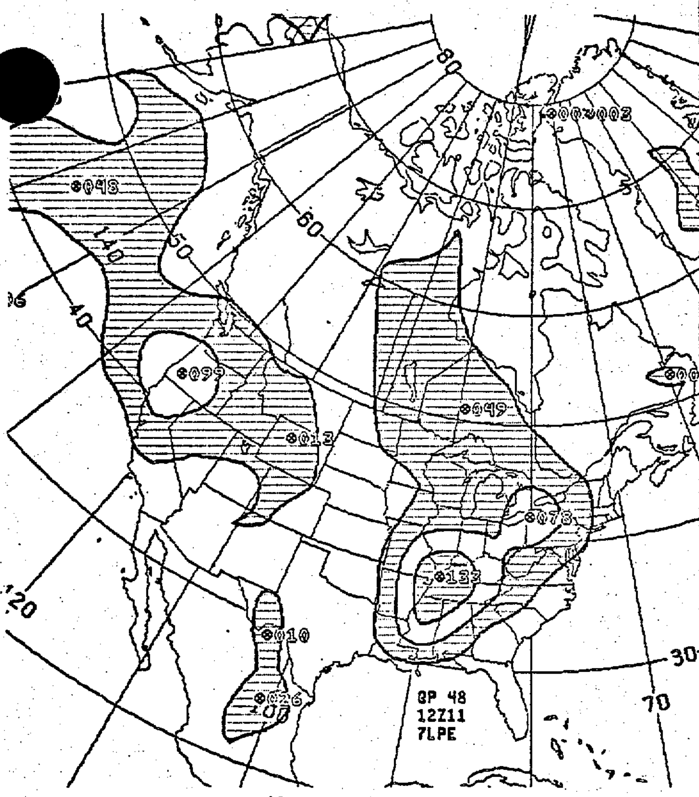


3c. 4TH order — version #2

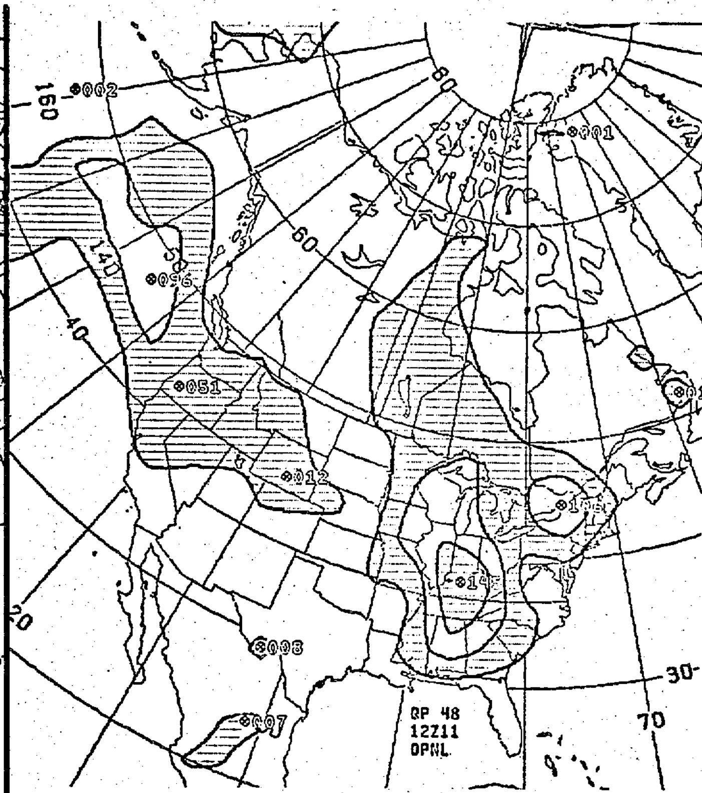


3d. 4TH order — version #3

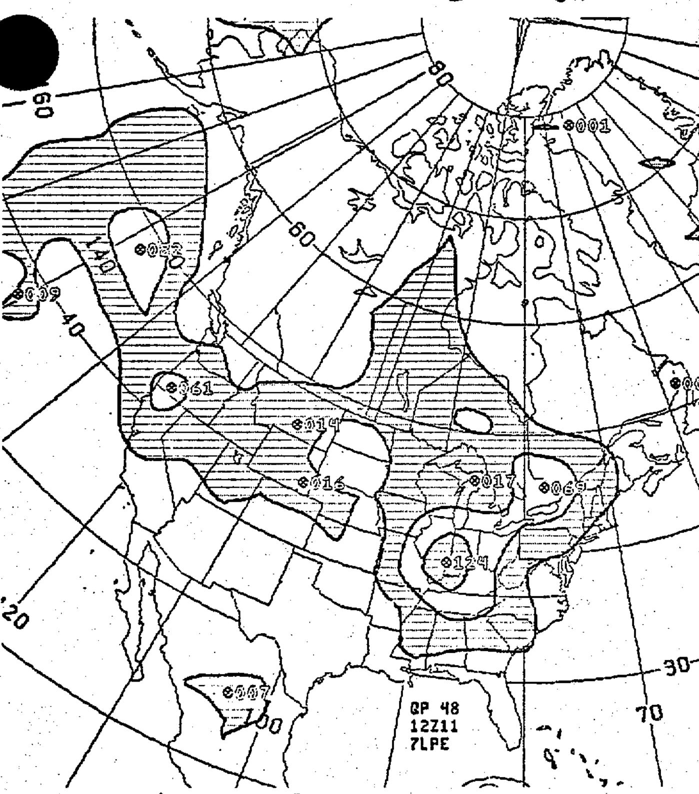
48 hr 500 mb height error (meters)
(forecast — observed) 9 Jan 75 12z



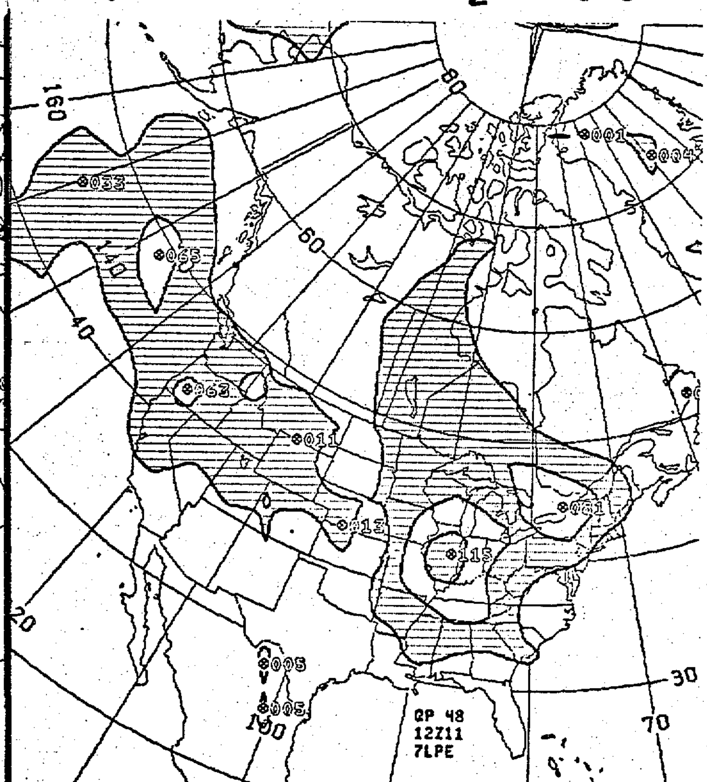
4a. 2[ND] order – 1 bedient



4b. 2[ND] order – ½ bedient

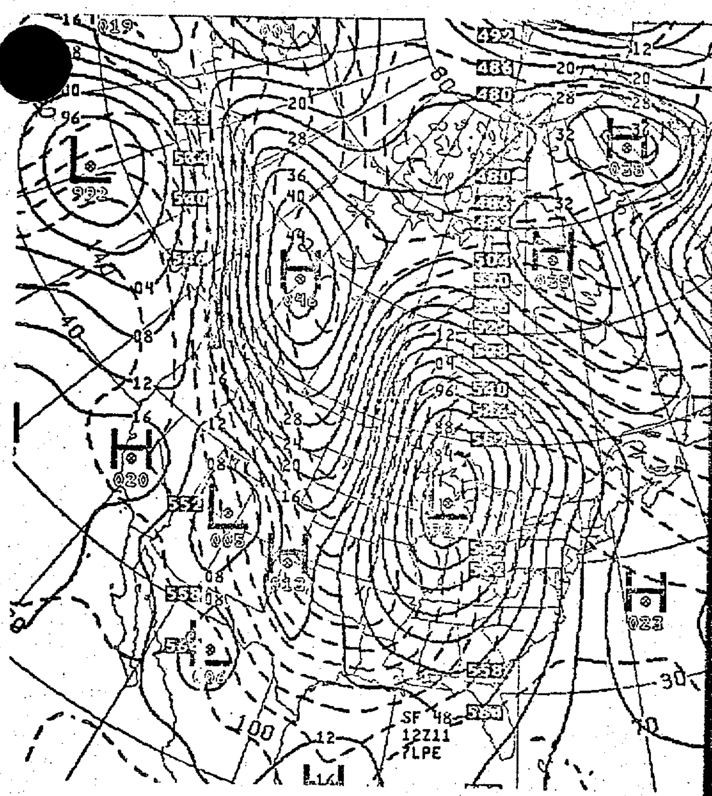


4c. 4[TH] order – Version #2

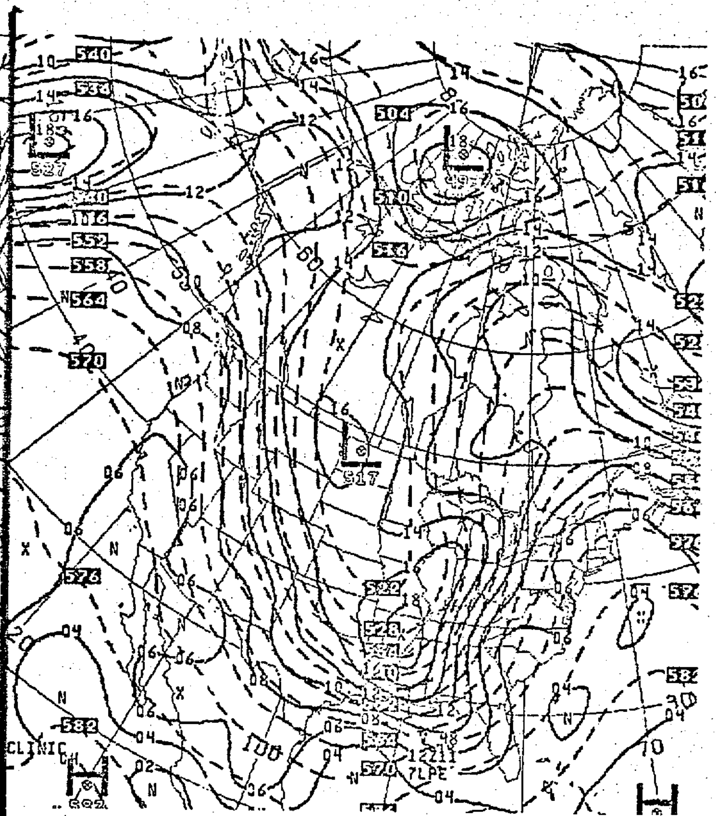


4d. 4[TH] order – Version #3

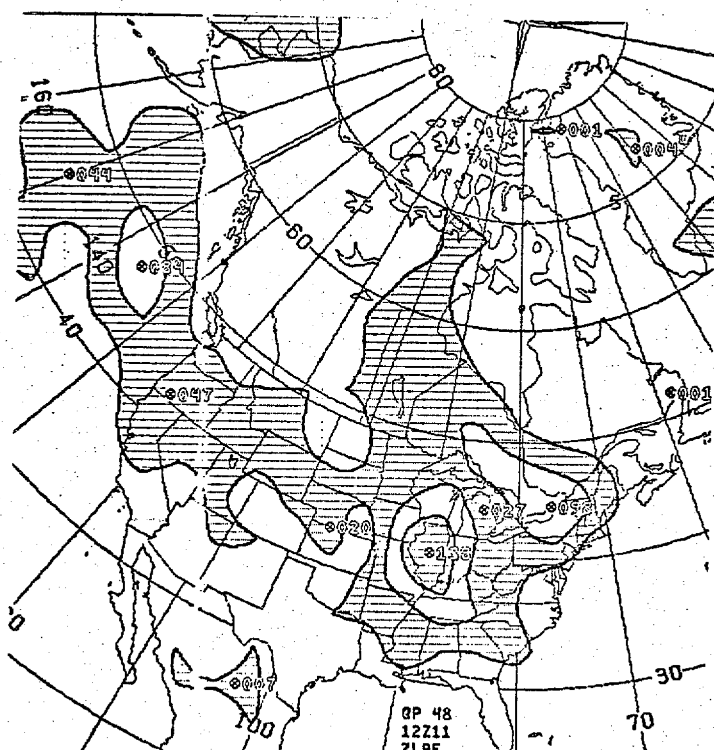
48HR FCST SFC PRECIP. AMOUNT VALID 12Z SAT 11 JAN 1975



5a. SFC/1000-500 Thickness



5b. 500 mb Hgts/Vorticity



5c. Precipitation Amount

48 hr FCST — 9 January 1975 12z — μ=.99405391

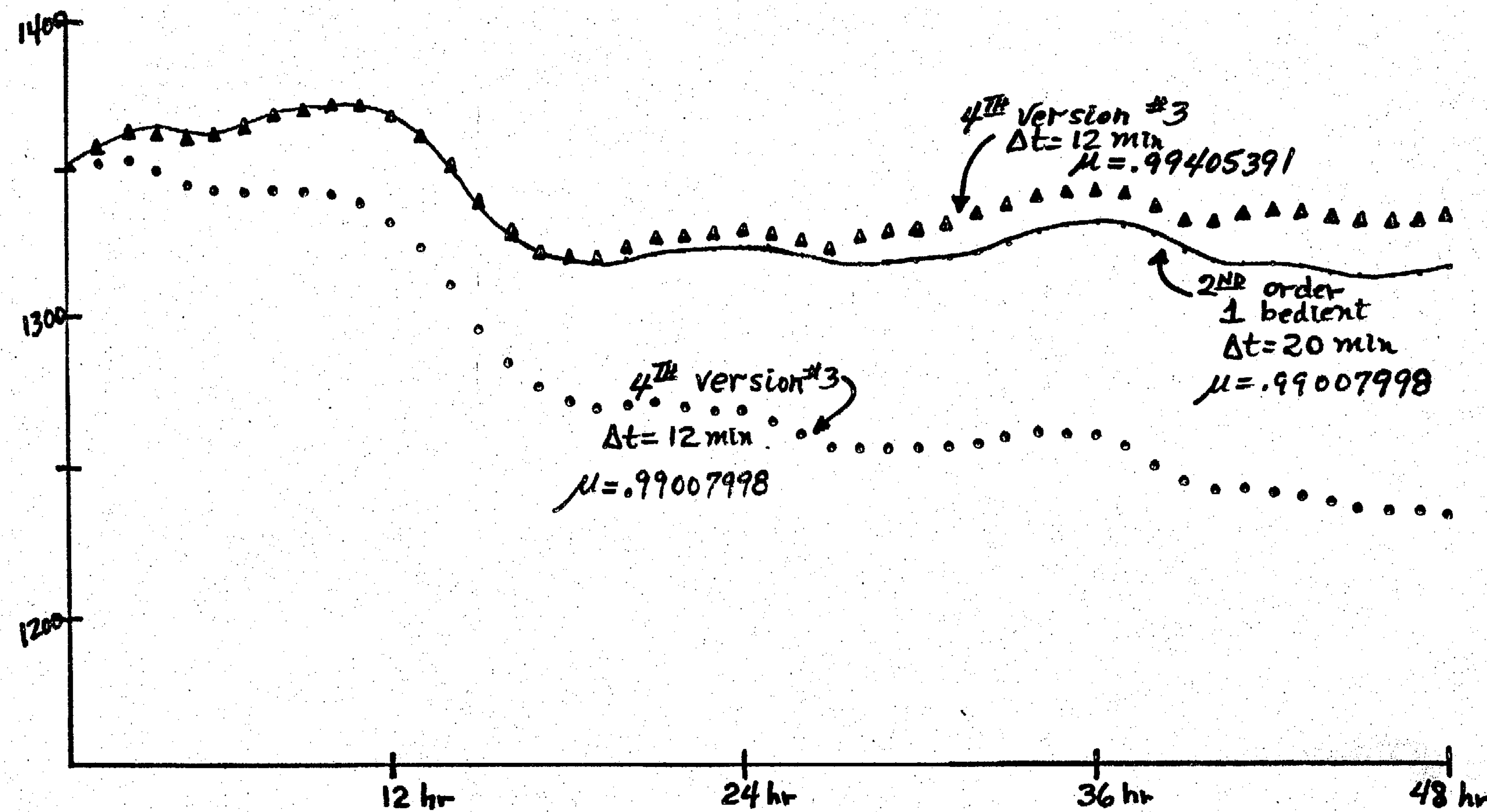


Fig 6. 74 PE Total Kinetic Energy

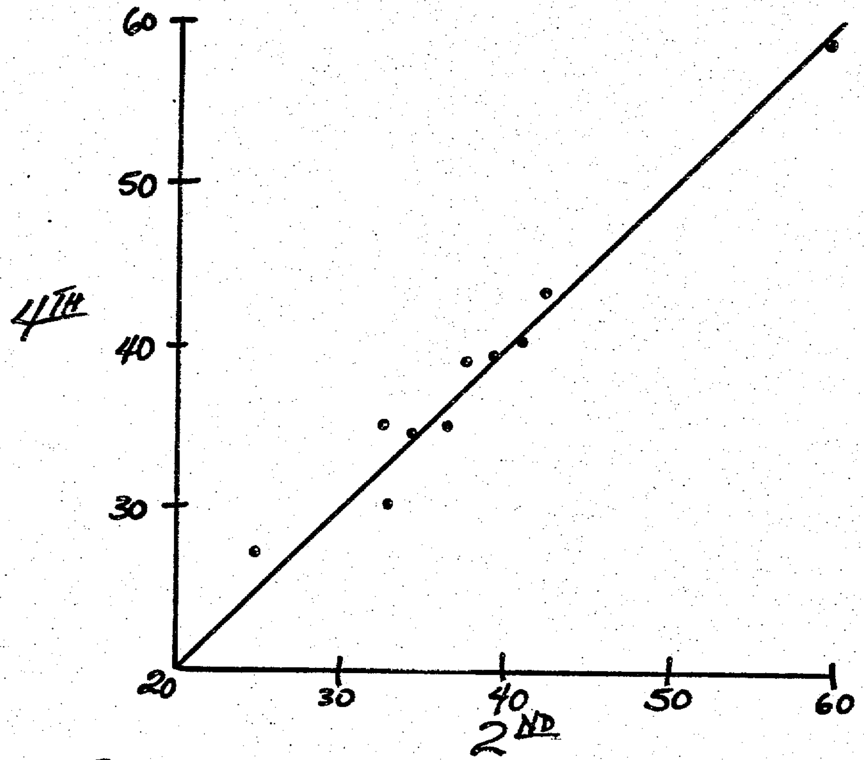


Fig 7a. 500 mb S1 scores (5 cases; 2 areas)

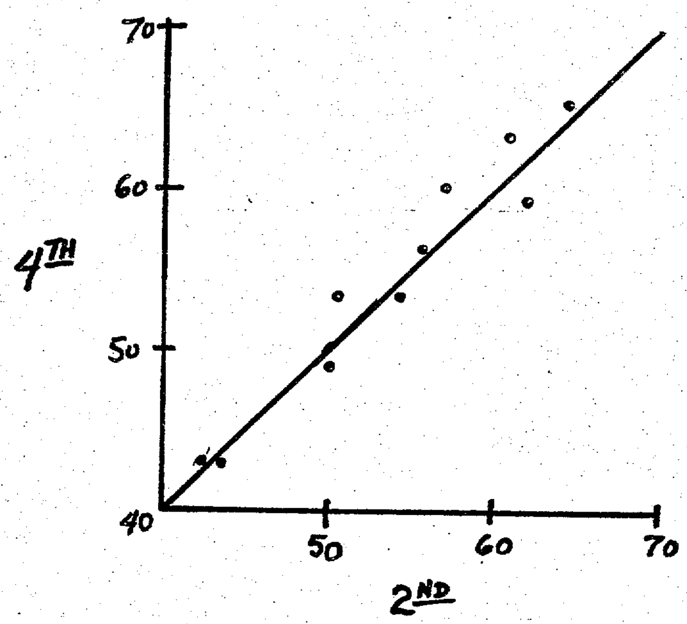
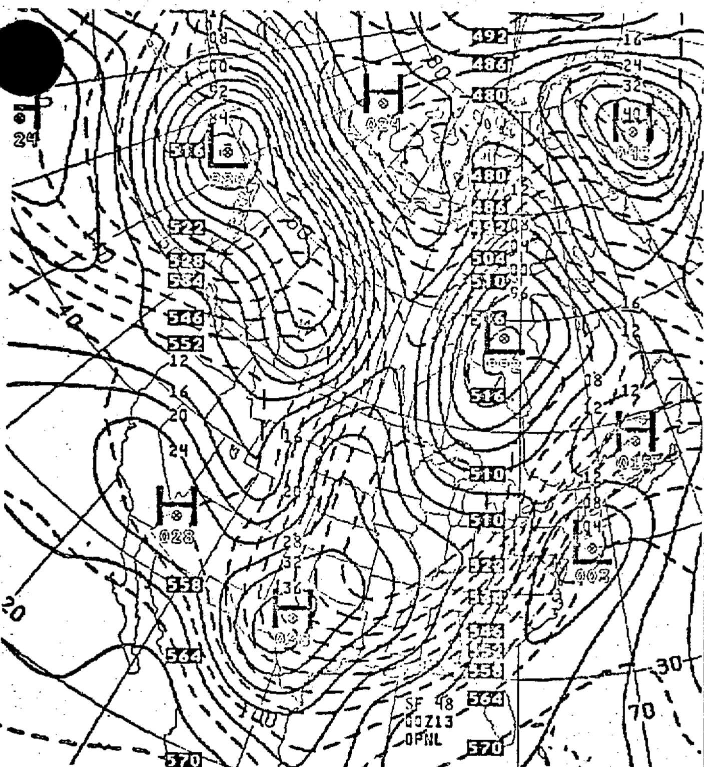
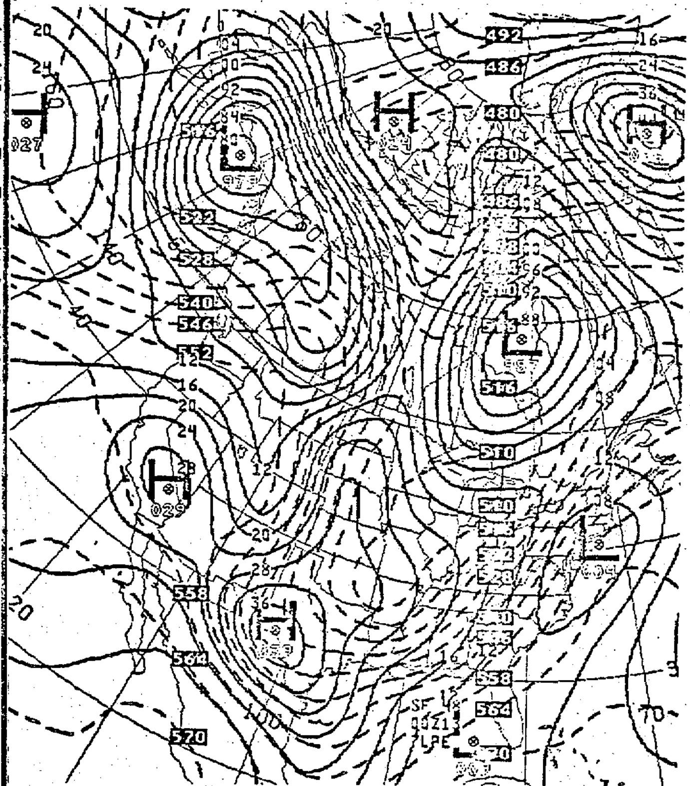


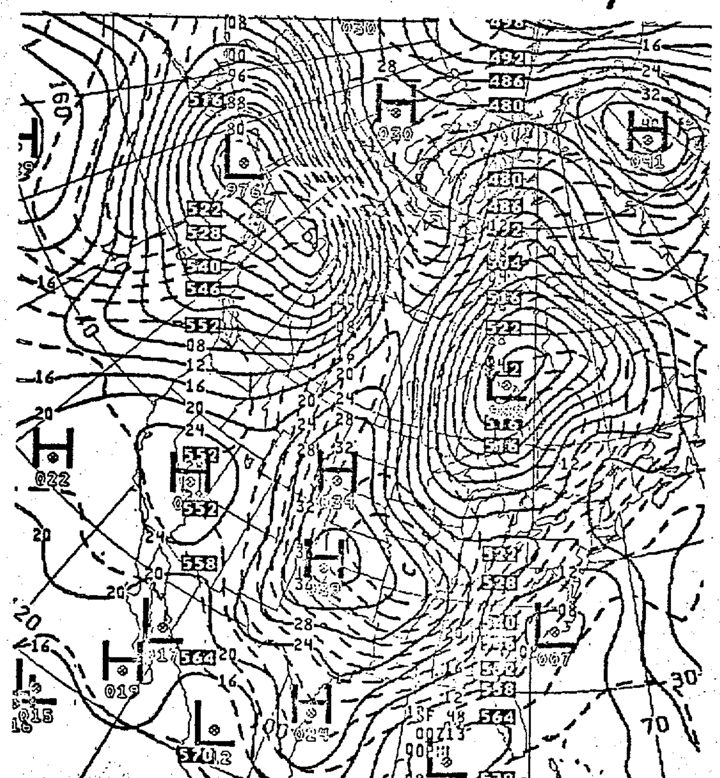
Fig 7b. 1000 mb S1 scores (5 cases; 2 areas)



8a. 7L-PE

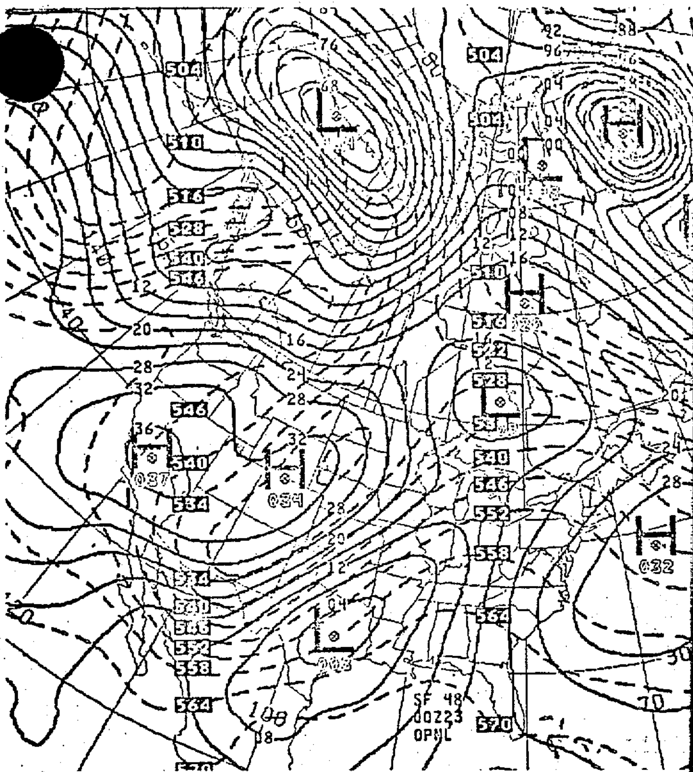


8b. 4th order 7L-PE

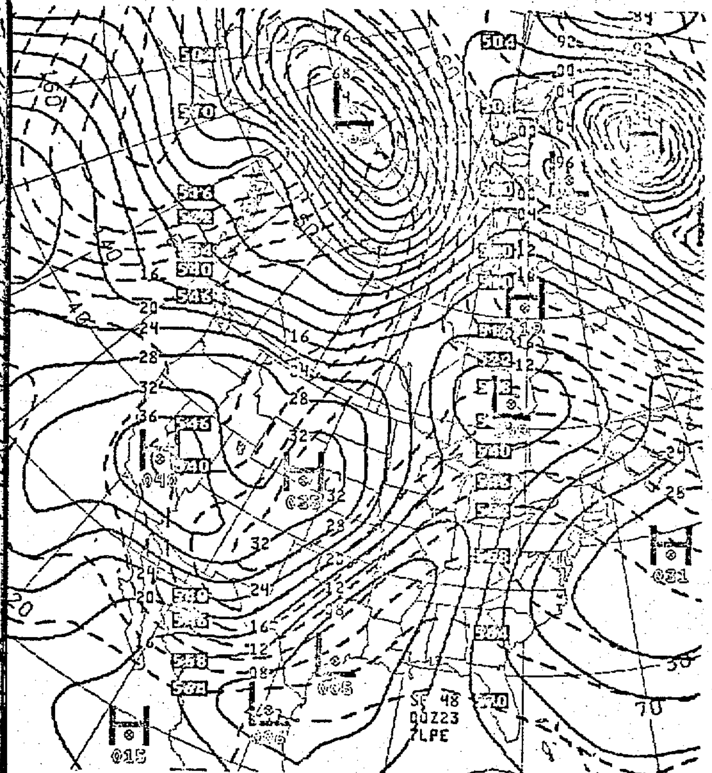


8c. 6L-PE

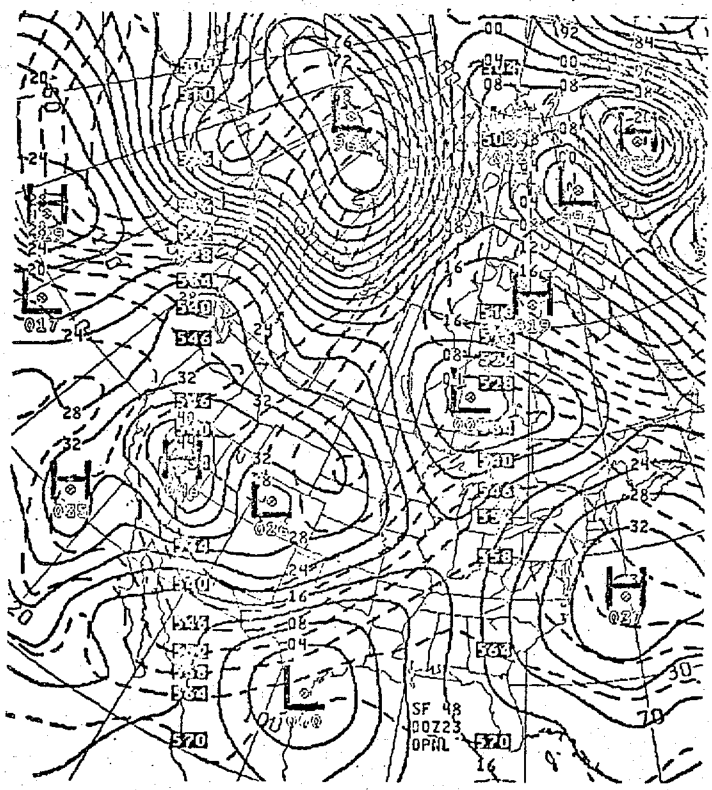
48HR FCST SFC/1000-500 THICKNESS VALID 00Z MON 13 JAN 1975



9a. 7L-PE

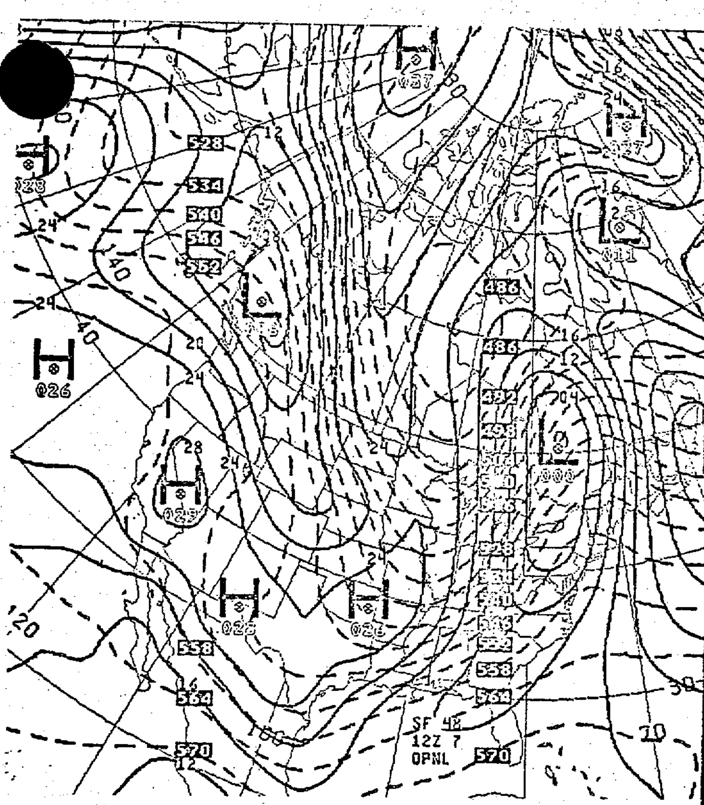


9b. 4th order 7L-PE

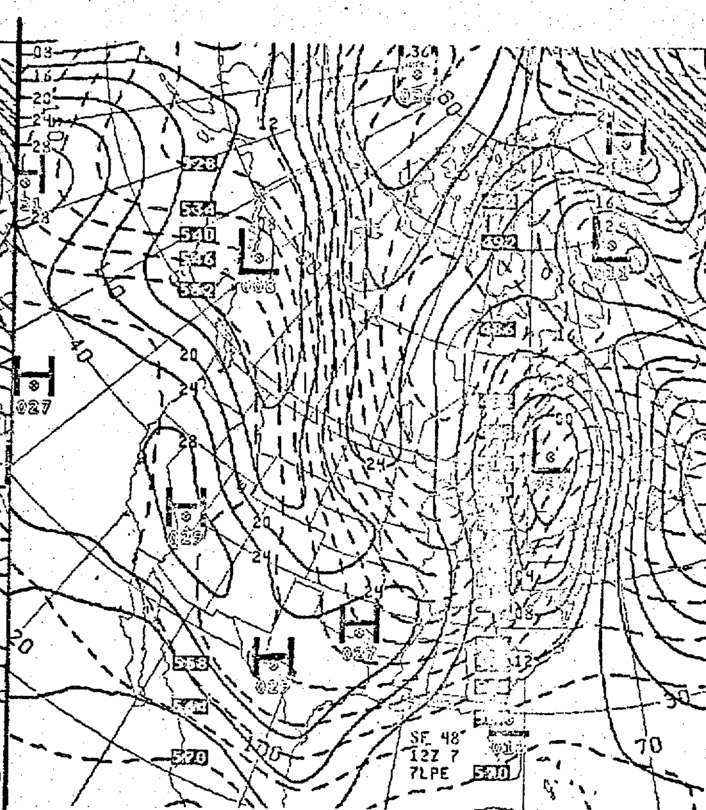


9c. 6L-PE

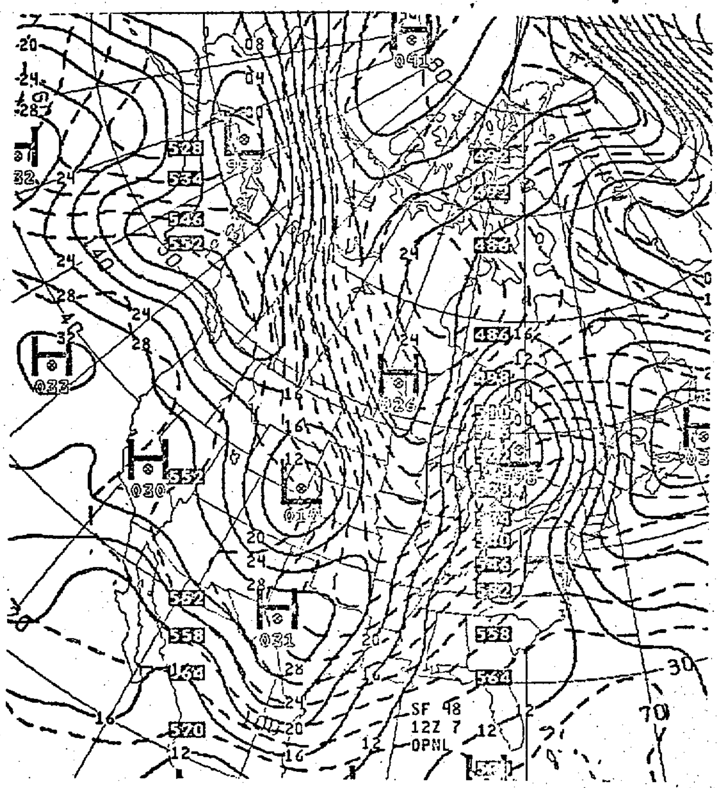
48HR FCST SFC/1000-500 THICKNESS VALID 00Z SUN 23 FEB 1975



10 a. 7L-PE

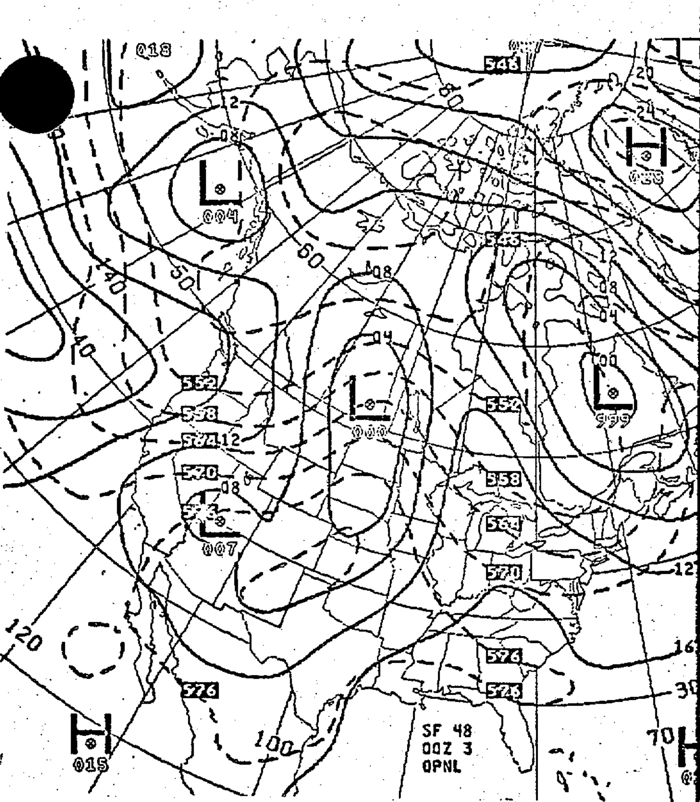


10b. 4th order 7L-PE

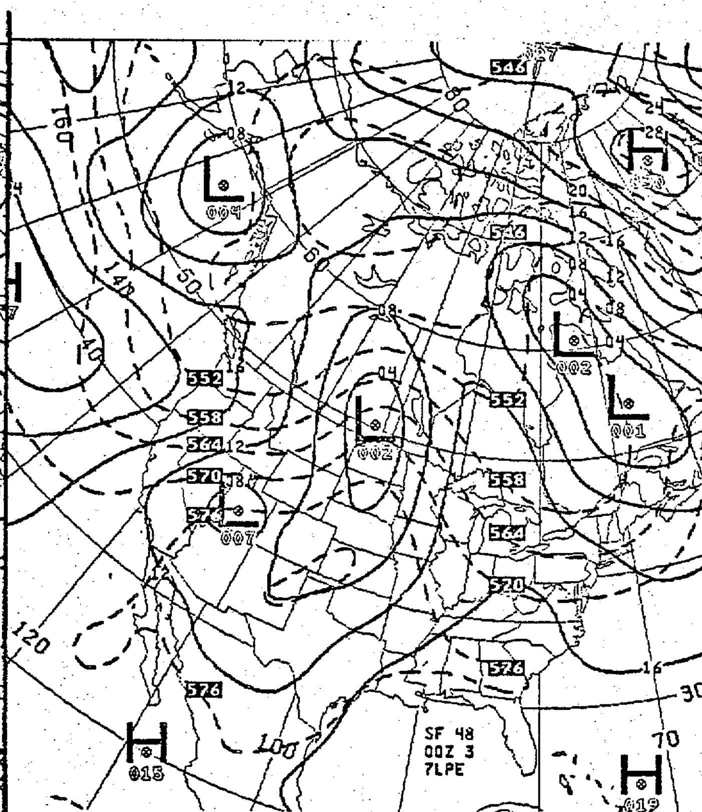


10c. 6L-PE

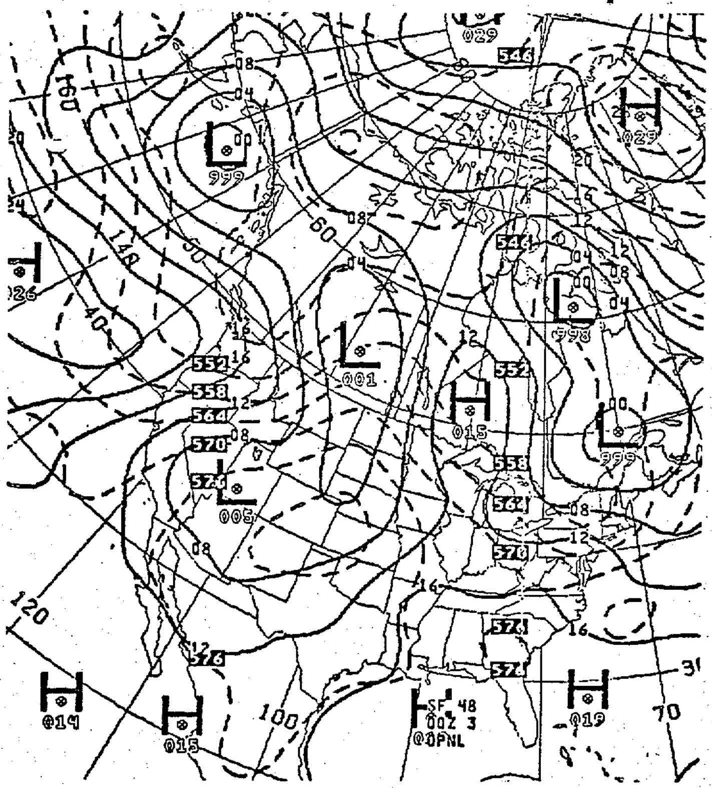
48HR FCST SFC/1000-500 THICKNESS VALID 12Z TUE 7 DEC 1976



11a. 7L-PE



11b. 4th order 7L-PE



11c. 6L-PE

48HR FCST SFC/1000-500 THICKNESS VALID 00Z SUN 3 JUL 1977