Magnetic Field in Curvilinear Coordinates[Br1]

Assuming midplane symmetry, the magnetic field is given to second order in x and y by,

$$B_x(x,y,s) = A_{11}y + A_{12}xy + \cdots$$

$$B_y(x,y,s) = A_{10} + A_{11}x + \frac{1}{2!}A_{12}x^2 + \frac{1}{2!}A_{30}y^2 + \cdots$$

$$B_s(x,y,s) = \frac{1}{(1+hx)}\left[\dot{A}_{10}y + \cdots\right.$$

where $A_{1,n}(s) = \partial^n B_y(s)/\partial x^n$, $A_{30} = -\ddot{A}_{10} - A_{12} - hA_{11}$ and $\dot{A}_{10} = dA_{10}/ds$.

Equations of Motion in Curvilinear Coordinates[Br1]

To second order in x, y and δ, the equations of motion in the above field are given by,

$$\ddot{x} + \left[h^2 - k\right]x = \left(2hk - h^3 + \frac{m}{2}\right)x^2 + \dot{h}x\dot{x} + \frac{1}{2}h\dot{x}^2 + h\delta$$
$$+\frac{1}{2}\left(\ddot{h} - hk - m\right)y^2 + \dot{h}y\dot{y} - \frac{h\dot{y}^2}{2} + \delta\left[2h^2 - k\right]x + \cdots$$

and

$$\ddot{y} + ky = -(m + 2hk)xy + \dot{h}x\dot{y} - \dot{h}\dot{x}y + h\dot{x}\dot{y} + \delta ky + \cdots$$

where

$$\dot{x} \equiv \frac{dx}{ds}, \qquad h \equiv \frac{1}{\rho}, \qquad k \equiv \frac{1}{B\rho}\frac{\partial B}{\partial x}, \qquad m \equiv \frac{1}{B\rho}\frac{\partial^2 B}{\partial x^2}$$