Title: As the Ball Rolls

Brief Overview:

Students will roll a ball up a ramp at various angles of incline. The CBL and the ultrasonic motion detector will measure the position of the ball relative to the detector at regular time intervals as it rolls up and then back down the ramp. After data collection, the TI-82 will be used for graphing the position and velocity of the ball. Students will analyze the graphs in relationship to each other, and fit each curve to a regression model.

Link to Standards:

- **Communication** Students will discuss the differences and similarities between the graphs of the motion curves.
- **Connections** Students will explore the link between physical motion and mathematics.
- **Algebra** Students will use the graphs and tables to select an appropriate equation.
- **Functions** Students will translate a table of values into functional notation.
- **Statistics** Students will select a regression model to fit the position and velocity curves.
- **Trigonometry** Students will use trigonometry to calculate the various elevations of the table.
- **Conceptual Underpinnings of Calculus** Students will explore the relationship between the slope of the position curve at a specific time to the velocity at that time.

Grade/Level:

Grades 9-12, Algebra II, Calculus

Duration/Length:

This activity is expected to take three or four class periods, anticipating one full period of introductory procedural instruction.

Prerequisite Knowledge:

Students should have working knowledge of the following:

- The Cartesian plane
- Quadratic functions and their graphs
- Trigonometric ratios of a right triangle
- Linear regression formula
- Definition of the derivative (Pre-Calculus and Calculus only)
- The TI-82 Graphing Calculator

Objectives:

Students will be able to:

- work cooperatively in groups to gather data using the CBL, ultrasonic motion detector, and TI-82 graphing calculator.
- identify the intercepts, extrema, and other key points of the motion curves.
- identify the general family of curves generated from the experiment.
- model the position curve, using the TI-82's quadratic regression function.
- approximate the slope and y-intercept of the linear regression model for the velocity curve, using the formula for line of best fit.
- recognize the relationship between both motion curves--graphically, symbolically, and numerically.
- represent and use formulas in a variety of equivalent forms.

Materials/Resources/Printed Materials:

- Rubber balls, between 1.5-4 inches in diameter
- Tables (at least 1.5 meters long)
- Meter sticks (two per table)
- Duct tape
- Bricks (or some other means of elevating one end of a table)
- Pencils and paper
- TI-82 with programs MOTION and PLOTS
- CBL, and ultrasonic motion detector
- Teacher instructions sheet
- Student worksheets----------**Roll, Roll, Roll Your Ball**
Will the (Curved) Shoe Fit?
Roll It Again, Sam

Development/Procedures:

- Set up, learn the procedures involved in tracking the motion of the ball, observe and practice the rolling of the ball up the "ramp."

- Divide class into groups of three or four students.

Then each group will:

- Record the angle of elevation of the table.

- Run the MOTION program to collect the required data.

- Draw conclusions from the two plots when displayed separately.

- Compare the characteristics of the two plots when shown together on the calculator, and hypothesize on their relationship to each other.

- Identify the best regression model for each plot.

- Repeat the experiment using two additional angles of elevation.

- Draw conclusions regarding how the elevation of the table affects the graphs.

- Verify relationship between two plots, regardless of angle of elevation of table.

Evaluation:

The teacher will rove from group to group, monitoring student participation and cooperation. Further evaluation will be based upon worksheet completion and accuracy of conclusions, as well as future demonstration of attained knowledge on quizzes or tests.

Extension/Follow Up:

1. The exercise may be repeated with longer ramps or on ramps with varying elevation.

2. The exercise may be modified by starting the ball at the top of the ramp and simply releasing it, and/or by placing the motion detector at the <u>bottom</u> of the ramp, rather than the top.

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Acknowledgment:

The MOTION and PLOTS programs were extracted from Brueningsen, Chris, and Krawiec, Wesley, Exploring Physics and Math with the CBL™ System, pp. 166, 17, with permission of the publisher, Copyright 1994, Texas Instruments Incorporated.

Teacher Instructions

Prior preparation for in-class demonstration:

---Place an ultrasonic motion detector at one end of a table. Stabilize the detector by taping it to the table.

---Starting about 1.5 feet from the detector, construct a “track,” using the meter sticks laid parallel to each other. The distance between the sticks should be chosen so that the rubber ball will roll smoothly between the sticks but not wobble from side to side. Tape the sticks to the table at several places to stabilize the track.

---Elevate one end of the table (the end with the motion detector) by placing a brick under each leg on that end.

---Confirm that the path of the motion detector coincides with the center of the ramp.

Perform the following in front of the entire class:

---Demonstrate rolling the ball up the ramp so that it almost reaches the top of the ramp.

---Have students take turns rolling the ball up the ramp, practicing the amount of force necessary, and noticing how much time the entire trip takes.

---Link TI-82 to CBL unit. Place the ultrasonic motion detector’s cord into the “sonic” port of the CBL. Turn on both units.

---Run program MOTION on the TI-82.

---Enter an appropriate time interval when prompted.

---Press [ENTER] as student rolls the ball up the ramp.

---After data collection, there will be a short delay, then the “PLOT OPTIONS” menu appears.

---Explain to students that they will set up and perform the experiment during the next class period, and will then proceed from this menu and complete their activity sheets.

---From the PLOT OPTIONS menu, they will select options 1, 2, and 3 only.

---To re-access the PLOT OPTIONS menu, run program PLOTS.

Student Activity #1
Roll, Roll, Roll Your Ball

1. Set up table and ramp as demonstrated by your teacher. Make certain that the distance between meter sticks is constant, and that there is 1.5 feet between the motion detector and the top of the ramp.

2. Calculate the angle of elevation of the table.

θ =

3. Practice rolling the ball up the ramp. Try to get the ball as far up the ramp as possible, but not beyond the end of the ramp. Notice the length of time that is necessary to go up and down the ramp.

4. Run the program MOTION on the TI-82 to collect the required data. One person should run the calculator, one should roll the ball, and the others should take notes and offer encouragement and constructive criticism.

5. After a brief delay, the PLOT OPTIONS menu will be displayed. From this menu, press [1], "Distance-Time." What does this graph portray? _____________
____
____

6. <u>Draw</u> the coordinate axes and <u>sketch</u> the graph below.

7. What is the <u>shape</u> of this graph? ____

8. Why does the graph of the ball's position take this shape? ___________________
____

9. What does the y-intercept of this graph represent? ___________________________
____

10. At what time does the curve hit its minimum value? ______________ What does the minimum value of the curve represent? ____________________________________

____

Describe the ball's motion at this particular time.________________________________

11. How does the slope of this curve behave throughout the curve? _______________

____

____

12. Press [2nd][QUIT] and run program PLOTS. This calls up the PLOT OPTIONS menu again. Press [2] for "Velocity-Time."

13. What does this graph portray? ____

____

14. Draw the coordinate axes and sketch the graph below.

15. What is the shape of this graph? _______________________________________

16. Why does the graph take this shape? ____________________________________

____

17. What does the y-intercept of this graph represent? _________________________

____

18. Explain why the velocity is negative at the beginning. ______________________

____

19. What is the x-intercept of this graph? _____________ What does the x-intercept represent in this case? ____

20. Why does the velocity then become positive? ____________________________

____

21. Press [2nd][QUIT] and run program PLOTS again.
    Press [3] to select "D-T & V-T".

22. What does this graph show? ____

23. While one member of your group slowly moves the ball along the ramp, have a second member trace along the position graph and a third member trace along the velocity graph. At five or more time intervals, verbally summarize the ball's position and velocity on the ramp in relation to the current locations on the graphs.

24. What is the relationship between the minimum value of distance and the x-intercept of velocity? ________________________ Will this relationship always happen? __________Explain. ____
____

25. What is the relationship between the velocity at any time, and the <u>slope</u> of the position curve at the same time? _________________ Will this relationship always happen? __________ Explain. ____
____

Student Activity #2

Will the (Curved) Shoe Fit?

1. Press [STAT] (CALC) [6] [2nd] [L_2] [,] [2nd] [L_4] [ENTER] to find the best quadratic model for the position curve. The calculator will display (eventually) the value of a, b, and c for the equation $y=ax^2+bx+c$. Record those values here

a= ________ b= __________ c=__________

2. Calculate the coordinates of the vertex of the model.

3. Does this match the location of the minimum of the actual position graph? ______

4. Press [Y=] [VARS] [5] (EQ) [7] . This enters in the regression equation into Y_1 (or whichever line you select). Press [GRAPH] to see how well it fits.

5. What degree function does the velocity curve resemble (regardless of what you know it should look like)? ____

6. Model the curve using the appropriate regression function on your TI-82. The keystrokes are: [STAT] (CALC) [5,6,7, or 8] [2nd] [L_2] [,] [2nd] [L_5].

7. Press [Y=] (move to empty line) [VARS] [5] (EQ) [7]. This enters the regression equation into Y_2 (or whichever line you select). Press [GRAPH] to see how well it fits.

Student Activity #3

Roll It Again, Sam

Repeat the ball-rolling activity two more times using steeper angles of elevation.

Record the following information for each additional experiment, while viewing PLOT OPTIONS #3 “D-T & V-T”:

Angle of elevation		
Shape of graphs in relation to previous experiment		
Value of y-intercept of position graph		
Value of y-intercept of velocity graph		
Time at which minimum distance occurred		
Value of x-intercepts of velocity graph		
Regression equation for position graph		
Regression equation for velocity graph		

On a separate sheet of graph paper, sketch the position and velocity graphs for each of your two additional experiments.

Write a paragraph that summarizes the effect of the increased elevation on the two graphs. Mention the width of the curves, the slope of the curves, and the intercepts. Be succinct.

How would you change the conditions of the experiment?

Why?

How do you think your changes would influence the outcome of the experiment and the corresponding graphs?