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Liboff 4.8 numerically looking at a solution
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4.8 Liboff
```

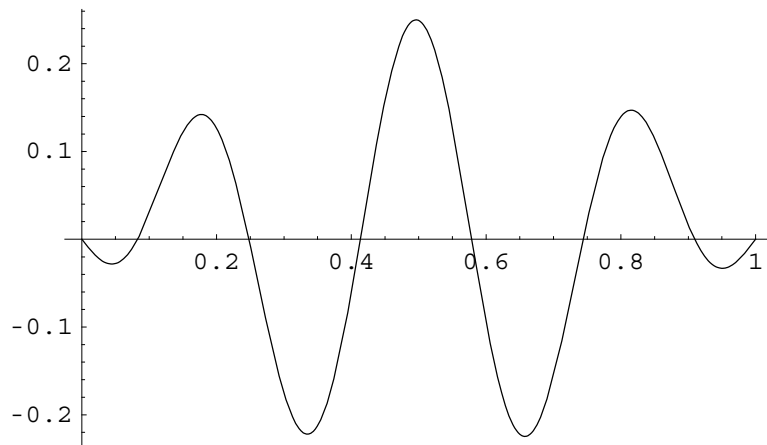
■ Pick values for a an k

```
a := 1.
```

```
k := 19.
```

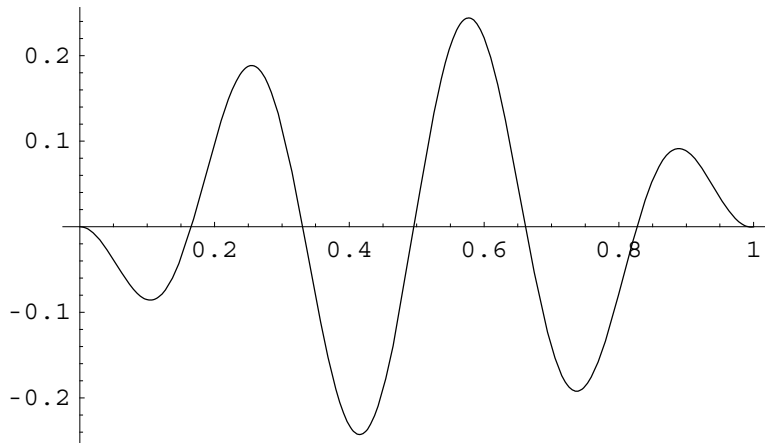
```
g[x_] := x*(x - a)*Exp[I*k*x]
```

```
Plot[Re[g[x]], {x, 0., 1.}]
```



```
- Graphics -
```

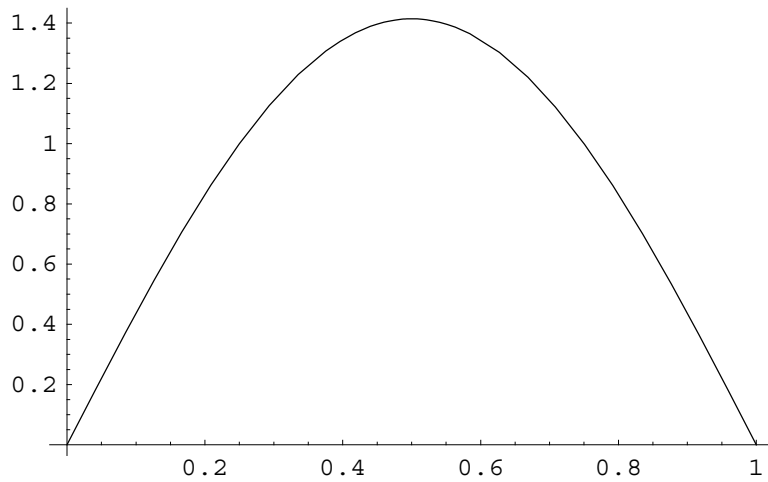
```
Plot[Im[g[x]], {x, 0., 1.}]
```



- Graphics -

```
f[x_, n_] := Sqrt(2/a) * Sin[n * Pi * x / a]
```

```
Plot[f[x, 1], {x, 0., 1.}]
```



- Graphics -

$$g(x) = x(x-a) e^{ikx} = \sum_{n=1}^{\infty} a_n \varphi_n(x)$$

$$\varphi_n(x) = \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right)$$

$a_n =$

$$\int_0^a g(x) \varphi_n(x) dx = \int_0^a x(x-a) e^{ikx} \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi x}{a}\right) dx = \sqrt{\frac{2}{a}} \int_0^a x(x-a) e^{ikx} \sin\left(\frac{n\pi x}{a}\right) dx$$

```
Integrate[x * (x - a) * Exp[I * k * x] * Sin[n * Pi * x / a], {x, 0, a}]
```

```
((6804.69 + 43096.4 i) n + (62.0126 - 1178.24 i) n^3 +
  ((-13187. + 41589.7 i) n + (115.279 - 1174.22 i) n^3 -
    (5.68434 × 10-14 + 1.42109 × 10-14 i) n^5) Cos[3.14159 n] +
  ((126793. + 33095.2 i) - (168.632 - 1112.43 i) n^2 - (96.3088 + 14.5994 i) n^4)
  Sin[3.14159 n]) / (361. + 0. n - 9.8696 n^2)^3
```

```
Simplify[%, Element[n, Integers]]
```

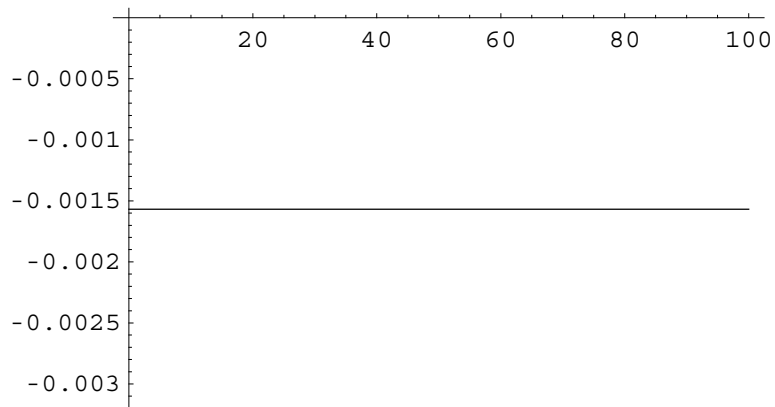
```
((6804.69 + 43096.4 i) n + (62.0126 - 1178.24 i) n^3 +
  ((-13187. + 41589.7 i) n + (115.279 - 1174.22 i) n^3 -
    (5.68434 × 10-14 + 1.42109 × 10-14 i) n^5) Cos[3.14159 n] +
  ((126793. + 33095.2 i) - (168.632 - 1112.43 i) n^2 - (96.3088 + 14.5994 i) n^4)
  Sin[3.14159 n]) / (361. + 0. n - 9.8696 n^2)^3
```

```
FullSimplify[%]
```

```
((6804.69 + 43096.4 i) n + (62.0126 - 1178.24 i) n^3 +
  ((-13187. + 41589.7 i) n + (115.279 - 1174.22 i) n^3 -
    (5.68434 × 10-14 + 1.42109 × 10-14 i) n^5) Cos[3.14159 n] +
  ((126793. + 33095.2 i) - (168.632 - 1112.43 i) n^2 - (96.3088 + 14.5994 i) n^4)
  Sin[3.14159 n]) / (361. + 0. n - 9.8696 n^2)^3
```

```
c[x_, n_] := ((7539.822368615503~ + 50265.48245743669~ i) n +
  (62.01255336059963~ - 1240.2510672119927~ i) n^3 +
  ((-48966.49974294103~ + 13628.99669651821~ i) n +
    (1106.9751108873543~ - 562.738278750181~ i) n^3 -
    (4.263256414560601~ *^-14 + 5.684341886080802~ *^-14 i) n^5)
  Cos[3.141592653589793~ n] + ((50686.00587850064~ + 152600.55310543464~ i) -
    (1081.2490157441935~ - 483.31302159348706~ i) n^2 -
    (39.75090270852479~ + 88.9291670371872~ i) n^4)
  Sin[3.141592653589793~ n]) / (400.~ + 0.~ n - 9.869604401089358~ n^2)^3
```

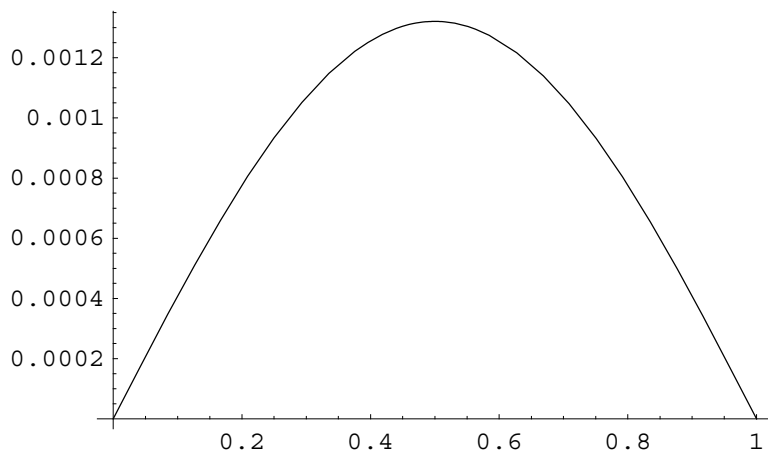
```
Plot[Re[c[x, 2]], {x, 0., 100.}]
```



- Graphics -

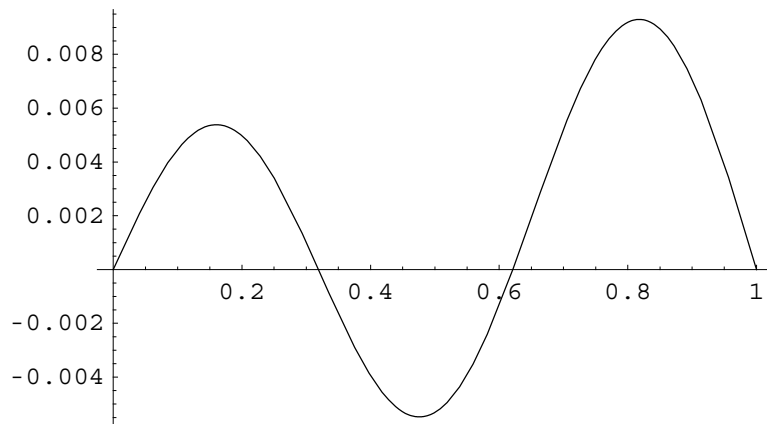
```
z[x_, n_] := c[x, n] * f[x, n]
```

```
Plot[Re[z[x, 1]], {x, 0., 1.}]
```



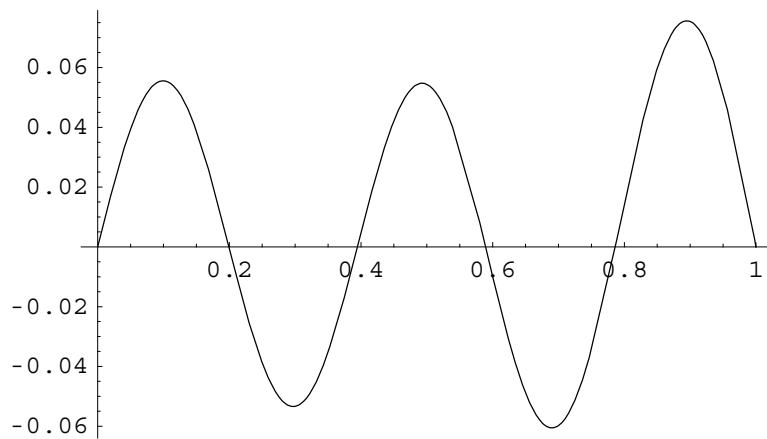
- Graphics -

```
Plot[Re[Sum[z[x, i], {i, 1, 3, 1}]], {x, 0., 1.}]
```



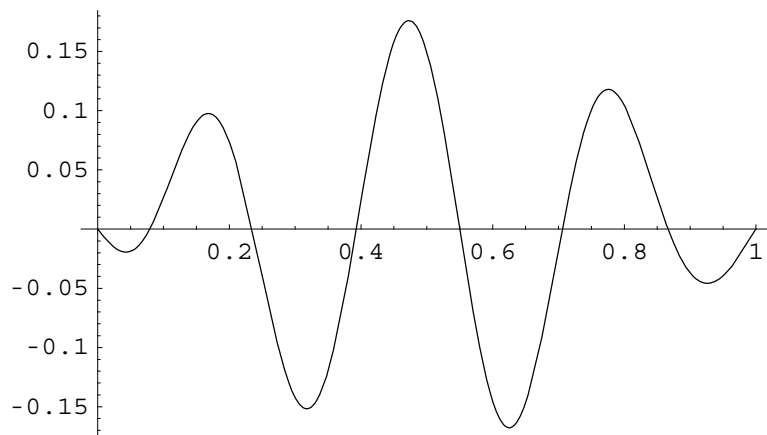
- Graphics -

```
Plot[Re[Sum[z[x, i], {i, 1, 5, 1}]], {x, 0., 1.}]
```



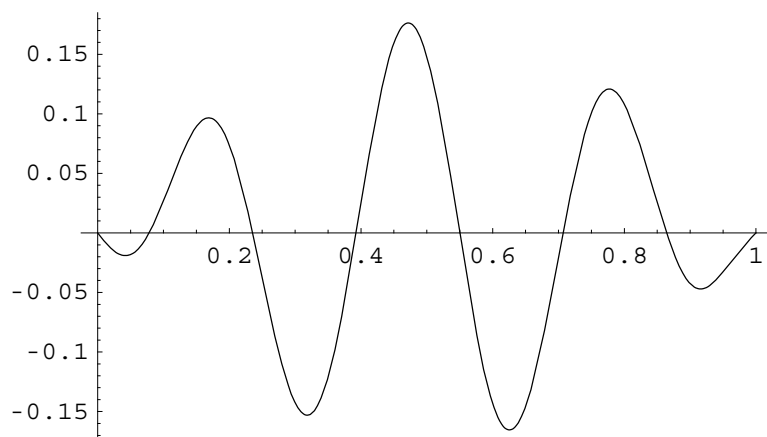
- Graphics -

```
Plot[Re[Sum[z[x, i], {i, 1, 10, 1}]], {x, 0., 1.}]
```



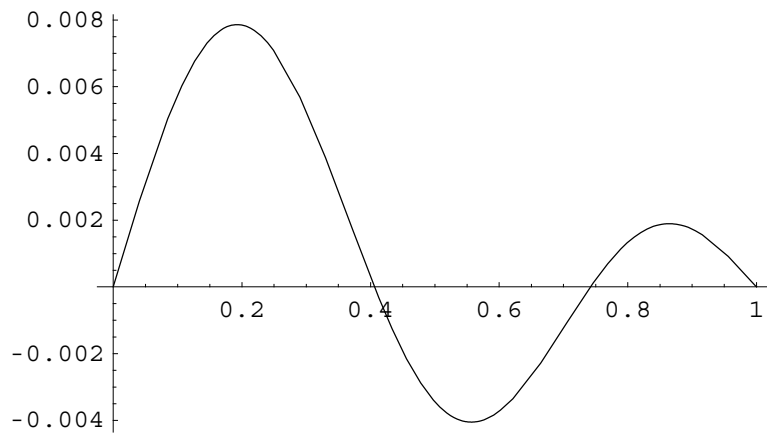
- Graphics -

```
Plot[Re[Sum[z[x, i], {i, 1, 20, 1}]], {x, 0., 1.}]
```



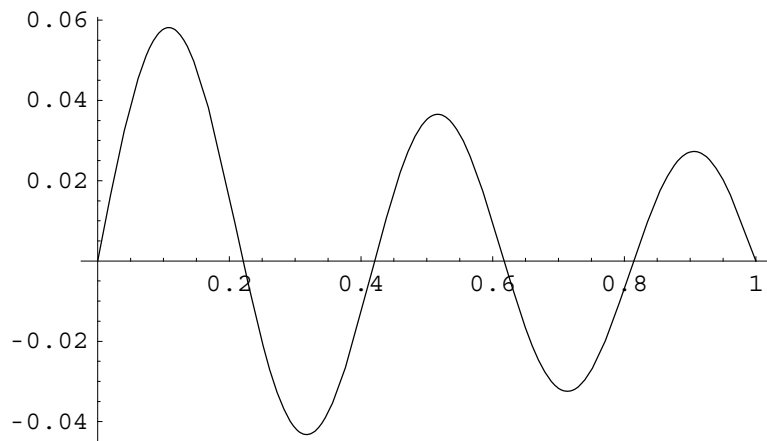
- Graphics -

```
Plot[Im[Sum[z[x, i], {i, 1, 3, 1}]], {x, 0., 1.}]
```



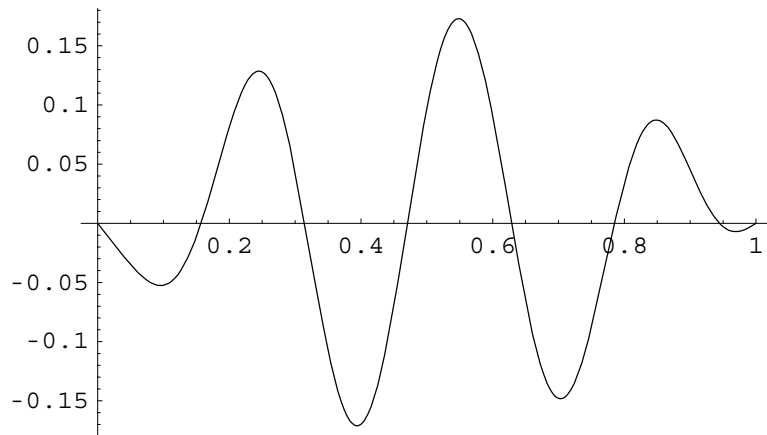
- Graphics -

```
Plot[Im[Sum[z[x, i], {i, 1, 5, 1}]], {x, 0., 1.}]
```



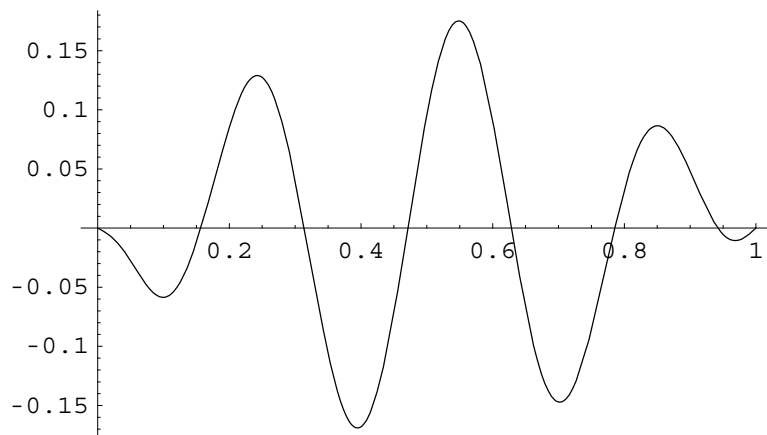
- Graphics -

```
Plot[Im[Sum[z[x, i], {i, 1, 10, 1}]], {x, 0., 1.}]
```



- Graphics -

```
Plot[Im[Sum[z[x, i], {i, 1, 20, 1}]], {x, 0., 1.}]
```



- Graphics -