Liboff 4.8

$$g(x) = x(x-a)\,e^{ikx} = \sum_{n=1}^{\infty} a_n\,\varphi_n(x)$$

$$\varphi_n(x) = \sqrt{\frac{2}{a}}\,\text{Sin}\left(\frac{n\pi x}{a}\right)$$

$a_n =$

$$\int_0^a g(x)\,\varphi_n(x)\,dx = \int_0^a x(x-a)\,e^{ikx}\sqrt{\frac{2}{a}}\,\text{Sin}\left(\frac{n\pi x}{a}\right)dx = \sqrt{\frac{2}{a}}\int_0^a x(x-a)\,e^{ikx}\,\text{Sin}\left(\frac{n\pi x}{a}\right)dx$$

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Integrate[x * (x - a) * Exp[I * k * x] * Sin[n * Pi * x / a], {x, 0, a}]
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$$\frac{1}{(a^2 k^2 - n^2 \pi^2)^3}\left(a^3\left(2 a^2 k^2 (3 + i a k)\, n\pi + 2(1 - i a k)\, n^3 \pi^3 + e^{i a k}\left(2 i n \pi\left(a^2 k^2 (3 i + a k) + (i - a k)\, n^2 \pi^2\right)\text{Cos}[n\pi] + \left(a^3 k^3 (2 i + a k) + 6 i a k n^2 \pi^2 - n^4 \pi^4\right)\text{Sin}[n\pi]\right)\right)\right)$$

```
Simplify[%, Element[n, Integers]]
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$$\frac{1}{(-a^2 k^2 + n^2 \pi^2)^3}\left(2 a^3 n\pi\left(3 a^2\left(-1 + (-1)^n e^{i a k}\right)k^2 - i a^3\left(1 + (-1)^n e^{i a k}\right)k^3 + \left(-1 + (-1)^n e^{i a k}\right)n^2 \pi^2 + i a\left(1 + (-1)^n e^{i a k}\right)k\, n^2 \pi^2\right)\right)$$

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FullSimplify[%]
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$$\frac{1}{(-a^2 k^2 + n^2 \pi^2)^3}\left(2 a^3 n\pi\left(-i a^2 k^2 (-3 i + a k) + (-1 + i a k)\, n^2 \pi^2 + (-1)^n e^{i a k}\left(a^2 k^2 (3 - i a k) + (1 + i a k)\, n^2 \pi^2\right)\right)\right)$$

$$a_n = \sqrt{\frac{2}{a}}\;\frac{1}{(-a^2 k^2 + n^2 \pi^2)^3}\left(2 a^3 n\pi\left(-i a^2 k^2 (-3 i + a k) + (-1 + i a k)\, n^2 \pi^2 + (-1)^n e^{i a k}\left(a^2 k^2 (3 - i a k) + (1 + i a k)\, n^2 \pi^2\right)\right)\right)$$