QCD Loop Corrections to Top Production and Decay

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Abstract. The subject of this talk is the computation of virtual QCD corrections to top production and decay at e^+e^- colliders. We examine the resonant behavior of the amplitudes that dominate the cross section and discuss the double pole approximation (DPA). The theoretical framework is similar to that which has been successfully applied for QED corrections to W pair production at LEP II.

INTRODUCTION

An e^+e^- collider with center-of-mass energy at and above the top threshold promises to provide a clean environment in which to perform precision studies of the top quark. It is conceivable that at such a machine the study of top can be performed with a precision similar to that achieved in the study of the W boson at LEP II. This means order % (and better) measurements of differential cross sections for processes involving the top quark. Such a precision in measuring experimental quantities implies the need for a like precision in our theoretical understanding of these processes. This in turn requires the inclusion of radiative corrections in our predictions.

Our aim is the computation of QCD corrections to the full production and decay process (that is, we do not treat the top quarks as on-shell particles). Therefore, besides corrections to particular subprocesses (production or decay), we also take into account interference (or nonfactorizable) corrections. Similar computations have been performed for the W pair production process [1]. We discuss similarities and differences between the two cases.

FRAMEWORK

The process of interest is, at tree level,

$$e^+e^- \to \ t\ \bar{t}\ \to b\ W^+\ \bar{b}\ W^- \tag{1}$$

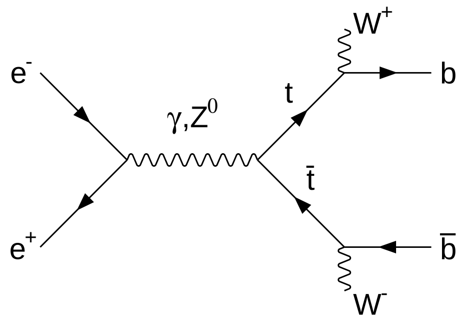

FIGURE 1. Tree level diagram for process (1)

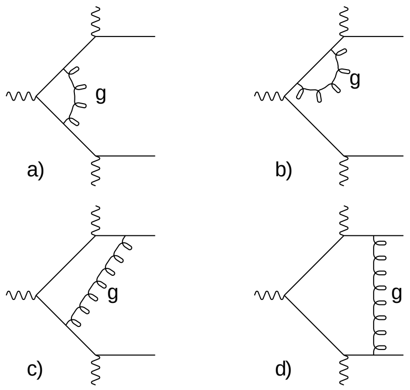

FIGURE 2. Virtual corrections to (1); a) and b) : corrections to vertex and fermion self energy; c) and d) : interference-type corrections

whose Feynman diagram is represented in Fig. 1. Note that there are many diagrams contributing to the final state $b\ W^+\ \bar{b}\ W^-$; however, in the region of the phase space where $p_t^2,\ p_{\bar{t}}^2 \approx m_t^2$, (1) dominates the amplitude (due to the two resonant top propagators).

Virtual corrections to (1) can be roughly divided into two classes : corections to particular subprocesses – like vertex and fermion self-energy diagrams (Figs. 2a and 2b respectively) – and interference-type corrections (as in Figs. 2c, 2d). For computational purposes, we consider six separate contributions :

$$M_1 = M_{t\bar{t}} + M_{tb} + M_{\bar{t}\bar{b}} + M_{b\bar{t}} + M_{t\bar{b}} + M_{b\bar{b}} \tag{2}$$

where the first three terms correspond to the three vertex corrections (in which we have included in a suitable way the fermion self energy corrections), and the last three terms come from the interference diagrams. The exact evaluation of these partial amplitudes is a difficult task, especially for the interference part. However, we don't need the exact results; these amplitudes are also dominated by terms which are enhanced by two resonant top propagators. Hence, we will restrict ourselves to the computation of these doubly resonant terms (double pole approximation).

As an example of how this approximation works, let's look at the production-top decay interference amplitude [1] :

$$M_{b\bar{t}} = \bar{u}(b) \left[(-ig_s^2) \int \frac{d^4k}{2\pi^4}\ \frac{1}{k^2}\ \gamma^\mu\ \frac{\hat{p}_b - \hat{k} + m_b}{(p_b-k)^2 - m_b^2}\ \hat{\epsilon}_{W^+}\ \frac{\hat{p}_t - \hat{k} + m_t}{(p_t-k)^2 - \bar{m}_t^2}\ \hat{a} \right.$$
$$\left. \frac{-\hat{p}_{\bar{t}} - \hat{k} + m_t}{(p_{\bar{t}}+k)^2 - \bar{m}_t^2}\ \gamma_\mu \right]\ \frac{-\hat{p}_{\bar{t}} + m_t}{p_{\bar{t}}^2 - \bar{m}_t^2}\ \hat{\epsilon}_{W^-}\ v(\bar{b}) \tag{3}$$

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1) here, $\bar{m}_t^2 = m_t^2 - im_t\Gamma_t$

The doubly resonant terms can be extracted with the help of a simple observation: if the virtual gluon in the loop is hard, then the quantity in brackets will be well behaved, and the overall resonant behavior for this amplitude is $1/(p_{\bar{t}}^2 - \bar{m}_t^2)$. This means that the doubly resonant terms are entirely due to soft virtual gluons. Therefore, we can use the extended soft gluon approximation (ESGA), which means neglecting the $\hat{k}$ terms in the the numerator of (3) (see also [2], [3]). Then, with the help of some standard Dirac algebra manipulations, we obtain :

$$M_{b\bar{t}}(DPA + ESGA) = \frac{\alpha_s}{4\pi} M_0 \ * \ (-4p_b p_{\bar{t}})(p_t^2 - \bar{m}_t^2) \ D^0_{b\bar{t}} \tag{4}$$

where M_0 is the tree level matrix element, and $D^0_{b\bar{t}}$ is the scalar 4-point function corresponding to the production-top decay interference diagram [2].

Let's take a look at the resonant behavior of $M_{b\bar{t}}$. The $(p_t^2 - \bar{m}_t^2)$ factor in the above equation cancels a pole in M_0, and the D_0 function has a logarithmic singularity when the $\bar{t}$ goes on-shell. Therefore, the resonant behavior of $M_{b\bar{t}}$ is of type *pole* $\times$ *log* rather than double pole. Similar results and behavior are obtained for the other interference terms.

The results for the purely non-factorizable corrections (another name for the interference diagrams 2c, 2d) for the $t\ \bar{t}$ production and decay process are completely analogous to the results obtained for the same type of diagrams in the W pair production case [2]. However, this similarity does not hold for the vertex corrections. Consider, for example, the vectorial part of the $t\ \bar{t}$ production vertex :

$$\delta\Gamma^\mu_V = \frac{\alpha_s}{4\pi} \int \frac{d^4k}{i\pi^2} \frac{1}{k^2} \gamma^\nu \frac{\hat{p}_t - \hat{k} + m_t}{(p_t - k)^2 - \bar{m}_t^2} \gamma^\mu C_V \frac{-\hat{p}_{\bar{t}} - \hat{k} + m_t}{(p_{\bar{t}} + k)^2 - \bar{m}_t^2} \gamma_\nu$$

Upon integration, we can express the result in terms of 8 form factors :

$$\delta\Gamma^\mu_V = \frac{\alpha_s}{4\pi} C_V \ [\gamma^\mu F_2 + (\hat{p}_t - m_t)\gamma^\mu F_4 + \gamma^\mu(\hat{p}_{\bar{t}} + m_t)F_6 + (\hat{p}_t - m_t)\gamma^\mu(\hat{p}_{\bar{t}} + m_t)F_8$$
$$+ (p_t - p_{\bar{t}})^\mu F_1 + (\hat{p}_t - m_t)(p_t - p_{\bar{t}})^\mu F_3 + \ldots] \tag{5}$$

In the on-shell case, only the F_2 (electric dipole) and F_1 (magnetic dipole momentum) form factors contribute. We might expect that in the double pole approximation we can drop the other terms, too, since the $(\hat{p}_t - m_t)$ terms will cancel a pole (or both) in the amplitude. However, the form factors themselves have a logarithmic resonant behavior when either one particle (for the decay vertices) or both (for the production vertex) go on-shell. It follows that in expression (5) we have to keep the terms which contain F_1 to F_6 (we can drop the F_7 and F_8 terms), and we do not have factorization (virtual corrections being proportional to the tree level

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[2]

$$D^0_{b\bar{t}} = \int \frac{d^4k}{i\pi^2} \frac{1}{k^2 + i\epsilon} \frac{1}{k^2 - 2kp_b} \frac{1}{(p_t - k)^2 - \bar{m}_t^2} \frac{1}{(p_{\bar{t}} + k)^2 - \bar{m}_t^2}$$

matrix element) anymore. This is different from what happens in the W pair production process, where in DPA factorization holds even for the vertex corrections. This difference is due to the fact that in our process the intermediate particles are fermions, and not bosons.

An issue which needs some attention is the gauge invariance of our result. A complete computation, taking into account all the diagrams, including the non-resonant ones, would give a gauge invariant answer; however, by restricting ourselves to a subset of diagrams, we may lose this property. In the W pair production case, gauge invariance in DPA is guaranteed by the fact that the off-shell corrections are proportional to the on-shell result. This is no longer true here; however, we have checked that our results are gauge invariant in the approximation used (that is, up to singly resonant terms).

As the last topic, we will mention only briefly the treatment of infrared singularities. In the computation of the virtual corrections, the only place they appear is in the self-energies of the b and $\bar{b}$ quarks and in the decay-decay interference diagram 2d. Once we take into account the soft gluon radiation, these virtual singularities will cancel against the ones coming from real gluon radiation from the b and $\bar{b}$ quarks.

CONCLUSIONS

The subject of this paper concerns QCD corrections to top production and decay at a linear collider. We have discussed in some detail the implementation of the double pole approximation in the evaluation of the dominant diagrams which contribute to this process. Similarities and differences to the computation of QED radiative corrections to W pair production at LEP have been mentioned.

This is work in progress, and we expect to have numerical results soon. The final aim of our work is a complete next to leading order QCD computation of top production and decay at an e^+e^- collider.

I wish to thank my adviser Prof. Lynne Orr and Dr. Doreen Wackeroth for support offered and many helpful discussions on the subject of virtual corrections.

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