STABILITY BOUNDARIES FOR COMMAND AUGMENTATION SYSTEMS

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INTRODUCTION

For statically unstable aircraft, there is an increased need to understand the effects of control saturation. The longitudinal mode of unstable aircraft usually has a single unstable pole, whereas the lateral-directional mode may have two real poles or a pair of complex-conjugate poles in the right half plane. Reduced-order models are often used to determine the stabiliy and performance of the aircraft approximately. This study examines the effects of control saturation using reduced-order models.

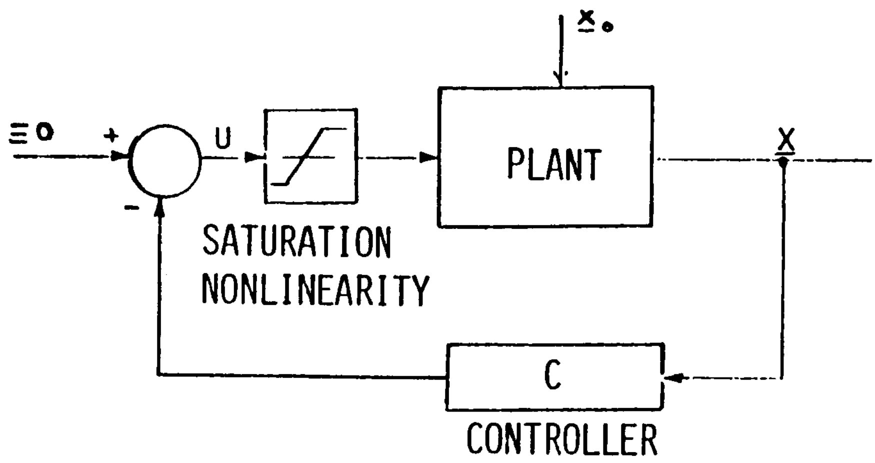
The stability boundaries for command augmentation systems are determined for three types of singularities: saddle-point, unstable nodes, and unstable foci. Control saturation imposes bounds on command vectors for which equilibrium can be attained. For the cases of saddle-point and unstable nodes, the region of stability reduces to zero for command vectors which demand a steady value of control exceeding the control saturation limits. In the case of unstable foci, the region of attraction does not gradually reduce in size, but at some point it breaks abruptly.

OVERVIEW

- INTRODUCTION
- PREVIOUS WORK
- COMMAND AUGMENTATION SYSTEMS
- CONCLUSIONS

STABILITY AUGMENTATION SYSTEM

A block diagram of the system under consideration is shown. It consists of a dynamic system to be controlled (the "plant"), a feedback controller, and a saturating element on one or more of the controls. Command inputs can be ignored in the stability analysis of this constant-coefficient system.



$$\dot{\underline{x}} = F\underline{x} + G\underline{u}$$

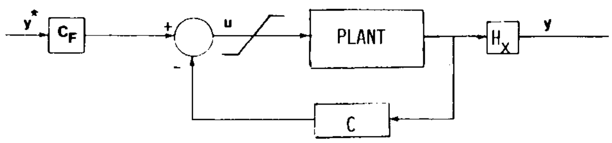
$$\underline{u} = -C\underline{x}$$

$$|\underline{u}| = \,<\underline{u}_M$$

COMMAND AUGMENTATION SYSTEM

Specific inputs and outputs must be considered in the command augmentation system. Defining y as an output vector that is a linear combination of state vector components, y* is the desired value of the output. The resulting closed-loop system is described by an ordinary differential equation, whose equilibrium state and control vectors, x* and u*, can be related to the desired output.

The steady-state value of control is independent of feedback gain, which can be obtained either from open- or closed-loop dynamics. For open-loop unstable systems, feedback is mandatory to achieve stability and command equilibrium. The linear feedback regulator provides satisfactory transient response to meet performance specifications otherwise not obtainable. The state equilibrium depends only on the open-loop dynamics and control magnitude. A steady-state control u* exists only for the "nonsingular" command vector.



$$\dot{\underline{x}} = F\underline{x} + G\underline{u} \qquad \text{OUTPUT:}$$

$$\underline{u} = C_B\,\underline{x} + C_F\,\underline{Y}^* \qquad \underline{Y}^* = H_x\,\underline{x}^*$$

CLOSED-LOOP

$$\dot{\underline{x}} = (F - G\,C_F)\,\underline{x} + G\,C_F\,\underline{Y}^*$$

EQUILIBRIUM

$$\underline{x}^* = -(F - G\,C_F)^{-1}\,G\,C_F\,\underline{Y}^*$$

$$\underline{u}^* = -\,C_B\,(F - G\,C_F)^{-1}\,G\,C_F\,\underline{Y}^* + C_F\,\underline{Y}^*$$

WHERE

$$C_F = S_{22} + C\,S_{12}$$

$$S_{22} = -\,H_xF^{-1}\,G$$

$$S_{12} = (-F^{-1}G)\,S_{22}$$

$$C_B = -\,C$$

OPEN LOOP EQUILIBRIUM:

$$\underline{x}^* = -\,F^{-1}\,G\,\underline{u}^*$$

$$\underline{Y}^* = H_x\,\underline{x}^* = -\,H_xF^{-1}\,G\,\underline{u}^*$$

$$\underline{u}^* = (-H_xF^{-1}\,G)^{-1}\,\underline{Y}^*$$

$$= -\,(H_xF^{-1}\,G)^{-1}\,H_x\,\underline{x}^*$$

COMMAND AUGMENTATION SYSTEM (CONCLUDED)

IT CAN BE SHOWN THAT

$$\underline{u}^* = - C_B(F - G\ C_F)^{-1}\ G\ C_F\ \underline{Y}^* + C_F\ \underline{Y}^*$$

$$= (-H_X F^{-1}\ G)^{-1}\ \underline{Y}^*$$

- $\underline{U}^*$ INDEPENDENT OF GAIN C
- C DETERMINES RESPONSE
- $\underline{X}^*$ LOCATION DETERMINED BY STATE EQUATION
- $\underline{U}^*$ EXISTS FOR "NONSINGULAR COMMAND" EQUILIBRIUM

EFFECTS OF CONTROL SATURATION

The control saturation limits and open-loop dynamics determine the minimum and maximum values of state equilibrium. Saturation prevents the system from attaining the desired equilibrium and response, and it imposes bounds on achievable command vector y*. To avoid saturation, desired state equilibrium points must lie within the unsaturated region. However, this still does not guarantee that the trajectories would not enter a saturated-control region, for some initial conditions and/or commands. This is mainly determined by the eigenvectors in the unsaturated region. Thus, saturation enforces bounds on the command vectors for which equilibrium could actually be attained without saturation. Equilibrium cannot be attained for the command vectors for which the state equilibrium point is located in the saturated region.

Feedback gain C alters the response of the system, but it does not affect u*. The command vectors for which equilibrium can be attained are independent of feedback gain. H_X does not affect the state equilibrium or the feedback gain. It changes the prefilter gain which shapes input to achieve the desired equilibrium. u* changes with the command vector; hence, the saturation boundaries change with commands.

$-\underline{U}_M < \underline{U}^* < +\underline{U}_M$: CONTROL SATURATION

$\underline{x}^* = -F^{-1} G \underline{u}^*$

$\underline{X}-U_M < \underline{X}^* < \underline{X}+U_M$: BOUNDS ON STATE EQUILIBRIUM

$\underline{Y}^* = Hx\, \underline{x}^*$:

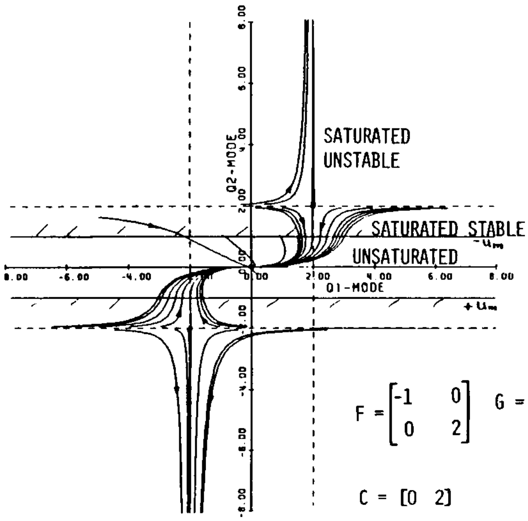
BOUNDS ON ACHIEVABLE COMMANDS

$\underline{Y}-U_M < \underline{Y}^* < \underline{Y}+U_M$:

- DEPENDENCE OF STATE EQUILIBRIUM ON $F^{-1} G$
- EFFECTS OF VARIATIONS OF C, H_X

STABILITY BOUNDARIES FOR THE SADDLE-POINT: MCE CASE

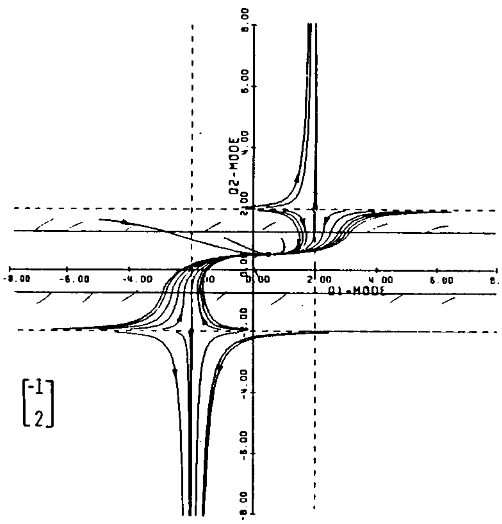
The stability boundaries for the minimum-control-energy (MCE) case are shown on the next three figures. Note the variations in saturation boundaries and location of x* with changes in the command vector. The region of stability remains unchanged. Equilibrium can be achieved only for those command vectors for which the equilibrium point lies within the saturated region. Invariance of stability boundaries with changes in command command vectors is a unique result for the saddle-point MCE case.



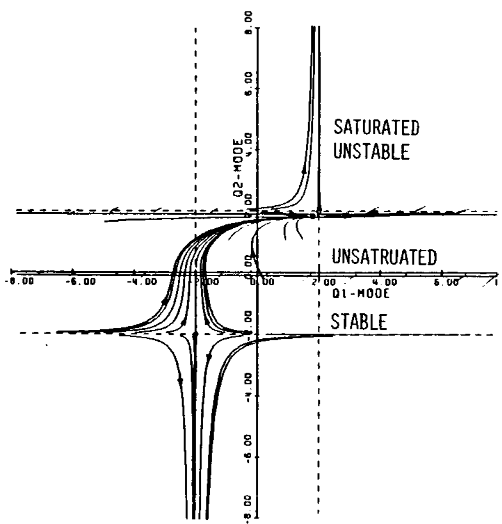
$$F = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix} \quad G = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

$C = [0 \;\; 2]$

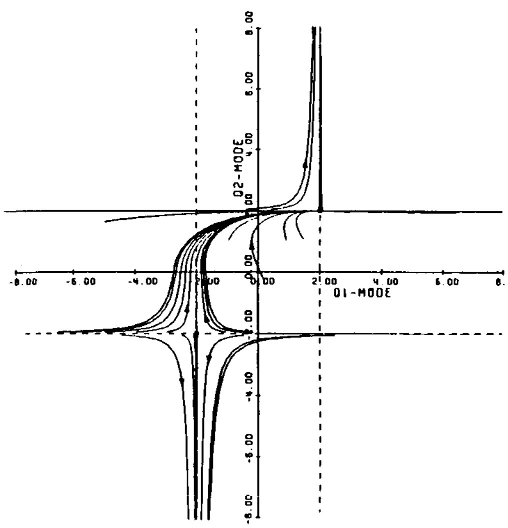
Y = 0.0



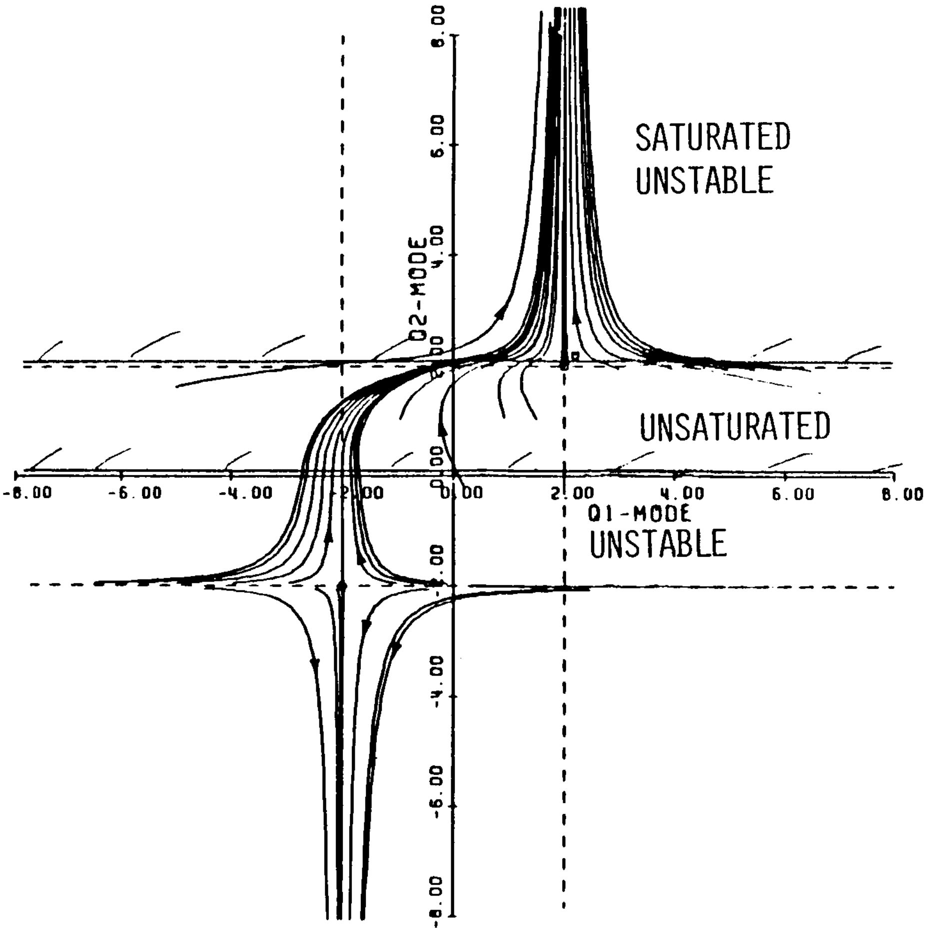
Y = 0.5



Y = 1.8



Y = 2.0

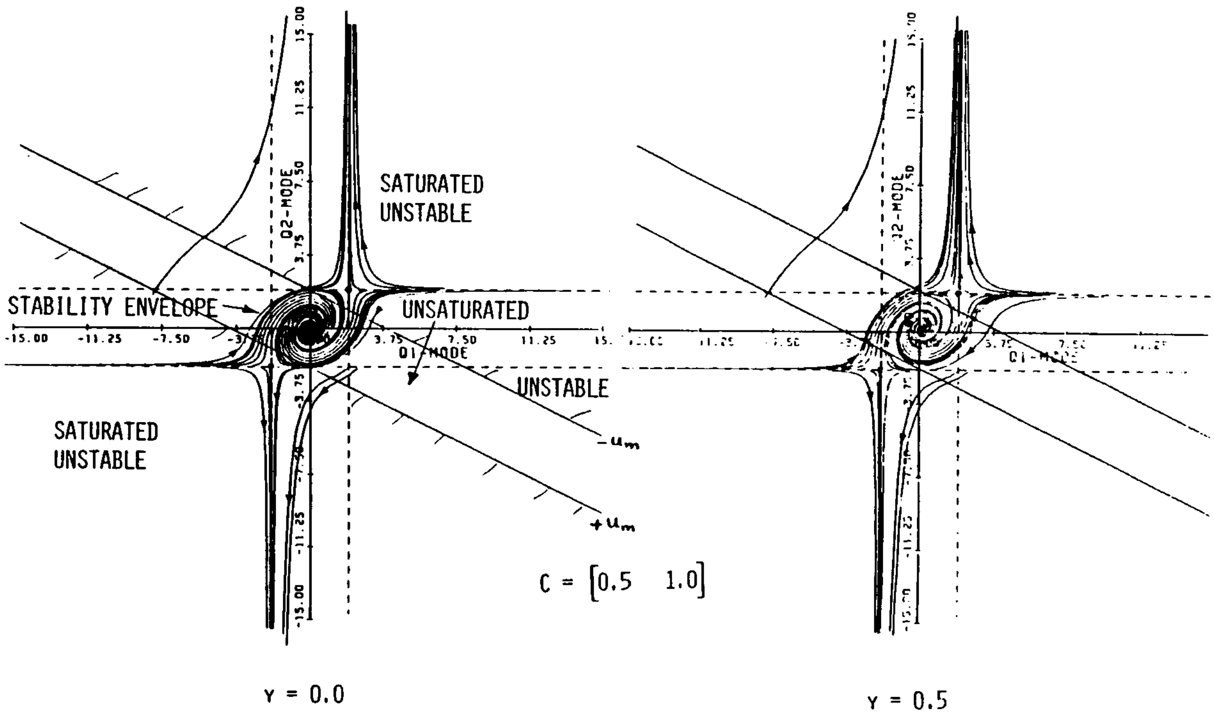

SATURATED
UNSTABLE
UNSATURATED
UNSTABLE
Q2-MODE
Q1-MODE
-8.00
-6.00
-4.00
-2.00
0.00
2.00
4.00
6.00
8.00

Y = 2.2

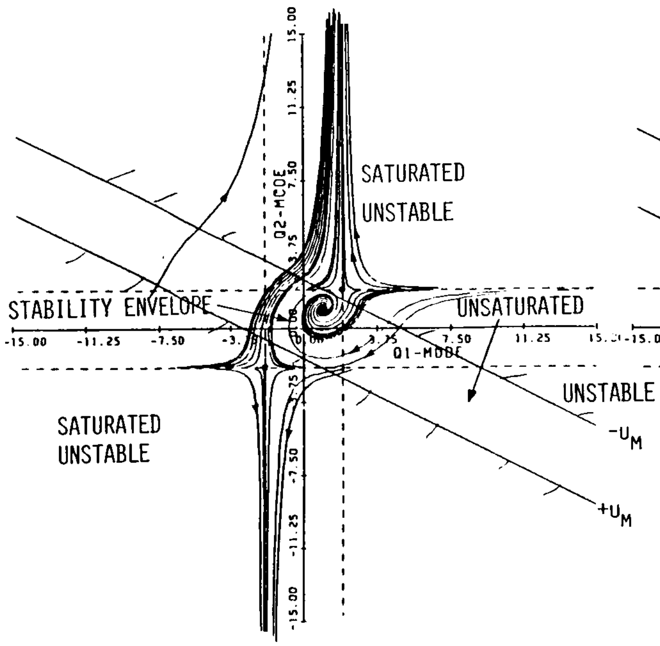
STABILITY BOUNDARIES FOR THE SADDLE-POINT: LOW GAIN CASE

The stability boundaries are shown on the next three figures, where unlike the minimum-control-energy (MCE) case, the stability boundaries change with commands. The locations of equilibrium points in the saturated region do not change with the command vectors. The region of stability is biggest for zero command, i.e., the maximum region of stability is achieved for the stability augmentation case. For non-zero command vectors, the stability region shrinks. It reduces to zero when the desired equilibrium control exceeds the saturation limits.

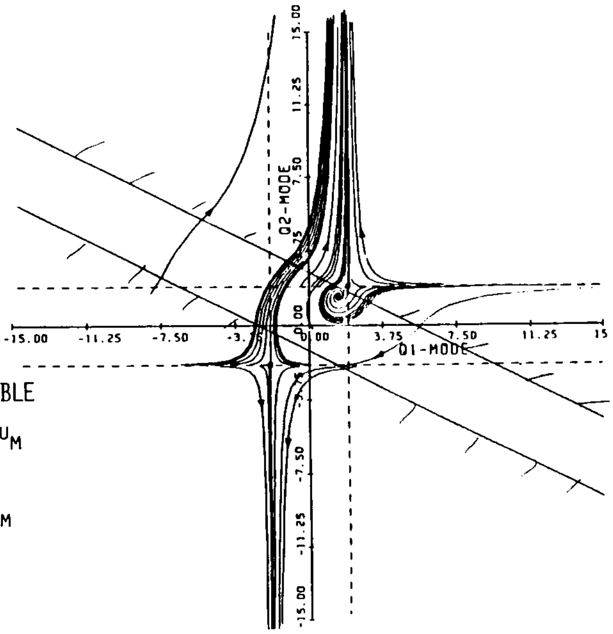
For each command vector, trajectories seek separate equilibrium points; hence, the trajectories starting from the same initial conditions follow entirely different paths in the phase plane. For this reason, markedly different time-histories for different command vectors are obtained, though the system eigenvalues/eigenvectors remain unchanged.



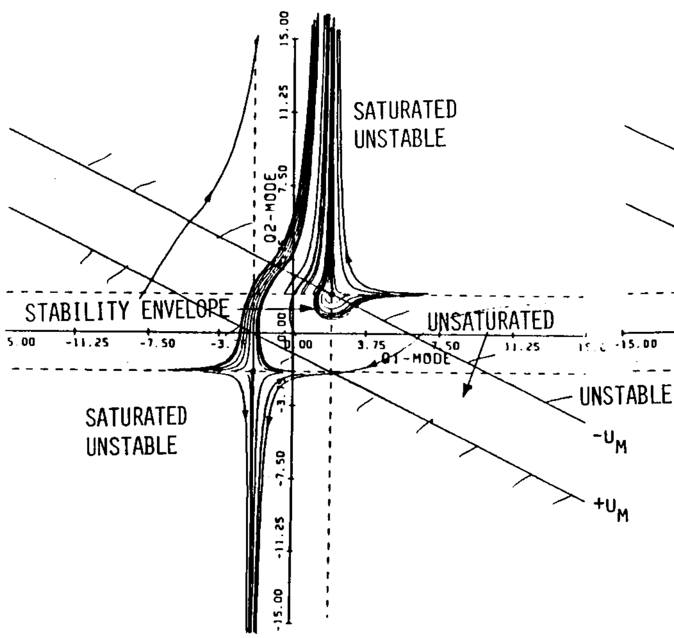
LOW-GAIN CASE (CONCLUDED)



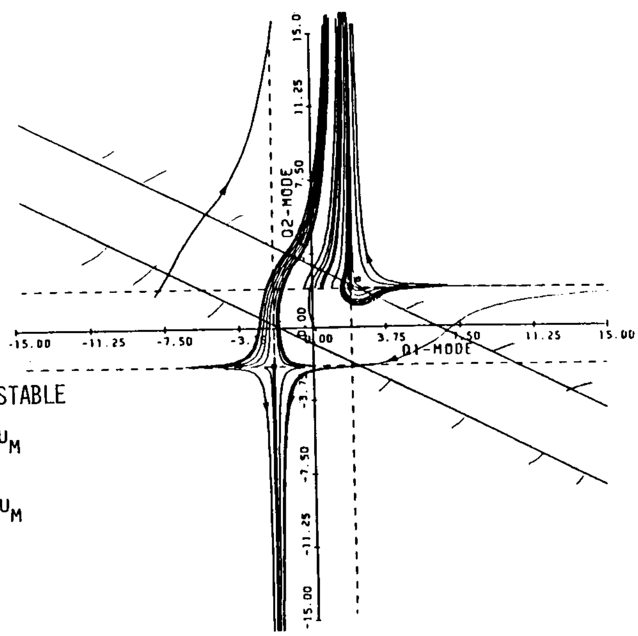
Y=1.5



Y = 1.8



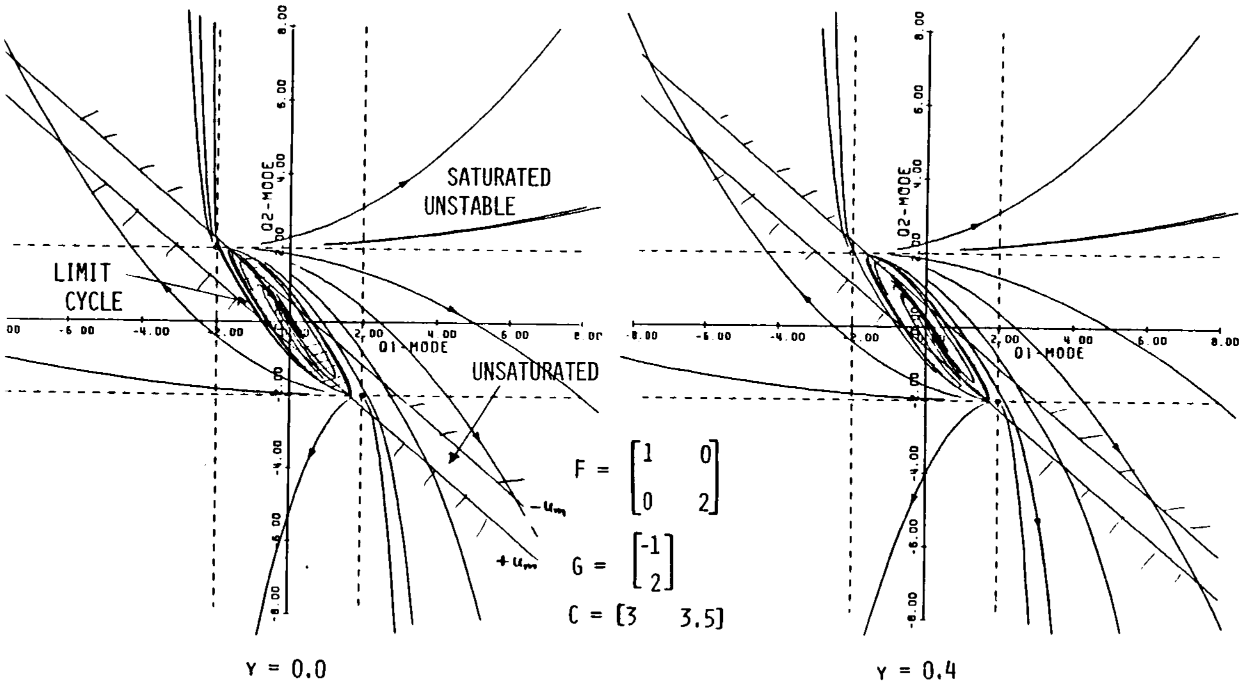
Y = 2.0



Y = 2.4

STABILITY BOUNDARIES FOR UNSTABLE NODES

The next three figures show the stability boundaries for the case of unstable nodes for various command vectors. As noticed before, the saturation boundaries here also change with changes in commands. The sizes of limit cycles, which represent the saturation boundaries, also change. The region of stability is biggest for the stability augmentation system (SAS) case. For increasing command magnitudes, the steady-state equilibrium point moves away from the origin. The region of stability shrinks, and eventually it reduces to zero for commands that require $u^* \geqslant |u_m|$.



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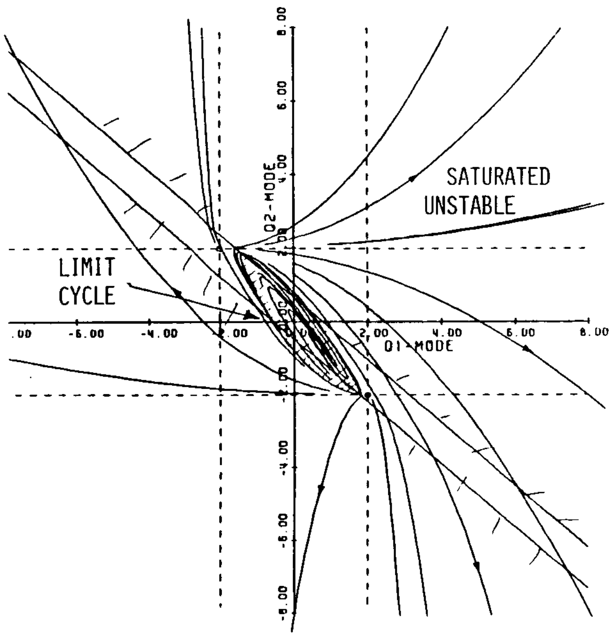
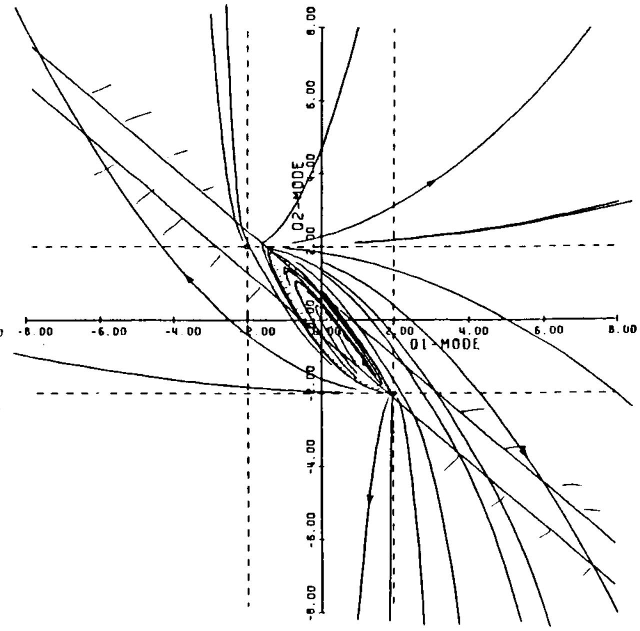
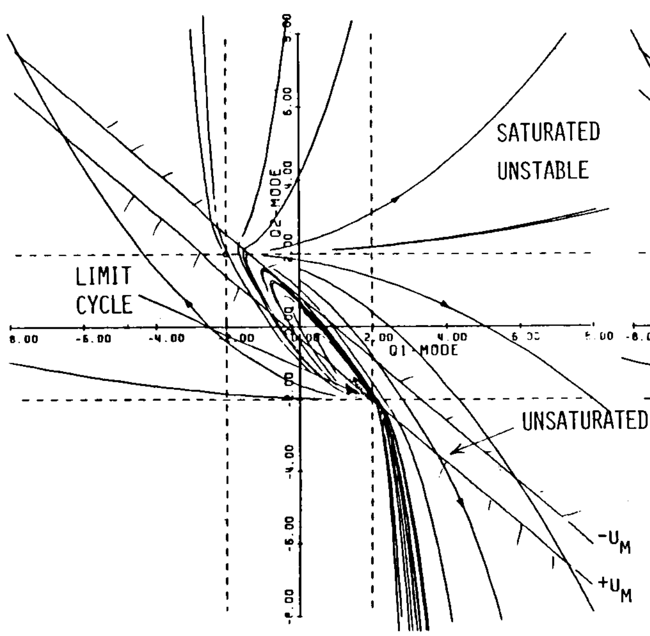
UNSTABLE MODES (CONCLUDED)



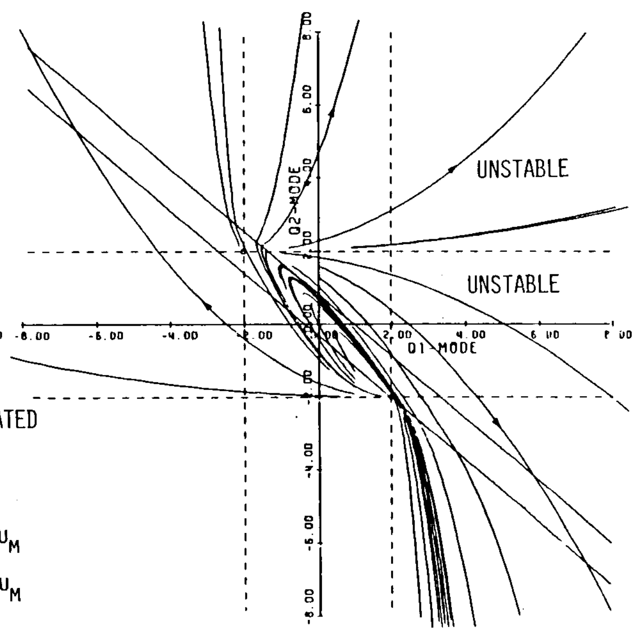
$\gamma = 0.8$



$\gamma = 1.2$



$\gamma = 1.9$



$\gamma = 2.0$

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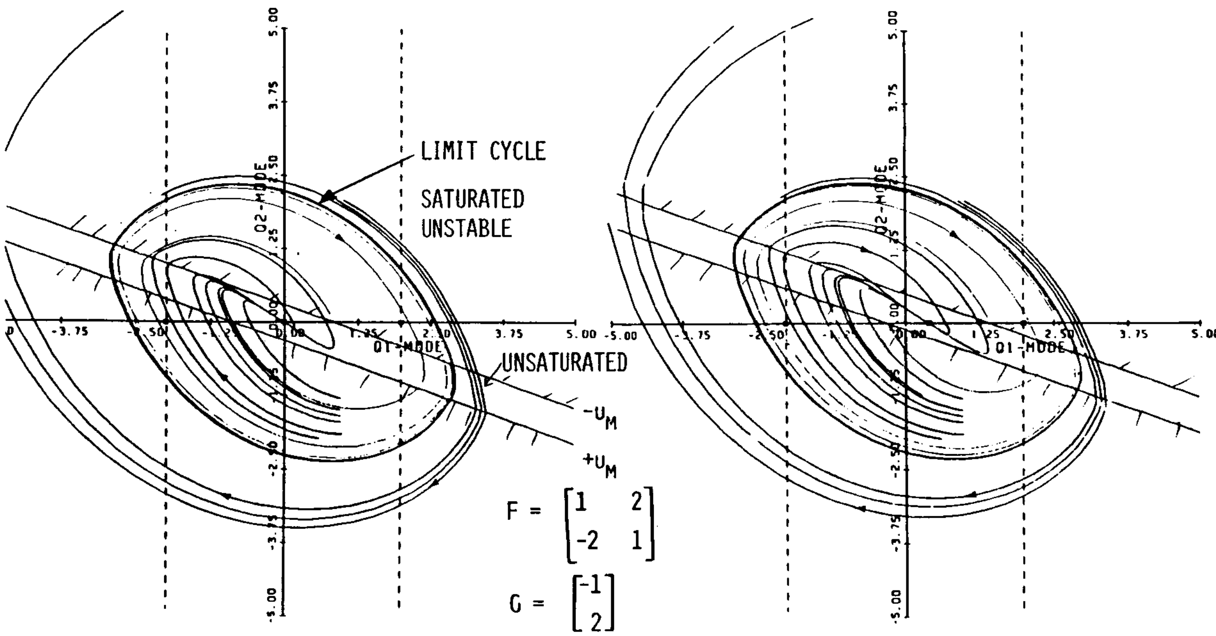
STABILITY BOUNDARIES OF UNSTABLE FOCI

The stability boundaries for the case of unstable foci are unstable limit cycles, as shown on the next six figures. Apparently, there is little variation in the size of stability region with commands. The stable equilibrium point moves to the right of the origin with increasing command values, and the saturation boundaries also shift. At control saturation limits, this equilibrium point lies on the saturation boundary, but the region of stablity does not shrink to zero, contrary to the cases of saddle-point and unstable nodes.

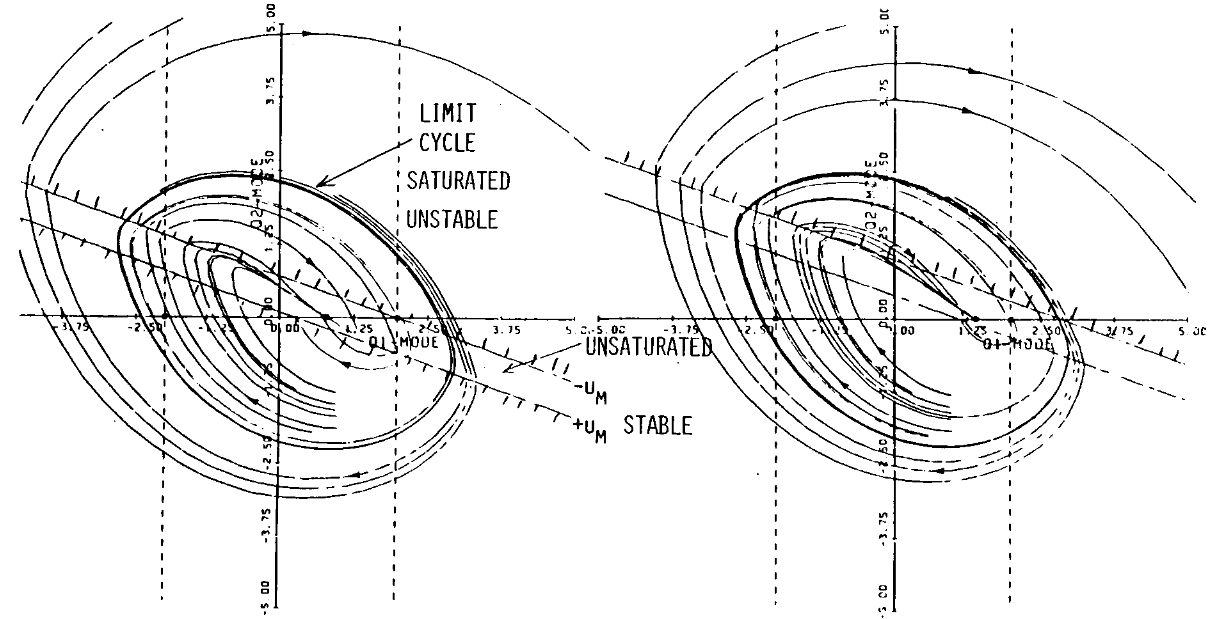
Further increase in commands moves the desired equilibrium point farther to the right, and another limit cycle emerges. The trajectories within it converge to this new limit cycle; those within the original limit cycle also converge to it. Thus, the new limit cycle is stable. This "inner" limit cycle grows with increase in commands, until it co-incides with the "outer" limit cycle. Any further increase in command results in breaking of the closed stability region (or "bursting" of the limit cycle), making the entire region unstable. This peculiar result in the case of unstable foci is under further investigation.



$F = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$

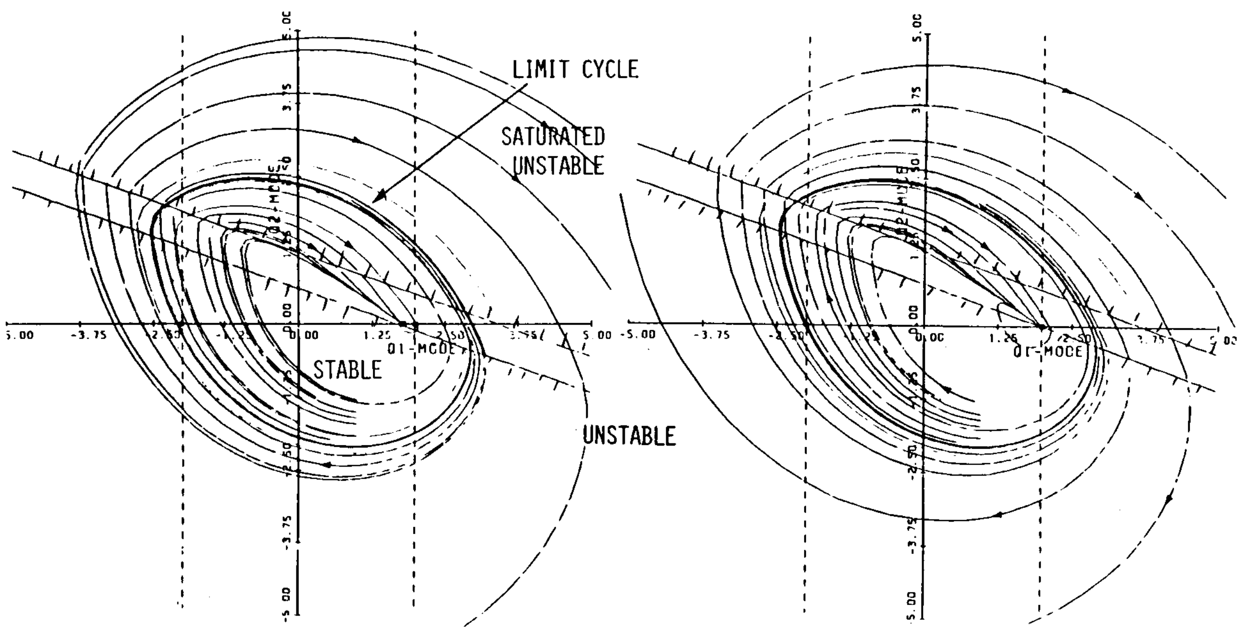
$G = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$

$Y = 0.0$ $C = [0.95 \quad 2.27]$ $Y = 0.4$



γ = 0.8

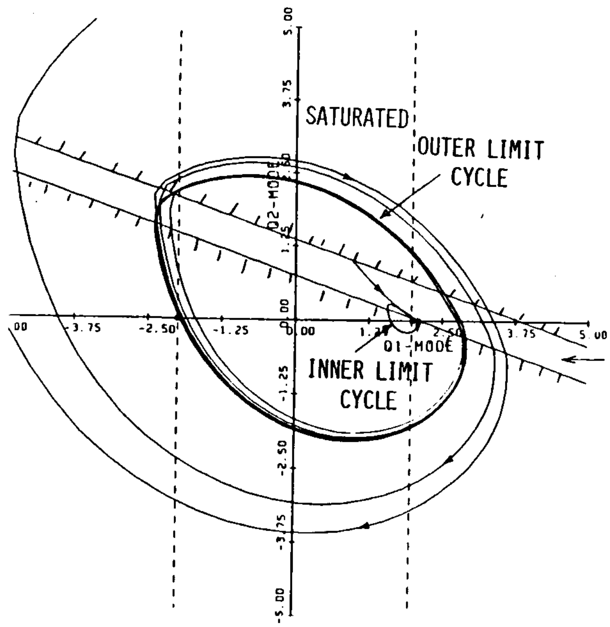
γ = 1.4



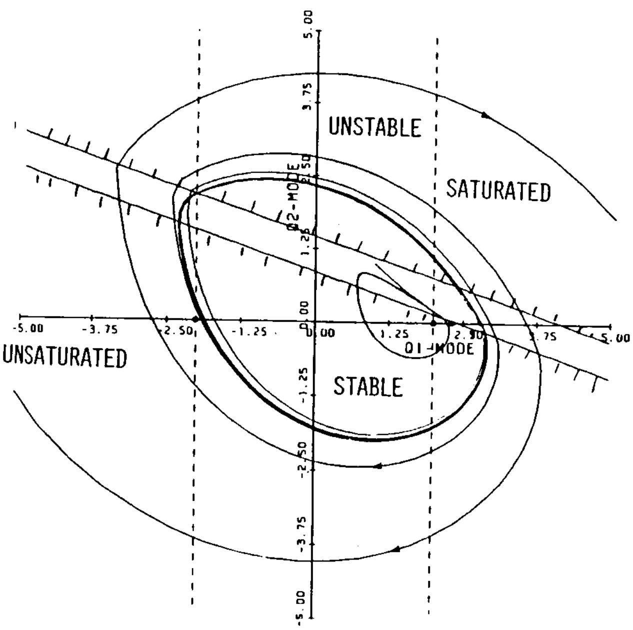
γ = 1.8

γ = 2.0

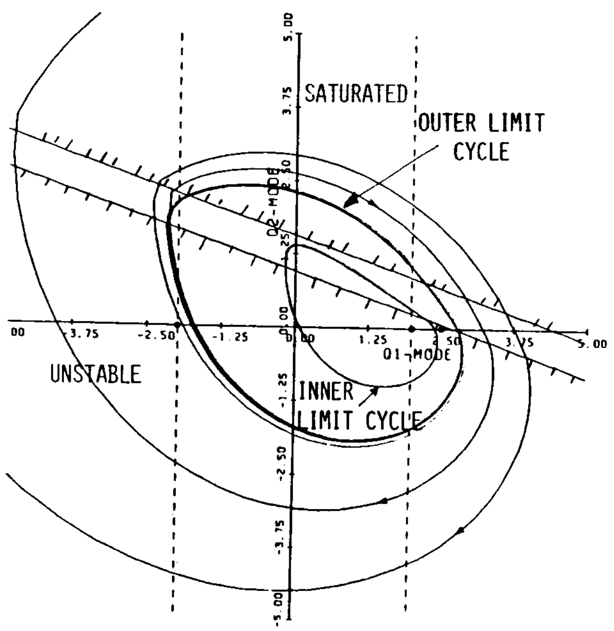
UNSTABLE FOCI (CONTINUED)



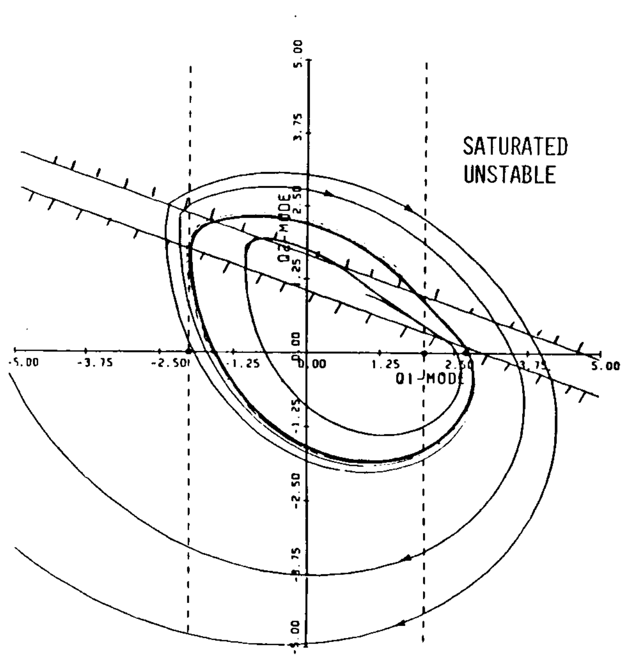
Y = 2.1



Y = 2.3

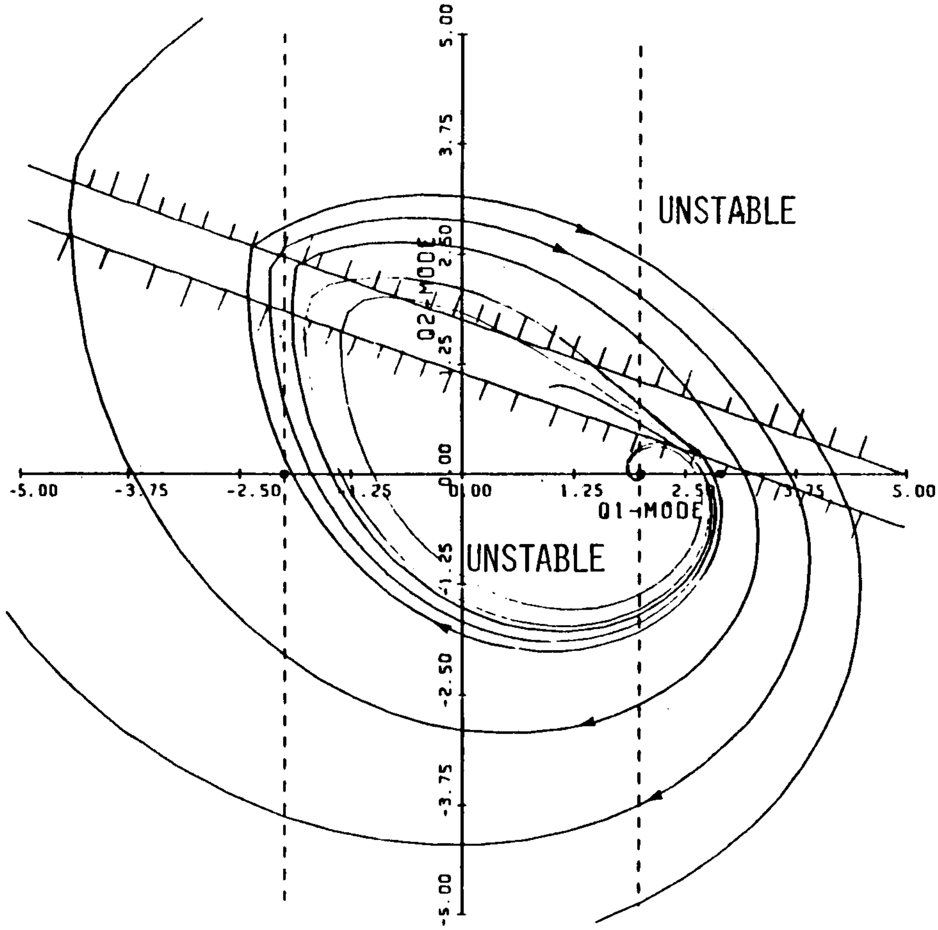


Y = 2.5



Y = 2.7

UNSTABLE FOCI (CONCLUDED)



Y = 2.9

CONCLUSIONS AND FUTURE WORK

The Stability Augmentation System (SAS) is a special case of the Command Augmentation System (CAS). Control saturation imposes bounds on achievable commands. The state equilibrium depends only on the open-loop dynamics and control deflection. The control magnitude to achieve a desired command equilibrium is independent of the feedback gain. A feedback controller provides the desired response, maintains the system equilibrium under disturbances, but it does not affect the equilibrium values of states and control.

The saturation boundaries change with comands, but the locations of the equilibrium points in the saturated region remain unchanged. Nonzero command vectors yield saturation boundaries that are asymmetric with respect to the state equilibrium. Except for the saddle-point case with MCE control law, the stability boundaries change with commands. For the cases of saddle-point and unstable nodes, the region of stability decreases with increasing command magnitudes; it is reduced to zero for commands that require steady-state control $u^* \geqslant |u_m|$.

The regions of stability are biggest for the SAS. In the case of unstable foci, the region of stability does not vanish at $u^* = u_m$. An "inner" limit cycle is obtained, which grows with increase in commands until it coalesces with the "outer" limit cycle. Any further increase in command breaks this closed stability boundary. For a fixed degree of stability, different commands cause markedly different responses because they seek different equilibrium states.

- STABILITY AUGMENTATION SYSTEM: A SPECIAL CASE OF COMMAND AUGMENTATION SYSTEM

- CONTROL SATURATION LIMITS ACHIEVABLE COMMANDS

- DEPENDENCE OF STATE EQUILIBRIUM ON OPEN-LOOP DYNAMICS AND CONTROL

- STATE EQUILIBRIUM, STEADY-STATE CONTROL: INDPENDENT OF FEEDBACK GAIN

- FEEDBACK: TO ACHIEVE DESIRED RESPONSE: MAINTAIN EQUILIBRIUM IN PRESENCE OF DISTURBANCE

CONCLUSIONS

- SATURATION BOUNDARIES CHANGE WITH COMMAND VECTORS
- STABILITY BOUNDARIES CHANGE EXCEPT FOR MCE CASE
- BIGGEST REGION OF STABILITY FOR SAS
- REGION OF STABILITY REDUCES TO ZERO FOR $Y = \pm Y_{MAX}$ (EXCEPT IN THE CASE OF UNSTABLE FOCI)
- MARKEDLY DIFFERENT "LOOKING" RESPONSES FOR DIFFERENT COMMAND VECTORS

FUTURE WORK

- STABILITY BOUNDARIES FOR TWO-INPUT COMMAND AUGMENTATION SYSTEM
- DESIGN OF LATERAL-DIRECTIONAL COMMAND AUGMENTATION SYSTEM
- VARIATION OF STABILITY BOUNDARIES WITH FLIGHT CONDITIONS