Why Interference Fragmentation Functions ?

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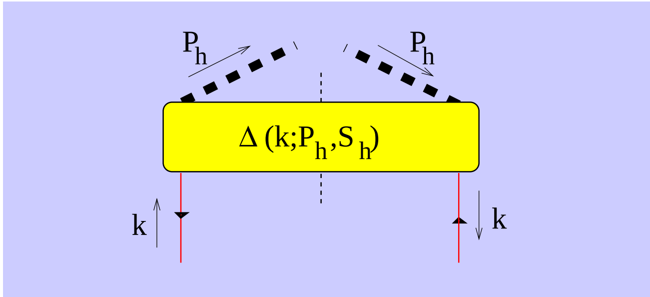
- ♣ transverse momentum dependent FF$^{q\to h\,X}$
- ♠ problems to model (interference) FF$^{q\to h\,X}$
- ♣ two hadron interference FF$^{q\to h_1\,h_2\,X}$

based on work with:
Andrea Bianconi, Sigfrido Boffi, Marco Radici,
Daniel Boer, Piet Mulders, Joao Rodrigues

Fragmentation Functions describe $q \to h + X$



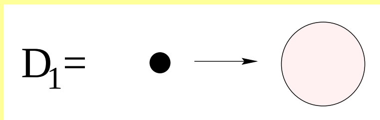
hadronic ME of bilocal quark operator Soper, Collins, Jaffe

quark-quark correlation function (one hadron)

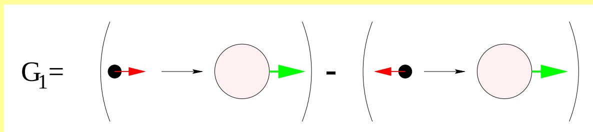
$$\Delta_{ij}(k, P_h, S_h) = \sum_X \int \frac{d^4x}{(2\pi)^4}\, e^{ik\cdot x} \times \langle 0|\psi_i(x)|P_h, S_h; X\rangle \langle P_h, S_h; X|\overline{\psi}_j(0)|0\rangle$$

$$\Delta^{[\Gamma]}(z) = \frac{1}{4z}\int dk^+\, d^2\mathbf{k_T}\ \mathrm{Tr}(\boldsymbol{\Delta}\ \boldsymbol{\Gamma})\Big|_{k^-=P_h^-/z}$$

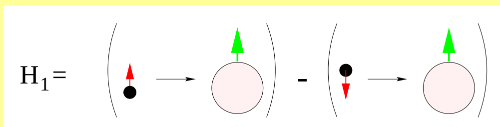
$$\Delta^{[\gamma^-]}(z) = D_1(z)$$



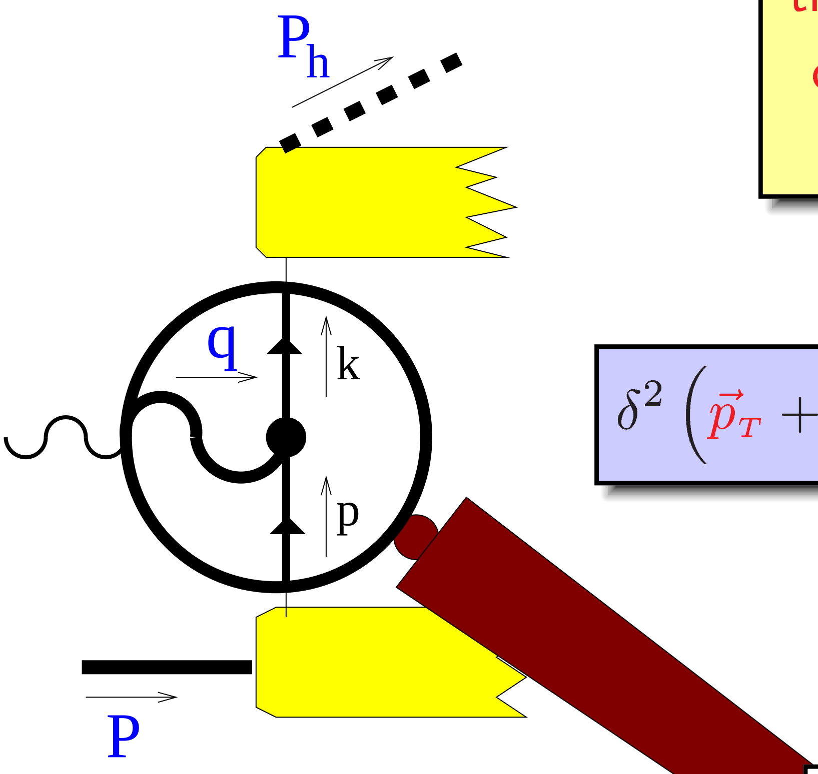
$$\Delta^{[\gamma^-\gamma_5]}(z) = \lambda_h\ G_1(z)$$



$$\Delta^{[i\sigma^{\alpha -}\gamma_5]}(z) = S_{hT}^{\alpha}\ H_1(z)$$

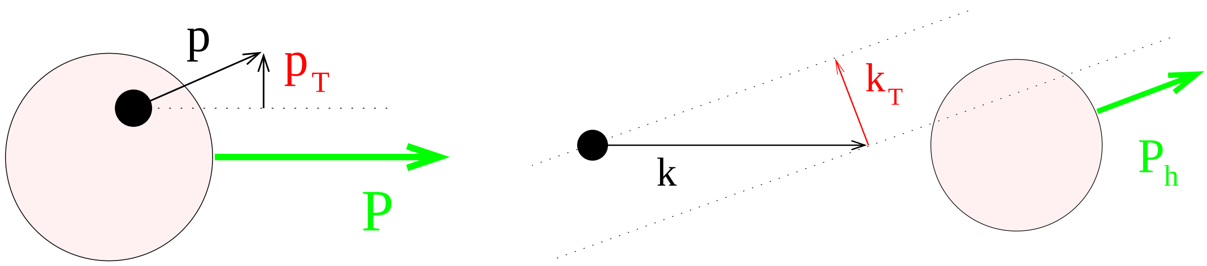


with λ_h helicity, S_{hT}^{α} transv.comp. of spin vector transverse momentum of quarks relative to parent hadrons



$$\delta^2\left(\vec{p}_T + \vec{q}_T - \vec{k}_T\right)$$

δ-fct. at the vertex couples transverse momentum components $\Longrightarrow$ $\dfrac{d\sigma}{d\ldots d^2\vec{q}_T}$ sensitive to $\vec{p}_T$, $\vec{k}_T$



$$\int d^2p_T\, f_1(x, \vec{p}_T^{\,2}) = f_1(x) \qquad \int d^2k'_T\, D_1(x, \vec{k}_T^{\prime\,2}) = D_1(x)$$

FF dependent on parton transverse momentum:

$$\Delta^{[\gamma^-]}(z_h,\vec{k}_T) = D_1(z_h,\vec{k}_T'^2) + \frac{\epsilon_{T\,ij}k_T^i S_{hT}^j}{M_h} D_{1T}^\perp(z_h,\vec{k}_T'^2)$$

$$\Delta^{[\gamma^-\gamma_5]}(z_h,\vec{k}_T) = \lambda_h\, G_{1L}(z_h,\vec{k}_T'^2) + \frac{\vec{k}_T'\cdot\vec{S}_{hT}}{M_h} G_{1T}(z_h,\vec{k}_T'^2)$$

$$\Delta^{[\mathrm{i}\sigma^{i-}\gamma_5]}(z_h,\vec{k}_T) = S_{hT}^i\, H_1(z_h,\vec{k}_T'^2) + \frac{\epsilon_T^{ij}k_{Tj}}{M_h} H_1^\perp(z_h,\vec{k}_T'^2) + \frac{\lambda_h k_T^i}{M_h} H_{1L}^\perp(z_h,\vec{k}_T'^2) + \frac{\left(k_T^i k_T^j - \frac{1}{2}\vec{k}_T^2\delta_{ij}\right) S_{hT}^j}{M_h^2} H_{1T}^\perp(z_h,\vec{k}_T'^2)$$

with $\vec{k}_T' = -z_h\vec{k}_T$

8 FFs at leading order

$\boxed{FF(z)}$ dependent on momentum fraction z only

quark spin	$\Delta^{[\Gamma]}$	T_N-even U	T_N-even L	T_N-even T	T_N-odd / FSI U	T_N-odd / FSI L	T_N-odd / FSI T
u	γ^-	$D_1(z)$	–	–	–	–	–
ℓ	$\gamma^-\gamma_5$	–	$G_1(z)$	–	–	–	–
t	$i\sigma^{i-}\gamma_5$	–	–	$H_1(z)$	–	–	–

$\boxed{FF(z,k_T)}$ with transverse momentum dependence

quark spin	$\Delta^{[\Gamma]}$	T_N-even U	T_N-even L	T_N-even T	T_N-odd / FSI U	T_N-odd / FSI L	T_N-odd / FSI T
u	γ^-	D_1	–	–	–	–	$D_{1T}^\perp$
ℓ	$\gamma^-\gamma_5$	–	G_{1L}	G_{1T}	–	–	–
t	$i\sigma^{i-}\gamma_5$	–	$H_{1L}^\perp$	$H_{1T}, H_{1T}^\perp$	$H_1^\perp$	–	–

$\boxed{\text{blue FF} = \text{chiral odd}}$ possible 'partner for transversity' there are problems to model (interference) FF$^{q\to h\,X}$

"T_N-odd" used in the sense:

"...would be forbidden by time reversal invariance if there were no FSI ..."

necessary condition for non-zero T_N-odd FF:

interference of two channels with different phase

possible models

after hadronization

FSI between hadrons

before hadronization

parton's 'feel' the presence of other partons

$\to$ dressed quark propagator

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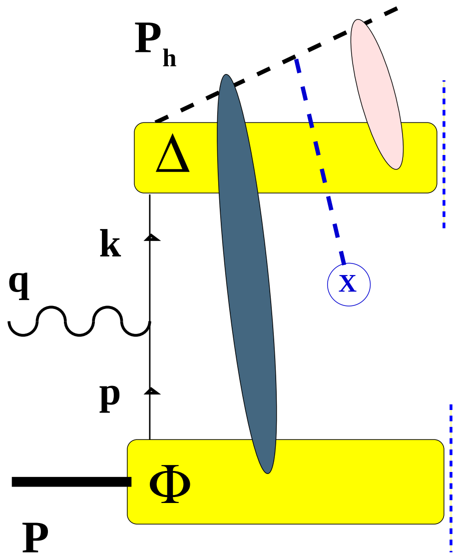
problems to model FSI leading to T_N-odd FF

possible residual interactions between hadrons in the final state



dark blob factorization breaking

dashed line averaged external potential

$\rightarrow$ breaks rotational and translational invariance

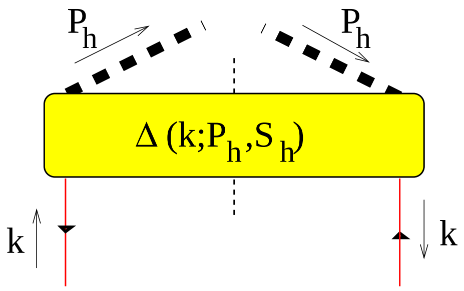
unless simplified model (not ‘rich’ enough for T_N-odd FF)

light blob interaction with residual fragments in jet

$\rightarrow$ requires non-trivial microscopic modifications of the hadron wave function

otherwise, it can be reabsorbed in the vertex

properties of correlation function



hermiticity:

$$\gamma_0\,\Delta^\dagger(k;P_h)\,\gamma_0 = \Delta(k;P_h)$$

parity invariance:

$$\gamma_0\,\Delta(\tilde{k};\tilde{P}_1,\tilde{P}_2)\,\gamma_0 = \Delta(k;P_1,P_2)$$

time-reversal invariance:(if applicable)

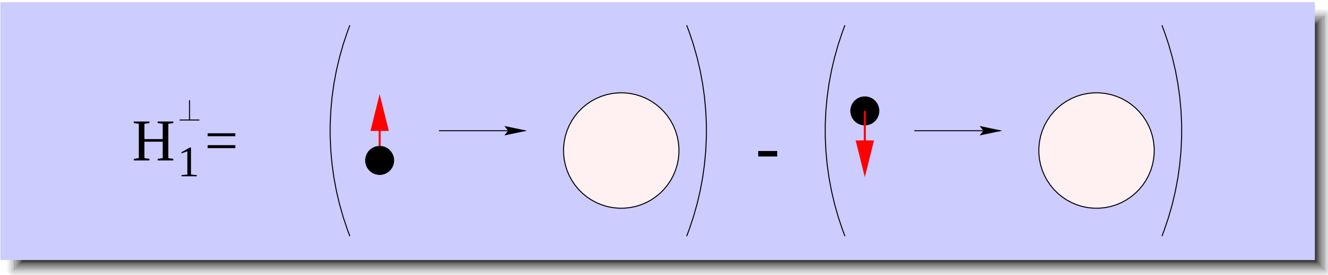
$$\left(\gamma_5 C\,\Delta(\tilde{k};\tilde{P}_1,\tilde{P}_2)\,C^\dagger\gamma_5\right)^* = \Delta(k;P_1,P_2)$$

where $\tilde{k} = (k^0, -\vec{k})$ and $C = i\,\gamma^2\,\gamma^0$.

most general ansatz (for $S_h = 0$)

$$\Delta = B_1 M_h + B_2\,\not{P}_h + B_3\,\not{k} + \frac{B_4}{M_h}\sigma_{\mu\nu}P_h^\mu k^\nu$$

for instance 'Collins function' $H_1^\perp$ is T_N-odd

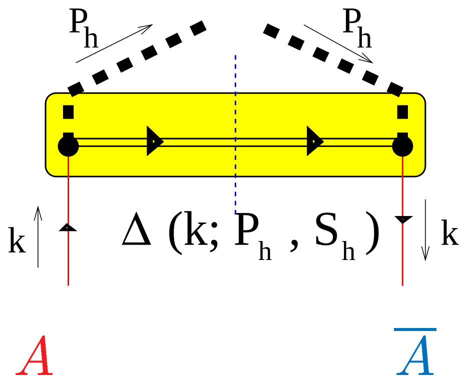


$$\begin{cases} \Delta = B_1 M_h + B_2 \not{P}_h + B_3 \not{k} + \dfrac{B_4}{M_h}\sigma_{\mu\nu}P_h^\mu k^\nu \\ \Delta^{[i\sigma^{i-}\gamma_5]} = \ldots + \dfrac{\epsilon_T^{ij} k_{Tj}}{M_h} H_1^\perp \end{cases}$$

$$\Downarrow$$

$$H_1^\perp(z_h, \vec{k}_T'^{\,2}) = \frac{P_h^-}{z_h}\int dk^+ \int dk^-\ \delta\left(k^- - \frac{P_h^-}{z_h}\right)\ [-B_4]$$

and would be forbidden by time-reversal invariance if there were no final state interactions



for illustration only:

simple toy model:
scalar spectator

$$\Delta \equiv \tilde{\Delta}\ \delta\left((P_h - k)^2 - M_{\mathrm{D}}^2\right)$$

$$\tilde{\Delta} = \frac{-i}{\not{k} - m}\ \underbrace{\mathrm{u}(P_h)\ \overline{\mathrm{u}}(P_h)}_{(\not{P}_h + M_h)}\ \frac{-i}{\not{k} - m}$$

$$= A_1 M_h + A_2 \not{P}_h + A_3 \not{k} + \frac{(A_4 = 0)}{M_h} \sigma_{\mu\nu} P_h^\mu k^\nu$$

- assume: $A \to A + \mathrm{e}^{\mathrm{i}\phi} A$ (uniform damping)

$$\Rightarrow \quad \tilde{\Delta} = 2(1 + \cos\phi)\ A\overline{A} \ \Rightarrow\ A_4 = 0$$

phase is necessary but not sufficient !

- assume: $A \to A + \mathrm{e}^{\mathrm{i}\phi}\ \not{k} A$

$$\Rightarrow \quad \tilde{\Delta} = (A_1 + B_1 \cos\phi) M_h + A_2 \not{P}_h + (A_3 + B_3 \cos\phi)\ \not{k} + \frac{B_4 \sin\phi}{M_h} \sigma_{\mu\nu} P_h^\mu k^\nu$$

sufficient (but can be redefined into the vertex !)

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two hadron interference FF

- two (!!) leading hadrons detected in the same jet
- interference of two channels with different phases can produce T_N-odd FF

J.C. Collins and G.A. Ladinski, hep-ph/9411444

two pions (chiral sigma model)

- independent production
- production via σ-'resonance'

R.L. Jaffe, Xuemin Jin, Jian Tang,

P.R.L. **80** (98) 1166; P.R. **D57** (98) 5920

two pions (known phase shifts)

- s-wave 'resonance' (σ)
- p-wave resonance (ρ)

A.Bianconi, S.Boffi, R.Jakob, M.Radici,

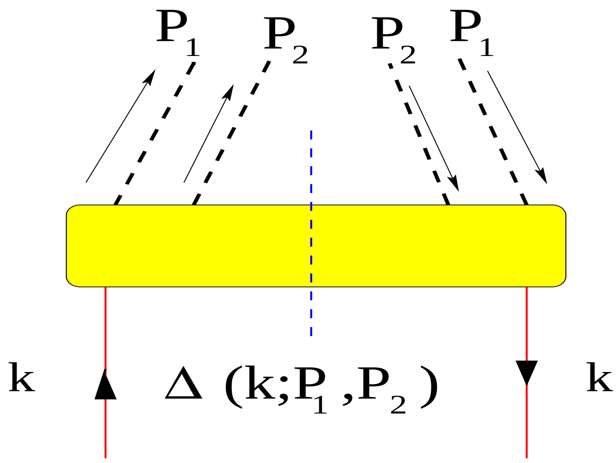
PRD 62 (2000) 034009 and 034008

pion & proton (spectator model)

- indepedent production
- production via ρ-resonance

two-hadron FF

definition of hadronic matrix element



$$\Delta(k;P_1,P_2) = \sum_X \int \frac{d^4\xi}{(2\pi)^4} \int \frac{d^4 P_X}{(2\pi)^4}\, e^{ik\cdot\xi}$$
$$\times \langle 0|\psi_i(\xi)|h_1,h_2,X\rangle\langle X,h_1,h_2|\overline{\psi}_j(0)|0\rangle$$

much easier to model the FSI between the two hadrons

(but not with the rest of the jet)

properties and more details → talk by **Marco Radici**