EDUCATIONAL MODULE

TITLE:

Designing A Wind Turbine
The Power of Math

AUTHOR:

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GRADE LEVEL/SUBJECT:

Upper Level Secondary School Students / Algebra in Context

CURRICULUM STANDARD (from *Benchmarks for Science Literacy* 1993):

- Mathematical modeling aids in technological design by simulating how a proposed system would theoretically behave. (2B 9-12#1)
- Mathematics provides a precise language for science and technology—to describe objects and events, to characterize relationships between variables, and to argue logically. (2B 9-12#3)
- Developments in science or technology often stimulate innovations in mathematics by presenting new kinds of problems to be solved. In particular, the development of computer technology (which itself relies on mathematics) has generated new kinds of problems and methods of work in mathematics. (2B 9-12#4)
- Developments in mathematics often stimulate innovations in science and technology. (2B 9-12#5)
- Some work in mathematics is much like a game—mathematicians choose an interesting set of rules and then play according to those rules to see what can happen. The more interesting the results, the better. The only limit on the set of rules is that they should not contradict one another. (2C 9-12#1)
- Much of the work of mathematicians involves a modeling cycle, which consists of three steps: (1) using abstractions to represent things or ideas, (2) manipulating the abstractions according to some logical rules, and (3) checking how well the results match the original things or ideas. If the match is not considered good enough, a new round of abstraction and manipulation may begin. The actual thinking need not go through these processes in logical order but may shift from one to another in any order. (2C 9-12#2)

From http://www.project2061.org/tools/benchol/bolframe.html

OVERVIEW:

In summary, this activity intends to challenge students to design and market a wind turbine. The students will work as teams and present their final product to the class in an attempt to "sell" their turbine. Mathematics, technology, economics, and science (physics) will be central to this activity.

PURPOSE:

The purpose of this activity is to provide students with authentic applications of mathematics. Additionally, the activity will demonstrate the interdisciplinary relationship between mathematics, technology, economics, and science in real world situations.

LEARNING OBJECTIVES:

Students will be able to:

- Use power equations to find power outputs.
- Explain the relationship between wind speed, rotor diameter, and power output.
- Graph tabular data with and without Excel.
- Explain tables and graphs.
- Verify equations in Excel.
- Calculate material costs.
- Design a marketable product.
- Present the product that they have designed for a consumer.

VOCABULARY:

Science related:

- Power
- Energy
- Wind Turbine
- Wind Speed
- Rotor Diameter
- Tower Height
- Energy Resources
- Alternative Energy Sources
- Anemometer
- Efficiency

Mathematics related:

- Independent Variable
- Dependent Variable
- Domain
- Range
- Power Curve
- Percentages

Technology related:

- Spreadsheet
- Microsoft Excel
- Excel Formulas

Economics related:

- Production Costs
- Expenditures

RESOURCES/MATERIALS:

- Computers with Internet Access (probably 8-12)
- Microsoft Excel software
- Microsoft Excel program
- Computer projector for presentations
- Calculators
- Graph paper

NECESSARY PRIOR KNOWLEDGE/REVIEW:

From math classes:

- Independent and dependent variables
- Graphing using tabular data
- How to read an x/y graph
- Domain and range

From concurrent classes:

- Technology
  1. Data Entry using Microsoft Excel
  2. Graphing using Microsoft Excel
  3. Formula input using Microsoft Excel

- Science
  1. Energy and power basics
  2. Energy efficiency
  3. Possible energy sources
  4. Alternative energy sources and their benefits

- Economics
  1. Determining costs and expenditures
  2. Marketing a product

MAIN ACTIVITIES (3 days):

Step #1: Wind Power Considerations (90 minute "blocked" class) (Day 1)

Preparatory Activity (homework assignment)

- Read articles provided in class: **Reusable News**, Volume 1, Issues 3&4

Introduction (20 minutes)

- Introduce new topic : wind energy and mathematics
- Wind Technology power point presentation

- Vocabulary to cover/review during presentation
    1. Energy (more in next presentation)
    2. Electricity
    3. Public service companies
    4. Utility grid
    5. Energy systems
    6. Hybrid systems
    7. Efficiency
    8. Battery
    9. Inverter
- Wind Power, Electricity, & Mathematics presentation
- Vocabulary to cover/review during presentation
    1. Power
    2. Work
    3. Energy
    4. Wind turbine
    5. Power coefficient
    6. Energy
    7. Electricity
- Leave formula slide on overhead for the remainder of the class

Small Group Discussion (30 minutes)

- Break students into groups of 4
- Have students take out assignment sheet (provide if necessary)
- Circulate as groups discuss the questions
- Have groups hand in answers to questions

Assignment Discussion and Preparation (30 minutes)

- Hand out The Power of Wind: Designing a Wind Turbine assignment
- Hand our rubric for the assignment
- Have students divide into groups of two and choose a computer (necessary program already installed on computers)
- Math Applications with Energy(Wind)
- Explain each part of the assignment in detail
- Repeat the instructions again and ask for questions
- Have students begin assignment
- Circulate and answer any other questions

Conclusion of Class (10 minutes)

- Clarify main assignment again
- Answer questions
- Provide troubleshooting tips

Step #2: Marketing Wind Power (Day 2)

Review (10 minutes)

- Clarify main assignment again

- Answer any questions necessary
- Review equations necessary for the assignment

Guest Speaker (20 minutes)

- Have a guest speaker come in and describe their home wind turbine system; have them describe their reasoning, problems, etc.
- Question and answer session with guest speaker
- If speaker not available, provide a video with similar information

Surfing the Web (20 minutes)

- In groups of two (already assigned) have students visit the following web site (or download and provide): http://www.windpower.dk/tour/index.htm
- Using a projector, navigate the web site with the students
- Recommended sections: 1, 2.1, 2.7, 2.9, 2.10, 2.11, 2.14, 3.7, 4.1, 5.1 and 10.1

Group Project (35 minutes)

- Have students work on the assignment **The Power of Wind: Designing a Wind Turbine**
- Circulate and provide feedback to each group
- Answer questions

Conclusion of Class (5 minutes)

- Remind students that the assignment and presentation will be done tomorrow at the end of class. Each group will be given 5 minutes to present their turbine. Time will be provided tomorrow in the beginning of class to finish up.

Step #3: Presenting the Turbines (Day 3)

Review (10 minutes)

- Review the assignment and answer questions

Finishing the Project (35 minutes)

- Have students complete the Excel project along with the word problem provided
- Have students prepare materials to present to the class using Excel (projector provided)
- Circulate and answer any final questions

Presentations (45 minutes)

- Have each group present the turbine they have created, show power curves with explanations, and argue for the appropriateness of their turbine in lieu of the word problem.

Conclusion of Class

- Congratulate students on a job well done
- No homework

Reusable News, Volume 1, Issues 3&4

http://www.pinnaclet.com/reusable/news1/news1.pdf
http://www.pinnaclet.com/reusable/news2/news2.pdf

Assignment Part 1: Read the handouts titled as above. Think about the following questions:

1. What is a renewable energy source? List three renewable energy sources.
2. Why aren't renewable energy sources used more often than they are now?
3. What were the first uses of wind energy (i.e. windmills)?
4. What are some of the difficulties using wind energy? Name at least two.
5. What are some of the benefits of wind energy? Name two.
6. What country uses more energy (any kind) per person than any other?

Assignment Part 2: In class, get into groups of 4. Elect a recorder for your group. Discuss these questions and have the recorder write down the consensual conclusions on a separate sheet of paper. Hand in one copy of the answers to the questions per group. Points will be assigned as follows:

4	3	2	1
a. Actively participated in a group. b. Contributed to the discussion. c. Demonstrated completion of first part of assignment. d. Attended class and had name on group product. e. Group correctly answered the above questions.	a. Contributed to the discussion. b. Demonstrated completion of first part of assignment. c. Attended class and had name on group product. d. Group correctly answered half of the above questions.	a. Demonstrated completion of first part of assignment. b. Attended class and had name on group product. c. Group correctly answered half of the above questions.	a. Attended class and had name on group product.

Wind Technology

The Hows, Whys, and Whats

The quick-quick version!

What causes wind?

Wind:

- is the movement of air
- is created by the uneven heating of the Earth's atmosphere
- is created by warmer air rising and cooler air sinking causing the flow of air

How does wind power work?

Wind:

- creates usable power by spinning the blades of a turbine

The wind turbine:

- transfers this power from the rotating blades into a generator that creates electricity!

Why should I use wind energy?

Wind energy:

- takes advantage of existing renewable energy resources
- does not generate harmful emissions
- reduces dependence on non-renewable, polluting fossil fuels
- avoids the high costs of connecting to the utility grid in remote locations

What things affect how much power my wind system can generate?

The power output (meaning how much power is generated) from a wind turbine system depends on:

- the wind resources in the region
- the amount of power lost by the turbine while changing wind power to electrical power
- the height of the tower
- the type of turbine (including weight and rotor diameter)
- the other stuff in the system

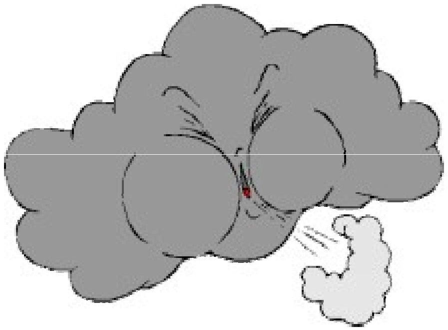
What components make up a wind energy system?

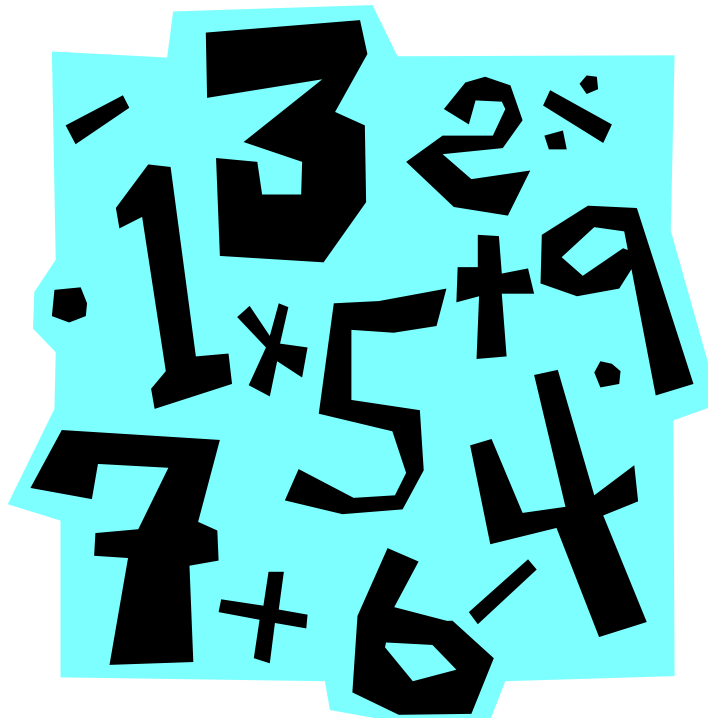
- Turbine components:
  1. Rotor (motor & blades)
  2. Generator
  3. Tail
- Tower
- Battery and/or inverter
- Power load (like the amount of power it takes to run your computer for a certain length of time)

Questions or comments?

Wind Power

Electricity





& Mathematics

Otherwise Known as "Why Do We Have to Learn This?"

Stuff we're going to talk about in this unit:

1. What does power have to do with math?
2. Are wind power and electrical power (electricity) the same thing?
3. What other math formulas do we need?
4. I keep hearing about energy. What's that?
5. How can we check our work?
6. What are some examples for us?
7. Can we have that one more time?

What does power have to do with math?

- By using math, it becomes easier to figure out how much power is available in a "moving" thing.
- Power is a rate that measures how much work you can do at one instant in time
- Using math, power can be calculated many different ways. For wind power, the calculations are → → → → → →

$P_w = (^1/_2)\ d \cdot A \cdot w^3$

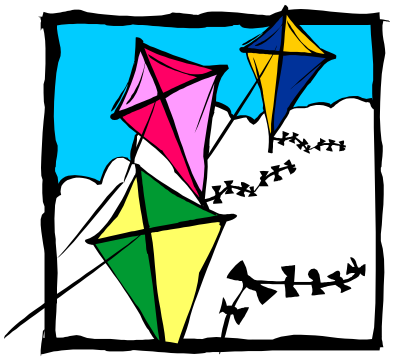
→ P_w = *wind power*

→ d = *air density*

→ A = *swept area*

→ w = *wind speed*

Are wind power and electrical power (electricity) the same thing?



$$P = c_p \cdot P_w$$

➜ *P = power from turbine*

➜ *c_p = power coefficient*

- Wind power cannot be changed into electrical power without some power being lost.
- The percentage of wind power that can be turned into electrical energy is called the power coefficient.
- We multiply the amount of power in the wind by this power coefficient in order to find the electrical power output (or power produced).

What other math formulas do we need?

How do we figure the swept area?

$$A = \pi \cdot (r^2)$$

➜ $r = (1/2)$ rotor diameter

How do we know what the wind speed is at different heights?

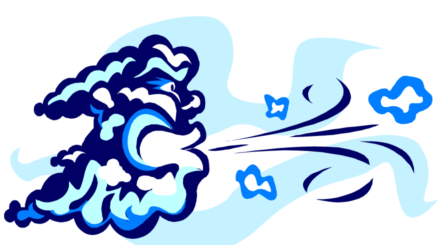
$$w = ((T/H)^s) \cdot v$$

➜ T = tower height

➜ H = anemometer height

➜ s = wind shear exp.

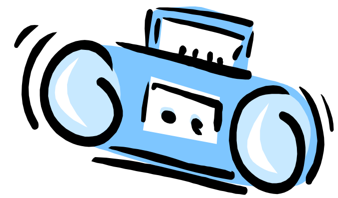
An anemometer is a device that measures wind speed at a specific height, usually 20m.



The wind shear exponent compensates for wind disturbances due to the roughness of the Earth's surface, usually $s \approx (1/7)$.

I keep hearing about energy. What's that?

- Energy is a way to figure out how much power you are using.
- Energy is simply the amount of power needed to run something (like a radio) for a certain amount of time (like an hour).
- Lots of people get power and energy mixed up… but we won't worry about it. Our main focus is POWER!



x

$$E = P \cdot t$$

➜ E = energy

➜ t = time

How can we check our work?

- ✠ It's easy to check our work if we pay attention to units.
- ✠ Units are what we use to illustrate different types of things with their values. For example, we use hours, minutes, and seconds to count time. Hours, minutes, and seconds are units!
- ✠ By using the units we know go with certain things, and by using math to cancel these (like variables) when we can (division), we can make sure that we are calculating the right things. For example, driving at 60 mph (m/h) for 1 hour yields an answer in MILES! $(m/h) \cdot (h) = m$

More on next slide →

More on checking our work →

Units we need:

Power → Watts (W)

Time → hours, seconds (h,s)

Distance → meters (m)
not miles !!!!!

Speed → meters/second
(m/s)

Area → meters ^{2} (m^2)

A note on density:

Density is a difficult concept. It measures how "thick" or "thin" the air is. The units of density use more science than we need so we are just going to say that the units of density are:

kilogram/meters^3

$\boxed{\text{Watt} = \text{kilogram} \cdot \text{meter/second}^3\ (\text{kg} \cdot (\text{m/s}^3))}$

What are some examples for us?

What if:

d = 1.225 ($kg \cdot s^2/m^4$), A = 20 (m^2), w = 5 (m/s).
Find the power in the wind.

Power equation:

$$P_w = (1/2)\, d \cdot A \cdot w^3$$

Units check: d A w^3

$$W = kg \cdot (m/s^3) = (kg/m^4) \cdot (m^2) \cdot (m^3/s^3)$$

Solution:

$$P_w = (1/2) \cdot d \cdot A \cdot w^3$$
$$= (1/2) \cdot 1.225\ (kg/m^4) \cdot 20\ (m^2) \cdot 5^3\ (m^3/s^3)$$
$$= (1/2) \cdot 1.225 \cdot 20 \cdot 125\ (kg \cdot (m/s^3))$$
$$= 1531.25\ W$$

Another example...

What if:

$d = 1.225\ (kg/m^4)$

r = 3m (1/2 rotor diameter!),

w = 5m/s

c_p =.3 (no units!)

Equations we need:

$P_w = (^1/_2)\ d \cdot A \cdot w^3$

$P = c_p \cdot P_w$

$A = \pi \cdot (r^2)$

Solution:

1. Find the Area $A = \pi \cdot (r^2) = \pi \cdot (3m) = \pi \cdot 9m^2 = \boxed{28.27m^2}$
2. Use the Area from #1 to find the power in the wind:

$P_w = (^1/_2)\ d \cdot A \cdot w^3 = (^1/_2) \cdot 1.225\ (kg/m^3) \cdot 28.27m^2 \cdot 125(m^3/s^3)$

$= (^1/_2) \cdot 1.225 \cdot 28.27 \cdot 125\ (kg \cdot (m^2/s^3)) \boxed{= 2164.42\ W}$

3. Now use the power coefficient to find the electrical power:

$P = c_p \cdot P_w = (.3) \cdot (2164.42\ W) = \boxed{649.33\ W}$

Can we have that one more time?

$$P_w = (1/2)\, d \cdot A \cdot w^3$$

➜ P_w = wind power

➜ d = air density

➜ A = swept area

➜ w = wind speed

$$P = c_p \cdot P_w$$

➜ P = power from turbine

➜ c_p = power coefficient

And the rest…..

$A = \pi \cdot (r^2)$

➜ $r = (^1/_2)$ rotor diameter

$w = ((T/H)\hat{}s)*v$

➜ T = tower height

➜ H = anemometer height

➜ s = wind shear exp.

All in one shot!

$P_w = (1/2)\, d \cdot A \cdot w^3$

➜ P_w = wind power

➜ d = air density

➜ A = swept area

➜ w = wind speed

$P = c_p \cdot P_w$

➜ P = power from turbine

➜ c_p = power coefficient

$E = P \cdot t$

➜ E = energy

➜ t = time

$A = \pi \cdot (r^2)$

➜ $r = (1/2)$ rotor diameter

w = ((T/H)^s)*v

➜ T = tower height

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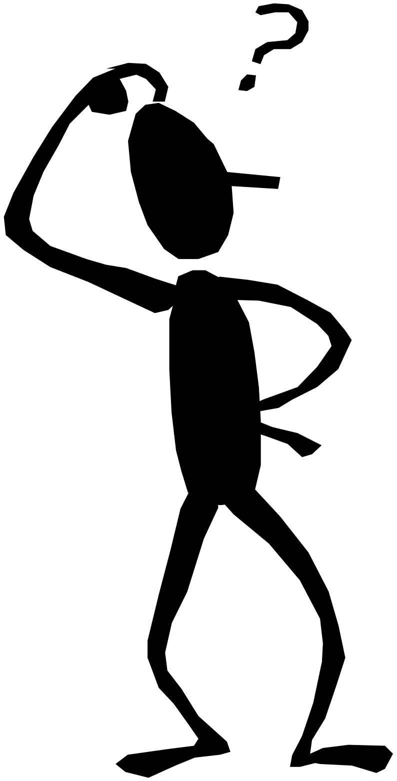
Questions?



The Power of Wind
Designing a Wind Turbine

The Washington family is looking for alternative power sources for their home. They are interested in wind power but haven't decided on the turbine type that would be best for them. They live in the township of Jennings, CA, which is at sea level and has an average annual wind speed of **5.4m/s** measured from a height of **20 meters** above sea level. They want the turbine that they purchase to produce between **1000 & 2000 Watts** (1-2 kW) in rated power.

Your wind turbine company is interested in procuring the business of this family. There are other turbine companies also interested and you need to design a turbine that will meet the needs of this household while also being as cost-effective as possible (meaning that it should be a reasonable cost but also be of good quality!). The materials you will need, along with the costs, are listed below. You will need to design the turbine. The power coefficient of your turbine will be 30% (which means you turbine will only be able to capture 30% of the energy supplied by the wind!).

Use the spreadsheet provided in class to design this turbine, provide power curves to help "sell" your product, and present these results to the Washington family (or the class) on the last day of this module. Be prepared to "defend" your design.

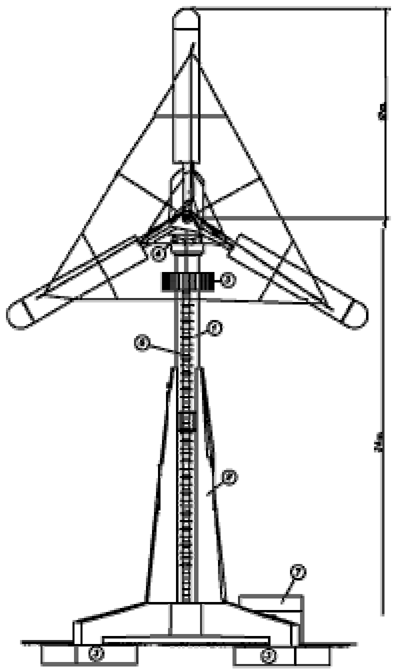
Materials available

1. Blades (1/2 of rotor diameter) of the following sizes are available for the given cost (assume 3 blades):
    a. Each blade of 3.0m length - $200.00
    b. Each blade of 3.5m length - $250.00
    c. Each blade of 4.0m length - $300.00
2. Towers of the following heights and prices are available:
    a. tower of 15m - $1500
    b. tower of 25m - $2500
    c. tower of 35m - $3500
3. Turbine motor and operational equipment lifetime information and prices (this involves a judgement call… would you rather have your turbine last longer or cost less… be prepared to debate your choice):
    a. 5 year lifetime expectancy - $2000
    b. 10 year lifetime expectancy - $2500
    c. 20 year lifetime expectancy - $3000

Necessary Wind Technology Formulas

Variables Used	Constants	Formulas	Miscellaneous
P = Turbine Power	d = 1.225	P_w = (1/2)*d*A*(w^3)	* means multiply
P_w = Wind Power	c_p = 0.2	P = c_p*(1/2)*d*A*(w^3)	^ means to the power of
E = Energy	s = (1/7)	E = P*t	
t = time	s = 3.14	A = *(r^2)	
d = air density	H = 20.0m	r = 1/2 rotor diameter	
A = Swept Area	v = 5.4 m/s	w = ((T/H)^s)*v	
r = radius = 1/2 rotor diameter			
c_p = power coefficient			
w= wind speed (velocity)			
v = original wind speed at H			
T = tower height			
H = anemometer height			
s = 3.14			
s = wind shear exponent			

Sample Presentation Finished Product
Turbine Name
Designers' Names



Picture of Final Product Here

Power Output at Average Annual Wind Speed:
Rotor Diameter:
Tower Height:
Lifetime:
Total Cost of Turbine: