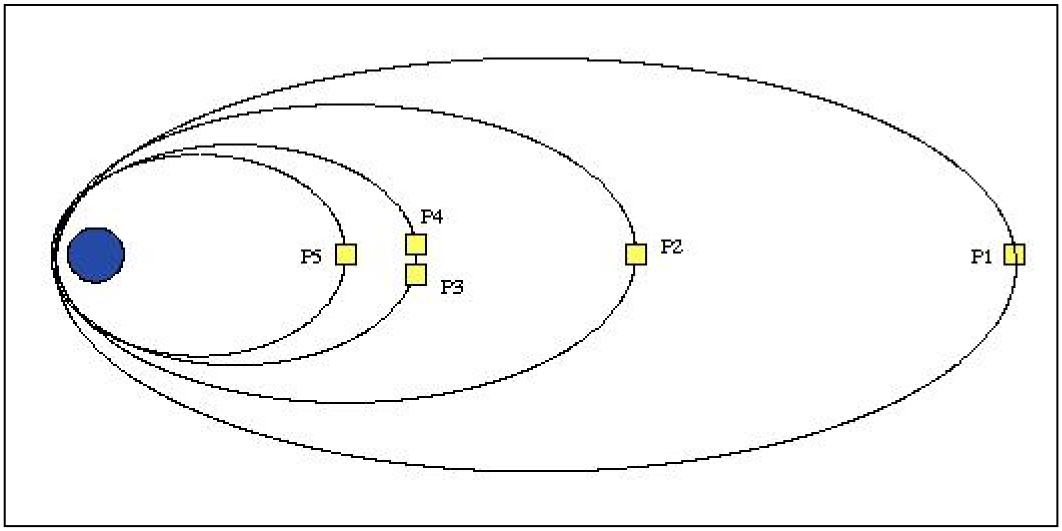
A Problem in Satellite Synchrony



The THEMIS program has launched five satellites that will orbit earth at different distances to study Earth's magnetic field. The goal is to have them line up a few times each year so that they can study how magnetic storms are triggered and evolve in time.

The following problems will be easier to solve if you use a spreadsheet, but they CAN be worked out by hand by finding the Least Common Multiples. Can you answer them both ways?

Problem 1 - Suppose you had two satellites orbiting Earth in the same plane. The closer-in satellite had a period of 4 hours, and the second satellite had a period of 8 hours. Looking down at Earth from the North Pole, suppose you started the satellites together at the same longitude, say 100 degrees West. How many hours later would they both be together again?

Problem 2 - Suppose the faster satellite had a period of 120 minutes, and the slower satellite had a period of 17 hours. After how many hours would they come together again in longitude?

Problem 3 - Suppose you had three satellites with periods of 2 hours, 4 hours and 6 hours. Using the Greatest Common Multiple, find how long would it take for them to return to their original line-up at the same longitude.

Problem 4: The THEMIS mission uses five satellites with periods of about 19 hours, 24 hours, 24 hours, 48 hours, and 96 hours, how many hours would it take for them all to get together again at the same longitude? Give your answer in days.

Answer Key:

Problem 1 - Suppose you had two satellites orbiting Earth. The closer-in satellite had a period of 4 hours, and the second satellite had a period of 8 hours. Looking down at Earth from the North Pole,
suppose you started the satellites together at the same longitude, say 100 degrees West. How many hours later would they both be together again?

Answer: Create a time line and mark of the times, starting from 00:00, when the satellites will return to West 100.
Satellite A 00:00………04:00…………08:00………..12:00……16:00………..20:00……..24:00…etc
Satellite B 00:00………………………..08:00……………………16:00……………………..24:00…etc

Another method: If you know the periods of the two satellites, look for the Lowest Common Multiple between them, which in this case is 8 hours.

Problem 2 - Suppose the faster satellite had a period of 2 hours, and the slower satellite had a period of 17 hours. After how many hours would the come together again in longitude?

In hours:
Sat A: 0, 2, 4, 6, 8, 10 ,12, 14, 16, 18, 20, …., 30, 32, 34, 36, 38, ………50, 52, 54, …..66, 68, 70, ….
Sat B: 0, 17, 34, 51, ………… 68, ….

Answer: Students can use the spreadsheet approach, or sketch out a timeline. The satellite periods, in hours, are 2 hours and 17 hours. The LCM, in this case, is 34 hours. So every 34 hours, the satellites will return to the same longitude they started at in synchrony.

Teachers; another, more algebraic method is as follows:
The angular speed of the two satellites is A: 360 degrees/2 hours = 180 degrees/hour and B: 360 degrees/17 hours = 21.176 degrees/hour. As time goes on, the speed with which the faster one pulls ahead of the slower one is given by V = 180 degrees/hr - 21.176 degrees/hour = 158.824 degrees/hour. How much time does it take for the angular distance between the satellites to equal exactly 360 degrees or an exact integer multiple (360 ,720, 1080, etc) ?

32hr x 21.176 d/hr = 677.63d ,
34hr x 21.176 d/hr = 719.98 d.

So, after 34 hours, the angular distance between the two satellites is 720 degrees, with is 2 x 360, so the satellites meet up at the same longitude they started from.

Problem 3: Suppose you had three satellites with periods of 2 hours, 4 hours and 6 hours. How long would it take for them to return to their original line-up at the same longitude?

Answer: The prime factors are 2, 2x2 and 2x3 so the GCM is 2x2x3 = 12, so after 12 hours the satellites will come together again.

Problem 4 - The THEMIS mission uses five satellites with periods of about 19 hours, 24 hours, 24 hours, 48 hours, and 96 hours, how many hours would it take for them all to get together again at the same longitude? Give your answer in days.

Answer: Find the prime factors 19: 19 , 24: 2x2x2x3 , 48: 2x2x2x2x3 , 96: 2x2x2x2x2x3
42: then 19 x 2x2x2x2x2 x3 = 1824 hours. Then to check: 1824/19 = 96, 1824/24 = 76, 1824/48 = 38, 1824/96 = 19. So, it will take 1,824 hours or 76 days!