Title: Investigating Special Right Triangles and Area of a Regular Polygon

Brief Overview:

This learning unit is designed for students to investigate special right triangle properties (for both 30°– 60°– 90° and 45°– 45°– 90° triangles). Students will develop the general formula for the area of any regular polygon using Geometer's Sketchpad and apply special right triangle properties to find the area of equilateral triangles, squares, and hexagons.

NCTM Content Standard/National Science Education Standard:

Geometry

- Analyze characteristic properties of two- and three-dimensional geometric shapes and develop mathematical arguments about geometric relationships
- Use visualization, spatial reasoning, and geometric modeling to solve problems.

Grade/Level:

High School Geometry

Duration/Length:

Four class periods (85 – 90 minutes each). Lessons 1 and 2 should take one class period each and Lesson 3 should take about two class periods.

Student Outcomes:

Students will:

- State and apply the 45° – 45° – 90° triangle theorem.
- State and apply the 30° – 60° – 90° triangle theorem.
- State and apply the formula for the area of a regular polygon.

Materials and Resources:

- Geometer's Sketchpad
- Student worksheets (activity sheets, homework sheets, and assessment sheets)
- Teacher Resource Materials
- Teacher notes

Development/Procedures:

Lesson 1

Preassessment – Students will review the Pythagorean Theorem, area of a triangle, and simplifying radicals by completing ***Warm Up 1***

Launch – Present the word problem on ***Warm Up 1.1*** to students and give them time to come up with a solution. Let several students explain their solutions. Then tell students that you will now go to the computer lab to explore 45° – 45° – 90° triangle relationships.

Teacher Facilitation/Student Application –

- Have students use Geometer's Sketchpad to complete Part I of ***Worksheet 1.1***.
- Monitor students in constructing their 45° – 45° – 90° triangles in Part I.
- Use the think-pair-share cooperative learning strategy to answer the questions in Part II.
- Discuss with students their observations. Let the students state verbally their conjectures about the properties of 45° – 45° – 90° triangles.
- Instruct students to measure the lengths of each side of their triangle.
- Have students use the tabulate feature on Geometer's Sketchpad to generate five different triangles with integer lengths for AB and AC and enter their values in the first three columns of the table in Part III.
- Have students calculate the values for the fourth column using the Pythagorean Theorem and direct them to give their answers in simplified radical form.
- Then have students study the pattern and use it to complete the last two rows of the table in Part III.
- Have students compare their answers in the first, second, and fourth columns of the table and make a conjecture.
- Lead a discussion of the students' observations and connect them to the 45° – 45° – 90° triangle theorem.
- Distribute ***Worksheet 1.2*** and have students complete it with a partner.
- After students have completed ***Worksheet 1.2***, go over the correct solutions and answer any student questions.
- If time does not permit for completing this worksheet in class, then assign the remaining problems for homework.
- If all students successfully complete ***Worksheet 1.2***, then assign ***Worksheet 1.3*** for homework.

Embedded Assessment –

- Students are asked to construct a 45° – 45° – 90° triangle in order to verify their conjectures. Students should notice that it is an isosceles triangle with congruent sides opposite congruent angles.
- Students are asked to construct five different 45° – 45° – 90° triangles and record the side lengths in a table. Then they are asked to compare their decimal values to the corresponding radical values in order to verify the 45° – 45° – 90° triangle theorem. Students should recognize that the decimal and radical values are equivalent.
- Students are asked to complete practice ***Worksheet 1.2***. Students should be able to use the 45° – 45° – 90° triangle theorem to find the missing lengths.

Reteaching/Extension –

- For those who have not completely understood the lesson, they are encouraged to complete ***Worksheet 1.3***.
- For those who have understood the lesson, have them help other students who are struggling.

Lesson 2

Preassessment – Students will review the 45° – 45° – 90° triangle theorem by completing ***Warm Up 2***.

Launch – Present the word problem on ***Warm Up 2.1*** to students and give them time to come up with a solution. Let several students explain their solutions. Then tell students that you will now go to the computer lab to explore 30° – 60° – 90° triangle relationships.

Teacher Facilitation/ Student Application –

- Have students use Geometer's Sketchpad to complete Part I of ***Worksheet 2.1***.
- Monitor students in constructing their 30° – 60° – 90° triangles in Part I.
- Use the think-pair-share cooperative learning strategy to answer the questions in Part II.
- Discuss with students their observations. Let the students state verbally their conjectures about the properties of 30° – 60° – 90° triangles.
- Instruct students to measure the lengths of each side of their triangle.

- Have students use the tabulate feature on Geometer's Sketchpad to generate five different triangles with integer lengths for AB and AC and enter their values in the first three columns of the table in Part III.
- Have students compare their answers in the first, second, and fourth columns of the table and make a conjecture.
- Lead a discussion of the students' observations and connect them to the 30° – 60° – 90° triangle theorem.
- Distribute ***Worksheet 2.2*** and have students complete it with a partner.
- After students have completed ***Worksheet 2.2***, go over the correct solutions and answer any student questions.
- If time does not permit for completing this worksheet in class, then assign the remaining problems for homework.
- If all students successfully complete ***Worksheet 2.2***, then assign ***Worksheet 2.3*** for homework.

Embedded Assessment –

- Students are asked to construct a 30° – 60° – 90° triangle in order to verify their conjectures. Students should notice that it is a right triangle whose hypotenuse is twice as long as the shortest side.
- Students are asked to construct five different 30° – 60° – 90° triangles and record the side lengths in a table. Then they are asked to compare their decimal values to the corresponding radical values in order to verify the 30° – 60° – 90° triangle theorem. Students should recognize that the decimal and radical values are equivalent.
- Students are asked to complete practice ***Worksheet 2.2***. Students should be able to use the 30° – 60° – 90° triangle theorem to find the missing lengths.

Reteaching/Extension –

- For those who have not completely understood the lesson, they are encouraged to complete ***Worksheet 2.3***.
- For those who have understood the lesson, have them help other students who are struggling.

Lesson 3

Preassessment – Students will review the properties of the incenter of a triangle and how to construct it by completing ***Warm Up 3***.

Launch – Present the word problem on ***Warm Up 3.1*** to students and give them time to come up with a solution. Let several students explain their solutions. Then tell students that you will now go to the computer lab to explore the area of a regular polygon formula.

Teacher Facilitation/Student Application –

- Have students use Geometer's Sketchpad to complete Part I of ***Worksheet 3.2***.
- Monitor students in constructing their equilateral triangles and finding the area using the area formula in Part I. Make sure that students have measured both the altitude and the base of their triangle and that they have entered their areas in the table.
- Have students complete steps #1 – 3 in Part II. Make sure that students are finding the incenter by constructing the angle bisectors of their triangle.
- Have students complete #4 – 6 in Part II. Be aware that students may need to change the color of the original triangle to white so that the colors will show up on the interior triangles.
- Introduce students to the definition of the apothem of a regular polygon.
- Have students work in pairs to answer the last two questions in Part II.
- Discuss with students their observations. Make sure that students connect the apothem to the height of an interior triangle and the sum of the b's to the perimeter of the triangle.
- Work through all problems in Part III of with students. Make sure students identify and record each step for these problems so that they can use this worksheet as a study guide.
- Distribute ***Worksheet 3.2*** and have students complete it with a partner.
- After students have completed ***Worksheet 3.2***, go over the correct solutions and answer any student questions.
- If time does not permit for completing this worksheet in class, then assign the remaining problems for homework.

- If all students successfully complete ***Worksheet 3.2***, then assign ***Worksheet 3.3*** for homework.

Embedded Assessment –

- Students are asked to compare the areas they entered into the table in Part II of ***Worksheet 3.2***. Students should notice that the areas are all the same.
- Students are asked to define the sum of all the b's in the equation. Students should be able to recognize that this sum is the perimeter of the original equilateral triangle.
- Students are asked to identify h_2 in the equation. Students should know that this is the apothem of the original equilateral triangle.
- Students are asked to complete Part III of ***Worksheet 3.2***. Students should be able to use the formula for the area of a regular polygon to find the area for each problem.

Reteaching/Extension –

- For those who have not completely understood the lesson, they are encouraged to complete ***Worksheet 3.3***.
- For those who have understood the lesson, have them help other students who are struggling.

Summative Assessment – See Area of a Regular Polygon Assessment Worksheet.

Authors:

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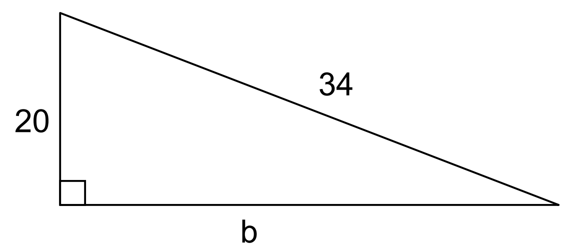
Warm Up 1

Name: ______________________________ Date: _______________

1) Find the length of the missing side of a right triangle with one leg (a) = 20 and hypotenuse (c) = 34.



2) Find the area of the triangle pictured in problem #1.

3) Simplify $\sqrt{\frac{7}{3}}$

4) Simplify $\frac{12}{\sqrt{2}}$

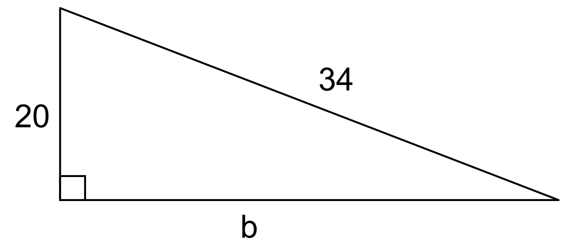
5) Simplify $\sqrt{72}$

Warm Up 1

Name: KEY Date: ________________

1) Find the length of the missing side of a right triangle with one leg (a) = 20 and hypotenuse (c) = 34.

$$a^2+b^2=c^2$$
$$20^2+b^2=34^2$$
$$400+b^2=1156$$
$$b^2=756$$
$$\boldsymbol{b = 27.5 \textit{ units}}$$



2) Find the area of the triangle pictured in problem #1.

$$A=\frac{1}{2}bh$$
$$A=\frac{1}{2}(20)(27.5)$$
$$A=\frac{1}{2}(550)$$
$$\boldsymbol{A = 275 \textit{ units}^2}$$

3) Simplify $\sqrt{\frac{7}{3}}$

$$\frac{\sqrt{7}}{\sqrt{3}}\bullet\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{21}}{\sqrt{9}}=\frac{\sqrt{\mathbf{21}}}{\mathbf{3}}$$

4) Simplify $\frac{12}{\sqrt{2}}$

$$\frac{12}{\sqrt{2}}\bullet\frac{\sqrt{2}}{\sqrt{2}}=\frac{12\sqrt{2}}{\sqrt{4}}=\frac{12\sqrt{2}}{2}=\mathbf{6}\sqrt{\mathbf{2}}$$

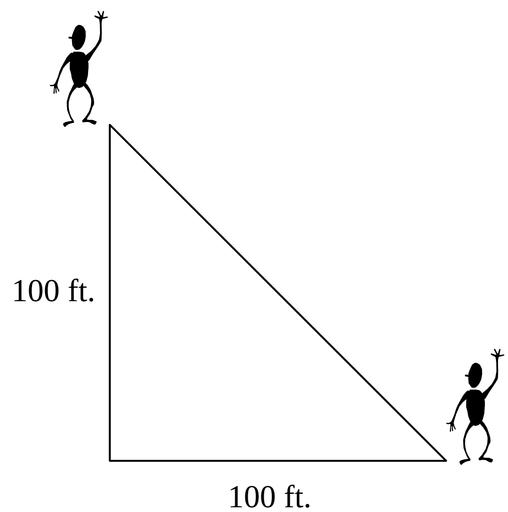
5) Simplify $\sqrt{72}$

$$\sqrt{9\bullet4\bullet2}=3\bullet2\sqrt{2}=\mathbf{6}\sqrt{\mathbf{2}}$$

Warm Up 1.1

Name: ______________________________ Date: _________________

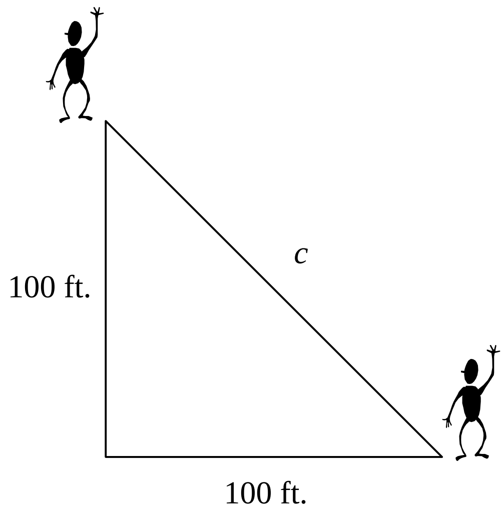
You and your friend had lunch together and then you both left the restaurant on foot. You headed due north and your friend went due east. After you both walked 100 feet you noticed that you had your friend's cell phone, so you called your friend who had your cell phone and decided that you would both walk toward each other along the shortest path possible. What is the sum of the distance you both have to walk in order to reach each other?



Warm Up 1.1

Name: ________________KEY___________ Date: ________________

You and your friend had lunch together and then you both left the restaurant on foot. You headed due north and your friend went due east. After you both walked 100 feet you noticed that you had your friend's cell phone, so you called your friend who had your cell phone and decided that you would both walk toward each other along the shortest path possible. What is the sum of the distance you both have to walk in order to reach each other?



You must use the Pythagorean Theorem to find the distance between you and your friend.

$$a^2 + b^2 = c^2$$
$$100^2 + 100^2 = c^2$$
$$10000 + 10000 = c^2$$
$$20000 = c^2$$
$$\sqrt{20000} = c$$
$$100\sqrt{2} = c$$

Worksheet 1.1

Name: ________________KEY____________ Date: _________________

PART I
Use Sketchpad to construct a 45°- 45°- 90° triangle. Follow the instructions below.

1. Construct a line segment and label the endpoints A and B.
2. Double click on point A.
3. Select line segment AB and point B.
4. Go to the Transform menu and select Rotate.
5. Verify that the degree measurement is 90°. If it is not 90°, then enter that value. Click the Rotate button.
6. Label the third point C.
7. Construct line segment CB.

PART II
What properties can you identify about the triangle you just constructed?

1.
2.
3.
4.
5.

PART III
Use the Tabulate feature in Sketchpad to complete the table below.

AB	AC	BC (decimal)	BC (radical)
1	1	1.41	
2.7			
5.3			

What conjecture can you make about the relationship between the values in the third and fourth columns above?

Worksheet 1.1

Name: KEY Date: ____________

PART I
Use Sketchpad to construct a 45°– 45°– 90° triangle. Follow the instructions below.

1. Construct a line segment and label the endpoints A and B.
2. Double click on point A.
3. Select line segment AB and point B.
4. Go to the Transform menu and select Rotate.
5. Verify that the degree measurement is 90°. If it is not 90°, then enter that value. Click the Rotate button.
6. Label the third point C.
7. Construct line segment CB.

PART II
What properties can you identify about the triangle you just constructed?

1. Isosceles Triangle
2. Right Triangle
3. Line Segment BC is the hypotenuse
4. AC = AB
5. m<B = m<C = 45°
6. m<A = 90°

PART III
Use the Tabulate feature in Sketchpad to complete the table below.

AB	AC	BC (decimal)	BC (radical)
1	1	1.41	$1\sqrt{2}$
2.7	2.7	3.82	
5.3	5.3	7.5	

Answers may vary.

What is the relationship between the lengths of the legs and the length of the hypotenuse in radical form of any 45° – 45° – 90° triangle?

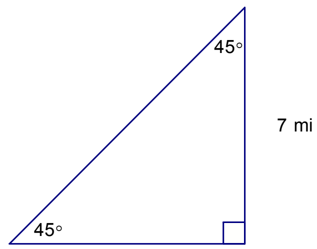
The sides of a 45° – 45° – 90° triangle are in the ratio of 1:1:$\sqrt{2}$.

Worksheet 1.2

Name: ________________________________ Date: ________________

Find all the missing sides in the 45°– 45°– 90° triangles below.

1)



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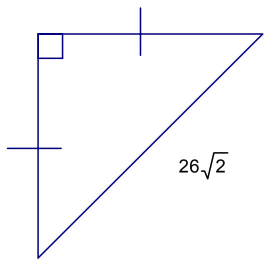

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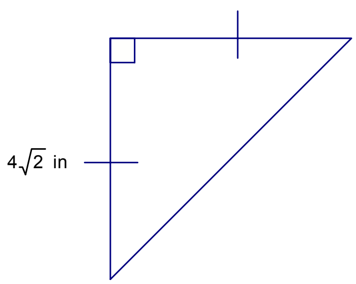
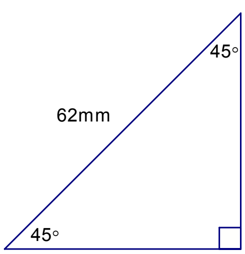


4)



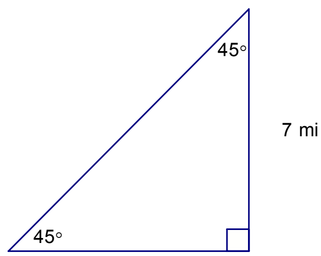
5) A baseball is hit to the third baseman. How far does he have to throw the ball to get to the first baseman to get the batter out? The distance from home plate to both third and first base is 90 feet.

Worksheet 1.2

Name: ______________KEY______________ Date: _______________

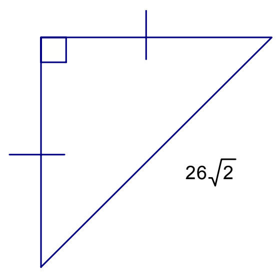
Find all the missing sides in the 45°– 45°– 90° triangles below. Give answers in radical form.

1)



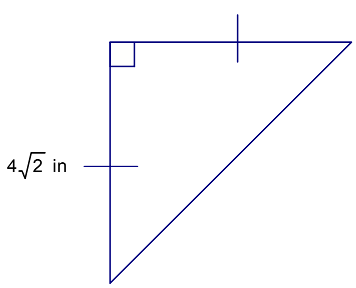
Hypotenuse length = $7\sqrt{2}$ mi.
Other missing side length = 7 mi.

2)



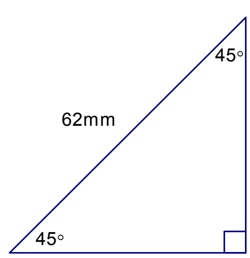
Both missing side lengths = 26.

3)



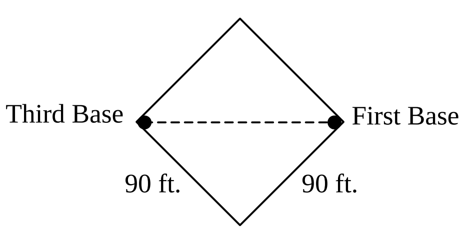
Hypotenuse length = 8 in.
Other missing side length = $4\sqrt{2}$ in.

4)



Both missing side lengths = $31\sqrt{2}$ mm.

5) A baseball is hit to the third baseman. How far does he have to throw the ball to get to the first baseman to get the batter out? The distance from home plate to both third and first base is 90 feet.



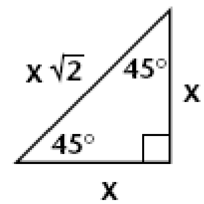
Hypotenuse length = $90\sqrt{2}$ ft.

Worksheet 1.3

Name: ______________________________ Date: _______________

RETEACHING

Using the 45° - 45° - 90° Triangle Theorem

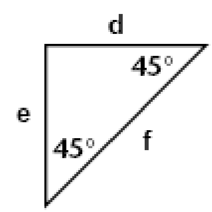


A 45° - 45° - 90° triangle is a right triangle in which both acute angles have measure 45°. In a 45° - 45° - 90° triangle, if the length of each leg is x, the length of the hypotenuse is $x\sqrt{2}$. If the length of the hypotenuse is x, then the length of each leg is $\frac{x}{\sqrt{2}}$ or $\frac{x\sqrt{2}}{2}$.

Example:
The legs of a 45° - 45° - 90° triangle are each 15 centimeters long. Find the length of the hypotenuse to the nearest tenth of a centimeter.

Solution:
Refer to the 45° - 45° - 90° triangle in the figure above. Since x = 15,
$h = x\sqrt{2} = 15\sqrt{2} \approx 21.2cm$

The length of one side of a 45° - 45° - 90° triangle is given. Find the lengths of the other two sides in simplest radical form.

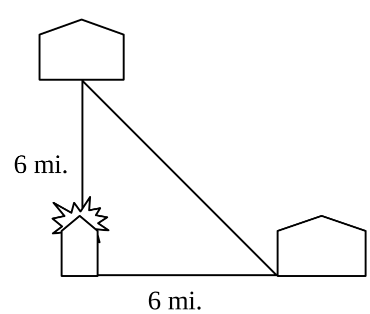


1. d = 13, e = __________, f = __________

2. d = __________, e = 4.5, f = __________

3. d = __________, e = __________, f = $9\sqrt{2}$

4. Two fire stations respond to the same burning house. One station is located due east of the house and the other station is located due north. If both fire stations are 6 miles from the burning house, how far apart are the two fire stations?

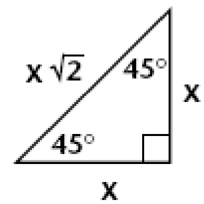


Worksheet 1.3

Name: KEY Date: ____________

RETEACHING

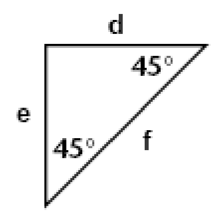
Using the 45° - 45° - 90° Triangle Theorem



A 45° - 45° - 90° triangle is a right triangle in which both acute angles have measure 45°. In a 45° - 45° - 90° triangle, if the length of each leg is x, the length of the hypotenuse is $x\sqrt{2}$. If the length of the hypotenuse is x, then the length of each leg is $\frac{x}{\sqrt{2}}$ or $\frac{x\sqrt{2}}{2}$.

> *Example:*
> The legs of a 45° - 45° - 90° triangle are each 15 centimeters long. Find the length of the hypotenuse to the nearest tenth of a centimeter.
>
> *Solution:*
> Refer to the 45° - 45° - 90° triangle in the figure above. Since x = 15,
> $h = x\sqrt{2} = 15\sqrt{2} \approx 21.2cm$

The length of one side of a 45° - 45° - 90° triangle is given. Find the lengths of the other two sides in simplest radical form.



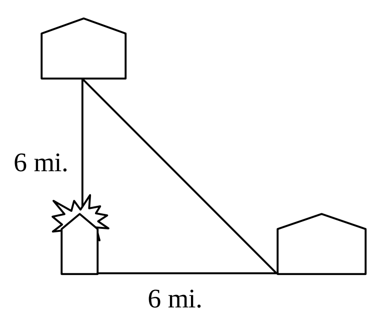
1. d = 13, e = 13, f = $13\sqrt{2}$

2. d = 4.5, e = 4.5, f = $4.5\sqrt{2}$

3. d = 9, e = 9, f = $9\sqrt{2}$

4. Two fire stations respond to the same burning house. One station is located due east of the house and the other station is located due north. If both fire stations are 6 miles from the burning house, how far apart are the two fire stations?

   The distance between the two fire stations is $6\sqrt{2}$

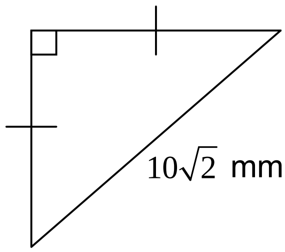


Warm Up 2

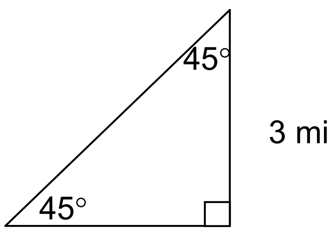
Name: ______________________________ Date: _______________

Find the missing side lengths in the 45°– 45°– 90° triangles below. <u>Show all work!!!</u>

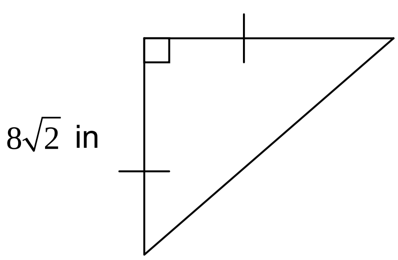
1)



2)



3)

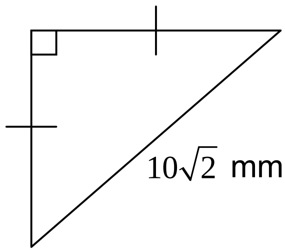


Warm Up 2

Name: KEY   Date: ________________

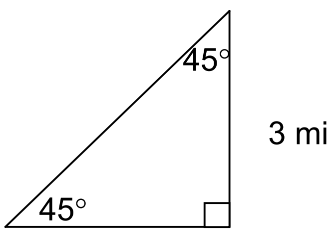
Find the missing side lengths in the 45°– 45°– 90° triangles below. <u>Show all work!!!</u>

1)



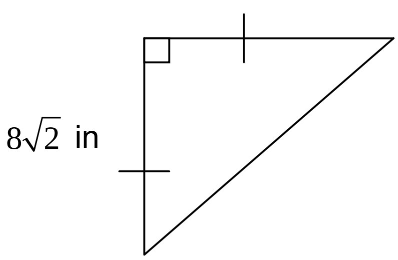
Both missing side lengths = 5 mm.

2)



Hypotenuse = $3\sqrt{2}$ mi.
Other missing side length = 3 mi.

3)

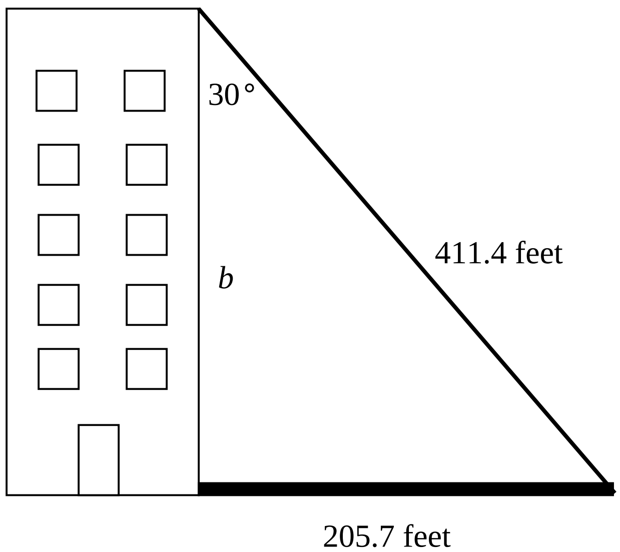


Hypotenuse = 16 in.
Other missing side length = $8\sqrt{2}$ in.

Warm Up 2.1

Name: Date:

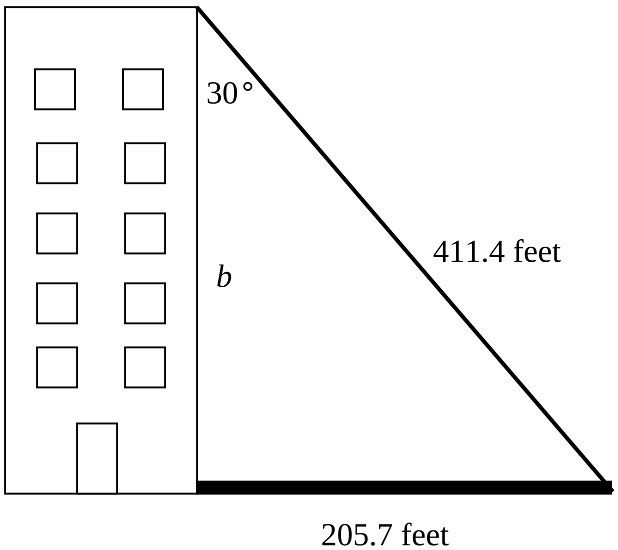
If an FBI agent wants to climb a rope to get to the top of a building where a suspect is holding someone hostage, how much rope does he need to use in order to reach the top of the building if the building casts a 205.7 foot shadow at a 30° angle to the ground?



Warm Up 2.1

Name: KEY Date:

If an FBI agent wants to climb a rope to get to the top of a building where a suspect is holding someone hostage, how much rope does he need to use in order to reach the top of the building if the building casts a 205.7 foot shadow at a 30° angle to the ground?



$$a^2 + b^2 = c^2$$
$$205.7^2 + b^2 = 411.4^2$$
$$42312.49 + b^2 = 169249.96$$
$$b^2 = 169249.96 - 42312.49$$
$$b^2 = 126937.47$$
$$b = \sqrt{126937.47}$$
$$b = 356.28$$

Worksheet 2.1

Name: ________________________________ Date: ________________

PART I
Use Sketchpad to construct a 30°- 60°- 90° triangle. Follow the instructions below.

1. Construct a line and label two points on that line A and B.
2. Double click on point B.
3. Select line AB, go to the Transform menu and select Rotate.
4. Verify that the degree measurement is 90°. If it is not 90°, then enter that value. Click the Rotate button.
5. Double click on point A.
6. Select line AB, go to the Transform menu and select Rotate.
7. Change the degree measurement to 30° and click the Rotate button.
8. Construct the point of intersection at the third vertex and label it point C.

PART II
What properties can you describe about the triangle you just constructed?

1.
2.
3.
4.

PART III
Use the Tabulate feature in Sketchpad to complete the table below.

AC	BC	AB (decimal)	AB (radical)
4	2	3.46	
5.1			$3.8\sqrt{3}$
7.6			$7.6\sqrt{3}$

What conjecture can you make about the relationship between the values in the third and fourth columns above?

Worksheet 2.1

Name: ____KEY____ Date: ____________

PART I

Use Sketchpad to construct a 30°- 60°- 90° triangle. Follow the instructions below.

1. Construct a line and label two points on that line A and B.
2. Double click on point B.
3. Select line AB, go to the Transform menu and select Rotate.
4. Verify that the degree measurement is 90°. If it is not 90°, then enter that value. Click the Rotate button.
5. Double click on point A.
6. Select line AB, go to the Transform menu and select Rotate.
7. Change the degree measurement to 30° and click the Rotate button.
8. Construct the point of intersection at the third vertex and label it point C.

PART II

What properties can you describe about the triangle you just constructed?

1. Right Triangle
2. Scalene Triangle
3. m<B = 90°
4. m<A = 30°
5. m<C = 60°
6. AC = 2 • BC (hypotenuse is twice the shorter leg)

PART III

Use the Tabulate feature in Sketchpad to complete the table below.

AC	BC	AB (decimal)	AB (radical)
4	2	3.46	$2\sqrt{3}$
5.1	2.55	4.42	$2.55\sqrt{3}$
7.6	3.8	6.58	$3.8\sqrt{3}$

Answers may vary.

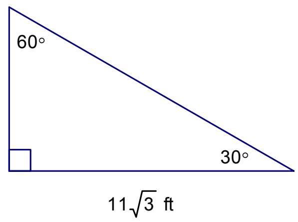
What conjecture can you make about the relationship between the values in the third and fourth columns above?

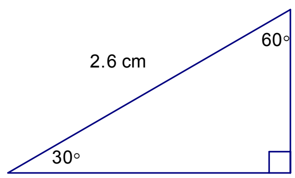
The sides of a 30°- 60°- 90° triangle are in the ratio of 2:1: $\sqrt{3}$.

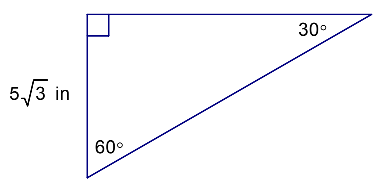
Worksheet 2.2

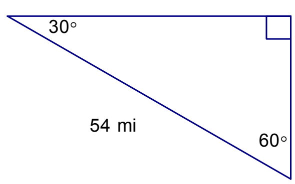
Name: ______________________________ Date: ________________

Find all the missing sides in the 30°– 60°– 90° triangles below.

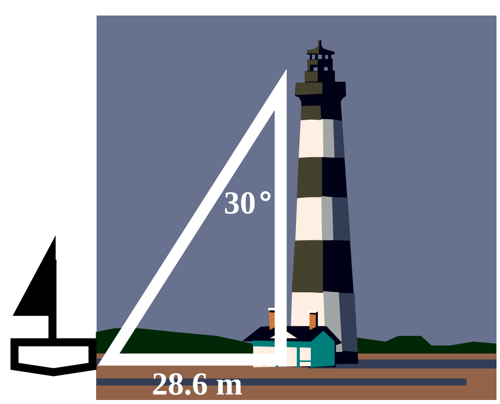
1) 

2) 

3) 

4) 

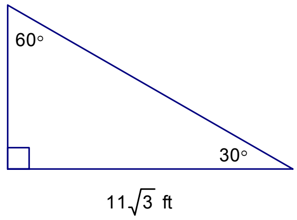
5) A sailboat is 28.6 meters from the base of the lighthouse. A person standing at the top of the lighthouse looks down at the sailboat at an angle of 30°. How tall is the lighthouse? What is the distance along the line of sight between the person at the top of the lighthouse and the sailboat?



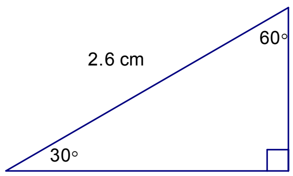
Worksheet 2.2

Name: ________________KEY______________ Date: ________________

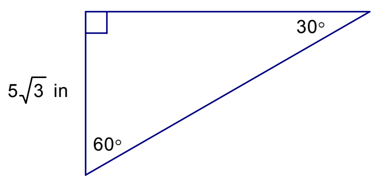
Find all the missing sides in the 30°– 60°– 90° triangles below.

1) 

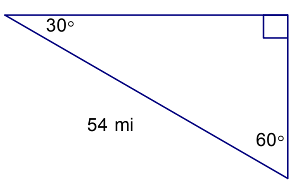
The short leg = 11 ft.
The hypotenuse = 22 ft.

2) 

The short leg = 1.8 cm.
The long leg = $1.8\sqrt{3}$ cm.

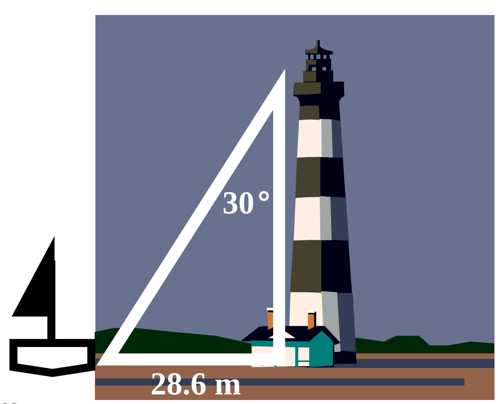
3) 

The short leg = 5 in.
The hypotenuse = 10 in.

4) 

The short leg = 27 mi.
The long leg = $27\sqrt{3}$ mi.

5) A sailboat is 28.6 meters from the base of the lighthouse. A person standing at the top of the lighthouse looks down at the sailboat at an angle of 30°. How tall is the lighthouse? What is the distance along the line of sight between the person at the top of the lighthouse and the sailboat?



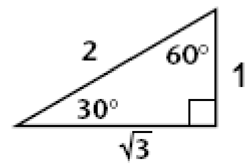
The height of the lighthouse is $28.6\sqrt{3}$.
The distance between the person and the sailboat is 57.2 m.

Worksheet 2.3

Name: ______________________________ Date: ______________

RETEACHING

Using the 30° - 60° - 90° Triangle Theorem



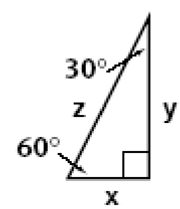
A 30° - 60° - 90° triangle is a right triangle in which the acute angles have measures of 30° and 60°. The shorter leg of the triangle is opposite the 30° angle and the longer leg is opposite the 60° angle. In a 30° - 60° - 90° triangle, if the length of the shorter leg is x, then the length of the longer leg is $x\sqrt{3}$, and the length of the hypotenuse is 2x.

Example:
The longer leg of a 30° - 60° - 90° triangle is 5 inches long. Find the length of the hypotenuse in simplest radical form.

Solution:
Refer to the 30° - 60° - 90° triangle in the figure above. Since $x\sqrt{3}$ = 5,

$$x = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3} \textit{ and } h = 2x = \frac{10\sqrt{3}}{3} \textit{ inches.}$$

The length of one side of a 30° - 60° - 90° triangle is given. Find the lengths of the other two sides in simplest radical form.



1. x = 4, y = __________, z = __________

2. x = $3\sqrt{3}$, y = __________, z = __________

3. x = __________, y = $7\sqrt{3}$, z = __________

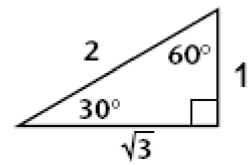
4. x = __________, y = 18, z = __________

Worksheet 2.3

Name: ________KEY________ Date: ________________

RETEACHING

Using the 30° - 60° - 90° Triangle Theorem



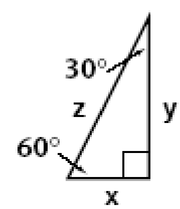
A 30° - 60° - 90° triangle is a right triangle in which the acute angles have measures of 30° and 60°. The shorter leg of the triangle is opposite the 30° angle and the longer leg is opposite the 60° angle. In a 30° - 60° - 90° triangle, if the length of the shorter leg is x, then the length of the longer leg is $x\sqrt{3}$, and the length of the hypotenuse is 2x.

Example:
The longer leg of a 30° - 60° - 90° triangle is 5 inches long. Find the length of the hypotenuse in simplest radical form.

Solution:
Refer to the 30° - 60° - 90° triangle in the figure above. Since $x\sqrt{3}$ = 5,

$$x = \frac{5}{\sqrt{3}} = \frac{5\sqrt{3}}{3} \text{ and } h = 2x = \frac{10\sqrt{3}}{3} \text{ inches.}$$

The length of one side of a 30° - 60° - 90° triangle is given. Find the lengths of the other two sides in simplest radical form.



1. x = 4, y = ____ $4\sqrt{3}$ ____, z = ____8____

2. x = $3\sqrt{3}$, y = ____9____, z = ____ $6\sqrt{3}$ ____

3. x = ____7____, y = $7\sqrt{3}$, z = ____14____

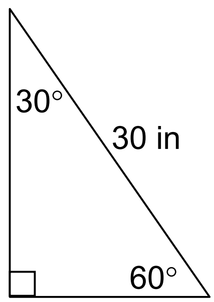
4. x = ____ $6\sqrt{3}$ ____, y = 18, z = ____ $12\sqrt{3}$ ____

Warm Up 3

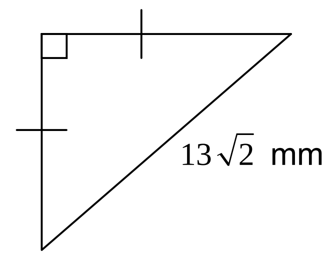
Name: ______________________________ Date: ______________

For #1 – 4, find the missing lengths of each side of the special right triangles.

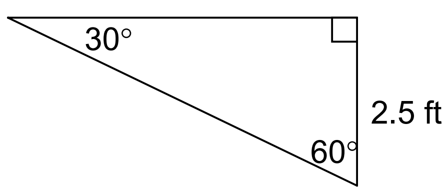
1)



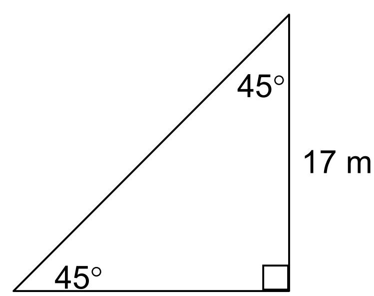
2)



3)



4)



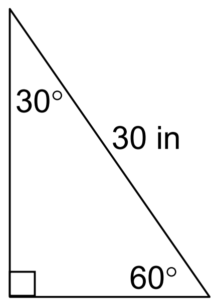
5) Define the incenter of a triangle.

Warm Up 3

Name: ________________KEY____________ Date: ________________

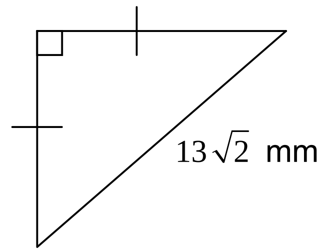
For #1 – 4, find the missing lengths of each side of the special right triangles.

1)



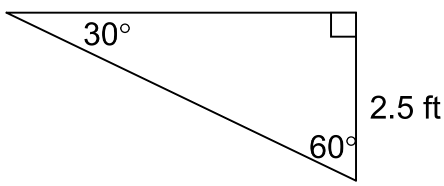
The short leg = 15 in.
The long leg = $15\sqrt{3}$ in.

2)



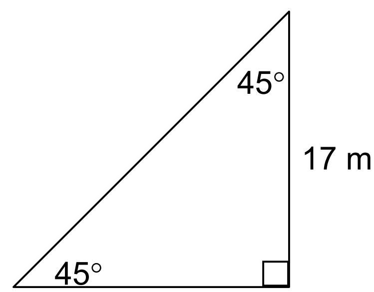
Both missing side lengths = 13 mm.

3)



The hypotenuse = 5 ft.
The long leg = $2.5\sqrt{3}$ ft.

4)



The hypotenuse = $17\sqrt{2}$ mi.
The other missing side length = 17 mi.

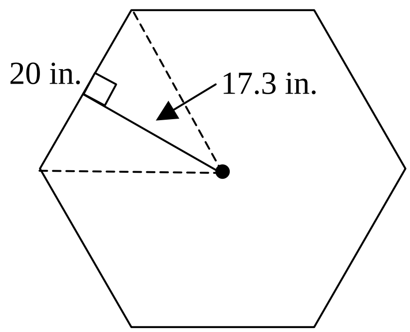
5) Define the incenter of a triangle.

The incenter of a triangle is the point where all the angle bisectors of the triangle meet. It is also the center of the inscribed circle. It is also the only point in the triangle that is equidistant from all the sides of the triangle.

Warm Up 3.1

Name: ______________________________ Date: _________________

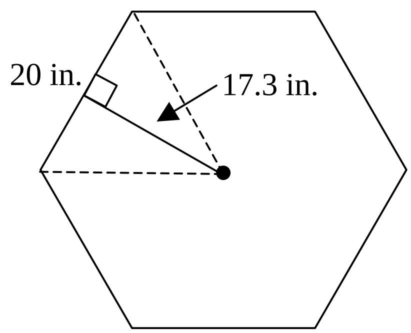
How much glass will need to be cut to make a regular hexagonal window 20 inches on each side for a bathroom window?



Warm Up 3.1

Name: KEY Date:

How much glass will need to be cut to make a regular hexagonal window 20 inches on each side for a bathroom window?



A = ½ bh
= ½ (20)(17.3)
A = 173 sq. in.

Multiply the area times the number of triangles. Since there are six equilateral triangles inside the hexagon, multiply the area that we just found by 6.

Therefore, the area of the hexagon is 438 sq. in.

Worksheet 3.1

Name: ________________________________ Date: ________________

PART I

Use Sketchpad to construct an equilateral triangle and find its area. Follow the instructions below.

8. Construct an equilateral triangle and label the vertices A, B and C.
9. Find the area of ΔABC and enter that value in the table below.
10. Construct an altitude through one of the vertices.
11. Place a point at the intersection of the altitude and the base. Label this point D.
12. Find the area by using the area formula for a triangle and enter that value in the table below.

PART II

Use Sketchpad to explore finding the area of a regular polygon using the area of a polygon formula. Follow the instructions below.

1. Construct the incenter of the triangle from above and label it point E.
2. Hide the angle bisectors.
3. Construct the three line segments connecting the incenter to each vertex.
4. Select different colors for the three interior triangles.
5. Find the area of each interior triangle.
6. Calculate the sum of the three areas and enter this number in the table below. Compare this area to the other areas in the table.

Area from Sketchpad	**Area of Triangle Formula for ΔABC**	**Sum of the Area of Triangle Formula for the Smaller Triangles**	**Area of a Polygon Formula**
Measure/Area	$A = \frac{1}{2} bh_1$	$A = \frac{1}{2} bh_2 + \frac{1}{2} bh_2 + \frac{1}{2} bh_2$	$A = \frac{1}{2} ap$

Notes:

In the third column, if the b's were added together, what would that sum represent?

In the third column, what do all the h_2's represent?

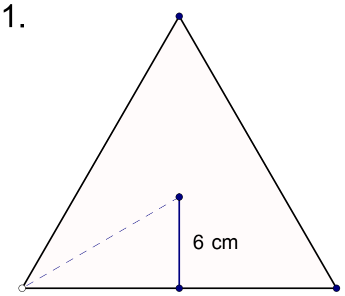
Worksheet 3.1

Name: ______________________________ Date: ________________

PART III

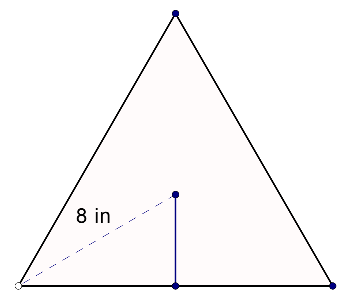
Guided Practice: Find the area of each figure using the formula A = ½ ap.

1.



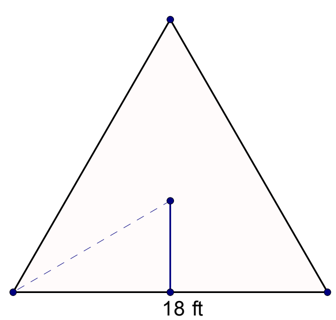
A = ____________

2.



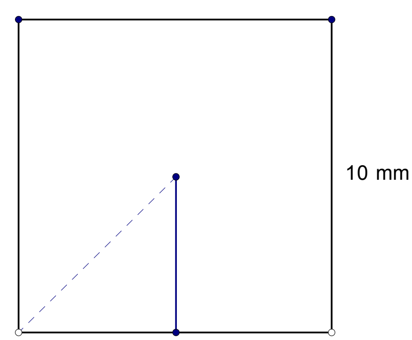
A = ____________

3.



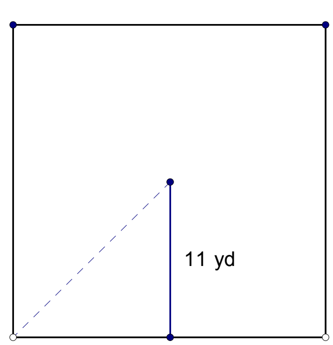
A = ____________

4.



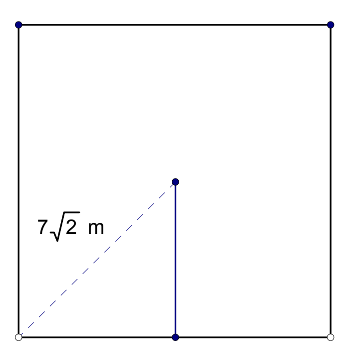
A = ____________

5.



A = ____________

6.



A = ____________

Worksheet 3.1

Name: _______________KEY_______________ Date: ________________

PART I

Use Sketchpad to construct an equilateral triangle and find its area. Follow the instructions below.

13. Construct an equilateral triangle and label the vertices A, B and C.
14. Find the area of ΔABC and enter that value in the table below.
15. Construct an altitude through one of the vertices.
16. Place a point at the intersection of the altitude and the base. Label this point D.
17. Find the area by using the area formula for a triangle and enter that value in the table below.

PART II

Use Sketchpad to explore finding the area of a regular polygon using the area of a polygon formula. Follow the instructions below.

7. Construct the incenter of the triangle from above and label it point E.
8. Hide the angle bisectors.
9. Construct the three line segments connecting the incenter to each vertex.
10. Select different colors for the three interior triangles.
11. Find the area of each interior triangle.
12. Calculate the sum of the three areas and enter this number in the table below. Compare this area to the other areas in the table.

Area from Sketchpad	Area of Triangle Formula for ΔABC	Sum of the Area of Triangle Formula for the Smaller Triangles	Area of a Polygon Formula
Measure/Area	$A = \frac{1}{2} bh_1$	$A = \frac{1}{2} bh_2 + \frac{1}{2} bh_2 + \frac{1}{2} bh_2$	$A = \frac{1}{2} ap$
Answers may vary, but all values in the table should be equal.			

Notes:

The apothem is the perpendicular distance from the center of a regular polygon to one of its sides. (we use the letter *a* to represent the apothem)

$A = \frac{1}{2} bh_2 + \frac{1}{2} bh_2 + \frac{1}{2} bh_2$
$A = \frac{1}{2} (bh_2 + bh_2 + bh_2)$
$A = \frac{1}{2} (3b) (h_2)$

In the third column, if the b's were added together, what would that sum represent?

The sum of all the b's added together is the perimeter of the triangle.

In the third column, what do all the h_2's represent?

The h_2's represent the apothem of the triangle.

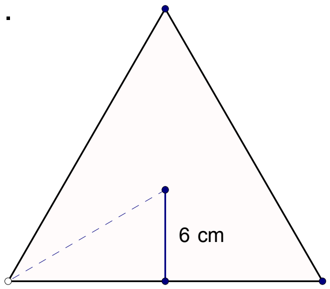
Worksheet 3.1

Name: ________________KEY________________ Date: _________________

PART III

Guided Practice: Find the area of each figure using the formula A = ½ ap.

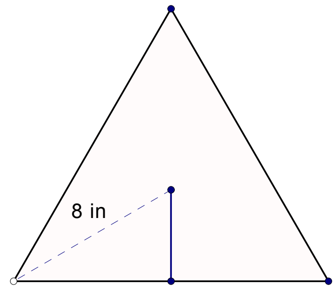
1.



A = $\underline{\quad 108\sqrt{3}\text{ cm}^2\quad}$

A = $\frac{1}{2} \bullet 6 \bullet 36\sqrt{3}$ cm^2

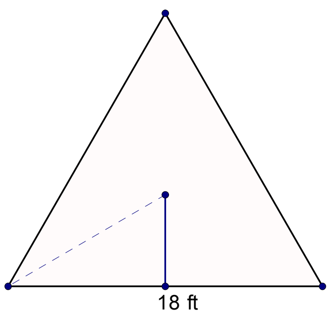
2.



A = $\underline{\quad 48\sqrt{3}\text{ in}^2\quad}$

A = $\frac{1}{2} \bullet 4 \bullet 24\sqrt{3}$ in^2

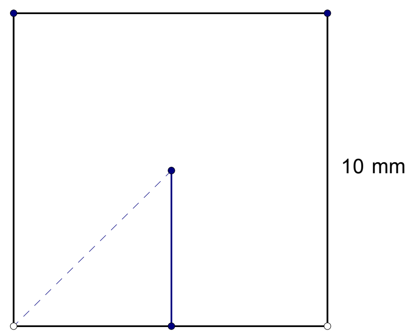
4.



A = $\underline{\quad 36\sqrt{3}\text{ ft}^2\quad}$

A = $\frac{1}{2} \bullet 3\sqrt{3} \bullet 24$ ft^2

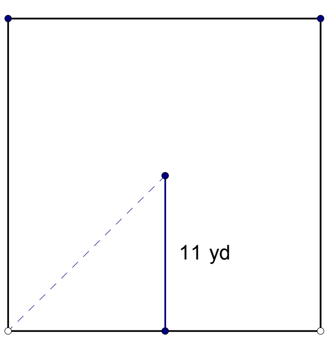
4.



A = $\underline{\quad 25\text{ mm}^2\quad}$

A = $\frac{1}{2} \bullet 5 \bullet 10$ mm^2

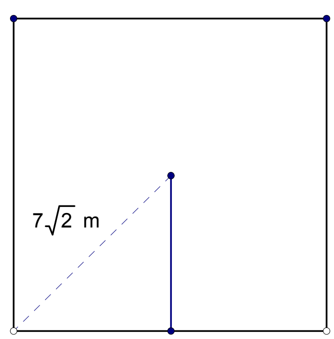
5.



A = $\underline{\quad 484\text{ yd}^2\quad}$

A = $\frac{1}{2} \bullet 11 \bullet 88$ yd^2

6.



A = $\underline{\quad 98\text{ m}^2\quad}$

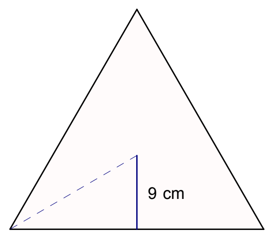
A = $\frac{1}{2} \bullet 7 \bullet 56$ m^2

Worksheet 3.2

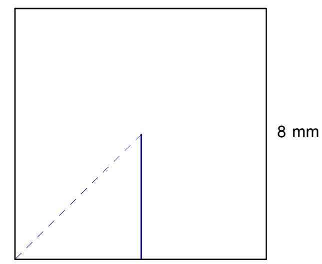
Name: ______________________________ Date: ______________

Find the area of each regular polygon below using the formula A = ½ap.

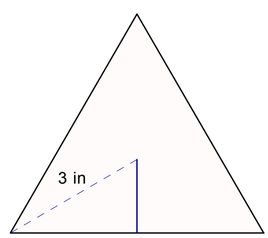
1)



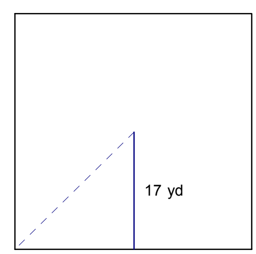
4)



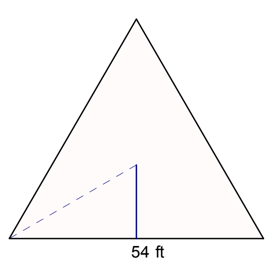
2)



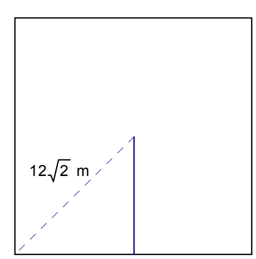
5)



3)



6)

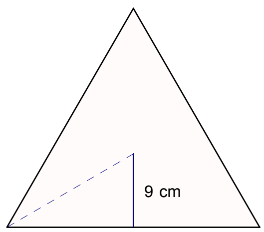


Worksheet 3.2

Name: ____________KEY_____________________ Date: _________________

Find the area of each regular polygon below using the formula A = ½ap.

1)

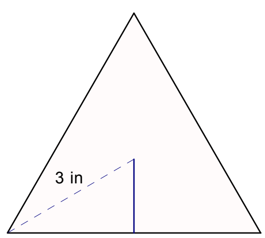


a = 9; p = $3 \bullet 9\sqrt{3} = 27\sqrt{3}$

A = $\frac{1}{2} \bullet 9 \bullet 27\sqrt{3}$

A = $121.5\sqrt{3}$ cm^2

2)

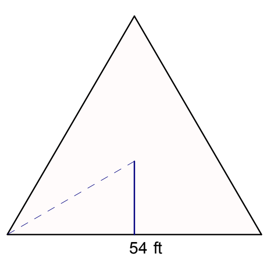


a = 1.5; p = $3 \bullet 1.5\sqrt{3} = 4.5\sqrt{3}$

A = $\frac{1}{2} \bullet 1.5 \bullet 4.5\sqrt{3}$

A = $3.375\sqrt{3}$ in^2

3)

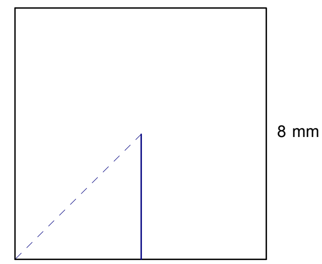


a = $9\sqrt{3}$; p = 162

A = $\frac{1}{2} \bullet 9\sqrt{3} \bullet 162$

A = 729 ft^2

4)

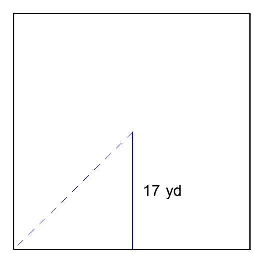


a = 4; p = 32

A = $\frac{1}{2} \bullet 4 \bullet 32$

A = 64 mm^2

5)

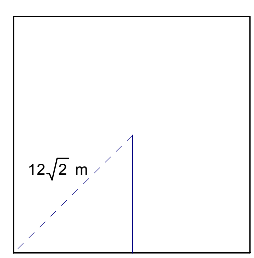


a = 17; p = 136

A = $\frac{1}{2} \bullet 17 \bullet 136$

A = 1156 yd^2

6)



a = 12; p = 96

A = $\frac{1}{2} \bullet 12 \bullet 96$

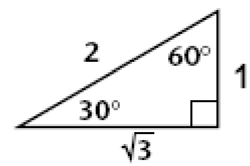
A = 576 yd^2

Worksheet 3.3

Name: ______________________________ Date: _______________

RETEACHING

Finding the area of a regular polygon

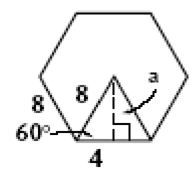


The apothem of a regular polygon is the length of a perpendicular segment from the center of the polygon to a side. The apothem can be used to determine the area of the polygon. The area of a regular polygon with apothem *a* and perimeter *p* is A = ½*ap*.

<u>*Example*</u>:
Find the area of a regular hexagon with sides that are 8 centimeters long.

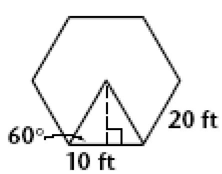
<u>*Solution*</u>:
Use the 30° - 60° - 90° Triangle Theorem to determine the apothem. In the 30° - 60° - 90° triangle shown, the length of the shorter leg is 4, so the apothem is $4\sqrt{3}$.

Since the perimeter is 6 • 8 = 48, A = ½ • $4\sqrt{3}$ • 48 = $96\sqrt{3}$.



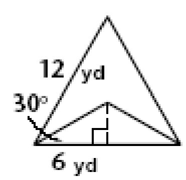
Find the area of each regular polygon. Give your answer in simplest radical form.

1.



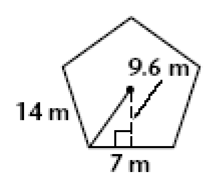
____

2.



____

3.



____

4. A regular octagon with sides that are 15 mm long and an apothem of 18.1mm

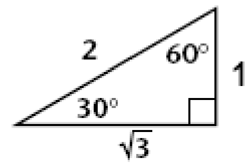
____

Worksheet 3.3

Name: ________________KEY________________ Date: _________________

RETEACHING

Finding the area of a regular polygon



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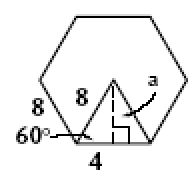
Example:
Find the area of a regular hexagon with sides that are 8 centimeters long.

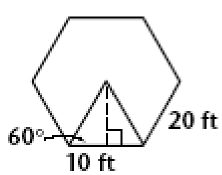
Solution:
Use the 30° - 60° - 90° Triangle Theorem to determine the apothem. In the 30° - 60° - 90° triangle shown, the length of the shorter leg is 4, so the apothem is $4\sqrt{3}$.

Since the perimeter is 6 • 8 = 48, A = ½ • $4\sqrt{3}$ • 48 = $96\sqrt{3}$.



Find the area of each regular polygon. Give your answer in simplest radical form.

1.

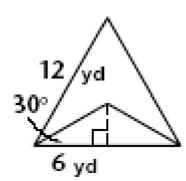


$A = 600\sqrt{3}$ ft²

$$A = \frac{1}{2}(10\sqrt{3})(120)$$

$$A = \frac{1}{2}(1200\sqrt{3})$$

2.

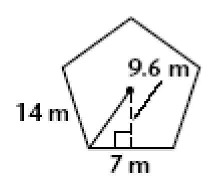


$A = 36\sqrt{3}$ yd²

$$A = \frac{1}{2}(2\sqrt{3})(36)$$

$$A = \frac{1}{2}(72\sqrt{3})$$

3.



$A = 336\sqrt{3}$ m²

$$A = \frac{1}{2}(9.6)(70)$$

$$A = \frac{1}{2}(672)$$

4. A regular octagon with sides that are 15 mm long and an apothem of 18.1mm

A = 1086 mm²

$$A = \frac{1}{2}(18.1)(120)$$

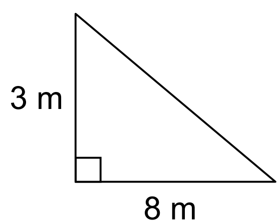
$$A = \frac{1}{2}(2172)$$

Geometry Partner Quiz

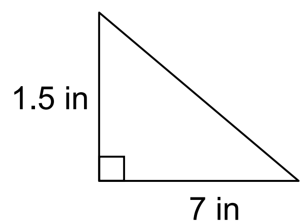
Student #1 ______________________ Student #2 ______________________

In order to get full credit, all problems must be completed showing each step. First, work out either problem #1a or #1b. Next, exchange papers with your partner and check each other's work for mistakes. Then, on the worksheet you have now, complete the problem directly beneath the one your partner just completed. Again, exchange papers with your partner and check each other's work for mistakes. Continue following these directions until you each complete 6 problems. Once you have worked out all the problems, make sure to copy all the work for each problem on both papers.

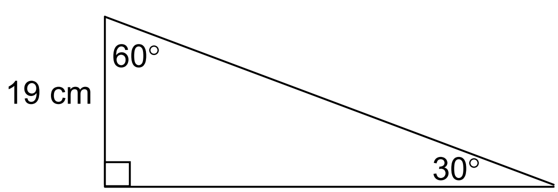
Quiz Problem #1a: Find the area of the triangle pictured below.



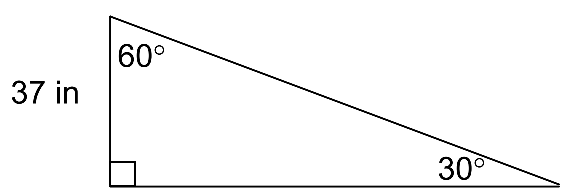
Quiz Problem #1b: Find area of the triangle pictured below.



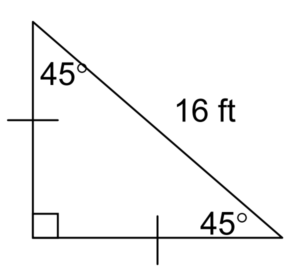
Quiz Problem #2a: Find all the missing lengths. Give answer in **radical form**.



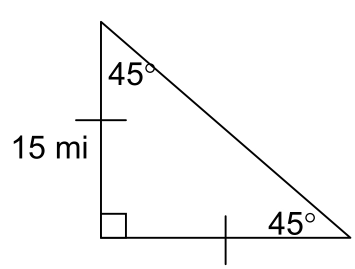
Quiz Problem #2b: Find all the missing lengths. Give answer in **radical form**.



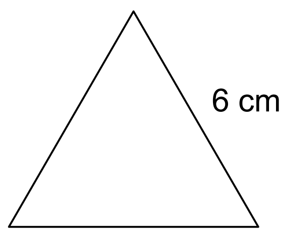
Quiz Problem #3a: Find all the missing lengths. Give answer in **radical form**.



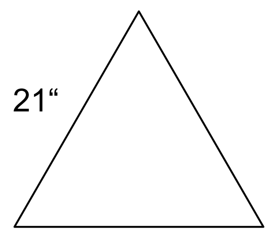
Quiz Problem #3b: Find all the missing lengths. Give answer in **radical form**.



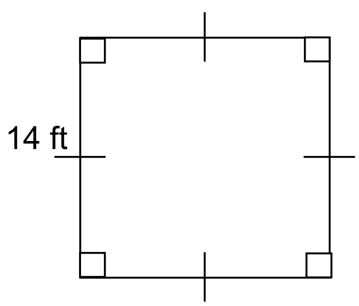
Quiz Problem #4a: Find the perimeter and area of the equilateral triangle pictured below.



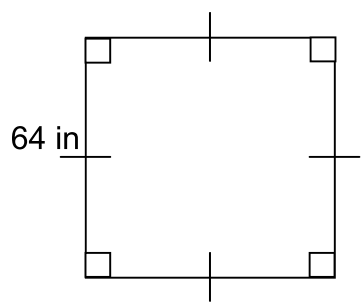
Quiz Problem #4b: Find the perimeter and area of the equilateral triangle pictured below.



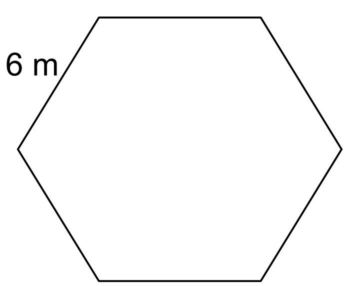
Quiz Problem #5a: Find the area of the square pictured below using the formula A = (½)ap, where a = apothem; p = perimeter.



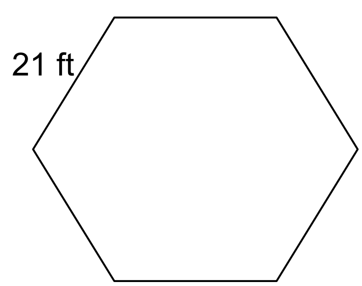
Quiz Problem #5b: Find the area of the square pictured below using the formula A = (½)ap, where a = apothem; p = perimeter.



Quiz Problem #6a: Find the perimeter and area of the regular hexagon pictured below.



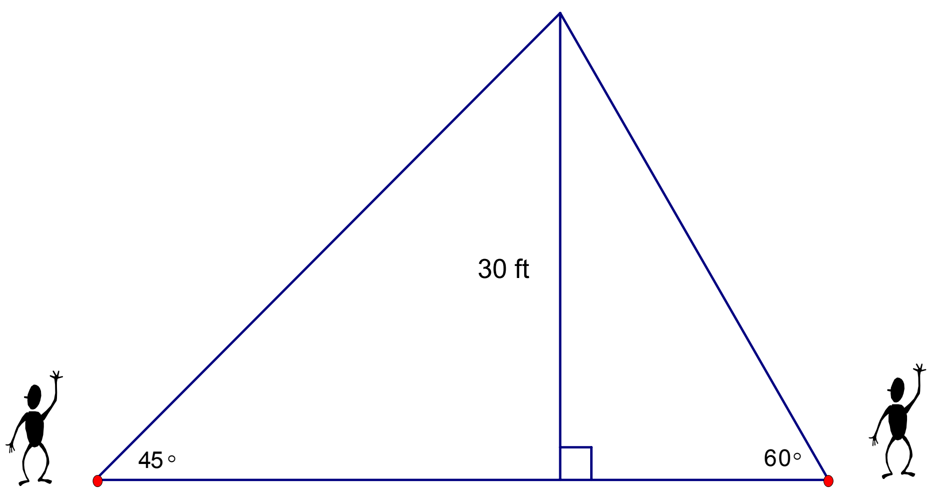
Quiz Problem #6b: Find the perimeter and area of the regular hexagon pictured below.



Name: ______________________________

BCR

Two surveyors are standing on opposite sides of a telephone pole at the two highlighted points shown below. The height of the telephone pole is 30 ft, the angle the wire on the left makes with the ground is 45°, and the angle the wire on the right makes with the ground is 60°.



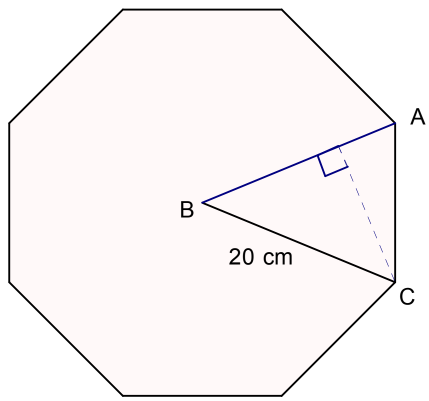
- Find the distance between the two surveyors in radical form. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

- What is the decimal form of the distance you found in the last problem?

Name: ______________________________

ECR

The octagon pictured below is the new size of the STOP signs that department of transportation plans to use next year. They need to know how much metal is in each sign so that they can purchase the correct sized metal sheets.



- Find the m<ABC. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

- Find the area of ΔABC. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

- Use your answer from the last problem to calculate the area of the STOP sign. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation

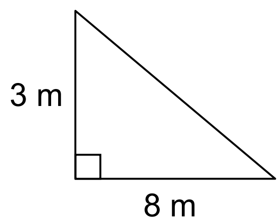
Geometry Partner Quiz

Student #1 ______________________ Student #2 ______________________

In order to get full credit, all problems must be completed showing each step. First, work out either problem #1a or #1b. Next, exchange papers with your partner and check each other's work for mistakes. Then, on the worksheet you have now, complete the problem directly beneath the one your partner just completed. Again, exchange papers with your partner and check each other's work for mistakes. Continue following these directions until you each complete 6 problems. Once you have worked out all the problems, make sure to copy all the work for each problem on both papers.

<table>
<tr>
<td>

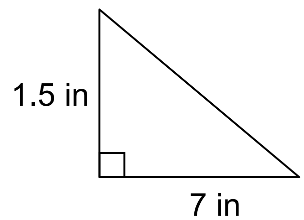
Quiz Problem #1a: Find the area of the triangle pictured below.



$A = \frac{1}{2} \bullet 8 \bullet 3 = 12\ m^2$

</td>
<td>

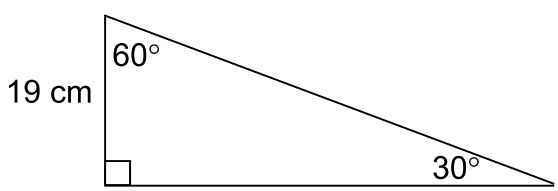
Quiz Problem #1b: Find area of the triangle pictured below.



$A = \frac{1}{2} \bullet 1.5 \bullet 7 = 12\ in^2$

</td>
</tr>
<tr>
<td>

Quiz Problem #2a: Find all the missing lengths. Give answer in **radical form**.



Hypotenuse =38 cm; Long leg = $19\sqrt{3}$cm

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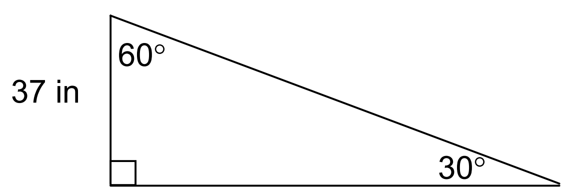
Quiz Problem #2b: Find all the missing lengths. Give answer in **radical form**.



Hypotenuse=74 in; Long leg = $37\sqrt{3}$

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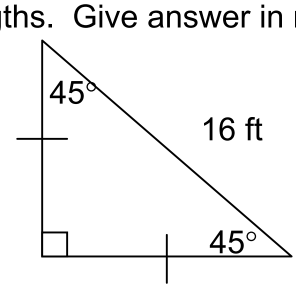
<tr>
<td>

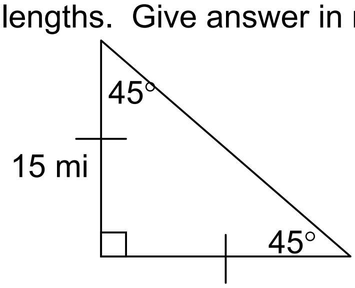
Quiz Problem #3a: Find all the missing lengths. Give answer in **radical form**.



Both of the missing lengths = $8\sqrt{2}$ ft.

</td>
<td>

Quiz Problem #3b: Find all the missing lengths. Give answer in **radical form**.



Hypotenuse= $15\sqrt{2}$ mi;Other side=15 mi

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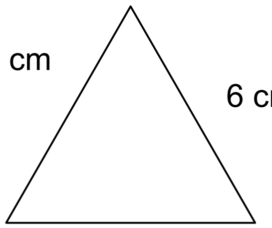
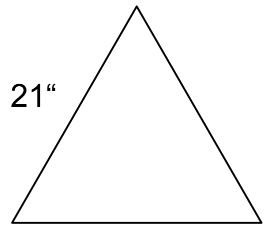
<table>
<tr>
<td>Quiz Problem #4a: Find the perimeter and area of the equilateral triangle pictured below.

$a = \sqrt{3}$ cm
P = 18 cm

6 cm

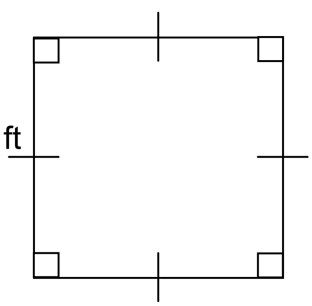
A = ½ ap

$A = \frac{1}{2} \bullet \sqrt{3} \bullet 18 = 9\sqrt{3}\ \text{cm}^2$</td>
<td>Quiz Problem #4b: Find the perimeter and area of the equilateral triangle pictured below.

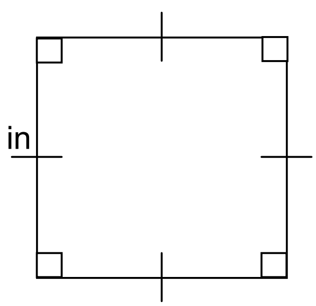
P = 66 “
21“

$a = 7\sqrt{3}$ “

A = ½ ap

$A = \frac{1}{2} \bullet 7\sqrt{3} \bullet 66 = 231\sqrt{3}\ \text{in}^2$</td>
</tr>
<tr>
<td>Quiz Problem #5a: Find the area of the square pictured below using the formula A = (½)ap, where a = apothem; p = perimeter.

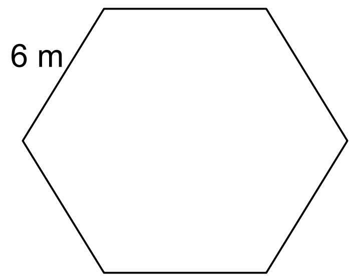

14 ft
a = 7 ft
P = 56 ft

$A = \frac{1}{2} \bullet 7 \bullet 56 = 484\ \text{ft}^2$</td>
<td>Quiz Problem #5b: Find the area of the square pictured below using the formula A = (½)ap, where a = apothem; p = perimeter.


64 in
a = 32 in
P = 128 in

$A = \frac{1}{2} \bullet 32 \bullet 128 = 4096\ \text{in}^2$</td>
</tr>
</table>

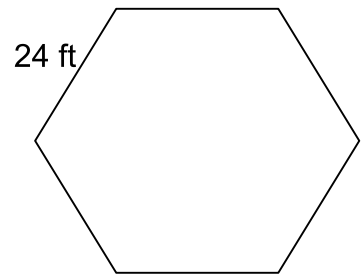
Quiz Problem #6a: Find the perimeter and area of the regular hexagon pictured below.



a = $\sqrt{3}$ cm; P = 6 • 6 = 36 cm

A = ½ • $\sqrt{3}$ • 36 = $18\sqrt{3}$ cm^2

Quiz Problem #6b: Find the perimeter and area of the regular hexagon pictured below.



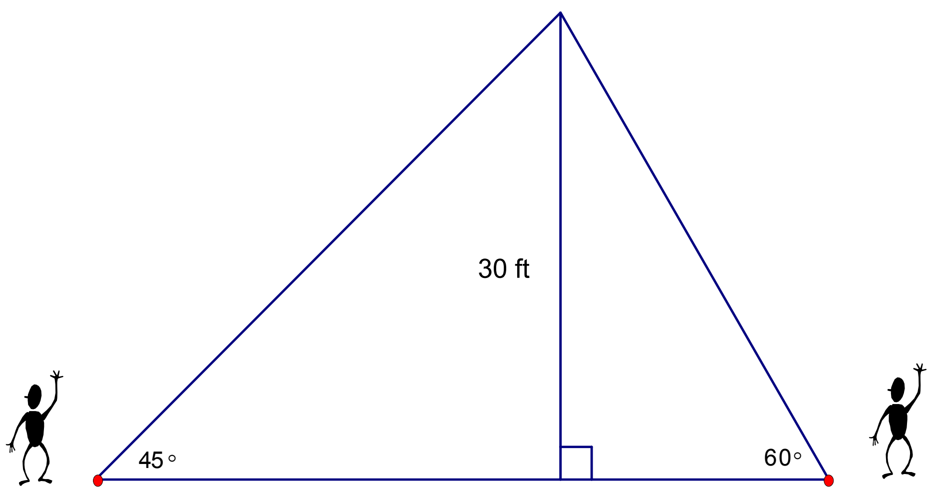
a = $4\sqrt{3}$ ft; P = 6 • 24 = 144 ft

A = ½ • $4\sqrt{3}$ • 144 = $288\sqrt{3}$ ft^2

Name: ______________________________

BCR

Two surveyors are standing on opposite sides of a telephone pole at the two highlighted points shown below. The height of the telephone pole is 30 ft, the angle the wire on the left makes with the ground is 45°, and the angle the wire on the right makes with the ground is 60°.



- Find the distance between the two surveyors in radical form. Use mathematics to explain how you determined your answer. Use words, symbols, or both in your explanation.

The distance from the surveyor on the left to the telephone pole = 30 ft since the triangle on the left is a 45° – 45° – 90° triangle so the legs are the same length.

The distance from the surveyor on the right to the telephone pole = $10\sqrt{3}$ ft since the triangle on the right is a 30° – 60° – 90° triangle so the short leg is $\frac{1}{\sqrt{3}}$ times the long leg.

The total distance between the two surveyors is the sum of these two values, or $30 + 10\sqrt{3}$ ft.

- What is the decimal form of the distance you found in the last problem?

The decimal form of the distance between the two surveyors = 47.32 ft.

Name: ______________________________

ECR

The octagon pictured below is the new size of the STOP signs that department of transportation plans to use next year. They need to know how much metal is in each sign so that they can purchase the correct sized metal sheets.