

ANALYSIS OF ENERGY CONSUMPTION OF TUNA SWIMMING IN  
CIRCULAR TANKS, AND HAVING ARTIFICIALLY INCREASED  
NEGATIVE BUOYANCY

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#### ABSTRACT

An analysis of the forces acting on fish swimming in curved paths is developed. Calculation procedures to include the influences of centrifugal forces on energy expenditure and swimming speeds are presented. This enables extrapolation of this data collected in round and annular tanks to natural situations.

Also included is an analysis of the changes in swimming speed, metabolic rates, etc., as a result of adding internal weights to negatively buoyant fish.

## 1. INTRODUCTION

Respiration and swimming speed experiments on tuna are carried out in round tanks at the Kewalo Research Facility of National Marine Fisheries Service (NMFS)/Southwest Fisheries Center (SWFC) Honolulu Laboratory. While this is probably the only test facility in the world capable of sustaining these large, active fish, the prevailing conditions introduce some artifacts which have to be removed, to better simulate behavior in the wild. This report provides a quantitative hydrodynamic analysis of the effects of confinement in round and annular tanks on the fish. A correction factor is introduced which enables simple application of the results obtained in these tanks, to situations in the open ocean. This correction factor shows the increase in thrust the fish has to apply, to stay in a curved trajectory.

Changes in the minimum swimming speeds of negatively buoyant scombrids due to swimming in the circular tank are also calculated, showing an inverse relation to turning radius. Finally hydrodynamic and energetic changes due to inserting weights into the fish are analyzed and equations for the change in thrust and energy requirements are developed.

## 2. CORRECTION FACTOR FOR SWIMMING IN A CURVED PATH

For a fish swimming horizontally at constant speed the force balance is, in that plane (excluding forces due to buoyancy and its compensation)

$$T = D \quad (1)$$

for swimming in a straight line where T is the thrust and D, the total hydrodynamic drag. Equation (1) describes the force balance in the

tangential direction also for curvilinear motion. However, an additional balance has to be made between the natural centrifugal force driving the fish to stay in a straight line and the countering centripetal force  $F$  causing it to remain on a curved path (see Figure 1A). This force is

$$F = \frac{m^1 U^2}{R} = \frac{V U^2}{R} (\rho_f + \rho_w \lambda) \quad (2)$$

where  $m^1$  is the fish virtual mass,  $\rho_f$ , the fish density (averaged),  $V$ , its volume,  $U$  the fish forward speed,  $\lambda$ , the longitudinal added mass coefficient, and  $R$  is the instantaneous radius of the curved path. The added mass coefficient  $\lambda$  is multiplied by the density of water  $\rho_w$  as the added mass is the volume of water dragged along with the fish. The hydrodynamic drag can be written as (Hoerner 1965)

$$D = \frac{1}{2} \rho_w V^{2/3} U^2 C_{Dt} \quad (3)$$

where  $C_{Dt}$  is the total drag coefficient based on the two-thirds power of the volume, as reference area. Here drag includes gill drag, induced drag due to pectoral fin extension and lift, etc. (see Magnuson 1978 for a description of the various components of drag).

To find the relative importance of the centripetal force, we divide (2) by (3), obtaining

$$\frac{F}{D} = \frac{2 V^{1/3}}{R C_{Dt}} \left( \frac{\rho_f}{\rho_w} + \lambda \right) \quad (4)$$

showing that the fish velocity does not influence the ratio of centrifugal to drag forces. Eq. (4) can now be used to obtain the correction to the force produced by the fish since the total force  $T_t$  the fish requires for swimming in a circular path in the horizontal plane is, by vectorial addition

$$T_t = \sqrt{D^2 + F^2} \quad (5)$$

or, dividing by  $D^2$  and applying (1) we obtain the required thrust increase as a function of the correction factor  $\frac{F}{D}$ .

$$\frac{T_t}{T} = \sqrt{1 + \left(\frac{F}{D}\right)^2} \quad (6)$$

The most complete set of data for estimation of  $D$  appears in Magnuson (1978, Table 6) for skipjack tuna. This data will now be used to obtain typical values of the correction factor. The total drag for a 44 cm skipjack tuna swimming at 66 cm/s was estimated there to be 19,780 dynes. The mass of a fish is approximately 1.67 kg (Nakamura and Uchiyama 1966) and with an average density of 1.09 (Magnuson 1978, Table 3), the fish volume is  $1,530 \text{ cm}^3$ . The drag coefficient  $C_{Dt}$  (which is different than Magnuson's drag coefficient because of the different reference area) is found, from equation (3), to be  $C_{Dt} = 0.067$  (with  $\rho_w = 1.025$ ).

Magnuson's (1978) data is partially based on the study of ram ventilation by Brown and Muir (1970) which was carried out at the Kewalo Research Facility in Honolulu holding tanks, which are of 7.3 m diameter. These allow a maximum path radius of about 300 cm. Recalling that the longitudinal added mass for a streamlined fish shape is  $\lambda \approx 0.2$  (Webb 1975) and substituting the above values into equation (4) we obtain

$$\frac{F}{D} = 2 \frac{(1530)^{1/3} (1.08 + 0.2)}{300 \cdot 0.067} = 1.47 \quad (7)$$

This means that the centripetal force is actually larger than the total drag force under these circumstances. The total force exerted by the

fish swimming in a circular path of 300 cm radius is thus, from equation (6)  $\frac{T_t}{T} = 1.78$ , i.e., 78% greater than that required for straight swimming at the same speed. The ratio  $F/D$  grows essentially proportionally to fish length so that larger fish expend an even greater proportion of energy in swimming in a curvilinear path.

Most other species of fish swim less efficiently than tuna, but being neutrally buoyant, do not have a contribution due to induced drag by the pectorals. This element, which is approximately 30% of the total drag appears only in negatively buoyant fish, so that the total drag coefficient  $C_{Dt}$  is changed very little, and the correction factor (4) can be applied to other, not necessarily negatively buoyant species.

### 3. CHANGES IN MINIMUM SPEED

Negatively buoyant fish, such as most of the scombrids have to swim continuously to maintain a horizontal course, by producing lift. The amount of lift required is equal to the submerged weight, which defines a hydrodynamical minimum swimming speed for horizontal motion (Magnuson 1970). At the minimum speed the pectoral fins produce the maximum possible lift, i.e., the fin's lift coefficient is  $C_{Lmax}$ . We can now estimate the influence of swimming in a curved path on the minimum swimming speed (see Figure 1B). Gradual turns during fast swimming are partially produced by banking the pectoral fins (Weihs 1972). Banking the fins reduces the vertical component of the force, since part of the hydrodynamic force is used to turn the fish. But the vertical force is prescribed by the weight, so that the swimming speed must be increased to obtain the required forces. Thus, the minimum swimming speed for

moving in a curved path is always higher than that for swimming along a straight line. This further increases the effort required to swim in circular tanks above and beyond the thrust correction found before (eq. 6).

At the minimum speed  $U_m$

$$W = \frac{1}{2} \rho_w A_f C_{Lmax} U_m^2 \quad (8)$$

where  $W$  is the submerged weight of the fish and  $A_f$  is the fin lifting area. If the fish has to turn (when it is confined to a round tank) and still wants, for energetic purposes, to stay at minimum speed, the pectoral fins are banked. The force produced on the fins is then (see Figure 1B)

$$P = \sqrt{W^2 + F^2} \quad (9)$$

or

$$\frac{P}{W} = \sqrt{1 + \left(\frac{F}{W}\right)^2} \quad (10)$$

$P$  is produced while the fins are at  $C_{Lmax}$  so that

$$P = \frac{1}{2} \rho_w A_f C_{Lmax} U_t^2 \quad (11)$$

where  $U_t$  is the minimum speed for a horizontal turn. This is a function of the turning radius, through  $F$  (see equation 2).

Substituting (11), (8), and (2) in (10) we have

$$\left(\frac{U_t}{U_m}\right)^2 = \sqrt{1 + \frac{m^2 U_t^2}{RW}} \quad (12)$$

or

$$\left(\frac{U_t}{U_m}\right)^4 = 1 + \frac{2m \left(1 + \lambda \frac{\rho_w}{\rho_f}\right)}{\rho_w R A_f C_{Lmax}} \left(\frac{U_t}{U_m}\right)^2 \quad (13)$$

from which

$$\frac{U_t}{U_m} = \left[ B \left( 1 + \sqrt{1 + \frac{1}{B^2}} \right) \right]^{1/2} \quad (14)$$

where

$$B = \frac{m \left( 1 + \lambda \frac{\rho_w}{\rho_f} \right)}{\rho_w C_{Lmax} A_f} \frac{1}{R}$$

From (14) we see that  $U_t > U_m$  for all finite values of  $R$  and that the minimum swimming speed grows as the radius decreases. For example, for the skipjack tuna described in Magnuson (1978) and used before,  $m = 1,670$  g,  $C_{Lmax} = 1.0$ ,  $A_f = 36$  cm<sup>2</sup> (Magnuson 1973),  $\rho_w = 1.022$ ,  $\lambda = 0.2$  and  $R = 300$  cm again. We obtain  $\frac{U_t}{U_m} = 1.094$ , i.e., the measured minimum speed in the round tank is about 10% higher than for the same fish moving in a straight line.

Using the allometric data of Magnuson (1973) we can obtain  $\frac{U_t}{U_m}$  as a function of fish size for various species. This appears in Figure 2 for skipjack tuna, for which  $m = 0.00490L^{3.36}$  and  $A_f = 0.00749L^{2.24}$  so that a slight dependence on fish length is retained, even after normalizing by the length.

The increase in minimum speed also is reflected in the total effort required for swimming in the round tank, as the force goes up as the square of the velocity.

The total thrust increase the fish has to produce is therefore

$$Z = \frac{T_t}{D} \cdot \left( \frac{U_t}{U_m} \right)^2 \quad (15)$$



which for our 44 cm skipjack tuna is  $Z = 1.77 \cdot (1.094)^2 = 2.12$  - i.e., over twice the minimum effort in the open sea.

An additional result of the centrifugal forces is the fact that the fish has to make asymmetric motions to produce them. While these motions may be masked by the oscillatory thrust motions, negatively buoyant species such as skipjack tuna will have to maintain a mean banking angle to produce P (equation 9). This angle can be obtained by banking the body and keeping symmetrical fin angles (as in Figure 1B), or by asymmetric deployment of the pectorals. An experimental program to study these angles by photography is underway at the present time. The predicted value of the banking angle (from the vertical) is obtained from (9) as

$$\tan \alpha = \frac{F}{W} \quad (16)$$

$$\tan \alpha = \frac{m(1+\lambda)U^2}{RW} \quad (17)$$

which for the skipjack tuna example above is  $12.1^\circ$ .

#### 4. EFFECTS OF ADDING INTERNAL WEIGHTS TO SKIPJACK TUNA

Adding internal weights to skipjack tuna has been suggested as a means of forcing the fish to swim at higher than their regular volitional swimming speeds. This is required to obtain information on the oxygen consumption of tuna at high swimming speeds. The tuna are expected to swim at higher speeds as they are assumed to be moving at the hydrodynamical minimum speed (i.e., at maximum lift coefficient). Increasing their submerged weight will increase the lift required for

horizontal swimming--and as the fins are already producing the highest lift possible at that speed--the swimming speed has to go up.

For horizontal swimming,

$$L = W \quad (18)$$

where  $W$  is the submerged weight, i.e.,

$$W = V(\rho_f - \rho_w) \quad (19)$$

changing the submerged weight to  $\tilde{W} = nW$  the lift has to change too, to  $\tilde{L} = W$  and therefore  $\tilde{L} = nL$  where the curly sign indicates the case of increased submerged weight. From (8)

$$L = W = \frac{1}{2} \rho_w A_f C_{Lmax} U_m^2 \quad (20)$$

and

$$\tilde{L} = nL = \frac{1}{2} \rho_w A_f C_{Lmax} \tilde{U}^2 \quad (21)$$

so that

$$\tilde{U} = \sqrt{n} U_m \quad (22)$$

here the angles of attack and sweep of the pectorals stay the same, and only the speed changes to accomodate the extra weight. Next, we examine the influence of added weights on the drag. The friction and form drag are dependent on velocity squared, therefore, they increase directly proportional to the weight ratio  $n$ , i.e.,

$$\tilde{D}_f = nD_f \quad (23)$$

The induced drag, resulting from lift is (Hoerner 1965)

$$D_i = \frac{L^2}{\frac{1}{2}\rho_w U^2 \pi b^2} \quad (24)$$

where  $b$  is the fin span

so that

$$\frac{\tilde{D}_i}{D_i} = \frac{\tilde{L}^2 U^2}{L^2 \tilde{U}^2} = \frac{n^2}{n} = n$$

or

$$\tilde{D}_i = n D_i \quad (25)$$

In a similar manner the drag induced by the hydrodynamic forces produced by the caudal fin is, as the mechanism is the same as (25), except the lift is replaced by the thrust

$$\tilde{D}_c = n D_c \quad (26)$$

Finally, the ram drag due to the gill through flow is directly proportional to the swimming speed (Magnuson 1978), i.e.,

$$\tilde{D}_g = \sqrt{n} D_g \quad (27)$$

The gill drag is approximately 20% of the total drag at typical speeds (Magnuson 1978) so that equation (23), and (25-27) can be combined, for the total drag

$$\frac{\tilde{D}_t}{D_t} \approx 0.8n + 0.2\sqrt{n} \quad (28)$$

The rate of working  $E$  required to swim at these minimum speeds, which is proportional to the oxygen consumption rate is obtained by

$$E = DU \quad (29)$$

so that

$$\frac{\tilde{E}}{E} = \frac{\tilde{D}\tilde{U}}{DU} = (0.8n + 0.2\sqrt{n})\sqrt{n} = 0.8n^{3/2} + 0.2n \quad (30)$$

i.e., the ratio of increase of oxygen consumption grows at a rate slightly less than proportional to the weight ratio to the  $3/2$  power. This relationship appears in Figure 3, which serves as a predictor for oxygen consumption rates for weighted fish as the energy requirements appear in the form of a ratio, thereby cancelling out the various transfer coefficients relating oxygen consumption to energy output. This figure also serves as a rough check on whether the fish was swimming at its minimum speed in both cases, as then the ratio of oxygen consumption should be on the curve in Figure 3.

A measured value of this ratio far below the predicted value indicates that the fish was moving at higher than minimum speed when unweighted. Less probable is a measured value higher than the prediction, which would imply that the fish is moving faster than necessary when weighted. Actually, agreement with the predicted oxygen consumption means only that the average lift coefficient for swimming under both weighted and natural conditions is equal, and both could be at above minimum speeds. (This, again is unlikely to occur in practice, due to energy conservation.)

The weighting experiments are carried out to force fish to swim at high speeds, but the information really required is the oxygen consumption at these same high speeds under unweighted (natural) conditions. This can again be obtained approximately by the same type of proportionality analysis. The weights are carried internally by the fish, and so the form and friction drag components are not changed by the presence of

the weight. Gill drag is not changed either, as the speed stays the same. The only component of drag different from equation (28) is the induced drag, as the lift required is now  $W$  and not  $nW$ . Thus, instead of equation (28), we can write for the ratio of drag at the higher speed  $D_2$  to that at the minimum speed

$$\frac{D_{2t}}{D_t} = 0.5n + 0.2\sqrt{n} + 0.3 \quad (31)$$

The induced drag component is no longer a function of the speed, as the lift stays constant for the unweighted fish see equation (27), and the span is reduced as the swimming speed goes up (by retracting the pectoral fins). Similarly to (30)

$$\frac{E_2}{E} \approx 0.5n^{3/2} + 0.2n + 0.3\sqrt{n} \quad (32)$$

so that we can eliminate the minimum speed energy requirements from (30) and (32) by dividing, i.e.,

$$\frac{E_2}{\tilde{E}} = \frac{0.5n^{3/2} + 0.2n + 0.3\sqrt{n}}{0.8n^{3/2} + 0.2n} \quad (33)$$

which enables an immediate estimate of the unweighted oxygen requirement at elevated speeds, with only the oxygen consumption of the weighted fish, and the submerged weight ratio as input parameters. The values of the ratio  $E_2/\tilde{E}$  versus  $n$  are plotted in Figure 4. When the fish is unweighted,  $n = 1$  and  $E_2/\tilde{E} = 1$ , while for very heavy weights ( $n \rightarrow \infty$ ) the ratio  $E_2/\tilde{E}$  tends asymptotically to 0.625.

Finally, it should be recalled that if the weighted swimming experiments take place in round tanks, the centrifugal corrections of sections 2 and 3 should also be applied.

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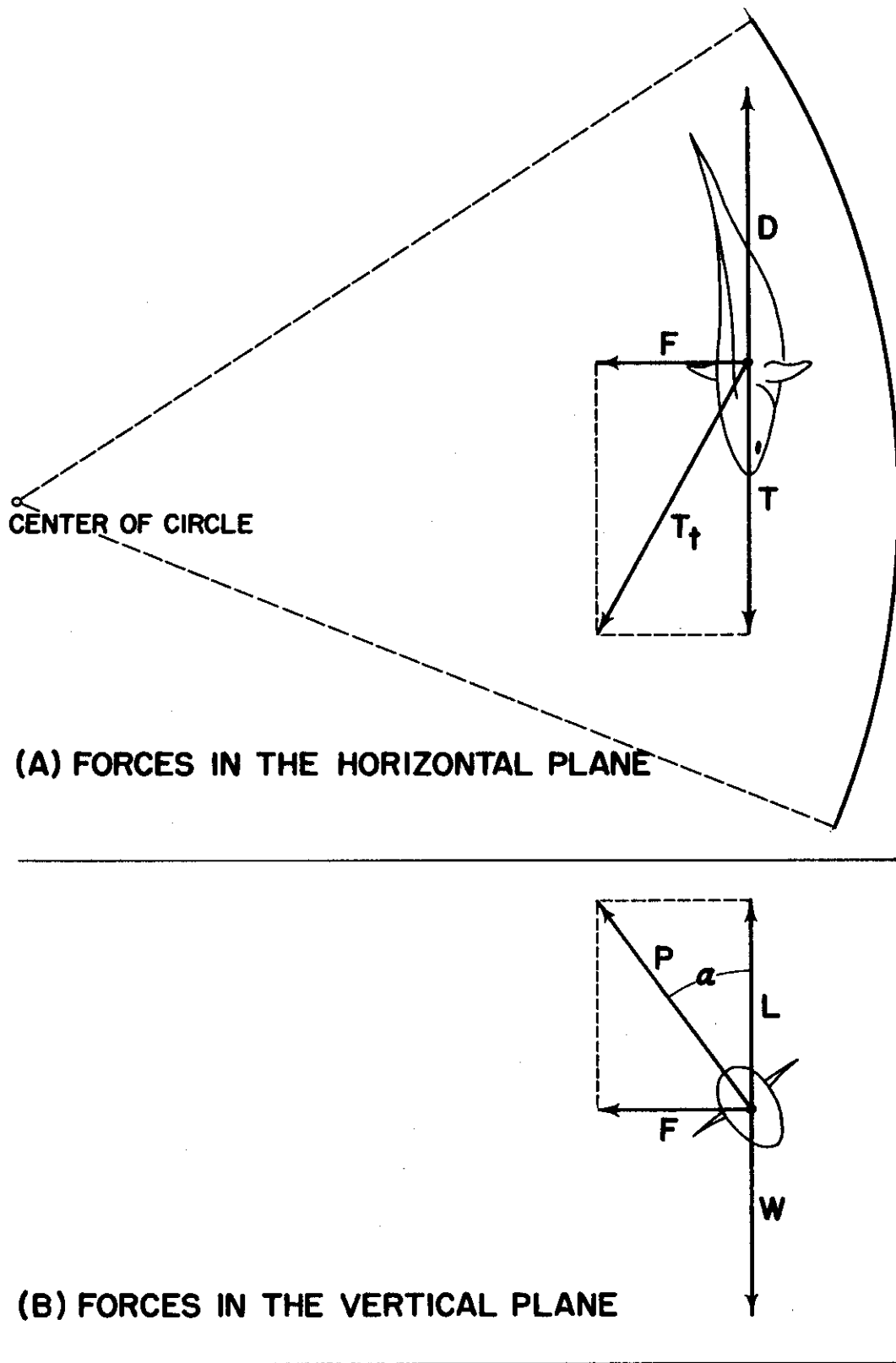


Figure 1. Schematic description of forces acting on the fish when swimming in a curved path. A. Forces in the horizontal plane. B. Forces in the vertical plane.



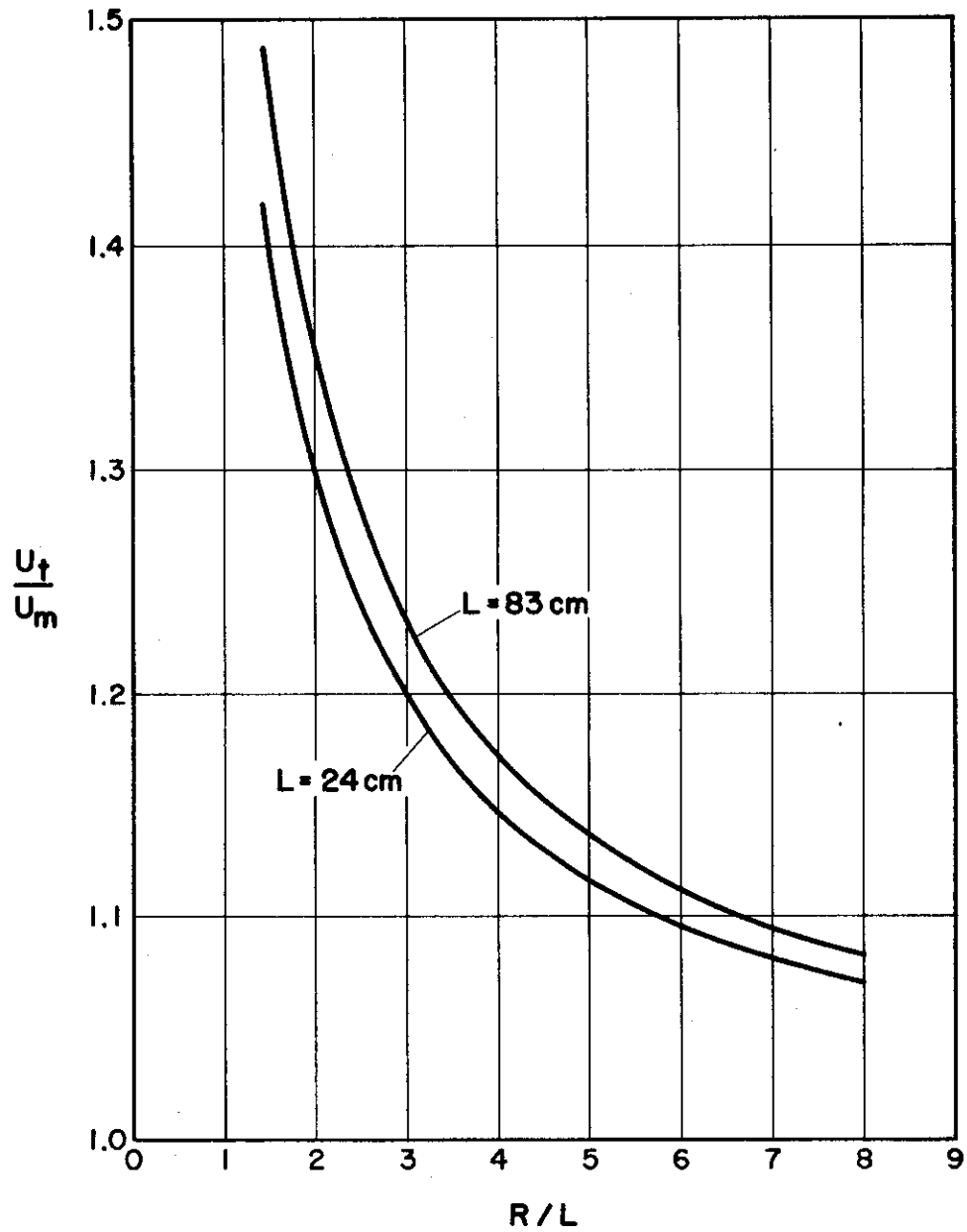


Figure 2. Ratio of minimum speed when moving in a circular path, to the minimum speed in straight-line swimming versus the ratio of turning radius to fish length, for skipjack tuna, Katsuwonus pelamis.

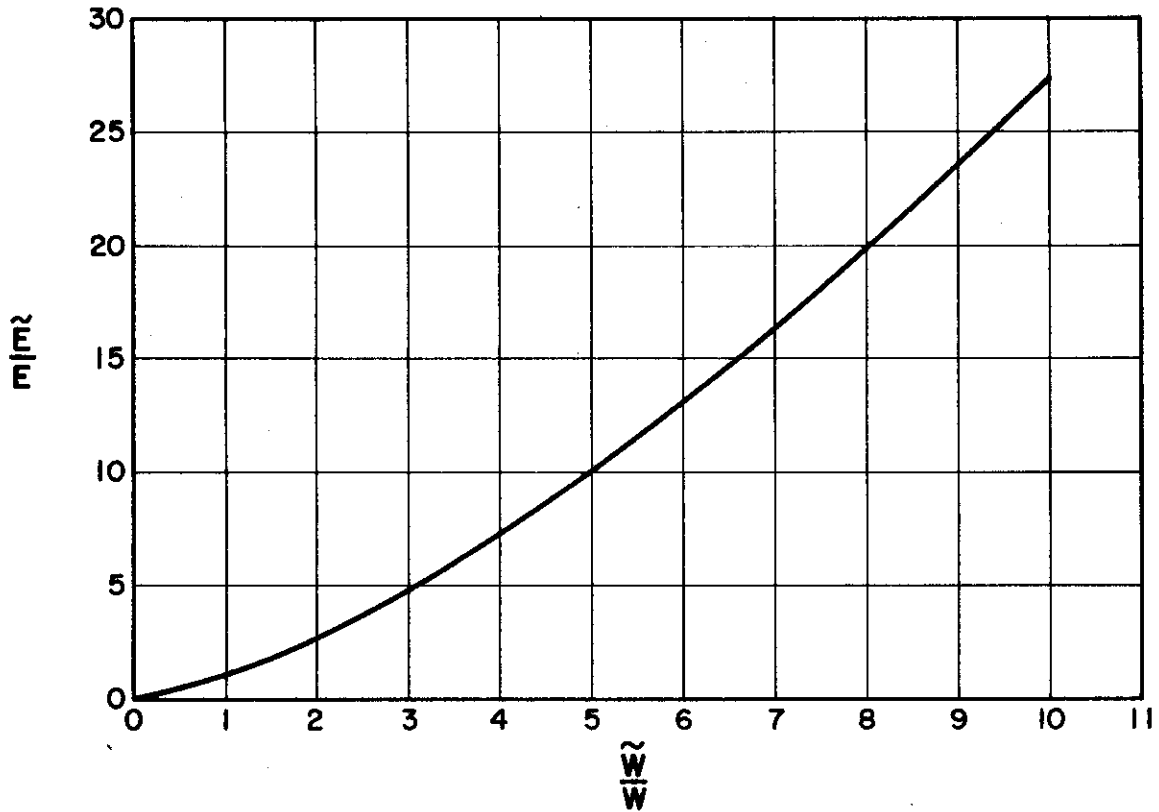


Figure 3.--Ratio of energy (= oxygen) consumption rates for a fish with added weights, to that of the same fish when unweighted versus the ratio of submerged weights of the fish with and without the added weights. It is assumed that the fish is moving at minimum speed in both cases.

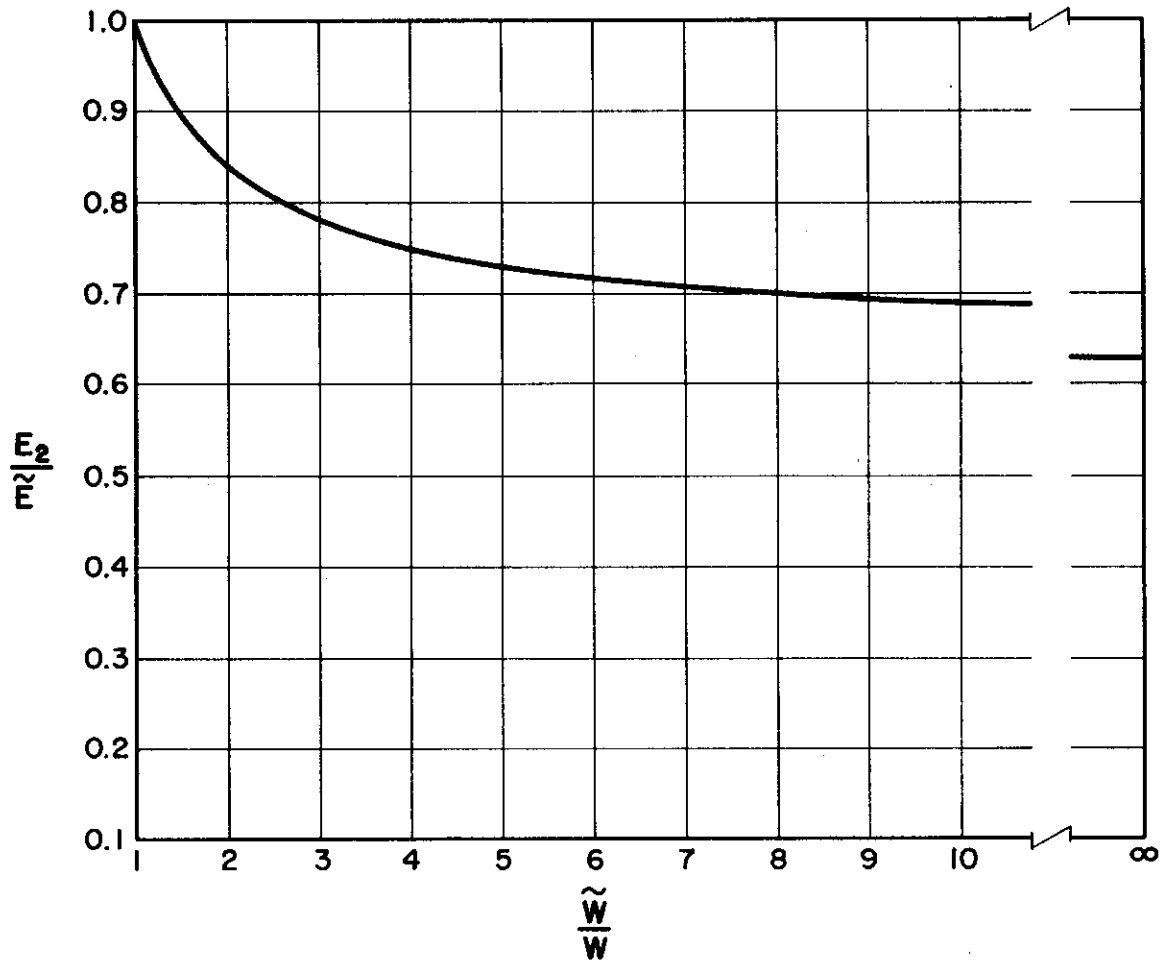


Figure 4. Ratio of energy (= oxygen) consumption rates for an unweighted fish to the rate for the fish moving at the same speed, when weighted, versus the ratio of submerged weights of the fish with and without the added weights.