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Development of Gear Technology and Theory of Gearing

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Foreword

Aeronautics, a principal research area for NASA from its inception as NACA, has also been the primary focus for the NASA Lewis Research Center since the 1940's. In the 1970's, the Army established drive systems technology as part of its research to support its ever increasing use of helicopters. This effort has continued to the present and is now a shared activity of the Army and NASA because of the need for civilian applications.

NASA and Army goals for this research are similar and emphasize reduced weight, noise, and cost with increased safety, life, and reliability. Meeting these competing goals requires that improvements be made in the way components are designed and manufactured. These goals are being met by in-house programs, contracts, and university grants, through which the talents of world-class university researchers have an impact on aerospace technology and products of the future.

Dr. Faydor L. Litvin, one of the innovators in the gear geometry field for many decades, best exemplifies the effective use of NASA funds for research. Dr. Litvin's methods and theories have been the catalyst to change the design and manufacture of gears so as to achieve major operational improvements in helicopter gear systems. He effected these improvements by applying principles based on the geometry of meshing gear surfaces to correct many problems associated with alignment and manufacturing errors that shift bearing contact and cause transmission errors.

This book presents recent developments in the theory of gearing and the modifications in gear geometry necessary to improve the conditions of meshing. Highlighted are low-noise gear drives that have a stable contact during meshing and a predesigned parabolic transmission error function that can handle misalignment during operation without sacrificing the low-noise aspects of operation.

This book also provides a comprehensive history of the development of the theory of gearing through biographies of major contributors to this field. The author's unique historical perspective was achieved by assiduous research into the lives of courageous, talented, and creative men who made significant contributions to the field of gearing. Very often they came from humble backgrounds, sought an education in the face of great obstacles, made personal sacrifices to attain goals, and worked hard for many years to fulfill their creative aspirations. The task of accumulating information about these men was extremely difficult, for many were deceased and facts existed only in family records, library archives, and their companies' files. Perhaps the most significant indication of this difficult task is the collection of portraits, many of which were obtained from family albums held by descendents many generations later. This collection is unique and exists nowhere else.

I believe that this book will be useful to students, engineers, and researchers who work in gearing and that it will help them to appreciate the genius of those who were pioneers in this field.

John J. Coy
NASA Lewis Research Center

Preface

There yet remains but one concluding tale
And then this chronicle of mine is ended. . .—A. S. Pushkin, "Boris Godunov," p. 19

This book consists of three chapters: chapter 1 presents the developments in the theory of gearing; chapter 2, the geometry and technology of gears; and chapter 3, the biographies of the inventors, scientists, and founders of gear companies. The goal of the third chapter is to credit the contributions made by our predecessors and to combine the separate historical pieces of the development of gear technology and gearing theory.

Although the author has tried to be objective in judging the history of the development of gear technique, it is not a certainty that he has achieved his desire. If this happened, it was not deliberate.

The history of gear development was the subject of research by H.-Chr. Graf v. Seherr-Thoss (1965), Darle W. Dudley (1969), and Dr. Hermann J. Stadtfeld (1993). In the present book, the author tried to complement these previous publications with new materials and hopes that the reader will find it enlightening.

The history of developments in any area, including gear technology and theory, is the history of creativity, which has often gone unrecognized during one's lifetime. The aspiration to create is a passion that enriches the life but requires unconditional devotion. Usually, creativity is associated with the arts (music, literature, painting), possibly because they have the greatest influence on our emotions. However, we do not realize the extent to which this passion conquers the daily activities of many in all levels of society. The desire of gifted persons to create is the driving force in their lives, bringing them joy and suffering and often no fame. For, Fame, a capricious goddess, does not award in the proper time and may not award at all.

My sympathy is for those who failed to achieve recognition for their accomplishments, and I share Dostoyevsky's philosophy that suffering is necessary for spiritual achievement, but the price to be paid is sometimes too high. However, an individual who gives his heart to create should not look for fame. This was expressed with great emotion by Pasternak (1960) in his famous verse "To Be That Famous Is Hardly Handsome":

Creation's aim—yourself to give,
Not loud success, appreciation.
To mean round nothing—shames to live,
On all men's lips an empty sermon.

I sympathize with the heroes of Pasternak's verse.

Although J. Henry Fabre's area of activity was different from the author's, he is an example of someone whose life deeply touched the author. Fabre (1949) was a famous entomologist who exemplified the unconditional devotion that creativity required. He was very poor, and struggled to support his family. However, every day he was in the field to observe the social life and habits of insects—the subject of his research. He made a modest confession that all he wanted was "a bit of land, oh, not so very large, but fenced in" where he could study insects. Fabre puzzled not only his neighbors but also his contemporaries by spending his time studying insects, but through his devotion to this subject, he became the founder of entomology. Unfortunately, Fabre became known only at the end of his long life (1823 to 1915), and he received a pension for only his last 5 years.

Another example is the Italian mechanic and clockmaker Juanelo Torriano (1501 to 1575), the inventor of the first known gear cutting machine (Dudley, 1969). He spent 20 years on this project and paid a high price for his intensive work. He was sick twice and almost died before he successfully created his manually operated machine. Maybe his reward was self-satisfaction.

The theory and technology of gearing is a narrow branch of science, but the author believes that what was said about the creative process holds true here. Who knows how many sleepless nights an inventor, a scientist, or the founder of a new gear company had? How often did they ponder Hamlet's question "To be or not to be?"

Recognizing the achievements of one's predecessors is the best way to honor their work. This is one of the goals of this book written in memory of the founders of the gear industry and gear technology and theory.

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Contents

Chapter 1

Development of Theory of Gearing

1.1 Introduction

The theory of gearing is the branch of science related to differential geometry, manufacturing, design, metrology, and computerized methods of investigation. The first developers of the theory of gearing (Olivier and Gochman) related it to projective and analytical geometry. Later, with the development of gear technology and the application of computers to gearing, researchers modified it to the modern theory of gearing and extended its methodology and industrial applications.

One of the most important problems the theory of gearing considers is the conjugation of profiles in planar gearing and surfaces in spatial gearing. De la Hire, Poncelet and Camus (Seherr-Thoss, 1965) deserve credit for developing cycloidal gearing. Euler (1781), in addition to his tremendous contribution to mathematics and mechanics, proposed involute gearing that was later found to have a broad application in industry (fig. 1.1.1). The Swiss government issued currency with Euler's picture and an involute curve to celebrate his achievements. Olivier (1842) and Gochman (1886) developed the basic ideas underlying the conjugation of gear tooth surfaces and their generation. N.I. Kolchin (1949) applied Gochman's ideas to develop the geometry of modern gear drives.

Willis, Buckingham, Wildhaber, and Dudley are very well-known names in English-speaking countries. Willis (1841) proposed the law of meshing of planar curves. Buckingham's book (1963) became a well-known reference on gears. Wildhaber, an inventor who held many patents, developed the theory of hypoid gear drives and the Revacycle method (Wildhaber, 1926, 1946a, 1946b, 1946c, 1956). Dudley (1943, 1954, 1961, 1962, 1969, 1984, 1994) was the chief editor of the first edition of the *Gear Handbook,* the foremost work on gears at that time. He became a prominent gear expert and well known for his contributions to gear technology.

Figure 1.1.1.—Leonhard Euler.

[illegible] In the 1940's and 1950's Profs. Chrisanf F. Ketov

N. I. Kolchin, V.A. Gavrilenko, and others supervised the research activities of many graduate students. The author, one of these students, remembers with gratitude and respect his advisor, Prof. Ketov. A few of the many distinguished Russian researchers from this generation are V.N. Kudriavtsev, G.I. Sheveleva, L.V. Korostelev, Ya. S. Davidov, and M.L. Erikhov. Several research centers in Western countries have contributed to the areas of gear geometry, technology, and dynamics:

(1) The Gleason Works in Rochester, New York; the NASA Lewis Research Center in Cleveland, Ohio; The Ohio State University in Columbus, Ohio; the University of Illinois in Chicago, Illinois (United States)
(2) The Universities of Munich, Aachen, Stuttgart, and Dresden (Germany)
(3) Institute de L'Engrenage et des Transmissions and Cetim (France)
(4) The Universities of Laval and Alberta (Canada)

Important contributions to the theory of gearing have been provided by

(1) E. Buckingham (1963), E. Wildhaber (1926, 1946, 1956), D. Dudley (1943, 1954, 1961, 1962, 1969, 1984, 1991), M. Baxter (1961, 1973), T. Krenzer (1981), A. Seireg (1969), G. Michalek (1966), and Y. Gutman from the United States
(2) G. Niemann (1953), G.R. Brandner (1983, 1988), H. Winter, B. Hohn, M. Weck, and G. Bär (1991, 1997) from Germany
(3) H. Stadtfeld (1993, 1995) formerly of Switzerland but now in the United States
(4) G. Henriotte and M. Octrue from France
(5) C. Gosselin (1995) and J.R. Colbourne (1974, 1985) from Canada

This chapter presents the latest developments in the theory of gearing. These resulted from the work of the author and his fellow researchers at the University of Illinois Gear Research Laboratory in Chicago.

We are on the eve of the 21st century and can expect that technology such as the CNC (computer numerically controlled) and CCM (computer coordinate measurement) machines will substantially change existing gear geometry and gear technology.

1.2 Equation of Meshing

The generation of a gear tooth surface by the tool surface (the surface of a head-cutter, hob, shaper, rack-cutter, etc.) and the conjugation of gear tooth surfaces in line contact are based on the concept of the envelope to a family of surfaces (curves in two-dimensional space in the case of planar gearing). This topic is related to differential geometry and to the theory of gearing. Zalgaller's book (1975) significantly contributes to the theory of envelopes and covers the necessary and sufficient conditions for the envelope's existence. Simplified approaches to the solution of these problems have also been developed in the theory of gearing (Litvin, 1968, where ϕ is the generalized parameter of motion. When ϕ is fixed, equation (1.2.2) represents the surface Σ_1 in the S_2 coordinate system.

Envelope Σ_2 is in tangency with all surfaces of the family of surfaces (eq. (1.2.2)). Surface Σ_2 can be determined if vector function $\mathbf{r}_2(u,\theta,\phi)$ of the family of surfaces will be complemented by

$$f(u,\theta,\phi)=0 \tag{1.2.3}$$

In 1952 in the theory of gearing, equation (1.2.3) had received the term "equation of meshing" (Litvin, 1952). Several alternative approaches have since been proposed for the derivation of equation (1.2.3).

Approach 1. The method of deriving the equation of meshing was proposed in differential geometry and is based on the following considerations:

(1) Assume that equation (1.2.3) is satisfied with a set of parameters $P(u^\circ, \theta^\circ, \phi^\circ)$. It is given that $f \in C^1$ and that one of the three derivatives (f_u, f_θ, f_ϕ), say f_u, is not equal to zero. Then, in accordance with the theorem of implicit function system existence (Korn and Korn, 1968), equation (1.2.3) can be solved in the neighborhood of $P=(u^\circ,\theta^\circ,\phi^\circ)$ by function $u(\theta,\phi)$.

(2) Consider now that surface Σ_2 may be represented by vector function $\mathbf{r}_2(\theta,\phi,u(\theta,\phi))$ and that the tangents to Σ_2 may be represented as

$$\mathbf{T}_2=\frac{\partial \mathbf{r}_2}{\partial \theta}+\frac{\partial \mathbf{r}_2}{\partial u}\frac{\partial u}{\partial \theta}, \quad \mathbf{T}_2^*=\frac{\partial \mathbf{r}_2}{\partial \phi}+\frac{\partial \mathbf{r}_2}{\partial u}\frac{\partial u}{\partial \phi} \tag{1.2.4}$$

(3) The normal $\mathbf{N}_2^{(1)}$ to surface Σ_1 that is represented in S_2 is determined as

$$\mathbf{N}_2^{(1)}=\frac{\partial \mathbf{r}_2}{\partial u}\times\frac{\partial \mathbf{r}_2}{\partial \theta} \tag{1.2.5}$$

The subscript 2 in the designation $\mathbf{N}_2^{(1)}$ indicates that the normal is represented in S_2; the superscript (1) indicates that the normal to Σ_1 is considered.

(4) If the envelope Σ_2 exists, it is in tangency with Σ_1, and Σ_1 and Σ_2 must have a common tangent plane. A tangent plane $\Pi_2^{(2)}$ to Σ_2 is determined by the couple of vectors $\mathbf{T}_2$ and $\mathbf{T}_2^*$. A tangent plane $\Pi_2^{(1)}$ to Σ_1 is determined by the couple of vectors $\partial \mathbf{r}_2/\partial u$ and $\partial \mathbf{r}_2/\partial \theta$. Vector $\mathbf{T}_2$ lies in plane $\Pi_2^{(1)}$ already. Surfaces Σ_2 and Σ_1 will have a common tangent plane if vector $\partial \mathbf{r}_2/\partial \phi$ also lies in $\Pi_2^{(1)}$. The requirement that vectors $(\partial \mathbf{r}_2/\partial u, \partial \mathbf{r}_2/\partial \theta$ and $\partial \mathbf{r}_2/\partial \phi)$ belong to the same plane $\left(\Pi_2^{(1)}\right)$ is represented by

$$\left(\frac{\partial \mathbf{r}_2}{\partial u}\times\frac{\partial \mathbf{r}_2}{\partial \theta}\right)\cdot\frac{\partial \mathbf{r}_2}{\partial \phi}=f(u,\theta,\phi)=0 \tag{1.2.6}$$

Remember that (1.2.6) is the equation of meshing and that equations (1.2.2) and (1.2.6) considered simultaneously represent surface Σ_2, the envelope to the family of surfaces Σ_1.

Approach 2. This approach is based on the following considerations:

(1) The cross product in equation (1.2.6) represents in S_2 the normal to Σ_1 (see eq. (1.2.5)).

(2) The derivative $\partial \mathbf{r}_2/\partial \phi$ is collinear to the vector of the relative velocity $\mathbf{v}_2^{(12)}$, which is the velocity of a point of Σ_1 with respect to the coinciding point of Σ_2. This means that equation (1.2.6) yields

$$\mathbf{N}_2^{(1)}\cdot\mathbf{v}_2^{(12)}=f(u,\theta,\phi)=0 \tag{1.2.7}$$

(3) The scalar product in (1.2.7) is invariant to the coordinate systems S_1, S_f, and S_2. Thus

$$\mathbf{N}_\mathrm{i}^{(1)}\cdot\mathbf{v}_\mathrm{i}^{(12)}=f(u,\theta,\phi)=0 \quad (\mathbf{i}=1,f,2) \tag{1.2.8}$$

The derivation of the equation of meshing becomes more simple if $i = 1$, or $i = f$.

Equation (1.2.8) was proposed almost simultaneously by Dudley and Poritsky, Davidov, Litvin, Shishkov, and Saari. Litvin has proven that equation (1.2.8) is the necessary condition for the envelope's existence (Litvin, 1952 and 1989).

The determination of the relative velocity $\mathbf{v}^{(12)}$ can be accomplished using well-known operations applied in kinematics (see appendix A). In the case of the transformation of rotation between crossed axes, an alternative approach for determining $\mathbf{v}^{(12)}$ may be based on the application of the concept of the axis of screw motion (appendix B).

In the case of planar gearing, the derivation of the equation of meshing may be represented as

$$\mathbf{T}_i^{(1)} \times \mathbf{v}_i^{(12)} = \mathbf{0} \tag{1.2.9}$$

or

$$\left(\mathbf{T}_i^{(1)} \times \mathbf{v}_i^{(12)}\right) \cdot \mathbf{k}_i = 0, \quad (i = 1, 2, f) \tag{1.2.10}$$

where, $\mathbf{T}_i^{(1)}$ is the tangent to the generating curve, $\mathbf{v}_i^{(12)}$ is the sliding velocity, $\mathbf{k}_i$ is the unit vector of the z_i-axis (assuming that the planar curves are represented in plane (x_i, y_i)).

In the cases of planar gearing and gearing with intersected axes, the normal to the generating curve (surface) at the current point of tangency of the curves (surfaces) passes through (1) the instantaneous center of rotation for planar gearing (first proposed by Willis (1941)), and (2) the instantaneous axis of rotation for gears with intersected axes. The derivation of the equation of meshing for gearing with intersected axes is based on

$$\frac{X_i - x_i}{\mathbf{N}_{x_i}^{(1)}} = \frac{Y_i - y_i}{\mathbf{N}_{y_i}^{(1)}} = \frac{Z_i - z_i}{\mathbf{N}_{z_i}^{(1)}}, \quad (i = 1, 2, f) \tag{1.2.11}$$

where (X_i, Y_i, Z_i) are the coordinates of a current point of the instantaneous axis of rotation; (x_i, y_i, z_i) are the coordinates of a current point of the generating (driving) surface; $\mathbf{N}_{x_i}^{(1)}, \mathbf{N}_{y_i}^{(1)}, \mathbf{N}_{z_i}^{(1)}$ are the projections of the normal to surface Σ_1.

1.3 Basic Kinematic Relations

Basic kinematic relations proposed in Litvin (1968 and 1989) relate the velocities (infinitesimal displacements) of the contact point and the contact normal for a pair of gears in mesh.

The velocity of a contact point is represented as the sum of two components: in the motions *with* and *over* the contacting surface, respectively. Using the condition of continuous tangency of the surfaces in mesh, we obtain

$$\mathbf{v}_r^{(2)} = \mathbf{v}_r^{(1)} + \mathbf{v}^{(12)} \tag{1.3.1}$$

where $\mathbf{v}_r^{(i)}$ $(i = 1,2)$ is the velocity of a contact point in the motion over surface Σ_i. Similarly, we can represent the relation between the velocities of the tip of the contact normal

$$\dot{\mathbf{n}}_r^{(2)} = \dot{\mathbf{n}}_r^{(1)} + \left(\boldsymbol{\omega}^{(12)} \times \mathbf{n}\right) \tag{1.3.2}$$

where, $\dot{\mathbf{n}}_r^{(i)}$ $(i = 1,2)$ is the velocity of the tip of the contact normal in the motion over the surface (in addition to the translational velocity of the unit normal $\mathbf{n}$ that does not affect the orientation of $\mathbf{n}$), and $\boldsymbol{\omega}^{(12)}$ is the relative angular velocity of gear 1 with respect to gear 2.

The advantage in using equations (1.3.1) and (1.3.2) is that they enable the determination of $\mathbf{v}_r^{(2)}$ and $\dot{\mathbf{n}}_r^{(2)}$ without having to use the complex equations of the generated surface Σ_2.

By applying equations (1.3.1) and (1.3.2) for the solutions of the following most important problems in the theory of gearing, the application of the complex equations of Σ_2 has been avoided:

Problem 1: Avoidance of singularities of the generated gear tooth surface Σ_2
Problem 2: Determination of the principal curvatures, the normal curvatures, and the surface torsions of Σ_2
Problem 3: Determination of the dimensions and the orientation of the instantaneous contact ellipse

1.4 Detection and Avoidance of Singularities of Generated Surface

Generating tool surface Σ_1 is already free from singularities because the inequality $(\partial \mathbf{r}_1/\partial u) \times (\partial \mathbf{r}_1/\partial \theta) \neq 0$ was observed. Tool surface Σ_1 may generate surface Σ_2, which will contain not only regular surface points but also singular ones. The appearance of singular points on Σ_2 is a warning of the possible undercutting of Σ_2 in the generation process.

The discovery of singular points on Σ_2 may be based on the theorem proposed in Litvin (1968): a singular point M on surface Σ_2 occurs if at M the following equation is observed:

$$\mathbf{v}_r^{(2)} = \mathbf{v}_r^{(1)} + \mathbf{v}^{(12)} = \mathbf{0} \tag{1.4.1}$$

Equation (1.4.1) and the differentiated equation of meshing

$$\frac{\mathrm{d}}{\mathrm{d}t}[f(u,\theta,\psi)] = 0 \tag{1.4.2}$$

yield

$$\frac{\partial \mathbf{r}_1}{\partial u}\frac{\mathrm{d}u}{\mathrm{d}t} + \frac{\partial \mathbf{r}_1}{\partial \theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\mathbf{v}_1^{(12)} \tag{1.4.3}$$

$$\frac{\partial f}{\partial u}\frac{\mathrm{d}u}{\mathrm{d}t} + \frac{\partial f}{\partial \theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\frac{\partial f}{\partial \phi}\frac{\mathrm{d}\phi}{\mathrm{d}t} \tag{1.4.4}$$

Taking into the account that Σ_1 is a regular surface, we may transform (1.4.3) as

$$\left(\frac{\partial \mathbf{r}_1}{\partial u}\right)^2 \frac{\mathrm{d}u}{\mathrm{d}t} + \left(\frac{\partial \mathbf{r}_1}{\partial u}\right)\cdot\left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)\frac{\mathrm{d}\theta}{\mathrm{d}t} = -\left(\frac{\partial \mathbf{r}_1}{\partial u}\right)\cdot\left(\mathbf{v}_1^{(12)}\right) \tag{1.4.5}$$

$$\left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)\cdot\left(\frac{\partial \mathbf{r}_1}{\partial u}\right)\frac{\mathrm{d}u}{\mathrm{d}t} + \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)^2 \frac{\mathrm{d}\theta}{\mathrm{d}t} = -\left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)\cdot\left(\mathbf{v}_1^{(12)}\right) \tag{1.4.6}$$

Equations (1.4.4) to (1.4.6) form an overdetermined system of three linear equations in two unknowns: $\mathrm{d}u/\mathrm{d}t$, $\mathrm{d}\theta/\mathrm{d}t$. The rank of the system matrix is $r = 2$, which yields

$$g_1(u,\theta,\phi) = \begin{vmatrix} f_u & f_\theta & f_\phi \\ \left(\frac{\partial \mathbf{r}_1}{\partial u}\right)^2 & \left(\frac{\partial \mathbf{r}_1}{\partial u}\right)\cdot\left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right) & \left(\frac{\partial \mathbf{r}_1}{\partial u}\right)\cdot\left(\mathbf{v}^{(12)}\right) \\ \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)\cdot\left(\frac{\partial \mathbf{r}_1}{\partial u}\right) & \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)^2 & \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)\cdot\left(\mathbf{v}^{(12)}\right) \end{vmatrix} = 0 \tag{1.4.7}$$

We have assigned in $\mathbf{v}^{(12)}$ that $\mathrm{d}\phi/\mathrm{d}t = 1$ rad/sec.

Equations $f = 0$ and $g_1 = 0$ allow us to determine on Σ_1 curve L_1, which is formed by surface "regular" points that generate "singular" points on Σ_2. Using the coordinate transformation from S_1 to S_2, we may determine curve $\mathbf{L}_2$, which is formed by singular points on surface Σ_2. To avoid the undercutting of Σ_2, it is sufficient to limit Σ_2 and to eliminate $\mathbf{L}_2$ while designing the gear drive.

It is easy to verify that the theorem proposed above yields a surface normal $\mathbf{N}_2^{(2)}$ equal to zero at the surface Σ_2 singular point. The derivations procedure follows:

Step 1: Equation (1.4.7) means that

$$g_1(u,\theta,\phi)=\mathbf{N}_1^{(2)}\cdot\left(\frac{\partial\mathbf{r}_1}{\partial u}\times\frac{\partial\mathbf{r}_1}{\partial\theta}\right) \tag{1.4.8}$$

where

$$\mathbf{N}_1^{(2)}=\left(\frac{\partial\mathbf{r}_1}{\partial\theta}\times\mathbf{v}^{(12)}\right)f_u+\left(\mathbf{v}^{12}\times\frac{\partial\mathbf{r}_1}{\partial u}\right)f_\theta+\left(\frac{\partial\mathbf{r}_1}{\partial u}\times\frac{\partial\mathbf{r}_1}{\partial\theta}\right)f_\phi \tag{1.4.9}$$

Step 2: Taking into account that the generating surface Σ_1 performs its relative motion as a rigid body (Σ_1 is not changed during such a motion), we may represent in coordinate system S_2 the normal to surface Σ_2 as

$$\mathbf{N}_2^{(2)}=\left(\frac{\partial\mathbf{r}_2}{\partial\theta}\times\frac{\partial\mathbf{r}_2}{\partial\phi}\right)f_u+\left(\frac{\partial\mathbf{r}_2}{\partial\phi}\times\frac{\partial\mathbf{r}_2}{\partial u}\right)f_\theta+\left(\frac{\partial\mathbf{r}_2}{\partial u}\times\frac{\partial\mathbf{r}_2}{\partial\theta}\right)f_\phi \tag{1.4.10}$$

Step 3: Equation $g_1=0$ yields $\mathbf{N}_1^{(2)}=0$, since $\left(\frac{\partial\mathbf{r}_1}{\partial u}\times\frac{\partial\mathbf{r}_1}{\partial\theta}\right)\neq 0$ (see eq. (1.4.8)). Respectively, we obtain $\mathbf{N}_2^{(2)}=0$. This means that equation (1.4.10) also yields a normal to surface Σ_2 that is equal to zero.

Step 4: We may easily verify that the surface Σ_2 singularity equation can be represented in terms of Σ_2:

$$g_2(u,\theta,\phi)=\begin{vmatrix} f_u & f_\theta & f_\phi \\ \left(\frac{\partial\mathbf{r}_2}{\partial u}\right)^2 & \left(\frac{\partial\mathbf{r}_2}{\partial u}\right)\cdot\left(\frac{\partial\mathbf{r}_2}{\partial\theta}\right) & \left(\frac{\partial\mathbf{r}_2}{\partial u}\right)\cdot\left(\frac{\partial\mathbf{r}_2}{\partial\theta}\right) \\ \left(\frac{\partial\mathbf{r}_2}{\partial\theta}\right)\cdot\left(\frac{\partial\mathbf{r}_2}{\partial u}\right) & \left(\frac{\partial\mathbf{r}_2}{\partial\theta}\right)^2 & \left(\frac{\partial\mathbf{r}_2}{\partial\theta}\right)\cdot\left(\frac{\partial\mathbf{r}_2}{\partial\phi}\right) \end{vmatrix}=0 \tag{1.4.11}$$

However, equation (1.4.7) is much simpler and therefore preferable.

We illustrate the phenomenon of undercutting with the generation of a spur involute gear by a rack-cutter (figs. 1.4.1 to 1.4.3). The rack-cutter (fig. 1.4.1) consists of a straight line 1 that generates the involute curve of the gear, a straight line 2 that generates the dedendum circle of the gear, and the rack fillet that generates the gear fillet.

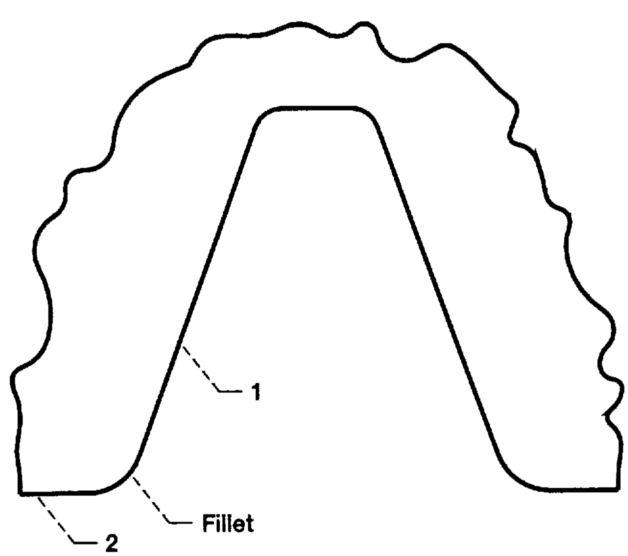


Figure 1.4.1.—Profiles of rack-cutter tooth.

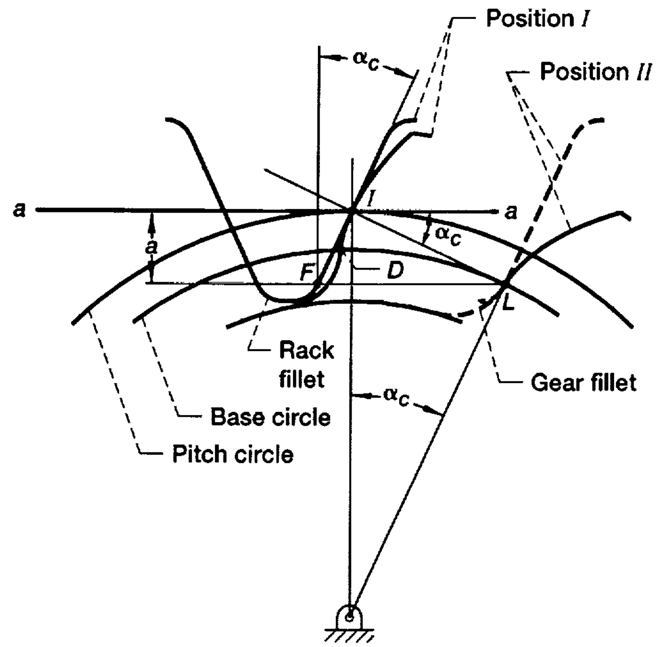


Figure 1.4.2.—Determination of limiting setting of a rack-cutter.

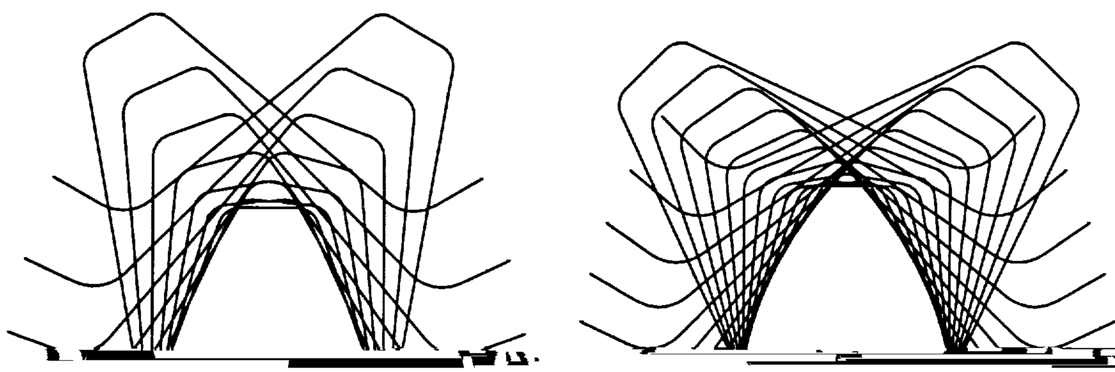


Figure 1.4.3(b) shows that the fillet of the rack-cutter has undercut the gear involute shape: the gear fillet and involute shape are no more in tangency but intersect each other. The undercutting occurred as a result of the magnitude of parameter setting a being too large.

Examples of the application of equation (1.4.7) in order to avoid undercutting are presented in Litvin (1989 and 1994).

1.5 Direct Relations Between Curvatures of Meshing Surfaces: Instantaneous Contact Ellipse

The solutions to these related problems are based on the application of the proposed kinematic relations discussed in section 1.3.

Direct relations between the curvatures of meshing gears are necessary for the local synthesis of gear tooth surfaces, the determination of an instantaneous contact ellipse, and other problems. The main difficulty in solving such problems is that the generated surface Σ_2 is represented by three related parameters, therefore making the determination of the curvatures of Σ_2 a complex problem. The approach proposed by Litvin (1968, 1969, 1994) enables one to overcome this difficulty because the curvatures of Σ_2 are expressed in terms of the curvatures of generating surface Σ_1 and the parameters of motion. Using this approach makes it possible to determine the direct relations between the principal curvatures and the directions of Σ_1 and Σ_2. An extension of this approach has enabled one to detemine the relations between normal curvatures of surfaces Σ_1 and Σ_2 and the torsions (Litvin, Chen, and Chen, 1995). The solution is based on the application of equations (1.3.1) and (1.3.2) and on the differentiated equation of meshing.

The developed equations allow one to determine the relationship between the curvatures of surfaces in line contact and in point contact.

The possibility of the interference of surfaces in point contact may be investigated as follows. Consider that two gear tooth surfaces designated Σ_p and Σ_g are in point tangency at point M. The principal curvatures and directions of Σ_p and Σ_g are known. The interference of surfaces in the neighborhood of point M will not occur if the relative normal curvature $\kappa_n^{(r)}$ does not change its sign in any direction in the tangent plane. Here,

$$\kappa_n^{(r)} = \kappa_n^{(p)} - \kappa_n^{(g)} \tag{1.5.1}$$

We are reminded that the normal curvature of a surface can be represented in terms of the principal curvatures in Euler's equation.

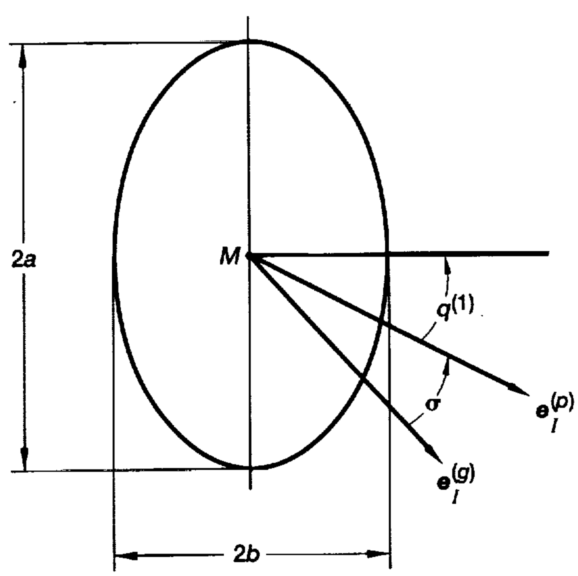


Figure 1.5.1.—Instantaneous contact ellipse.

The approach presented in Litvin (1994) and in Litvin, Chen, and Chen (1995) enables one to determine the dimensions of the contact ellipse and its orientation by knowing the principal curvatures and directions of the contacting surfaces and the elastic deformation of the tooth surfaces. An alternative approach is based on the application of the surface normal curvatures, their torsions, and the tooth elastic approach (Litvin, Chen, and Chen, 1995).

Figure 1.5.1 shows the instantaneous contact ellipse and its center of symmetry, which coincides with the point of tangency M; the unit vectors $\mathbf{e}_f^{(p)}$ and $\mathbf{e}_f^{(g)}$ of the respective principal directions; the major and minor axes of the contact ellipse $2a$ and $2b$; angle $q^{(1)}$, which determines the orientation of the contact ellipse with respect to the unit vector $\mathbf{e}_f^{(p)}$; angle σ, which is formed by unit vectors $\mathbf{e}_f^{(p)}$ and $\mathbf{e}_f^{(g)}$.

1.6 Sufficient Conditions for Existence of Envelope to Family of Surfaces

Sufficient Conditions for Existence of Envelope to Family of Surfaces Represented in Parametric Form

The sufficient conditions for the existence of an envelope to a family of surfaces guarantee that the envelope *indeed exists*, that it be in *tangency* with the surfaces of the family, and that it be a *regular* surface. These conditions are represented by the following theorem proposed by Zalgaller (1975) and modified by Litvin (1968, 1994) for application in the theory of gearing.

Theorem. Given a regular generating surface Σ_1 represented in S_1 by

$$\mathbf{r}_1(u,\theta) \in C^2, \quad \frac{\partial \mathbf{r}_1}{\partial u} \times \frac{\partial \mathbf{r}_1}{\partial \theta} \neq 0, \quad (u,\theta) \in E \tag{1.6.1}$$

The family Σ_ϕ of surfaces Σ_1 generated in S_2 is represented by $\mathbf{r}_2(u, \theta, \phi)$, $a < \phi < b$.

Suppose that at a point $M\ (u_0, \theta_0, \phi_0)$, the following conditions are observed:

$$\left(\frac{\partial \mathbf{r}_2}{\partial u} \times \frac{\partial \mathbf{r}_2}{\partial \theta}\right) \cdot \frac{\partial \mathbf{r}_2}{\partial \phi} = f(u,\theta,\phi) = 0, \qquad f \in C^1 \tag{1.6.2}$$

or

$$\left(\frac{\partial \mathbf{r}_1}{\partial u} \times \frac{\partial \mathbf{r}_1}{\partial \theta}\right) \cdot \mathbf{v}^{(12)} = f(u,\theta,\phi) = 0 \tag{1.6.3}$$

$$f_u^2 + f_\theta^2 \neq 0 \tag{1.6.4}$$

$$g_1(u,\theta,\phi) = \begin{vmatrix} f_u & f_\theta & f_\phi \\ \left(\frac{\partial \mathbf{r}_1}{\partial u}\right)^2 & \left(\frac{\partial \mathbf{r}_1}{\partial u}\right) \cdot \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right) & \left(\frac{\partial \mathbf{r}_1}{\partial u}\right) \cdot \left(\mathbf{v}^{(12)}\right) \\ \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right) \cdot \left(\frac{\partial \mathbf{r}_1}{\partial u}\right) & \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)^2 & \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right) \cdot \left(\mathbf{v}^{(12)}\right) \end{vmatrix} \neq 0 \tag{1.6.5}$$

Then, the envelope to the family of surfaces Σ_1 exists in the neighborhood of point M and may be represented by

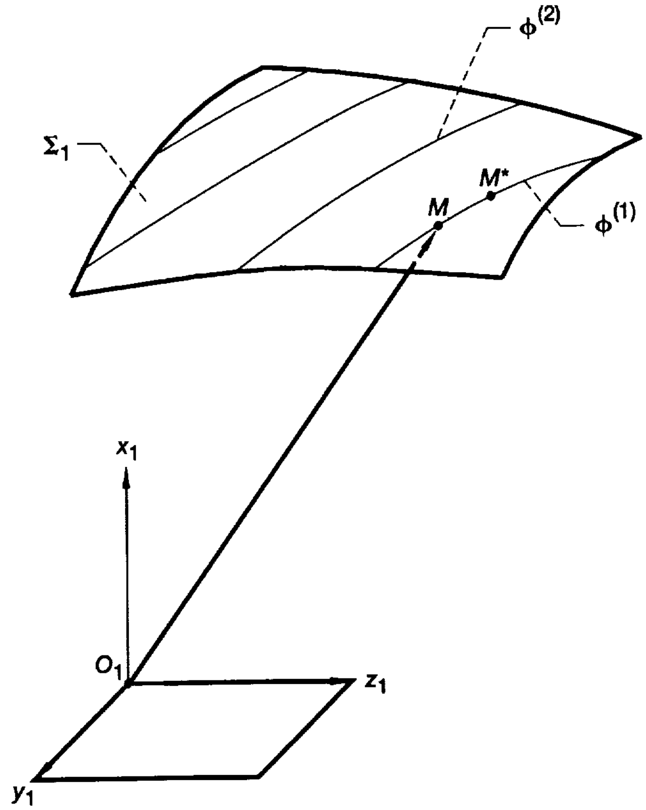


Figure 1.6.1.—Contact lines on tooth surface.

$$\mathbf{r}_2(u,\theta,\phi), \quad f(u,\theta,\phi)=0 \tag{1.6.6}$$

The contact lines of Σ_2 and Σ_1 are represented as

$$\mathbf{r}_2(u,\theta,\phi), \quad f(u,\theta,\phi)=0, \quad \phi=\text{Constant} \tag{1.6.7}$$

$$\mathbf{r}_1(u,\theta), \quad f(u,\theta,\phi)=0, \quad \phi=\text{Constant} \tag{1.6.8}$$

Figure 1.6.1 shows contact lines on surface Σ_1 determined by equations (1.6.8) while constant values of $\phi=(\phi^{(1)}, \phi^{(2)},\ldots)$ were taken.

The tangent to the contact line is represented in S_2 and S_1 by

$$\mathbf{T}_2=\frac{\partial \mathbf{r}_2}{\partial u}f_\theta-\frac{\partial \mathbf{r}_2}{\partial \theta}f_u \tag{1.6.9}$$

and

$$\mathbf{T}_1=\frac{\partial \mathbf{r}_1}{\partial u}f_\theta-\frac{\partial \mathbf{r}_1}{\partial \theta}f_u \tag{1.6.10}$$

The surface of action is the family of contact lines in the fixed coordinate system S_f that is rigidly connected to the frame. The surface of action is represented by

$$\mathbf{r}_f=\mathbf{r}_f(u,\theta,\phi), \quad f(u,\theta,\phi)=0 \tag{1.6.11}$$

where

$$\mathbf{r}_f(u,\theta,\phi) = \mathbf{M}_{f1}(\phi)\mathbf{r}_1(u,\theta) \quad (1.6.12)$$

The 4×4 matrix $\mathbf{M}_{f1}$ describes the coordinate transformation in transition from S_1 to S_f.

Sufficient Conditions for Existence of Envelope to Family of Surfaces Represented in Implicit Form

The family of generating surfaces in S_2 is

$$G(x,y,z,\phi) = 0, \quad G \in C^2, \quad (G_x)^2 + (G_y)^2 + (G_z)^2 \neq 0 \quad (1.6.13)$$

$$(x,y,z) \in A, \quad a < \phi < b$$

The theorem of sufficient conditions for the envelope existence proposed by Zalgaller (1975) states that at point M (u_0, θ_0, ϕ_0), the following requirements are observed:

$$G(x_0, y_0, z_0, \phi_0) = 0, \quad G_\phi = 0, \quad G_{\phi\phi} \neq 0,$$

$$\Delta = \left|\frac{D(G,G_\phi)}{D(x,y)}\right| + \left|\frac{D(G,G_\phi)}{D(x,z)}\right| + \left|\frac{D(G,G_\phi)}{D(y,z)}\right| \neq 0 \quad (1.6.14)$$

Thus, the envelope exists locally in the neighborhood of point M and is a regular surface:

$$G(x,y,z,\phi) = 0, \quad G_\phi(x,y,z,\phi) = 0 \quad (1.6.15)$$

The surface of action for the case just discussed can be represented by using equations similar to (1.6.11) and (1.6.12).

1.7 Envelope $\mathbf{E}_2$ to Contact Lines on Generated Surface Σ_2 and Edge of Regression

Sufficient Conditions for Existence of Envelope $\mathbf{E}_2$ to Contact Lines

Singular points on generated surface Σ_2 may form a curve that is the envelope (designated $\mathbf{E}_2$) to the family of contact lines (characteristics) on Σ_2. Envelope $\mathbf{E}_2$ is simultaneously the "edge of regression" of Σ_2, which means that envelope $\mathbf{E}_2$ is simultaneously the common line of two branches of Σ_2 determined by the same equation. If the conditions necessary for the existence of $\mathbf{E}_2$ as an envelope are not satisfied, singular points on generated surface Σ_2 just form an edge of regression.

Sufficient conditions for the existence of $\mathbf{E}_2$ to a family of contact lines on generated surface Σ_2 represented by an implicit function were determined by Favard (1957). In this work, it was proven that envelope $\mathbf{E}_2$ is also the edge of regression.

Our goal is to present the sufficient conditions for the existence of $\mathbf{E}_2$ to a family of contact lines on generated surface Σ_2 that is determined parametrically by three related parameters. In addition, we will show that $\mathbf{E}_2$, if it exists, is also the edge of regression.

Sufficient conditions for the existence of $\mathbf{E}_2$ are formulated by the following theorem based on investigations conducted by Zalgaller (1975), Zalgaller and Litvin (1977), and Litvin (1975).

Theorem. A family of generated surfaces Σ_ϕ is considered:

$$\mathbf{r}_2(u,\theta,\phi) \in C^3, \quad (u,\theta) \in G, \quad a < \phi < b \quad (1.7.1)$$

The family Σ_ϕ is generated by surface Σ_1 represented by

$$\mathbf{r}_1(u,\theta) \in C^3, \quad \frac{\partial \mathbf{r}_1}{\partial u} \times \frac{\partial \mathbf{r}_1}{\partial \theta} \neq 0 \tag{1.7.2}$$

The following conditions are observed at point $M(u_0, \theta_0, \phi_0)$:

$$f(u,\theta,\phi) = \left(\frac{\partial \mathbf{r}_1}{\partial u} \times \frac{\partial \mathbf{r}_1}{\partial \theta}\right) \cdot \mathbf{v}^{(12)} = 0 \tag{1.7.3}$$

$$g_1(u,\theta,\phi) = \begin{vmatrix} f_u & f_\theta & f_\phi \\ \left(\frac{\partial \mathbf{r}_1}{\partial u}\right)^2 & \left(\frac{\partial \mathbf{r}_1}{\partial u}\right)\cdot\left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right) & \left(\frac{\partial \mathbf{r}_1}{\partial u}\right)\cdot\left(\mathbf{v}^{(12)}\right) \\ \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)\cdot\left(\frac{\partial \mathbf{r}_1}{\partial u}\right) & \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)^2 & \left(\frac{\partial \mathbf{r}_1}{\partial \theta}\right)\cdot\left(\mathbf{v}^{(12)}\right) \end{vmatrix} = 0 \tag{1.7.4}$$

$$\begin{vmatrix} f_u & f_\theta \\ g_u & g_\theta \end{vmatrix} \neq 0 \tag{1.7.5}$$

$$\mathbf{H}_1 = \begin{vmatrix} \frac{\partial \mathbf{r}_1}{\partial u} & \frac{\partial \mathbf{r}_1}{\partial \theta} & \mathbf{v}^{(12)} \\ f_u & f_\theta & f_\phi \\ g_u & g_\theta & g_\phi \end{vmatrix} \neq 0 \tag{1.7.6}$$

Thus, the envelope $\mathbf{E}_2$ exists locally at point $M(u_0, \theta_0, \phi_0)$ and is within the neighborhood of M. Envelope $\mathbf{E}_2$ is a regular curve and is determined by

$$\mathbf{r}_2 = \mathbf{r}_2(u,\theta,\phi), \qquad f(u,\theta,\phi) = 0, \qquad g(u,\theta,\phi) = 0 \tag{1.7.7}$$

The tangent to $\mathbf{E}_2$ is collinear to tangent $\mathbf{T}_2$ to the contact line at point M of the tangency of $\mathbf{E}_2$ and $\mathbf{T}_2$. Envelope $\mathbf{E}_2$ does not exist if at least one of the inequalities ((1.7.5) and (1.7.6)) is not observed.

The above theorem was applied by F.L. Litvin, A. Egelja, M. De Donno, A. Peng, and A. Wang to determine the envelopes to the contact lines on the surfaces of various spatial gear drives.

Structure of Curve L on Generated Surface Σ_2 Near Envelope $\mathbf{E}_2$

We consider curve **L** on surface Σ_2 that starts at point M of envelope $\mathbf{E}_2$ to the contact lines. Since M is a singular point of surface Σ_2, the velocity $\mathbf{v}_r^{(2)}$ in any direction that differs from the tangent to $\mathbf{E}_2$ is equal to zero. Therefore, we may expect that M is the point of regression. A detailed investigation of the structure of curve **L** requires that the Taylor series be applied to prove that M is the point of regression and that envelope $\mathbf{E}_2$ is simultaneously the edge of regression.

Example 1.7.1. The generation of a helical involute gear by a rack-cutter is considered. The approach discussed above is applied to determine the envelope $\mathbf{E}_2$ to the contact lines on the generated screw surface Σ_2 and the edge of regression.

<u>Step 1</u>: We apply coordinate systems S_1, S_2, and S_f that are rigidly connected to the rack-cutter, the gear, and the frame, respectively (fig. 1.7.1).

Step 2: The generating surface is plane Σ_1 (fig. 1.7.2). The position vector $\overline{O_1M}$ of point M of the generating plane is

$$\overline{O_1M} = \overline{O_1A} + \overline{O_1B} \tag{1.7.8}$$

where $\left|\overline{O_1A}\right| = \theta$ and $\left|\overline{O_1B}\right| = u$.

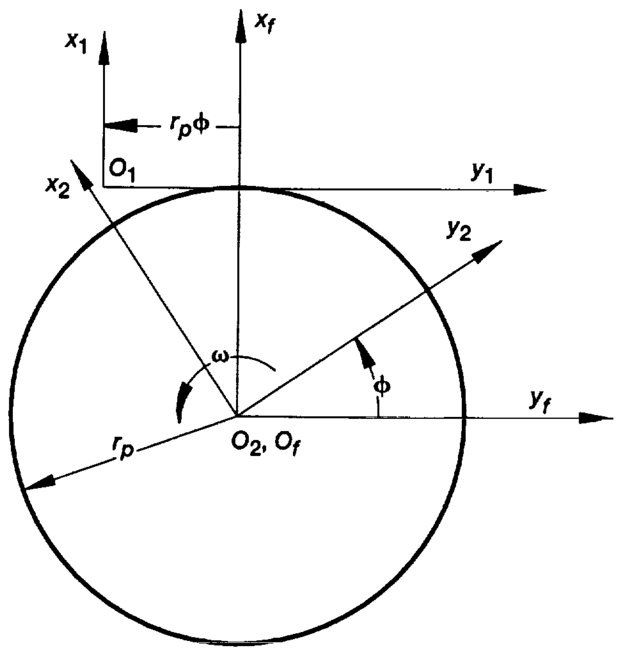


Figure 1.7.1.—Coordinate systems applied for generation of screw involute surface.

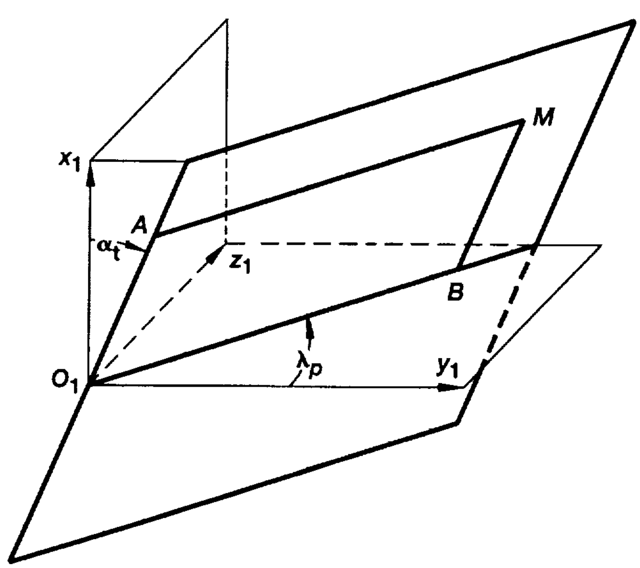


Figure 1.7.2.—Generating plane: $\left|\overline{O_1A}\right| = \theta$ and $\left|\overline{O_1B}\right| = u$.

Then, we obtain the following equation for the generating plane Σ_1:

$$\mathbf{r}_1(u,\theta)=\begin{bmatrix}\theta\cos\alpha_t\\ \theta\sin\alpha_t+u\cos\lambda_p\\ u\sin\lambda_p\\ 1\end{bmatrix} \qquad (1.7.9)$$

The normal $\mathbf{N}_1$ to Σ_1 is

$$\mathbf{N}_1=\frac{\partial\mathbf{r}_1}{\partial u}\times\frac{\partial\mathbf{r}_1}{\partial\theta}=\begin{bmatrix}-\sin\alpha_t\sin\lambda_p\\ \cos\alpha_t\sin\lambda_p\\ -\cos\alpha_t\cos\lambda_p\end{bmatrix} \qquad (1.7.10)$$

Here, α_t is the profile angle of the rack-cutter in the transverse section, and λ_p is the lead angle on the pitch

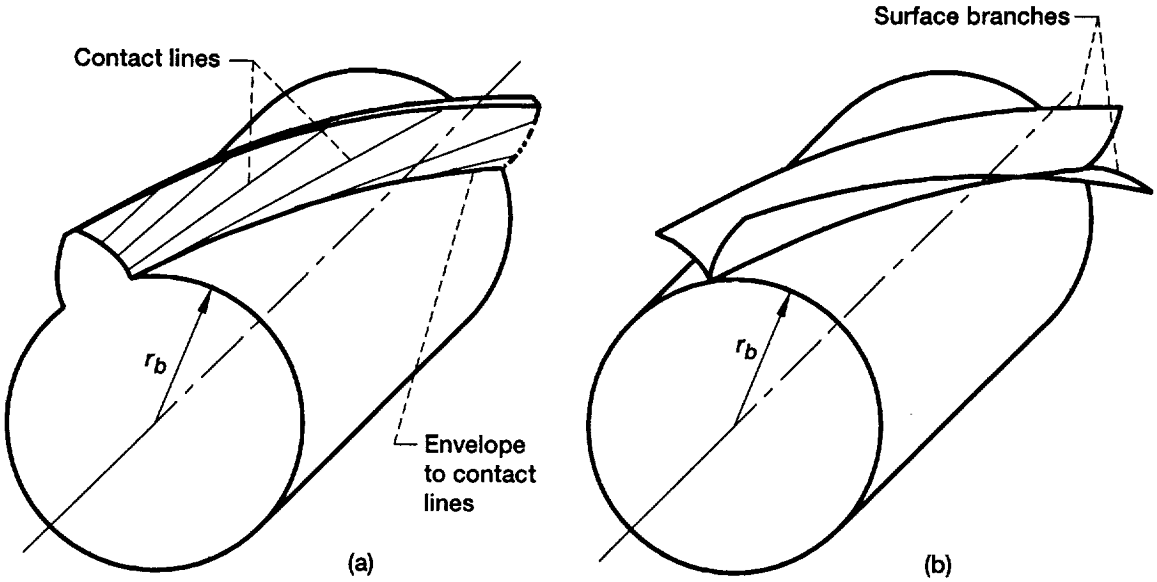


Figure 1.7.3.—Involute helical gear. (a) Contact lines and envelope to contact lines. (b) Surface branches.

Step 5: Equation (1.7.4) of singularities yields

$$g_1(u,\theta,\phi) = -u\cos\lambda_p \cos\alpha_t + r_p\phi\cos\alpha_t + r_p \sin\alpha_t = 0 \tag{1.7.21}$$

Step 6: The conditions for the existence of envelope $\mathbf{E}_2$ to contact lines on Σ_2 formulated by the above theorem are satisfied in the case discussed; particularly, there are observed inequalities (1.7.5) and (1.7.6). Therefore, envelope $\mathbf{E}_2$ indeed exists and is determined by

$$\mathbf{r}_2(u,\theta,\phi), \qquad f(u,\theta,\phi) = 0, \qquad g_1(u,\theta,\phi) = 0 \tag{1.7.22}$$

Equations (1.7.17) to (1.7.22) yield envelope $\mathbf{E}_2$, the helix on the base cylinder of radius r_b, and the lines of contact, tangents to the helix (fig. 1.7.3(a)) that is represented by

$$x_2 = r_b \cos(\alpha_t + \phi) \tag{1.7.23}$$

$$y_2 = r_b \sin(\alpha_t + \phi) \tag{1.7.24}$$

$$z_2 = p(\phi + \tan\alpha_t) \tag{1.7.25}$$

where $r_b = r_p \cos\alpha_t$, the screw parameter $p = r_p \tan\lambda_p$.

Two branches of the generated surface are shown in figure 1.7.3(b).

Step 7: The generation of a spur gear may be considered a particular case of the generation of a helical gear by taking $\lambda_p = 90°$. In such a case, envelope $\mathbf{E}_2$ to the contact lines does not exist since inequality (1.7.5) is not observed. Only the edge of regression exists, as shown in figure 1.7.4.

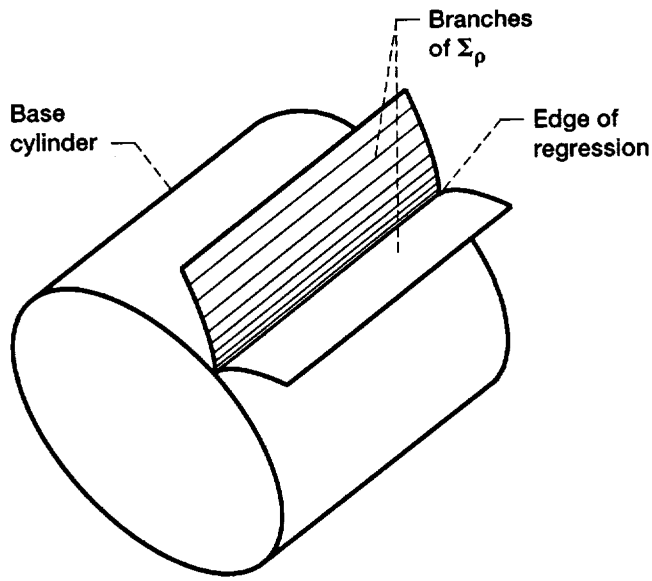


Figure 1.7.4.—Edge of regression of spur involute surface.

1.8 Necessary and Sufficient Conditions for Existence of Envelope $\mathbf{E}_1$ to Contact Lines on Generating Surface Σ_1

Necessary Condition

Contact lines on generating surface Σ_1 may also have an envelope $\mathbf{E}_1$. This envelope divides the generating surface into two parts: A, which is covered with contact lines, and B, which is empty of contact lines (fig. 1.8.1). Envelope $\mathbf{E}_1$ was discovered by Litvin (1975) and the necessary conditions for the existence of $\mathbf{E}_1$ were formulated. In addition, sufficient conditions for the existence of $\mathbf{E}_1$ are formulated in this book.

The family of contact lines on Σ_1 is represented in S_1 by the expressions

$$\mathbf{r}_1(u,\theta) \in C^2, \quad \frac{\partial \mathbf{r}_1}{\partial u} \times \frac{\partial \mathbf{r}_1}{\partial \theta} \neq 0, \quad f(u,\theta,\phi) = 0, \quad (u,\theta) \in G, \quad a < \phi < b \tag{1.8.1}$$

The necessary condition for the existence of envelope $\mathbf{E}_1$ is $f_\phi = 0$. The proof is based on the following considerations (Litvin, 1975):

(1) Vector $\delta\mathbf{r}_1$ of displacement along the tangent to a contact line may be represented by

$$\delta\mathbf{r}_1 = \frac{\partial \mathbf{r}_1}{\partial u} \delta u \times \frac{\partial \mathbf{r}_1}{\partial \theta} \delta\theta, \quad f_u \delta u + f_\theta \delta\theta = 0 \tag{1.8.2}$$

(2) Vector $\mathrm{d}\mathbf{r}_1$ of displacement along the tangent to envelope $\mathbf{E}_1$ may be represented by

$$\mathrm{d}\mathbf{r}_1 = \frac{\partial \mathbf{r}_1}{\partial u} \mathrm{d}u + \frac{\partial \mathbf{r}_1}{\partial \theta} \mathrm{d}\theta, \quad f_u \mathrm{d}u + f_\theta \mathrm{d}\theta + f_\phi \mathrm{d}\phi = 0 \tag{1.8.3}$$

(3) Vectors $\delta\mathbf{r}_1$ and $\mathrm{d}\mathbf{r}_1$ must be collinear if envelope $\mathbf{E}_1$ exists. This requirement is satisfied if $f_\phi \mathrm{d}\phi = 0$, which yields $f_\phi = 0$ ($\mathrm{d}\phi \neq 0$ since ϕ is a varied parameter of motion).

Sufficient Conditions

The generating surface Σ_1 is represented as

$$\mathbf{r}_1(u,\theta) \in C^2, \quad \frac{\partial \mathbf{r}_1}{\partial u} \times \frac{\partial \mathbf{r}_1}{\partial \theta} \neq 0, \quad (u,\theta) \in G, \quad a < \phi < b \tag{1.8.4}$$

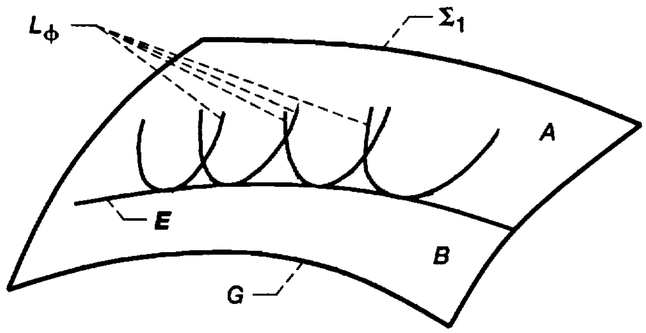


Figure 1.8.1.—Envelope to contact lines.

The following conditions are observed at a point (u_o,θ_o,ϕ_o) designated M:

$$f(u,\theta,\phi)=0 \tag{1.8.5}$$

$$f_\phi(u,\theta,\phi)=0 \tag{1.8.6}$$

$$f_{\phi\phi}\neq 0 \tag{1.8.7}$$

$$\begin{vmatrix} f_u & f_\theta \\ f_{\phi u} & f_{\phi\theta} \end{vmatrix}\neq 0 \tag{1.8.8}$$

Then, envelope $\mathbf{E}_1$ exists, is a regular curve, and is determined by

$$\mathbf{r}_1(u,\theta), \qquad f(u,\theta,\phi)=0, \qquad f_\phi(u,\theta,\phi)=0 \tag{1.8.9}$$

The proof of the theorem of sufficient conditions is based on the following procedure:

Step 1: Consider the system of equations (1.8.5) and (1.8.6) and apply the theorem of implicit equation system existence. We can solve these equations in the neighborhood of point (u_o,θ_o,ϕ_o) by functions $\{u(\phi),\theta(\phi)\}\in C^1$ since inequality (1.8.7) is observed. Then, we may determine a curve on surface Σ_1 to be

$$\mathbf{R}(\phi)=\mathbf{r}_1(u(\phi),\theta(\phi)) \tag{1.8.10}$$

Step 2: The tangent to curve $\mathbf{R}(\phi)$ is determined as

$$\frac{\partial\mathbf{R}}{\partial\phi}=\frac{\partial\mathbf{r}_1}{\partial u}\frac{\mathrm{d}u}{\mathrm{d}\phi}+\frac{\partial\mathbf{r}_1}{\partial\theta}\frac{\mathrm{d}\theta}{\mathrm{d}\phi} \tag{1.8.11}$$

Step 3: We may determine the derivatives $\mathrm{d}u/\mathrm{d}\phi$ and $\mathrm{d}\theta/\mathrm{d}\phi$ as follows: differentiating equations (1.8.5) and (1.8.6), we obtain

$$f_u\frac{\mathrm{d}u}{\mathrm{d}\phi}+f_\theta\frac{\mathrm{d}\theta}{\mathrm{d}\phi}=-f_\phi \tag{1.8.12}$$

$$f_{\phi u}\frac{\mathrm{d}u}{\mathrm{d}\phi}+f_{\phi\theta}\frac{\mathrm{d}\theta}{\mathrm{d}\phi}=-f_{\phi\phi} \tag{1.8.13}$$

Solving equations (1.8.12) and (1.8.13), we obtain $\mathrm{d}u/\mathrm{d}\phi$ and $\mathrm{d}\theta/\mathrm{d}\phi$.

Equations (1.8.5), (1.8.10), and (1.8.11) to (1.8.13) yield

$$\frac{\partial\mathbf{R}}{\partial\phi}=\frac{f_{\phi\phi}}{\begin{vmatrix} f_u & f_\theta \\ f_{\phi u} & f_{\phi\theta} \end{vmatrix}}\left(\frac{\partial\mathbf{r}_1}{\partial u}f_\theta-\frac{\partial\mathbf{r}_1}{\partial\theta}f_u\right) \tag{1.8.14}$$

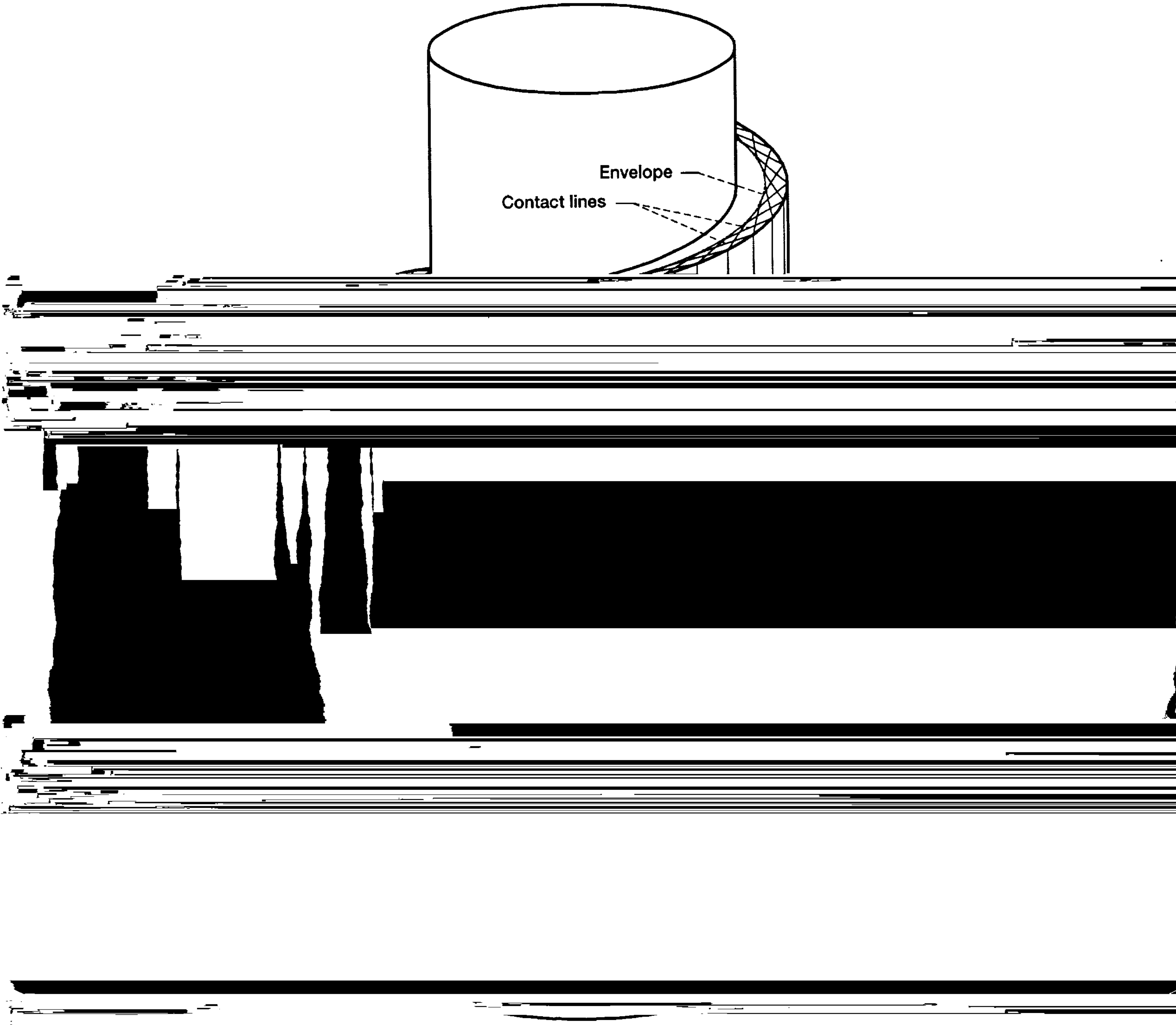


Figure 1.8.2.—Envelope to contact lines on worm surface.

where

$$\frac{\partial \mathbf{r}_1}{\partial u} f_\theta - \frac{\partial \mathbf{r}_1}{\partial \theta} f_u = \mathbf{T}_1 \tag{1.8.15}$$

is the equation of the tangent to the contact line.

Equations (1.8.14) and (1.8.15) confirm that $\mathbf{R}(\phi)$ is a regular curve (due to inequalities (1.8.7) and (1.8.8) and $\mathbf{T}_1 \neq 0$); the tangent $\partial \mathbf{R}/\partial \phi$ is collinear to tangent $\mathbf{T}_1$ to the contact line, and $\mathbf{R}(\phi)$ is indeed the envelope to the family of contact lines on Σ_1.

Note 1: Equation (1.8.5) represents a family of planar curves in the space of parameters (u, θ). Using an

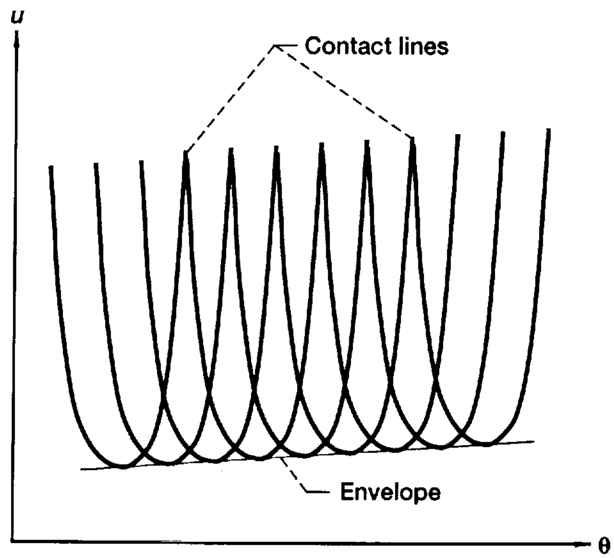


Figure 1.8.3.—Contact lines in space of surface parameters.

1.9 Axes of Meshing

Initial Considerations

The concept of the axes of meshing and their derivation was first presented by Litvin in 1955 and was then published in the works (Litvin, 1968, 1989). The revised and complemented concept is now presented in this book.

We consider that a gear drive transforms rotations with angular velocities $\boldsymbol{\omega}^{(1)}$ and $\boldsymbol{\omega}^{(2)}$ between crossed axes. The angular velocities $\boldsymbol{\omega}^{(1)}$ and $\boldsymbol{\omega}^{(2)}$ lie in parallel planes and form a crossing angle γ, with the shortest distance between $\boldsymbol{\omega}^{(1)}$ and $\boldsymbol{\omega}^{(2)}$ being E (fig. 1.9.1).

The relative motion of gear 1 with respect to gear 2 is represented by vector $\boldsymbol{\omega}^{(12)} = \boldsymbol{\omega}^{(1)} - \boldsymbol{\omega}^{(2)}$ and by vector moment $\mathbf{m}\left(-\boldsymbol{\omega}^{(2)}\right) = \overline{O_f O_2} \times \left(-\boldsymbol{\omega}^{(2)}\right)$.

A point M is the current point of tangency of gear tooth surfaces Σ_1 and Σ_2 if the following equation (the equation of meshing) is observed:

$$\mathbf{n} \cdot \mathbf{v}^{(12)} = \mathbf{n} \cdot \left\{ \left(\boldsymbol{\omega}^{(12)} \times \mathbf{r} \right) + \left[\overline{O_f O_2} \times \left(-\boldsymbol{\omega}^{(2)} \right) \right] \right\} = 0 \qquad (1.9.1)$$

where $\mathbf{r}$ is the position vector of M, and $\mathbf{n}$ is the unit normal to surface Σ_1. The normal $\mathbf{N}$ to surface Σ_1 instead of unit normal $\mathbf{n}$ may be applied in equation (1.9.1). Henceforth, we will consider that vectors in equation (1.9.1) and those derived below are represented in the fixed coordinate system S_f (fig. 1.9.1).

It is known from kinematics that the same relative motion will be provided if vectors $\boldsymbol{\omega}^{(1)}$ and $\boldsymbol{\omega}^{(2)}$ are substituted by vectors $\boldsymbol{\omega}^{(I)}$ and $\boldsymbol{\omega}^{(II)}$ and if the following conditions are observed:

(1) Vectors $\boldsymbol{\omega}^{(I)}$ and $\boldsymbol{\omega}^{(II)}$ lie in planes $\Pi^{(I)}$ and $\Pi^{(II)}$, which are parallel to $\boldsymbol{\omega}^{(1)}$ and $\boldsymbol{\omega}^{(2)}$.
(2) Vectors $\boldsymbol{\omega}^{(I)}$ and $\boldsymbol{\omega}^{(II)}$ are correlated and satisfy these equations:

$$\boldsymbol{\omega}^{(I)} - \boldsymbol{\omega}^{(II)} = \boldsymbol{\omega}^{(12)} \qquad (1.9.2)$$

$$\overline{O_f O}^{(I)} \times \boldsymbol{\omega}^{(I)} + \overline{O_f O}^{(II)} \times \left(-\boldsymbol{\omega}^{(II)} \right) = \overline{O_f O_2} \times \left(-\boldsymbol{\omega}^{(2)} \right) \qquad (1.9.3)$$

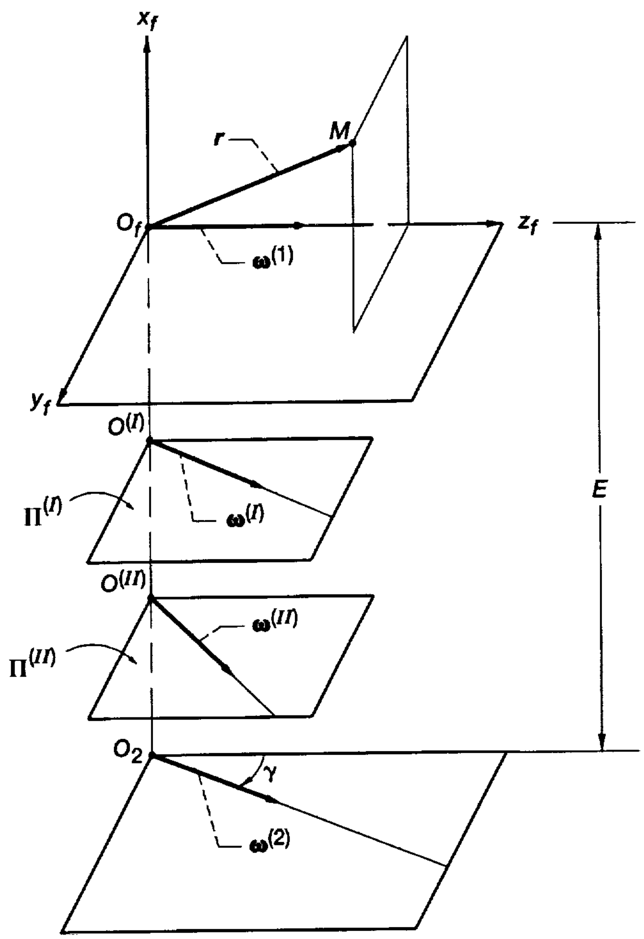


Figure 1.9.1.—Derivation of axes of meshing.

General Concept of Axes of Meshing

A manifold of couples of vectors $\omega^{(I)}$ and $\omega^{(II)}$ satisfy equations (1.9.2) and (1.9.3). We will consider a submanifold of vectors $\omega^{(I)}$ and $\omega^{(II)}$ that not only satisfy equations (1.9.2) and (1.9.3) but also satisfy the requirement that the common surface normal **N** (or the unit normal **n**) intersect the lines of action $L^{(I)}$ and $L^{(II)}$ of $\omega^{(I)}$ and $\omega^{(II)}$ (fig. 1.9.2).

If the normal (unit normal) at point M intersects at least one of the couple of lines $L^{(I)}$ and $L^{(II)}$, say $L^{(I)}$, it is easy to prove that two equations of meshing are satisfied and that the normal intersects the other line, $L^{(II)}$. The proof is based on these considerations:

(1) An equation of meshing similar to equation (1.9.1) can be represented as

$$\mathbf{n} \cdot \mathbf{v}^{(I,II)} = \mathbf{n} \cdot \left(\mathbf{v}^{(I)} - \mathbf{v}^{(II)}\right) = 0 \tag{1.9.4}$$

(2) If the normal intersects $L^{(I)}$, we can represent $\mathbf{v}^{(I)}$ by

$$\mathbf{v}^{(I)} = \boldsymbol{\omega}^{(I)} \times \boldsymbol{\rho}^{(I)} \tag{1.9.5}$$

where $\rho^{(I)}$ is a position vector drawn to M from any point on the line of action $L^{(I)}$. Not losing the generality, $\rho^{(I)}$ can be represented as $\overline{P^{(I)}M}$, where $P^{(I)}$ is the point of intersection of $L^{(I)}$ and the extended unit normal **n**.

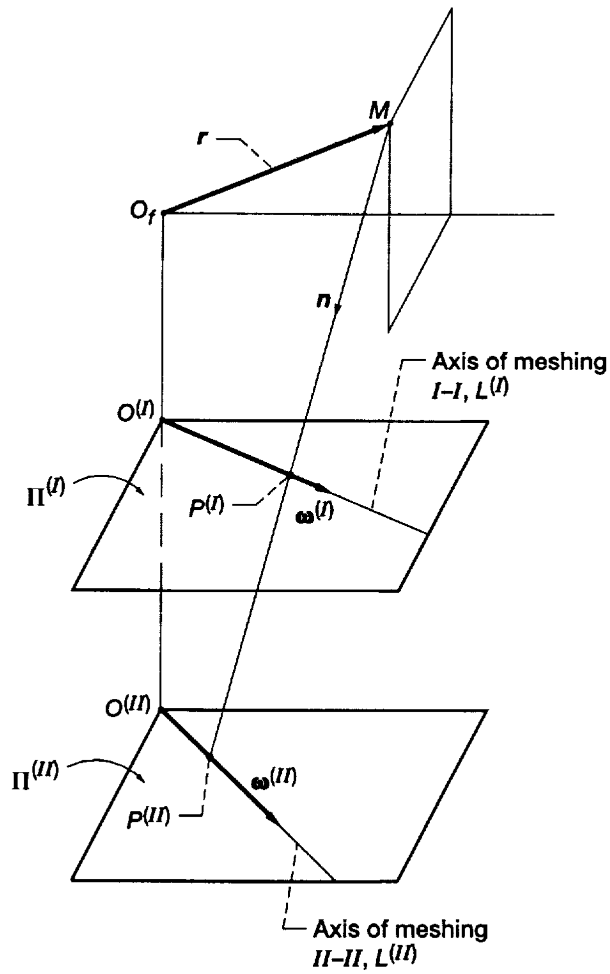


Figure 1.9.2.—Intersection of axes of meshing by the normal to the contacting surfaces.

Thus, we have

$$f_{\phi}(u,\theta,\phi)=0 \tag{1.8.6}$$

(3) Considering the scalar product $\mathbf{n}\cdot\mathbf{v}^{(I)}$, we obtain

$$\mathbf{n}\cdot\mathbf{v}^{(I)}=\mathbf{n}\cdot\left(\boldsymbol{\omega}^{(I)}\times\lambda^{(I)}\mathbf{n}\right)=0 \tag{1.9.7}$$

(4) Equations (1.9.4) and (1.9.7) yield

$$\mathbf{n}\cdot\mathbf{v}^{(II)}=0 \tag{1.9.8}$$

The foregoing discussions mean that (a) if the surface normal at point M (the candidate for the point of tangency of surfaces) intersects at least one line of the couple of lines $L^{(i)}$ ($i = I,II$), it intersects the other line as well; (b) two equations of meshing, (1.9.7) and (1.9.8), are satisfied simultaneously; and (c) point M is the point of tangency of the surfaces if at least one equation of meshing of the couple (1.9.7) and (1.9.8) is satisfied.

We call lines $L^{(I)}$ and $L^{(II)}$ the axes of meshing. However, we emphasize that the couple of lines $L^{(I)}$ and $L^{(II)}$ must satisfy not only equations (1.9.2) and (1.9.3) but also the requirement that $L^{(I)}$ and $L^{(II)}$ be intersected by the surface normal. Lines $L^{(I)}$ and $L^{(II)}$ that satisfy requirements (a) to (c) are called the axes of meshing.

Correlation Between Parameters of Axes of Meshing

Our goal is to prove that the parameters of the axes of meshing are correlated, that they depend on the position vector **r** and the surface normal **N**, and that the solution for determining the parameters of $L^{(I)}$ and $L^{(II)}$ is not unique.

The determination of the sought-for parameters of the axes of meshing is based on the following procedure:

(1) The axes of meshing lie in planes that are perpendicular to the x_f-axis (fig. 1.9.1), and we designate the sought-for parameters $X^{(i)}$, $K^{(i)}$ $(i = I, II)$. The algebraic parameter $X^{(i)}$ determines the location of $O^{(i)}$ on axis x_f; parameter $K^{(i)}$ is determined as

$$K^{(i)} = \frac{\omega_z^{(i)}}{\omega_y^{(i)}} \qquad (1.9.9)$$

(2) The requirement that the surface unit normal pass through the axes of meshing is presented in the following equations (Litvin, 1968):

$$\frac{X^{(i)} - x}{n_x} = \frac{Y^{(i)} - y}{n_y} = \frac{Z^{(i)} - z}{n_z} \quad (i = I, II) \qquad (1.9.10)$$

Here (fig. 1.9.2)

$$\overline{O_f P^{(i)}} = \left(X^{(i)}, Y^{(i)}, Z^{(i)} = K^{(i)} Y^{(i)}\right)(i = I, II); \quad \overline{O_f M} = \mathbf{r} = (x, y, z); \quad \mathbf{n} = (n_x, n_y, n_z)$$

where $P^{(i)}$ is the point of intersection of the normal with the axis of meshing.

After eliminating $Y^{(i)}$ in equations (1.9.10), we obtain

$$X^{(i)} K^{(i)} n_y - X^{(i)} n_z + K^{(i)}\left(y n_x - x n_y\right) + x n_z - z n_x = 0 \quad (i = I, II) \qquad (1.9.11)$$

Equation (1.9.11) yields

$$\left(X^{(I)} K^{(I)} - X^{(II)} K^{(II)}\right) n_y - \left(X^{(I)} - X^{(II)}\right) n_z + \left(K^{(I)} - K^{(II)}\right)\left(y n_x - x n_y\right) = 0 \qquad (1.9.12)$$

(3) Additional relations between parameters $X^{(i)}$, $K^{(i)}$ $(i = I, II)$ can be obtained by using equations (1.9.2) and (1.9.3). These equations yield the following four dependent scalar equations in the unknowns $\omega_y^{(I)}$ and $\omega_y^{(II)}$:

$$\omega_y^{(I)} - \omega_y^{(II)} = -m_{21} \sin\gamma \qquad (1.9.13)$$

$$K^{(I)} \omega_y^{(I)} - K^{(II)} \omega_y^{(II)} = 1 - m_{21} \cos\gamma \qquad (1.9.14)$$

$$X^{(I)} K^{(I)} \omega_y^{(I)} - X^{(II)} K^{(II)} \omega_y^{(II)} = E m_{21} \cos\gamma \qquad (1.9.15)$$

$$X^{(I)} \omega_y^{(I)} - X^{(II)} \omega_y^{(II)} = E m_{21} \sin\gamma \qquad (1.9.16)$$

While deriving these equations, we assign the value of $\boldsymbol{\omega}^{(1)}$ to be 1 rad/sec and $\boldsymbol{\omega}^{(2)}$ to be m_{21} rad/sec, where m_{21} is the gear ratio. The system of equations (1.9.13) to (1.9.16) in the unknowns $\omega_y^{(I)}$ and $\omega_y^{(II)}$ may exist if the rank of matrix

$$\begin{bmatrix} 1 & -1 & -m_{21}\sin\gamma \\ K^{(I)} & -K^{(II)} & 1-m_{21}\cos\gamma \\ X^{(I)}K^{(I)} & -X^{(II)}K^{(II)} & Em_{21}\cos\gamma \\ X^{(I)} & -X^{(II)} & Em_{21}\sin\gamma \end{bmatrix} \tag{1.9.17}$$

is two.

Taking into consideration that in matrix (1.9.17) the four respective determinants of the third order are equal to zero, we obtain after transformations the relations

$$X^{(I)} = \frac{Em_{21}\left(\cos\gamma - K^{(II)}\sin\gamma\right)}{1 - m_{21}\cos\gamma + K^{(II)}m_{21}\sin\gamma} \tag{1.9.18}$$

$$X^{(II)} = \frac{Em_{21}\left(\cos\gamma - K^{(I)}\sin\gamma\right)}{1 - m_{21}\cos\gamma + K^{(I)}m_{21}\sin\gamma} \tag{1.9.19}$$

(4) By analyzing the system of equations (1.9.12), (1.9.18), and (1.9.19), we can conclude the following:

(1) The parameters $X^{(i)}$ $K^{(i)}$ $(i = I, II)$ of the axes of meshing depend on the coordinates (x,y) of the contact point of the surfaces and on the components of the surface normal (unit normal).

(2) Three equations relate four parameters of the axes of meshing. The solution for the parameters of the axes of meshing is not unique, even when the point of tangency and the common normal to the contacting surfaces are considered known. This means that for any instant of meshing, there is a manifold of the axes of meshing. However, there are two particular cases of meshing when only a couple of the axes of meshing, but not a manifold of such axes, exist and the parameters of the axes of meshing do not depend on the contact point and the contact normal: case 1, where the rotation is performed between crossed axes and the surface of one of the gears is the helicoid; case 2, where a helicoid is generated by a peripheral milling (grinding) tool whose surface is a surface of revolution.

Case 1 has been applied in the analysis of the meshing of worm-gear drives with cylindrical worms, helicon drives, face-gear drives with crossed axes, and some types of spiroid gear drives. Case 2 has been applied in the generation of worms and helical gears by a peripheral cutting (grinding) disk.

Case 1 of axes of meshing.—The surface of one of the mating gears is a helicoid. To derive the parameters of the axes of meshing, we apply equations (1.9.12) and (1.9.13) to (1.9.16) and require that the sought-for parameters be independent with respect to the point of tangency of the mating surfaces.

In the case of a helicoid, we have the following relation (Litvin, 1968):

$$yn_x - xn_y = pn_z \tag{1.9.20}$$

where p is the screw parameter of the helicoid.

Equations (1.9.20) and (1.9.12) yield

$$\left(X^{(I)}K^{(I)} - X^{(II)}K^{(II)}\right)n_y - \left[\left(X^{(I)} - K^{(I)}p\right) - \left(X^{(II)} - K^{(II)}p\right)\right]n_z = 0 \tag{1.9.21}$$

Equation (1.9.21) shows that the parameters of the axes of meshing do not depend on components of the surface normal if these relations are observed:

$$X^{(I)}K^{(I)} = X^{(II)}K^{(II)} \tag{1.9.22}$$

$$X^{(I)} - K^{(I)}p = X^{(II)} - K^{(II)}p \tag{1.9.23}$$

Further derivations of $X^{(i)}$, $K^{(i)}$ $(i = I, II)$ are based on the application of equations (1.9.22) and (1.9.23) and the system of equations (1.9.13) to (1.9.16). The procedure for deriving the parameters follows:

Step 1: Considering equations (1.9.22), (1.9.13), and (1.9.15), we obtain

$$X^{(I)}K^{(I)} = X^{(II)}K^{(II)} = -E\cot\gamma \tag{1.9.24}$$

Step 2: Considering equations (1.9.14) and (1.9.16), we obtain after transformations

$$\left(X^{(I)} - pK^{(I)}\right)\omega_y^{(I)} - \left(X^{(II)} - pK^{(II)}\right)\omega_y^{(II)} = Em_{21}\sin\gamma - p(1 - m_{21}\cos\gamma) \tag{1.9.25}$$

Equations (1.9.25), (1.9.23), and (1.9.13) yield

$$\frac{Em_{21}\sin\gamma - p(1 - m_{21})}{X^{(i)} - pK^{(i)}} + m_{21}\sin\gamma = 0 \qquad (i = I, II) \tag{1.9.26}$$

Then, using equations (1.9.26) and (1.9.24), we obtain

$$\left(K^{(i)}\right)^2 - \left(\frac{E}{p} - \frac{1 - m_{21}\cos\gamma}{m_{21}\sin\gamma}\right)K^{(i)} + \frac{E\cot\gamma}{p} = 0 \tag{1.9.27}$$

Step 3: The final equations for the determination of parameters $X^{(i)}$, $K^{(i)}$ of the couple of the axes of meshing are

$$K^{(I)} = \frac{1}{2}\left(\frac{E}{p} - \frac{1 - m_{21}\cos\gamma}{m_{21}\sin\gamma}\right) + \frac{1}{2}\left[\left(\frac{E}{p} - \frac{1 - m_{21}\cos\gamma}{m_{21}\sin\gamma}\right)^2 - \frac{4E\cot\gamma}{p}\right]^{0.5} \tag{1.9.28}$$

$$X^{(I)} = -\frac{E\cot\gamma}{K^{(I)}} \tag{1.9.29}$$

$$K^{(II)} = \frac{1}{2}\left(\frac{E}{p} - \frac{1 - m_{21}\cos\gamma}{m_{21}\sin\gamma}\right) - \frac{1}{2}\left[\left(\frac{E}{p} - \frac{1 - m_{21}\cos\gamma}{m_{21}\sin\gamma}\right)^2 - \frac{4E\cot\gamma}{p}\right]^{0.5} \tag{1.9.30}$$

$$X^{(II)} = -\frac{E\cot\gamma}{K^{(II)}} \tag{1.9.31}$$

In the case of an orthogonal gear drive, we have $\gamma = \pi/2$. To determine the expression for $X^{(II)} = 0/0$, we use the Lopithal rule (Korn and Korn, 1968). Then, we obtain the following equations for the axes of meshing parameters:

$$K^{(I)} = \frac{E}{p} - \frac{1}{m_{21}} \tag{1.9.32}$$

$$X^{(I)} = 0 \tag{1.9.33}$$

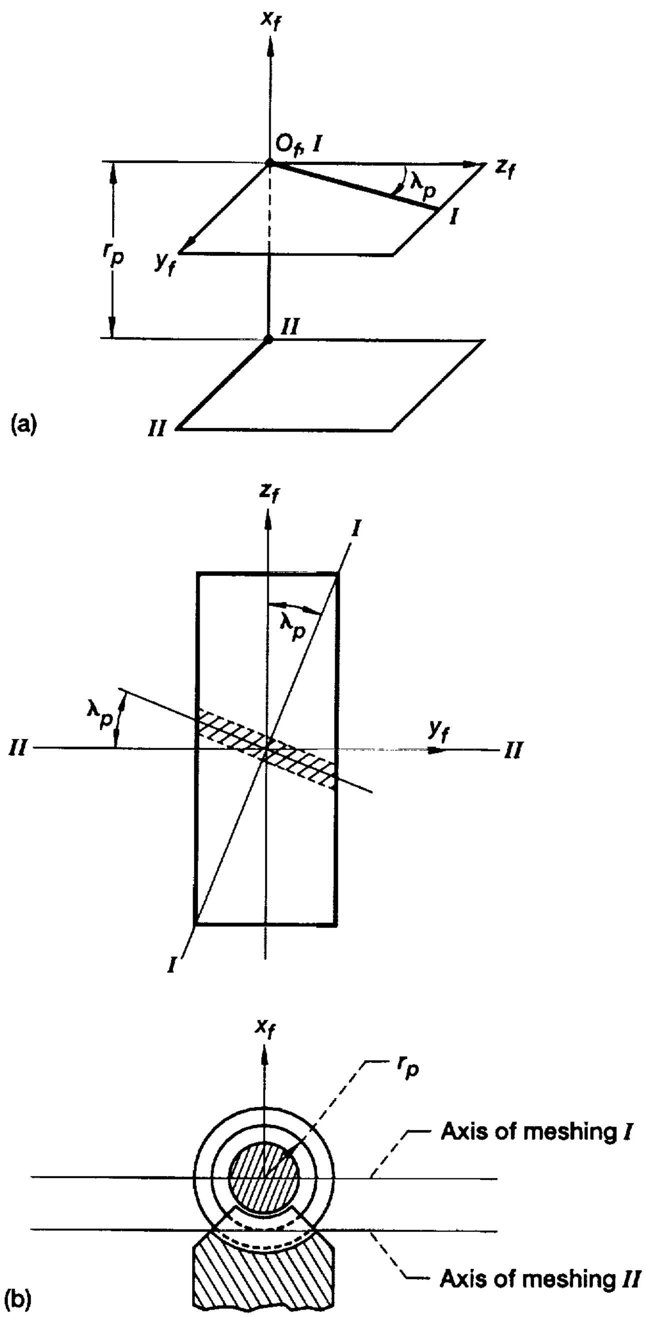


Figure 1.9.3.—Axes of meshing of orthogonal worm-gear drive. (a) In three-dimensional space. (b) In orthogonal projections.

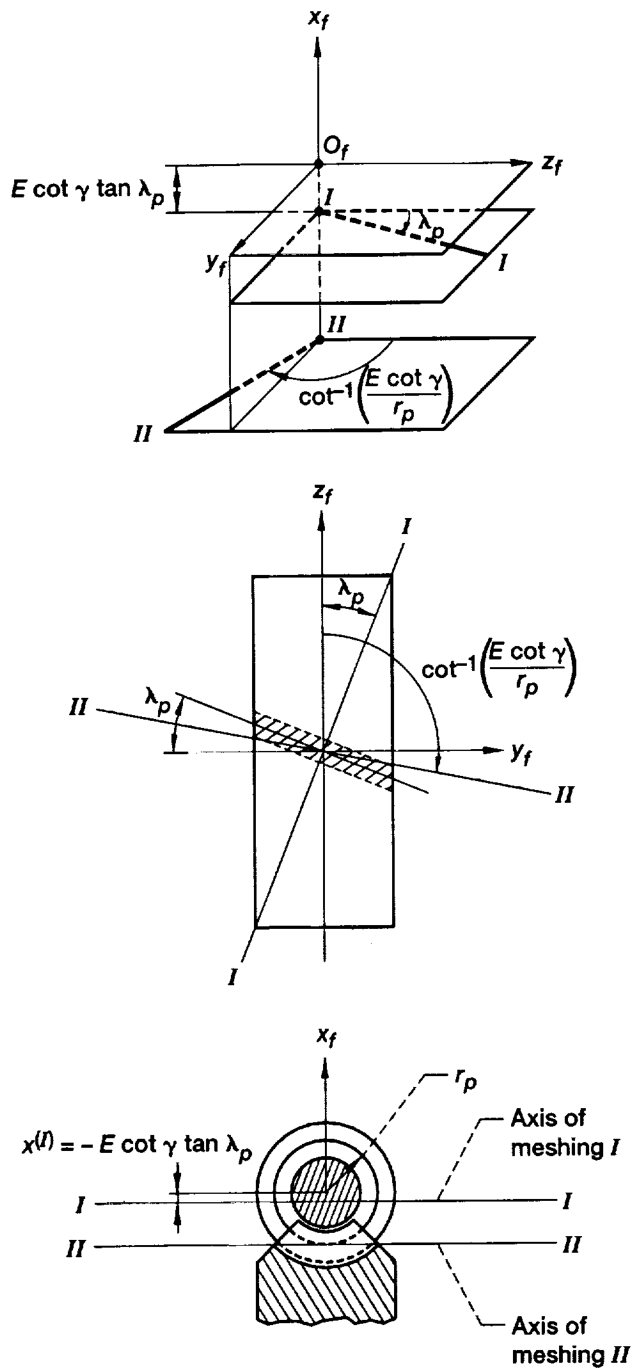


Figure 1.9.4.—Axes of meshing of nonorthogonal worm-gear drive with right-hand worm. (a) In three-dimensional space. (b) In orthogonal projections.

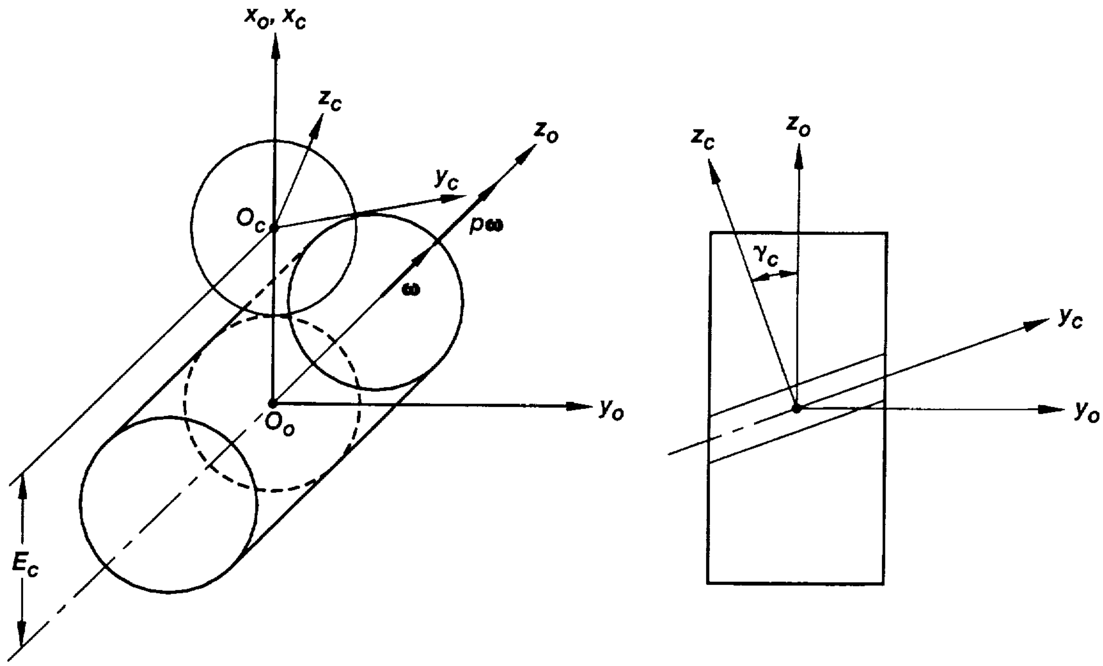


Figure 1.9.5.—Generation of worm by peripheral tool.

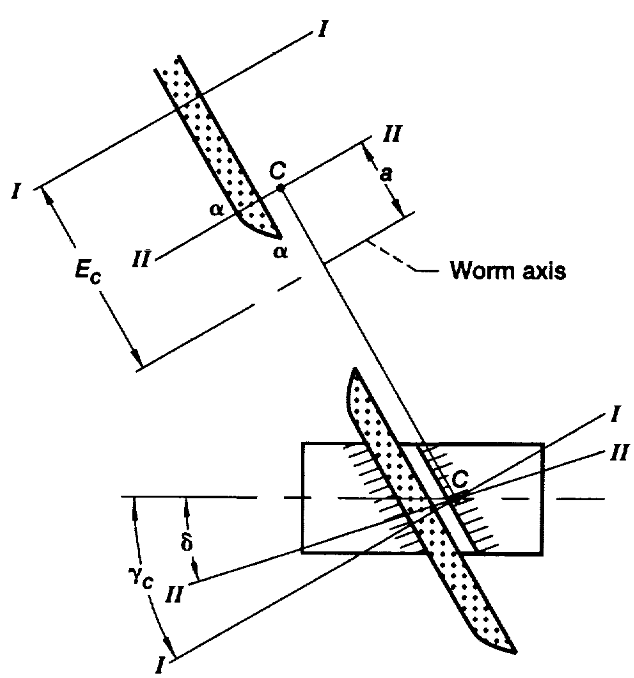


Figure 1.9.6.—Axes of meshing in case of helicoid generation.

$$K^{(II)} = 0 \tag{1.9.34}$$

$$X^{(II)} = -\left(E - \frac{p}{m_{21}}\right) \tag{1.9.35}$$

The axes of meshing for a worm-gear drive with a cylindrical worm are shown in figures 1.9.3 and 1.9.4.

For a face-gear drive with crossed axes and with a pinion as a spur gear, we have to take $p = \infty$.

In this case, all contact normals are perpendicular to the pinion axis, and one of the axes of meshing lies in the infinity.

Case 2 of axes of meshing.—We will consider the generation of a helicoid by a cutting or grinding disk. The installment of the tool is shown in figure 1.9.5. Coordinate system S_o is rigidly connected to the frame. The helicoid in the process of generation performs a screw motion about the z_o-axis with the screw parameter p. We may neglect the tool rotation since it is provided to obtain the desired cutting velocity and does not affect the process of generation. Therefore, we may consider that systems S_c and S_o are rigidly connected during the generation process. The crossing angle γ_c between the z_c- and z_o-axes is usually equal to the lead angle λ_p on the helicoid pitch cylinder. The shortest distance is E_c.

There are two axes of meshing in this case: I - I coincides with the tool axis; II - II lies in the plane that is perpendicular to the shortest distance E_c (fig. 1.9.6). The shortest distance between the helicoid axis and the axes of meshing II - II is

$$a = X_0^{(II)} = p \cot \gamma_c \tag{1.9.36}$$

1.10 Two-Parameter Enveloping

The motion of the generating surface Σ_1 is determined with two independent parameters designated (ϕ, ψ), and the family of surfaces Σ_1 is represented in S_2 as

$$\mathbf{r}_2(u,\theta,\phi,\psi) = \mathbf{M}_{21}(\phi,\psi)\mathbf{r}_1(u,\theta) \tag{1.10.1}$$

The two equations of meshing for the case of two-parametric enveloping are determined as

$$\mathbf{N}_1 \cdot \mathbf{v}_1^{(\phi)} = f(u,\theta,\phi,\psi) = 0\,, \quad \mathbf{N}_1 \cdot \mathbf{v}_1^{(\psi)} = g(u,\theta,\phi,\psi) = 0 \tag{1.10.2}$$

Here, $\mathbf{v}_1^{(\phi)} \equiv \mathbf{v}_1^{(12,\phi)}$, $\mathbf{v}_1^{(\psi)} \equiv \mathbf{v}_1^{(12,\psi)}$ represent the sliding velocity when the respective parameter of motion (ϕ or ψ) is fixed. The subscript 1 in equations (1.10.2) indicates that the respective vectors are represented in coordinate system S_1.

The two-parameter method of enveloping was discussed in Litvin, Krylov, and Erikhov (1975) and Litvin and Seol (1996). It can be successfully applied when the tool has a feed motion in the generation processes, such as hobbing, shaving, and grinding. We have to emphasize that in reality the generation of surfaces with feed motion is a one-parameter enveloping process because the two parameters of motion, ϕ and ψ, are related by the generating function $\psi(\phi)$. Using one-parameter enveloping makes it possible to determine the real surface Σ_2^* and its deviation from the theoretical envelope Σ_2 and to evaluate the influence of the feed motion (of function $\psi(\phi)$).

A detailed example of two-parameter enveloping is presented in appendix C.

1.11 Localization of Contact and Simulation of Meshing of Gear Tooth Surfaces

The localization of gear tooth surface contact is achieved when point contact instead of line contact of the surfaces is provided. This enables one to reduce the sensitivity of the gear drive to misalignment and to also avoid so-called edge contact. The localization can be achieved by the mismatch of gear tooth surfaces. The Gleason Works engineers have successfully developed spiral bevel gears and hypoid gears with point contact of the surfaces. A localized contact is provided for circular-arc helical gears (Novikov-Wildhaber gears) and can be achieved for other types of gear drives by gear tooth surface crowning.

We consider a great achievement to be the computerized simulation of meshing and of gear tooth surfaces in point contact accomplished by applying TCA (Tooth Contact Analysis) computer programs. The simulation of the meshing of gear tooth surfaces is based on the conditions of continuous tangency of gear tooth surfaces that are represented by the following equations

$$\mathbf{r}_f^{(1)}(u_1,\theta_1,\phi_1,q_j)=\mathbf{r}_f^{(2)}(u_2,\theta_2,\phi_2,q_j) \tag{1.11.1}$$

$$\mathbf{n}_f^{(1)}(u_1,\theta_1,\phi_1,q_j)=\mathbf{n}_f^{(2)}(u_2,\theta_2,\phi_2,q_j) \tag{1.11.2}$$

Equations (1.11.1) and (1.11.2) indicate that the contacting surfaces have at the current point of tangency common position vectors $\mathbf{r}_f^{(i)}$ and surface unit normals $\mathbf{n}_f^{(i)}$, $(i = 1,2)$. The coincidence of directions of the unit normals for both surfaces can be provided by the proper order of cofactors in the cross products $(\partial \mathbf{r}_f^{(i)}/\partial u_i) \times (\partial \mathbf{r}_f^{(i)}/\partial \theta_i)$, (i = 1,2). The gear tooth surfaces Σ_1 and Σ_2 are represented in the fixed coordinate systems S_f where the axes of gear rotation are located; (u_i, θ_i) $(i = 1,2)$ are the surface parameters; ϕ_1 and ϕ_2 are the angles of gear rotation; q_j $(j = 1,2,...)$ designate the parameters of assembly.

Equations (1.11.1) and (1.11.2) yield a system of only five independent nonlinear equations (in six unknowns) since $\left|\mathbf{n}_f^{(1)}\right| = \left|\mathbf{n}_f^{(2)}\right| = 1$. These equations are represented as

$$f_k(u_1,\theta_1,\phi_1,u_2,\theta_2,\phi_2)=0, \quad f_k \in C^1, \quad (k=1,5) \tag{1.11.3}$$

One of the unknowns, say ϕ_1, may be chosen as the input. Henceforth, we assume that equations (1.11.3) are satisfied at a point

$$P^0=\left(u_1^0,\theta_1^0,\phi_1^0,u_2^0,\theta_2^0,\phi_2^0\right) \tag{1.11.4}$$

and the Jacobian system at P^0 differs from zero. Thus,

$$\Delta_5=\frac{D(f_1,f_2,f_3,f_4,f_5)}{D(u_1,\theta_1,u_2,\theta_2,\phi_2)}\neq 0 \tag{1.11.5}$$

From the theorem of the existence of the implicit function system, it follows that equations (1.11.3) can be solved in the neighborhood of P^0 by functions

$$\left[u_1(\phi_1),\theta_1(\phi_1),u_2(\phi_1),\theta_2(\phi_1),\phi_2(\phi_1)\right]\in C^1 \tag{1.11.6}$$

By using equations (1.11.1) and (1.11.2) and functions (1.11.6), we can determine the paths of contact on surfaces Σ_1 and Σ_2 and the transmission function $\phi_2(\phi_1)$. The gear misalignment is simulated by the variation of assembly parameters q_j that will cause the shift in the paths of contact and the deviations of $\phi_2(\phi_1)$ from the transmission function of an aligned gear drive. Note that a unique solution of equations (1.11.3) by functions (1.11.6) exists only for the case of meshing by a point contact of surfaces. The Jacobian Δ_5 becomes equal to zero when the surfaces are in line contact.

This method of simulating the meshing of misaligned gear drives was proposed by Litvin and Guo (1962). We must credit The Gleason Works researchers who developed and applied in industry the TCA computer programs for hypoid gear and spiral bevel gear drives. Similar programs were developed later by Litvin and Gutman (1981).

The numerical solution of nonlinear equations (1.11.3) is an iterative process based on the application of computer programs (Dongarra et al., 1979 and Moré, 1980).

1.12 Equation of Meshing for Surfaces in Point Contact

The computerized simulation of the meshing of gear tooth surfaces described in section 1.11 does not require knowledge of the equation of meshing. However, it is possible to prove that equation (1.2.7) can also be extended to apply in the case of surfaces in point contact.

Consider an aligned gear drive when the assembly parameters are observed. The differentiation of equation (1.11.1) yields

$$\frac{\partial \mathbf{r}_f^{(1)}}{\partial u_1}\frac{du_1}{dt}+\frac{\partial \mathbf{r}_f^{(1)}}{\partial \theta_1}\frac{d\theta_1}{dt}+\frac{\partial \mathbf{r}_f^{(1)}}{\partial \phi_1}\frac{d\phi_1}{dt}=\frac{\partial \mathbf{r}_f^{(2)}}{\partial u_2}\frac{du_2}{dt}+\frac{\partial \mathbf{r}_f^{(2)}}{\partial \theta_2}\frac{d\theta_2}{dt}+\frac{\partial \mathbf{r}_f^{(2)}}{\partial \phi_2}\frac{d\phi_2}{dt} \tag{1.12.1}$$

It is easy to verify that the derivatives $(\partial \mathbf{r}_f^{(i)}/\partial u_i)$, $(\partial \mathbf{r}_f^{(i)}/\partial \theta_i)$ $(i = 1,2)$ lie in the common tangent plane for surfaces in tangency. Using the scalar product of the common surface normal $\mathbf{N}_f^{(i)}$ with both sides of equation (1.12.1), we obtain

$$\left(\frac{\partial \mathbf{r}_f^{(1)}}{\partial \phi_1}\frac{d\phi_1}{dt}-\frac{\partial \mathbf{r}_f^{(2)}}{\partial \phi_2}\frac{d\phi_2}{dt}\right)\cdot \mathbf{N}_f^{(i)}=0 \tag{1.12.2}$$

Taking into account that $(\partial \mathbf{r}_f^{(i)}/\partial \phi_i)$ $(d\phi_i/dt)$ is the velocity $\mathbf{v}_{tr}^{(i)}$ of the surface point (in transfer motion with the surface), we obtain

$$\mathbf{N}_f^{(1)}\cdot\left(\mathbf{v}_{tr}^{(1)}-\mathbf{v}_{tr}^{(2)}\right)=\mathbf{N}_f^{(1)}\cdot\mathbf{v}_f^{(12)}=0 \tag{1.12.3}$$

This is the proof that the equation of meshing can also be applied for the case of the point contact of surfaces.

Similar derivations performed for a misaligned gear drive yield

$$\mathbf{N}_f^{(1)}\cdot\left(\frac{\partial \mathbf{r}_f^{(1)}}{\partial \phi_1}d\phi_1-\frac{\partial \mathbf{r}_f^{(2)}}{\partial \phi_2}d\phi_2-\frac{\partial \mathbf{r}_f^{(2)}}{\partial q_j}dq_j\right)=0 \tag{1.12.4}$$

which allows us to investigate the influence of gear misalignment. However, this equation can be applied when the theoretical line of action (the set of contact points in the fixed coordinate system) is known. The influence of the misalignment errors can be determined directly by applying the TCA program.

1.13 Transition From Surface Line Contact to Point Contact

Instantaneous line contact of gear tooth surfaces may exist only in ideal gear drives without misalignment and manufacturing errors. Such errors cause the gear tooth surfaces to contact each other at a point at every instant instead of on a line. The set of contact points on the gear tooth surface forms the contact path. A current point of the contact path indicates the location of the center of the instantaneous contact ellipse. (Recall that because of the elastic deformation of the teeth, the contact is spread over an elliptical area.) The set of contact ellipses represents the bearing contact, which covers only a certain part of the tooth surface instead of the entire working tooth surface (in the case of the line contact of an ideal gear drive).

Our goals are to determine the following: (1) the contact path for a misaligned gear drive and (2) the transmission errors caused by misalignment. Such problems are important for those gear drives whose gear tooth surfaces are designed as mutually enveloping. Typical examples are a worm-gear drive with a cylindrical worm and involute helical gears with parallel axes.

Figure 1.13.1(a) shows two neighboring contact lines $L_1(\phi)$ and $L_2(\phi + d\phi)$ on surface Σ_1 of an ideal gear drive without misalignment. Surface Σ_1 is in instantaneous line contact with Σ_2; parameter ϕ is the generalized parameter of motion, and point M is a current point of contact line $L_1(\phi)$. The displacement from M to any point on $L_2(\phi + d\phi)$ can be performed in any direction if it differs from the tangent to line $L_1(\phi)$ at M. However, in the case of a misaligned gear drive with a point contact of surfaces Σ_1 and Σ_2, we have to determine (1) the *transition* point P on the contact line $L_1(\phi)$ (fig. 1.13.1(b)) (the transfer from line contact of the surfaces to point

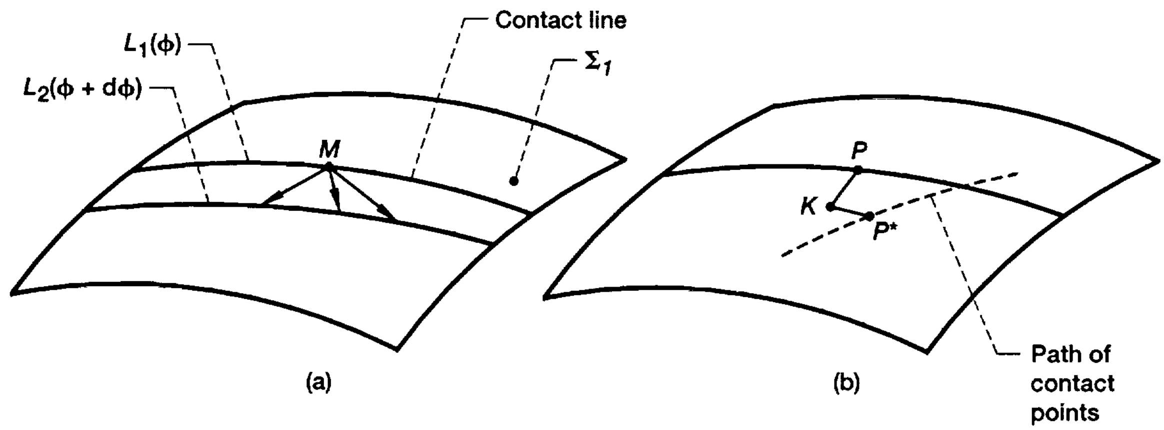


Figure 1.13.1.—For derivation of transition point. (a) Representation of two neighboring contact lines. (b) Transition from surface point P to P^* via K.

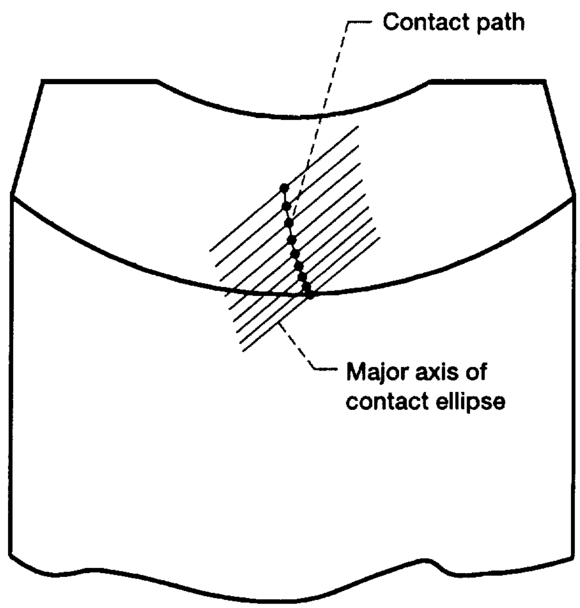


Figure 1.13.2.—Contact path of misalignment of worm-gear drive. Change of center distance, ΔE = 0.5 mm; change of shaft angle, $\Delta\gamma$ = 5'.

contact will occur in the neighborhood of P) and (2) the current point P^* of the real contact path. The direct determination of P^* is impossible because the Jacobian Δ_5 of the system of equations (1.12.3) of surface tangency is equal to zero. Therefore, it becomes necessary to determine an intermediate point K in the neighborhood of P (fig. 1.13.1(b)) where Δ_5 differs from zero. The determination of point K is based on the fact that vector $\overline{PK}$ is collinear to the vector that passes through two neighboring transition points. An equation to determine the transition point on a contact line $L_1(\phi)$ was proposed in Litvin (1994) and Litvin and Hsiao (1993). The Jacobian Δ_5 at point K differs from zero, and we can start the procedure of simulating the meshing of two surfaces in point contact.

Figure 1.13.2 shows the shift in the bearing contact in a misaligned worm-gear drive. Transmission errors due to misalignment will occur and may cause noise and vibration (see section 1.14).

1.14 Design and Generation of Gear Drives With Compensated Transmission Errors

Influence of Transmission Errors on Conditions for Transfer of Meshing

Experimental tests show that the level of noise and vibration depends on the level and shape of transmission errors caused by gear misalignment. Henceforth, we will assume that the gear tooth surfaces are mismatched and that they contact each other at every instant at a point. This precondition is important when designing low-noise gear drives, but it must be complemented with the requirement that one apply the predesigned parabolic function of transmission errors, which is represented as

$$\Delta\phi_2(\phi_1) = -a\phi_1^2 \tag{1.14.1}$$

It will now be shown that the application of such a function allows one to absorb transmission errors caused by gear misalignment, to avoid edge contact, and to improve the conditions for the transfer of meshing. Edge contact means curve-to-surface contact that may occur instead of surface-to-surface contact. In such a case, the curve is the edge of the gear tooth surface of one of the mating gears that is in mesh with the tooth surface of the mating gear. The transfer of meshing means that the continuous transformation of motions by a gear drive requires that a pair of teeth in mesh be changed for another pair.

Figure 1.14.1(a) shows that the transmission function $\phi_2(\phi_1)$ for an ideal gear drive is linear and is represented as

$$\phi_2(\phi_1) = \frac{N_1}{N_2}\phi_1 \tag{1.14.2}$$

where N_1 and N_2 are the gear tooth numbers. The contact ratio (the number of teeth being in mesh simultaneously) may be larger than 1 in an ideal gear drive. In reality, ideal gear drives do not exist because alignment errors cause transmission errors that substantially worsen the conditions for the transfer of motion. Figure 1.14.1(b) shows the transmission function $\phi_2(\phi_1)$ for a misaligned gear drive that is a piecewise nonlinear function for each cycle of meshing with worsened conditions for the transfer of meshing. The cycle of meshing is determined with the angles of rotation of the driving and driven gear represented as $\phi_1 = (2\pi/N_1)$ and $\phi_2 = (2\pi/N_2)$. The author and his fellow researchers at the University of Illinois investigated crowned involute helical gears, double-circular-arc helical gears, and hypoid gears. They found that the function of transmission errors $\Delta\phi_2(\phi_1)$ for misaligned gear drives usually has the shape shown in figure 1.14.2(a). The linear part of $\Delta\phi_2(\phi_1)$ is caused by gear misalignment; the nonlinear dashed part of $\Delta\phi_2(\phi_1)$ corresponds to the portion of the meshing cycle when the edge contact occurs. The second derivative of $\Delta\phi_2(\phi_1)$, and therefore the acceleration of the driven gear, makes a big jump at the transfer point A of the meshing cycle.

The author's approach is directed at improving the conditions for the transfer of meshing and is based on the application of a predesigned parabolic function (1.14.1) of transmission errors. Such a function is provided by the proper modification of gear tooth surfaces or by the stipulation of specific relations between the motions of the tool and the generating gear in the generation process. It will be shown next, that the simultaneous action of both transmission error functions, the predesigned one and that caused by misalignment (in fig. 1.14.2(a)), causes a resulting function of transmission errors that is again a parabolic function having the same slope as the initially predesigned parabolic function. The magnitude $\Delta\phi_{2\max}$ of the resulting maximal transmission errors (caused by the interaction of both functions shown in fig. 1.14.2(b)) can be substantially reduced. The level of the driven gear accelerations is reduced as well, and an edge contact, as a rule, can be avoided.

The transmission function for the gear drive, when the predesigned parabolic function of transmission errors is provided, is shown in figure 1.14.3(a). The predesigned parabolic function is shown in figure 1.14.3(b). It is important to recognize that the contact ratio for a misaligned gear drive with rigid teeth is equal to 1. However, the real contact ratio is larger than 1 because of the elastic deformation of the teeth. While investigating the correlation between the predesigned function of transmission errors and the elastic deformation of teeth, we have to consider the *variation* in the elastic deformation of the teeth during the meshing process, but not the whole value of the elastic deformation. It is assumed that the variation in elastic deformation is comparable to the level of compensated transmission errors.

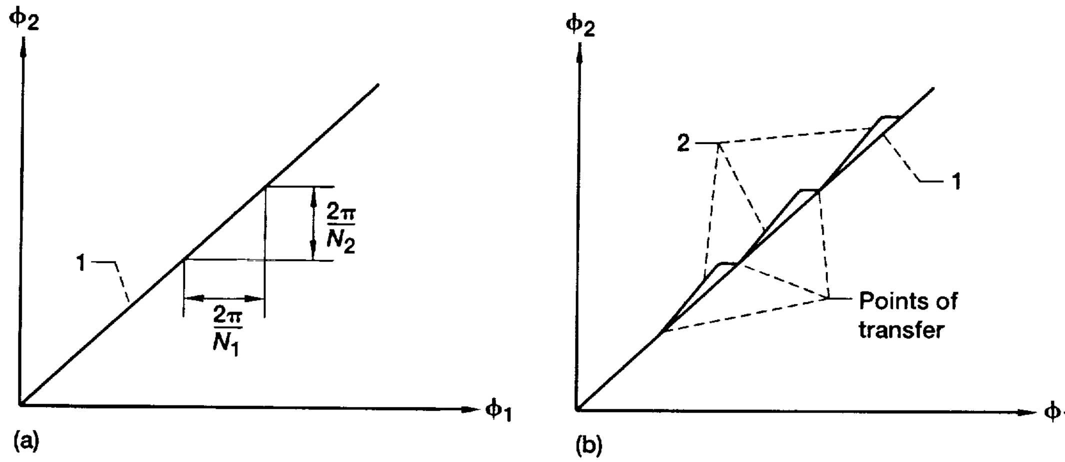


Figure 1.14.1.—Transmission functions of ideal and misaligned gear drives. (a) Ideal. (b) Misaligned.

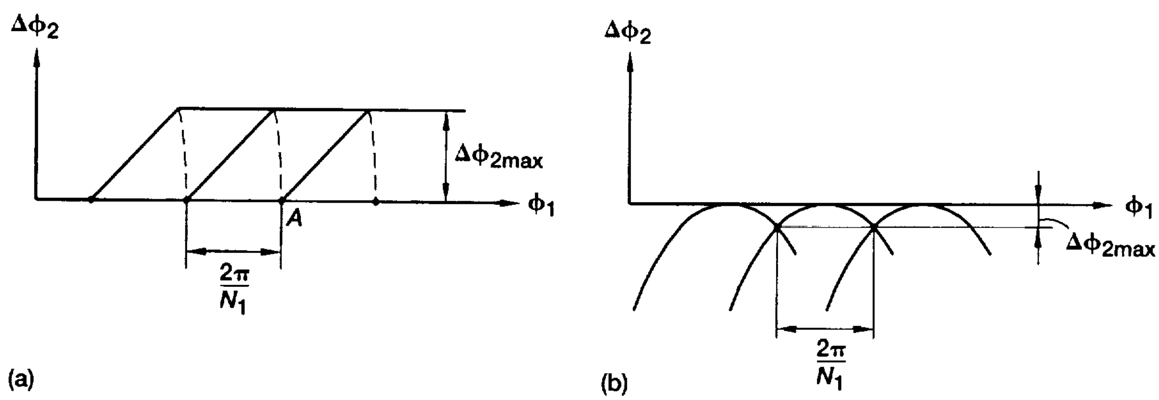


Figure 1.14.2.—Function of transmission errors for gears. (a) Existing geometry. (b) Modified geometry.

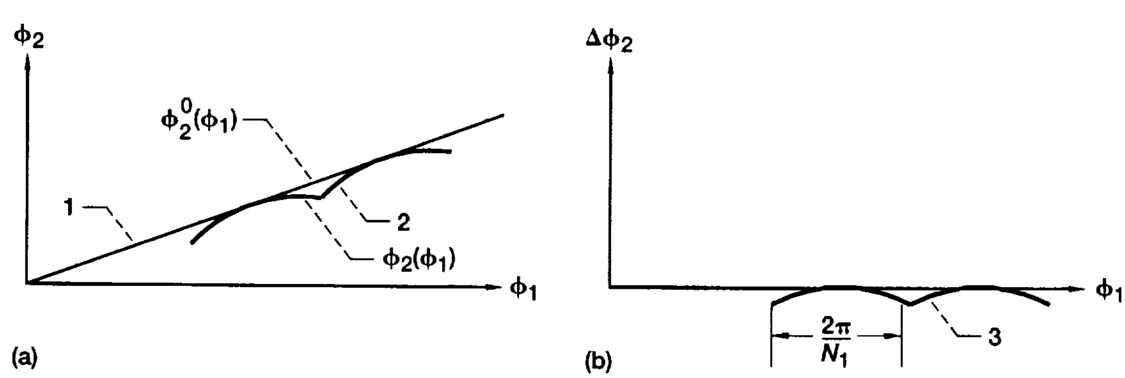


Figure 1.14.3.—Transmission function and function of transmission errors for misaligned gear drive. (a) 1, ideal transmission function; 2, transmission function for gears with modified geometry. (b) 3, predesigned parabolic function of transmission errors.

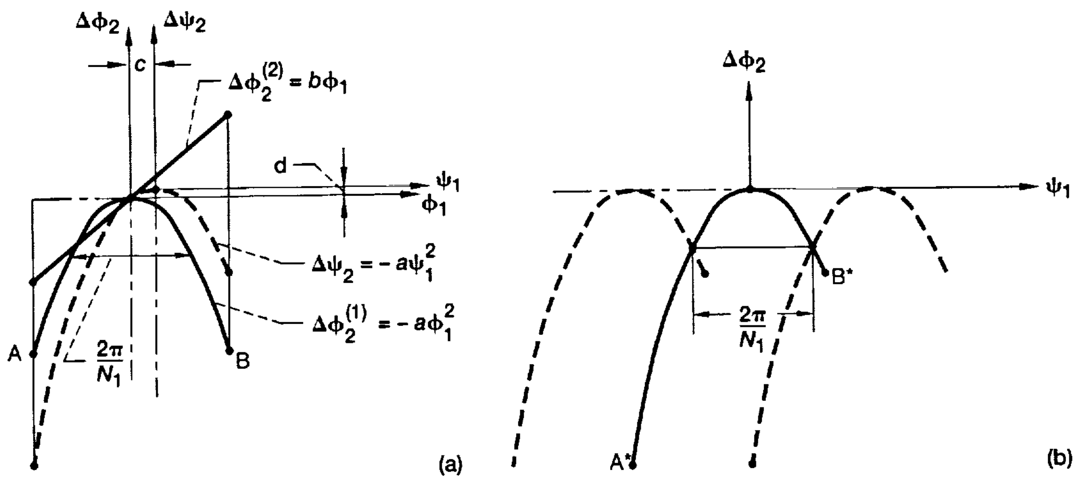


Figure 1.14.4.—Interaction of parabolic and linear functions. (a) Linear and parabolic functions of transmission errors. (b) Resulting function of transmission errors.

Interaction of Parabolic and Linear Functions of Transmission Errors

Figure 1.14.4(a) shows the interaction of two functions: (1) the linear function $\Delta\phi_2^{(1)} = b\phi_1$ caused by gear misalignment and (2) the predesigned parabolic function $\Delta\phi_2^{(2)} = -a\phi_1^2$ provided by the modification of the contacting gear tooth surfaces. Our goal is to prove that the linear function $\Delta\phi_2^{(1)}(\phi_1)$ will be absorbed because of the existence of the parabolic function $\Delta\phi_2^{(2)} = -a\phi_1^2$. To prove it, we consider the resulting function of transmission errors to be

$$\Delta\phi_2(\phi_1) = \Delta\phi_2^{(1)}(\phi_1) + \Delta\phi_2^{(2)}(\phi_1) = b\phi_1 - a\phi_1^2 \tag{1.14.3}$$

The proof is based on the consideration that equation (1.14.3) represents in a new coordinate system with axes $(\Delta\psi_2, \psi_1)$ (fig. 1.14.4(a)) the parabolic function

$$\Delta\psi_2 = -a\psi_1^2 \tag{1.14.4}$$

The axes of coordinate systems $(\Delta\psi_2, \psi_1)$ and $(\Delta\phi_2, \phi_1)$ are parallel but their origins are different. The coordinate transformation between the coordinate systems above is represented by

$$\Delta\psi_2 = \Delta\phi_2 - \frac{b^2}{4a}, \quad \psi_1 = \phi_1 - \frac{b}{2a} \tag{1.14.5}$$

Equations (1.14.3) and (1.14.5), considered simultaneously, yield equation (1.14.4). Thus, the linear function $\Delta\phi_2^{(1)}(\phi_1)$ is indeed absorbed because of its interaction with the predesigned parabolic function $\Delta\phi_2^{(2)}(\phi_1)$. This statement is in agreement with the transformation of equations of second-order curves discussed in the mathematics literature (Korn and Korn, 1968).

The difference between the predesigned parabolic function $\Delta\phi_2^{(2)}(\phi_1)$ and the resulting parabolic function $\Delta\psi_2(\psi_1)$ is the location of points (A^*, B^*) in comparison with (A,B). Figure 1.14.4(a) shows that the symmetrical location of (A,B) is turned into the asymmetrical location of (A^*, B^*). However, the interaction of several functions $\Delta\psi_2(\psi_1)$, determined for several neighboring tooth surfaces, provides a symmetrical parabolic function of transmission errors $\Delta\psi_2(\psi_1)$ as shown in figure 1.14.4(b). (The neighboring tooth surfaces enter into mesh in sequence.) The symmetrical shape of function $\Delta\psi_2(\psi_1)$ determined for several cycles of meshing can be achieved if the parabolic function $\Delta\phi_2^{(2)}(\phi_1)$ is predesigned in the area (fig. 1.14.4(a))

$$\phi_1(B) - \phi_2(A) \geq \frac{2\pi}{N_1} + \frac{b}{a} \tag{1.14.6}$$

The requirement (1.14.6), if observed, provides a continuous symmetrical function $\Delta\psi_2(\psi_1)$ for the range of the meshing cycle $\phi_1 = 2\pi/N_1$.

Appendix A
Parallel Transfer of Sliding Vectors

A sliding vector **a** is determined by its magnitude |**a**| and the line of its action A_0-A (fig. A.1), along which it can be moved. Examples of a sliding vector are forces and angular velocities. In the last case, the line of action A_0-A is the axis of rotation.

The parallel transfer of a sliding vector means that **a** can be substituted by $\mathbf{a}^* = \mathbf{a}$ and a vector moment

$$\mathbf{m} = \mathbf{R} \times \mathbf{a} \tag{A.1}$$

where **R** is a position vector drawn from point O of the line of action of $\mathbf{a}^*$ to any point of the line of action of **a**. It is easy to prove that the vector moment **m**, for instance, can be expressed as (fig. A.1)

$$\mathbf{m} = \overline{OA_0} \times \mathbf{a} \tag{A.2}$$

Taking into account that

$$\mathbf{R} = \overline{OA_0} + \overline{A_0A^*} = \overline{OA_0} + \lambda\mathbf{a} \tag{A.3}$$

where λ is a scalar factor, we obtain

$$\mathbf{m} = \mathbf{R} \times \mathbf{a} = \left(\overline{OA_0} + \lambda\mathbf{a}\right) \times \mathbf{a} = \overline{OA_0} \times \mathbf{a} \tag{A.4}$$

Figure A.2 is an example of a sliding vector as the angular velocity of rotation $\boldsymbol{\omega}$ about the axis A_0-A. The axis of rotation does not pass through the origin O of the considered coordinate system $S(x,y,z)$. The velocity of point M in rotation about A_0-A can be determined using the following procedure:

Step 1: We substitute vector $\boldsymbol{\omega}$ that passes through A_0 by a parallel and equal vector $\boldsymbol{\omega}^*$ that passes through O and the vector moment

$$\mathbf{m} = \overline{OA_0} \times \boldsymbol{\omega} \tag{A.5}$$

where **m** is a free vector that represents the velocity of translation.

Step 2: The motion of point M is represented now in two components: (1) as translation with the velocity **m** and (2) as rotation about OO^* with the angular velocity $\boldsymbol{\omega}^* = \boldsymbol{\omega}$.

Step 3: The velocity of rotation of point M about the axis $\overline{OO^*}$ is determined as

$$\mathbf{v}_{rot} = \boldsymbol{\omega}^* \times \overline{OM} \tag{A.6}$$

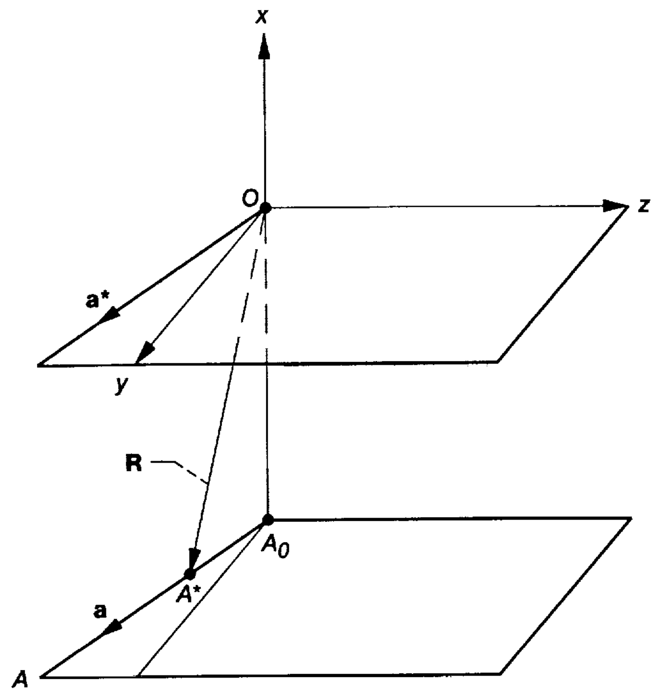


Figure A.1.—Representation of sliding vector.

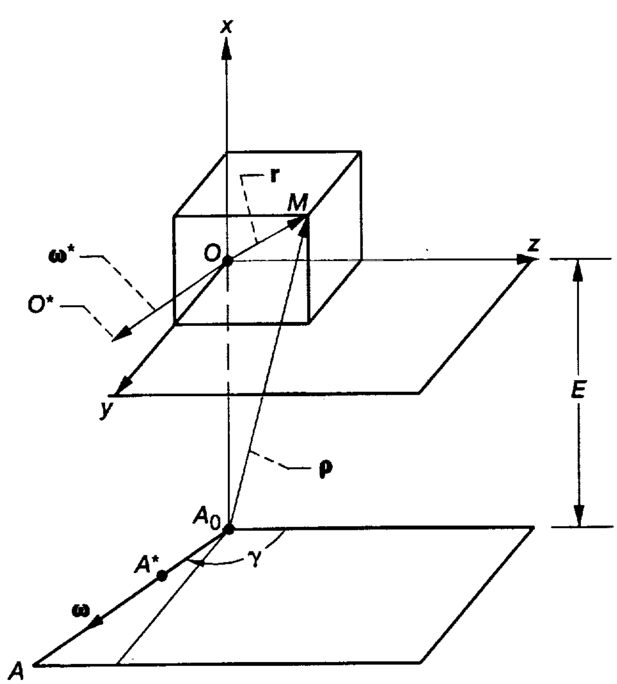


Figure A.2.—Substitution of sliding vector.

Step 4: The whole velocity of point M is determined as

$$\mathbf{v} = \left(\overline{OA_0} \times \boldsymbol{\omega}\right) + \left(\boldsymbol{\omega} \times \overline{OM}\right) = \boldsymbol{\omega} \times (\mathbf{r} - \mathbf{R}) \tag{A.7}$$

where $\mathbf{r} = \overline{OM}$ and $\mathbf{R} = \overline{OA_0}$.

Step 5: It is easy to verify that equation (A.7) may be interpreted as

$$\mathbf{v} = \boldsymbol{\omega} \times \boldsymbol{\rho} \tag{A.8}$$

where $\boldsymbol{\rho} = \mathbf{r} - \mathbf{R} = \overline{A_0M}$ represents the position vector drawn from point A_0 of the axis of rotation A_0-A to point M. A position vector $\boldsymbol{\rho}^*$ may be drawn to M from any point on the line of action A_0-A. For instance, we may consider that $\boldsymbol{\rho}^* = \overline{A^*M}$ (not shown in the figure) and represent $\mathbf{v}$ as

$$\mathbf{v} = \boldsymbol{\omega} \times \overline{A^*M} \tag{A.9}$$

Taking into account that

$$\overline{A^*M} = \overline{A_0^*A_0} + \overline{A_0M} = \lambda^* \, \mathbf{a} + \overline{A_0M} \tag{A.10}$$

we obtain

$$\mathbf{v} = \boldsymbol{\omega} \times \overline{A^*M} = \boldsymbol{\omega} \times \left(\lambda^* \, \mathbf{a} + \boldsymbol{\rho}\right) = \boldsymbol{\omega} \times \boldsymbol{\rho} \tag{A.11}$$

since

$$\boldsymbol{\omega} \times \lambda^* \, \mathbf{a} = 0 \tag{A.12}$$

because of the collinearity of vectors.

Equations (A.7) and (A.8) enable one to determine the velocity of point M by two alternate approaches. Henceforth, we will use the approach that is based on the substitution of sliding vector $\boldsymbol{\omega}$ with an equal vector $\boldsymbol{\omega}^*$ and the vector of translation $\mathbf{m}$.

Problem A.1. Represent analytically vector velocity $\mathbf{v}$ of point M by considering as given (fig. A.2)

$$\boldsymbol{\omega} = \omega[0 \;\; \sin\gamma \;\; \cos\gamma]^T \quad \overline{OA_0} = \mathbf{R} = [-E \;\; 0 \;\; 0]^T \quad \overline{OM} = [\quad]^T$$

Solution.

$$\mathbf{v} = \boldsymbol{\omega} \times (\mathbf{r} - \mathbf{R}) = \begin{vmatrix} \mathbf{i}_f & \mathbf{j}_f & \mathbf{k}_f \\ 0 & \omega\sin\gamma & \omega\cos\gamma \\ x + E & y & z \end{vmatrix} = \omega \begin{bmatrix} z\sin\gamma - y\cos\gamma \\ (x + E)\cos\gamma \\ -(x + E)\sin\gamma \end{bmatrix} \tag{A.13}$$

Appendix B
Screw Axis of Motion: Axodes

B.1 Screw Motion

Generally, the motion of a rigid body may be represented as a screw motion—rotation about and translation along an axis called the axis of screw motion.

Figure B.1.1 shows that gears 1 and 2 perform rotation about crossed axes with angular velocities $\boldsymbol{\omega}^{(1)}$ and $\boldsymbol{\omega}^{(2)}$. The instantaneous relative motion of gear 1 may be represented as a screw motion with parameter p about the S-S axis that lies in plane Π that is parallel to vectors $\boldsymbol{\omega}^{(1)}$ and $\boldsymbol{\omega}^{(2)}$. To determine the location of plane Π and the screw parameter p, we use the following procedure:

Step 1: Substitute vectors $\boldsymbol{\omega}^{(1)}$ and $-\boldsymbol{\omega}^{(2)}$ with equal vectors that lie in plane Π and with respective vector moments

$$\mathbf{m}_f^{(1)} = \left(\overline{O_s O_f}\right)_f \times \boldsymbol{\omega}_f^{(1)}, \qquad \mathbf{m}_f^{(2)} = \left(\overline{O_s O_2}\right)_f \times -\boldsymbol{\omega}_f^{(2)} \tag{B.1.1}$$

The subscript f indicates that vectors in equations (B.1.1) are represented in coordinate system S_f.

Step 2: The angular velocity in relative motion $\boldsymbol{\omega}_f^{(12)}$ is represented as (fig. B.1.2)

$$\boldsymbol{\omega}_f^{(12)} = \boldsymbol{\omega}_f^{(1)} + \left(-\boldsymbol{\omega}_f^{(2)}\right) = -\omega^{(2)} \sin\gamma \; \mathbf{j}_f + \left(\omega^{(1)} - \omega^{(2)} \cos\gamma\right)\mathbf{k}_f \tag{B.1.2}$$

The resulting vector moment is

$$\mathbf{m}_f^{(12)} = \mathbf{m}_f^{(1)} + \mathbf{m}_f^{(2)} = \left(\overline{O_s O_f} \times \boldsymbol{\omega}^{(1)}\right) + \overline{O_s O_2} \times -\boldsymbol{\omega}^{(2)} \tag{B.1.3}$$

Step 3: Vector moment $\mathbf{m}_f^{(12)}$ is a function of $\overline{O_s O_f} = X_f \, \mathbf{i}_f$. Our next goal is to make $\mathbf{m}_f^{(12)}$ collinear to $\boldsymbol{\omega}^{(12)}$, and this requirement can be represented by

$$\left(\overline{O_s O_f}\right)_f \times \boldsymbol{\omega}_f^{(1)} - \left(\overline{O_s O_2}\right)_f \times \boldsymbol{\omega}_f^{(2)} = p\boldsymbol{\omega}_f^{(12)} \tag{B.1.4}$$

Drawings of figure B.1.1 show that

$$\overline{O_s O_2} = \overline{O_f O_2} + \overline{O_s O_f} \tag{B.1.5}$$

Equations (B.1.4) and (B.1.5) yield

$$\left(\overline{O_s O_f}\right)_f \times \boldsymbol{\omega}_f^{(12)} - \left(\overline{O_f O_2}\right)_f \times \boldsymbol{\omega}_f^{(2)} = p\boldsymbol{\omega}_f^{(12)} \tag{B.1.6}$$

Step 4: The determination of $\overline{O_s O_f} = X_f \, \mathbf{i}_f$ is based on the following transformation of equation (B.1.6):

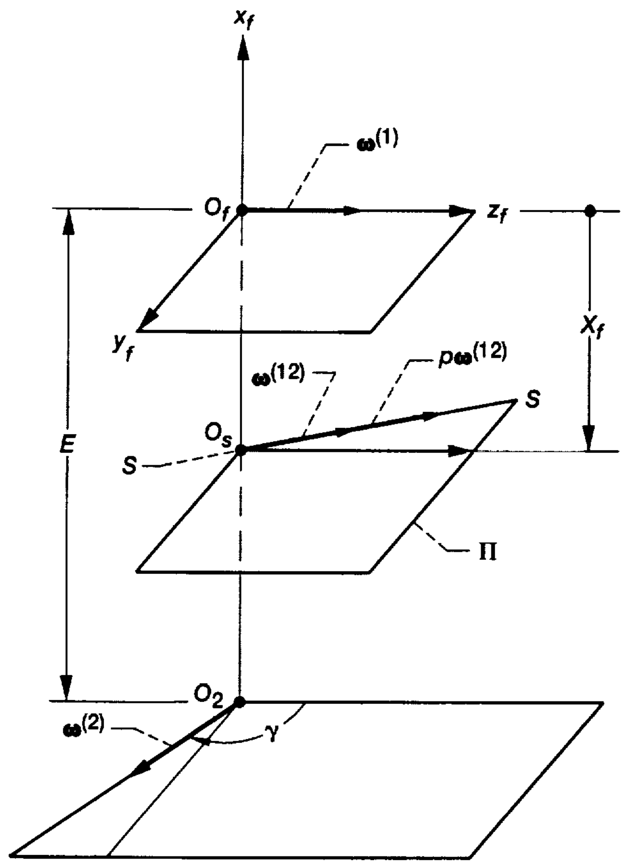


Figure B.1.1.—Screw axis of rotation.

$$\boldsymbol{\omega}_f^{(12)} \times \left[\left(\overline{O_s O_f} \right)_f \times \boldsymbol{\omega}_f^{(12)} \right] - \boldsymbol{\omega}_f^{(12)} \times \left[\left(\overline{O_f O_2} \right)_f \times \boldsymbol{\omega}_f^{(2)} \right] = p\left(\boldsymbol{\omega}_f^{(12)} \times \boldsymbol{\omega}_f^{(12)} \right) = 0 \qquad \text{(B.1.7)}$$

Equation (B.1.7) yields

$$\left(\boldsymbol{\omega}_f^{(12)} \right)^2 \overline{O_s O_f} - \left(\overline{O_f O_2} \right)_f \left(\boldsymbol{\omega}_f^{(12)} \cdot \boldsymbol{\omega}_f^{(2)} \right) = 0 \qquad \text{(B.1.8)}$$

since

$$-\boldsymbol{\omega}_f^{(12)} \left[\boldsymbol{\omega}_f^{(12)} \cdot \left(\overline{O_s O_f} \right)_f \right] = 0 \qquad \text{(B.1.9)}$$

$$-\boldsymbol{\omega}_f^{(2)} \left[\boldsymbol{\omega}_f^{(12)} \cdot \left(\overline{O_f O_2} \right)_f \right] = 0 \qquad \text{(B.1.10)}$$

The scalar products of vectors in equations (B.1.9) and (B.1.10) are equal to zero because of the perpendicularity of cofactor vectors.

Vectors of equation (B.1.8) are represented as (fig. B.1.1)

$$\boldsymbol{\omega}_f^{(12)} = -\omega^{(2)} \sin\gamma \, \mathbf{j}_f + \left(\omega^{(1)} - \omega^{(2)} \cos\gamma \right) \mathbf{k}_f$$

$$= \omega^{(1)} \left[-m_{21} \sin\gamma \, \mathbf{j}_f + \left(1 - m_{21} \cos\gamma \right) \mathbf{k}_f \right] \qquad \text{(B.1.11)}$$

where $m_{21} = \omega^{(2)}/\omega^{(1)}$

$$\overline{O_s O_f} = X_f \, \mathbf{i}_f \tag{B.1.12}$$

$$\overline{O_f O_2} = -E \, \mathbf{i}_f \tag{B.1.13}$$

$$\boldsymbol{\omega}_f^{(2)} = m_{21}\omega^{(1)}\left(\sin\gamma \, \mathbf{j}_f + \cos\gamma \, \mathbf{k}_f\right) \tag{B.1.14}$$

The orientation of $\boldsymbol{\omega}^{(12)}$ and axis *S-S* of screw motion is illustrated by the drawings of figure B.1.2. Equations (B.1.8) and (B.1.11) to (B.1.14) yield

$$X_f = E\frac{m_{21}(m_{21} - \cos\gamma)}{1 - 2m_{21}\cos\gamma + m_{21}^2} \tag{B.1.15}$$

Step 5: The determination of the screw parameter *p* is based on the following transformation of equation (B.1.7):

$$\boldsymbol{\omega}_f^{(12)} \cdot \left[\left(\overline{O_s O_f}\right)_f \times \boldsymbol{\omega}_f^{(12)}\right] - \boldsymbol{\omega}_f^{(12)} \cdot \left[\left(\overline{O_f O_2}\right)_f \times \boldsymbol{\omega}_f^{(2)}\right] = p\left(\boldsymbol{\omega}^{(12)}\right)^2 \tag{B.1.16}$$

Here

$$\boldsymbol{\omega}_f^{(12)} \cdot \left[\left(\overline{O_s O_f}\right)_f \times \boldsymbol{\omega}_f^{(12)}\right] = 0 \tag{B.1.17}$$

because of the collinearity of two cofactor vectors in the triple product.

Equations (B.1.16) and (B.1.17) yield the following expression for *p*:

$$p = E\frac{m_{21}\sin\gamma}{1 - 2m_{21}\cos\gamma + m_{21}^2} \tag{B.1.18}$$

For the case when the rotation of gear 2 is opposite to that shown in figure B.1.1, it is necessary to make m_{21} negative in equations (B.1.15) and (B.1.18). A negative value of X_f in equation (B.1.15) indicates that plane Π intersects the negative axis x_f. A negative value of *p* indicates that vector $p\boldsymbol{\omega}^{(12)}$ is opposite the direction shown in figure B.1.1.

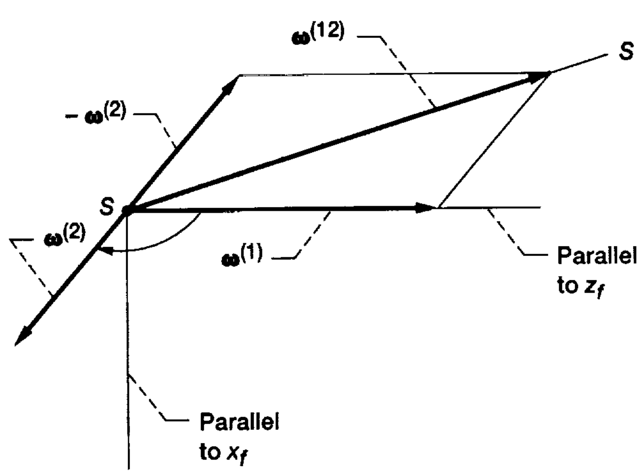


Figure B.1.2.—Relative angular velocity.

B.2 Axodes

For the case of rotation between crossed axes with a constant gear ratio, the axodes are two hyperboloids of revolution. The axode of gear i (i = 1,2) is formed as a family of instantaneous axes of screw motion that is generated in coordinate system S_i when gear i is rotated about its axis. An axode as a hyperboloid of revolution is shown in figure B.2.1. Two mating hyperboloids (fig. B.2.2) contact each other along a straight line that is the axis of screw motion. The relative motion of hyperboloids is rolling with sliding (about and along the axis of screw motion).

In the real design of gears with crossed axes, pitch surfaces instead of axodes are applied. In cases of worm-gear drives and hypoid gear drives, the pitch surfaces are two cylinders and two cones, respectively (Litvin, 1968, 1989). The point of tangency of pitch surfaces is one of the points of tangency of gear tooth surfaces.

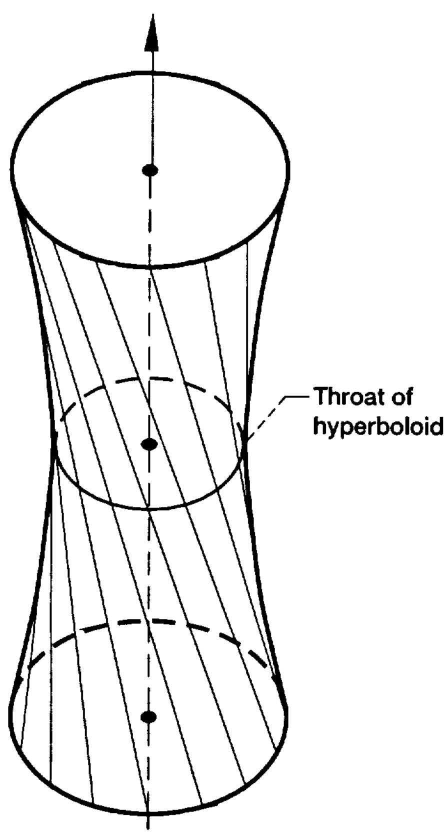


Figure B.2.1.—Hyperboloid of revolution.

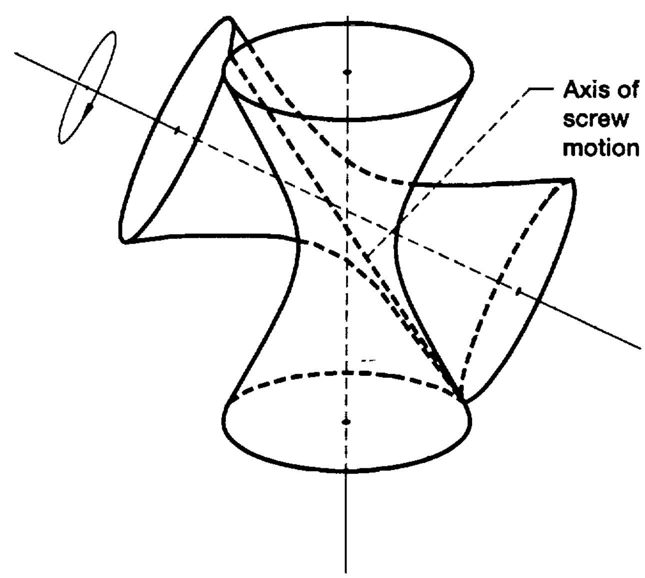


Figure B.2.2.—Mating hyperboloids.

Appendix C
Application of Pluecker's Coordinates and Linear Complex in Theory of Gearing

C.1 Introduction

This appendix covers the basic concepts of Pluecker's coordinates, Pluecker's equation for a straight line, and Pluecker's linear complex (Pluecker, 1865). It is shown that applying the linear complex enables one to illustrate the vector field in the screw motion and to interpret geometrically the equation of meshing and the two-parameter enveloping process.

Many scientists considered Pluecker's ideas to have been a significant contribution to the theory of line

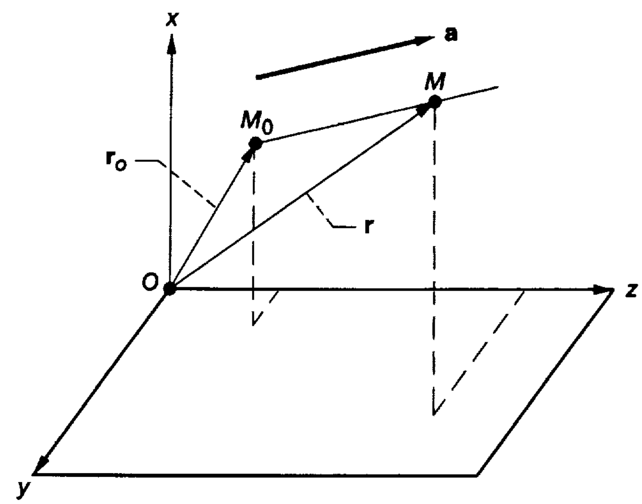


Figure C.2.1.—Representation of straight line.

C.3 Pluecker's Linear Complex for Straight Lines on a Plane

A plane may be determined by considering as given: (1) a point M_o through which passes the plane and (2) a normal **N** to the plane (fig. C.3.1).

A straight line that belongs to plane Π is represented as

$$\mathbf{r} = \mathbf{r}_o + \lambda\mathbf{a} \qquad (\lambda \neq 0), \quad \mathbf{N}\cdot\mathbf{a} = 0 \tag{C.3.1}$$

Equations (C.3.1) yield

$$\mathbf{N}\cdot(\mathbf{r} - \mathbf{r}_o) = 0 \tag{C.3.2}$$

and the plane may be represented by the equation

$$\mathbf{N}\cdot\mathbf{r} = d \tag{C.3.3}$$

where **N**, $\mathbf{r}_o$, and consequently, d are given.

We consider now a line ($\mathbf{g}$, $\mathbf{m}_g$), which belongs to the same plane if the following equation is observed (Merkin, 1962):

$$\mathbf{N}\cdot(\mathbf{g}\times\mathbf{m}_g) = d(\mathbf{g}\cdot\mathbf{g}) \tag{C.3.4}$$

Direct transformations of equation (C.3.4) yield

$$\mathbf{N}\cdot(\mathbf{g}\times\mathbf{m}_g) = \mathbf{N}\cdot[\mathbf{g}\times(\mathbf{r}\times\mathbf{g})] = \mathbf{N}\cdot[\mathbf{r}(\mathbf{g}\cdot\mathbf{g}) - \mathbf{g}(\mathbf{N}\cdot\mathbf{g})] = \mathbf{N}\cdot\mathbf{r}(\mathbf{g}\cdot\mathbf{g}) = d(\mathbf{g}\cdot\mathbf{g}) \tag{C.3.5}$$

We are reminded that $\mathbf{N}\cdot\mathbf{g} = 0$ since **g** must lie in plane Π.

Considering lines ($\mathbf{a}$, $\mathbf{m}_a$) and ($\mathbf{g}$, $\mathbf{m}_g$), we can verify that the couple of lines intersect each other or they are parallel if the following equation is observed:

$$\mathbf{a}\cdot\mathbf{m}_g + \mathbf{m}_a\cdot\mathbf{g} = 0 \tag{C.3.6}$$

The proof of this statement is based on the following:

(1) Direct transformations of equation (C.3.6) yield

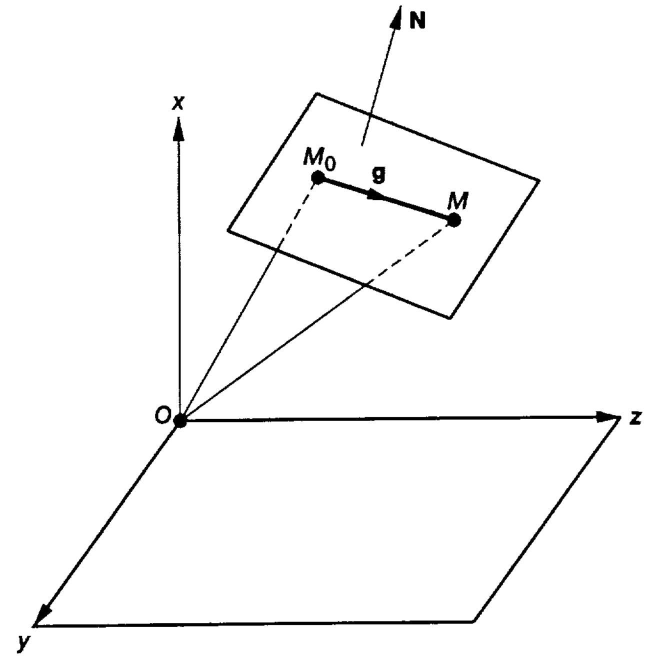


Figure C.3.1.—Representation of straight lines on a plane.

$$\mathbf{a} \cdot \left(\mathbf{r}_g \times g\right) + \left(\mathbf{r}_a \times \mathbf{a}\right) \cdot \mathbf{g} = \left(\mathbf{r}_a - \mathbf{r}_g\right) \cdot (\mathbf{a} \times \mathbf{g}) = 0 \qquad \text{(C.3.7)}$$

(2) Equation (C.3.7) is observed if

(a) $\mathbf{r}_a - \mathbf{r}_g = \mathbf{0}$, $\mathbf{a} \times \mathbf{g} \neq \mathbf{0}$, which means that the lines have a common point.
(b) $\mathbf{r}_a \neq \mathbf{r}_g$, $\mathbf{a} \times \mathbf{g} = \mathbf{0}$, which means that the two lines are parallel.

Equation (C.3.6) represents Pluecker's linear complex of all lines ($\mathbf{g}$, $\mathbf{m}_g$) defined by a given vector **a** and couple ($\mathbf{a}$, $\mathbf{m}_a$).

Line ($\mathbf{g}$, $\mathbf{m}_g$) has six Pluecker's coordinates (g_x, g_y, g_z, m_{gx}, m_{gy}, m_{gz}), but only three are independent because a line has only four independent Pluecker's coordinates and an additional relation between the coordinates is provided by equation (C.3.6).

C.4 Application to Screw Motion

Screw Motion

We consider the rotation of two gears about crossed axes. The relative motion may be determined as the screw motion represented by vector $\boldsymbol{\omega}_s$ and moment $\mathbf{m}_s = p\boldsymbol{\omega}_s$ (fig. C.4.1). Here, $\boldsymbol{\omega}_s$ and $\mathbf{m}_s$ are the angular velocity of rotation *about* and translation *along* the instantaneous axis of screw motion; p is the screw parameter.

Two coordinate systems S_1 and S_2 are rigidly connected to gears 1 and 2. Let us say that gear 1 is movable whereas gear 2 is held at rest. Point M of gear 1 will trace out in S_2 (in infinitesimal motion) the small arc of a helix. The velocity **v** of point M of gear 1 with respect to point M of gear 2 may be represented by vector equation

$$\mathbf{v} = \boldsymbol{\omega}_s \times \mathbf{r} + p\boldsymbol{\omega}_s \qquad \text{(C.4.1)}$$

where **r** is the position vector of point M. Two components of **v** represent the velocities in rotation about and translation along the screw axis.

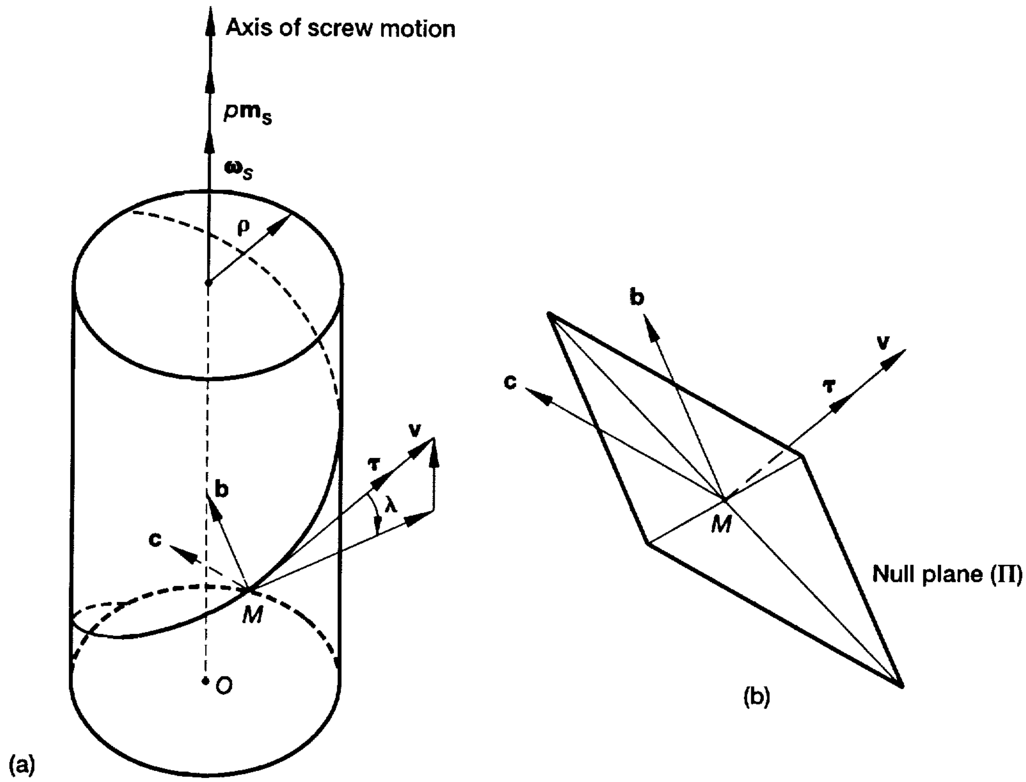


Figure C.4.1.—Presentation of relative velocity in screw motion. (a) Helix and trihedron τ, b, and c. (b) Null plane.

Helix and Trihedron τ, b, and c

The velocity **v** is directed along the tangent **τ** to the helix. Tangent **τ** and the screw axis form the angle (90° −λ) (fig. C.4.1) where

$$\tan\lambda = \frac{p\omega_s}{\omega_s\rho} = \frac{p}{\rho} \tag{C.4.2}$$

and ρ is the shortest distance of point M from the axis of screw motion.

The helix on a cylinder with radius ρ is represented as

$$\mathbf{r}(\theta) = \begin{bmatrix} \rho\cos(\theta+q) & \rho\sin(\theta+q) & p\theta \end{bmatrix} \tag{C.4.3}$$

where θ is the varied helix parameter; q and $\theta = 0$ determine in plane $z = 0$ the location of the helix reference point. The vector field of velocity **v** may be determined as the family of tangents to helices that are traced out on coaxial cylinders of various radii ρ. Any one of the family of helices is a spatial curve. A small arc of the helix is traced out in S_2 by point M of S_1, and it belongs to the osculating plane formed by tangent τ to the helix and the principal normal **c** to the helix (fig. C.4.2).

Trihedron **τ**, **b**, and **c** is formed by tangent **τ** to the helix, binormal **b**, and principal normal **c** (see definitions in Favard, 1957; Litvin, 1994). The unit vectors of the trihedron are represented by

$$\boldsymbol{\tau} = \frac{\mathbf{r}_\theta}{|\mathbf{r}_\theta|}, \qquad \mathbf{b} = \frac{\mathbf{r}_\theta \times \mathbf{r}_{\theta\theta}}{|\mathbf{r}_\theta \times \mathbf{r}_{\theta\theta}|}, \qquad \mathbf{c} = \mathbf{b} \times \tau \tag{C.4.4}$$

After derivations, we obtain

$$\boldsymbol{\tau} = \begin{vmatrix} -\sin(\theta+q)\cos\lambda \\ \cos(\theta+q)\cos\lambda \\ \sin\lambda \end{vmatrix} \tag{C.4.5}$$

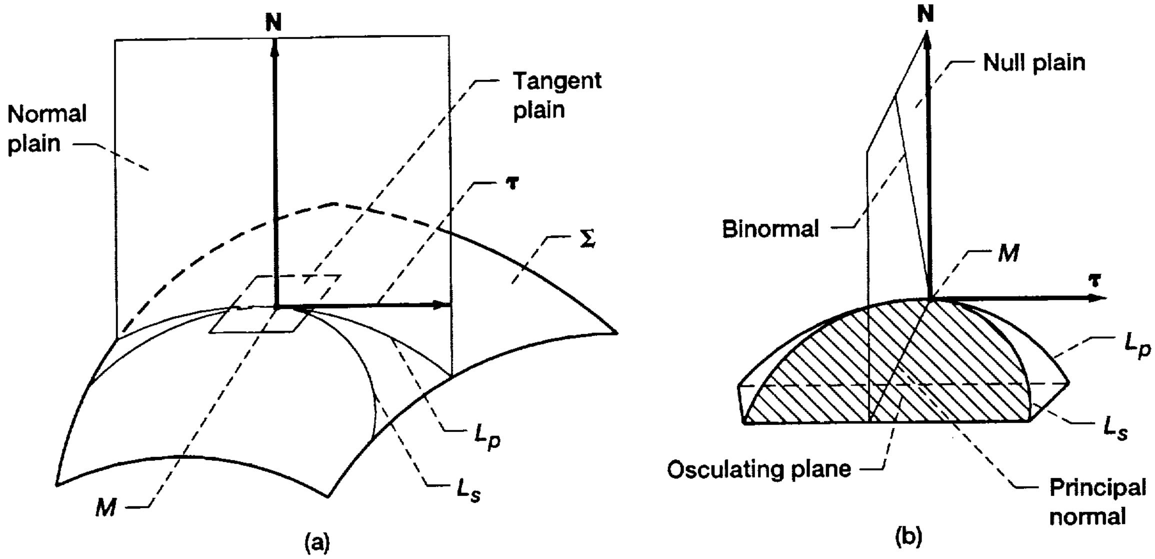


Figure C.4.2.—Illustration of orientation of null plane. (a) Normal plane and tangent plane. (b) Null plane and osculating plane.

$$\mathbf{b} = \begin{vmatrix} \sin\lambda\sin(\theta+q) \\ -\sin\lambda\cos(\theta+q) \\ \cos\lambda \end{vmatrix} \tag{C.4.6}$$

$$\mathbf{c} = \begin{vmatrix} -\cos(\theta+q) \\ -\sin(\theta+q) \\ 0 \end{vmatrix} \tag{C.4.7}$$

Null Plane, Null Axes, and Linear Complex in Screw Motion

We choose in coordinate system S_2 a point M and determine the velocity $\mathbf{v}$ of point M of coordinate system S_1 with respect to the screw motion by using equation (C.4.1). Then, we determine plane Π that passes through M and is perpendicular to $\mathbf{v}$ (fig. C.4.1(b)). Plane Π is called the null plane of point M and vice versa, and point M is called the null point (pole) of Π. Plane Π contains straight lines $(\mathbf{g}, \mathbf{m}_g)$ that pass through pole M. In accordance with the definition of plane Π, we have

$$\mathbf{v}\cdot\mathbf{g} = 0 \tag{C.4.8}$$

Equations (C.4.8) and (C.4.1) yield

$$\mathbf{m}_s\cdot\mathbf{g} + \boldsymbol{\omega}_s\cdot\mathbf{m}_g = 0 \tag{C.4.9}$$

where $\mathbf{m}_s = p\boldsymbol{\omega}_s$, $\mathbf{m}_g = \mathbf{r}\times g = \overline{OM}\times\mathbf{g}$. Equation (C.4.9) may be called Pluecker's linear complex for screw motion.

Any straight line $(\mathbf{g}, \mathbf{m}_g)$ through M that belongs to null plane Π is called a null axis. Particularly, the binormal to the helix is also a null axis.

C.5 Interpretation of Equation of Meshing of One-Parameter Enveloping Process

The basic form of the equation of meshing (see section 1.2) is represented by

$$\mathbf{N}_i \cdot \mathbf{v}_i^{(12)} = 0 \quad (i = 1, 2, f) \tag{C.5.1}$$

where $\mathbf{N}_i$ is the common normal to the contacting surfaces and $\mathbf{v}_i^{(12)}$ is the relative velocity. The scalar product of vectors is invariant to the applied coordinate systems S_1, S_2, and S_f that are rigidly connected to gears 1, 2, and the frame, respectively. Equation (C.5.1) is obtained from the conditions of tangency of the generating and enveloping surfaces.

The application of the concept of Pluecker's linear complex enables one to illustrate the equation of meshing in terms of the null axis and the null plane and the orientation of the normal **N** in the null plane. We now introduce two approaches for determining Pluecker's linear complex.

Approach 1. Since normal vector **N** to the contacting surfaces is perpendicular to the relative velocity $\mathbf{v}_s$, the normal ($\mathbf{N}$, $\mathbf{m}_N$) belongs to the null plane, and an equation similar to (C.4.9) yields the linear complex

$$\mathbf{m}_s \cdot \mathbf{N} + \boldsymbol{\omega}_s \cdot \mathbf{m}_N = 0 \tag{C.5.2}$$

where $\mathbf{m}_s = p\boldsymbol{\omega}_s$ and $\mathbf{m}_N = \overline{OM} \times \mathbf{N}$.

The orientation of the normal ($\mathbf{N}$, $\mathbf{m}_N$) in the null plane is shown in figure C.4.2.

Approach 2. The concept of using screw motion for the determination of relative velocity was presented in Approach 1 and in section C.4. We have considered as well that coordinate system S_1 rigidly connected to gear 1 performs a screw motion with respect to coordinate system S_2 rigidly connected to gear 2 while S_2 and gear 2 are held at rest. However, this concept is not the primary one to be used in kinematics and in the theory of gearing. The following derivations of Pluecker's linear complex are based on the concept that both gears perform rotations about their axes as shown in figure C.5.1. The derivation procedure follows:

Step 1: Rotation about crossed axes is performed with angular velocities $\boldsymbol{\omega}^{(1)}$ and $\boldsymbol{\omega}^{(2)}$ as shown in fig-

$$\boldsymbol{\omega} = \boldsymbol{\omega}^{(1)} - \boldsymbol{\omega}^{(2)}, \qquad \mathbf{m}_{\omega} = \mathbf{E} \times \left(-\boldsymbol{\omega}^{(2)}\right), \qquad \left(\boldsymbol{\omega} \equiv \boldsymbol{\omega}^{(12)}\right) \tag{C.5.3}$$

where $\mathbf{E} = \overline{O_f O_2}$.

Step 2. It is known from differential geometry and the theory of gearing that the generating surface and the envelope are in line contact at every instant. Figure C.5.1(b) shows that point M is a point of the line of contact, $\mathbf{N}$ is the common normal to the contacting surfaces at M, and $\mathbf{r} = \overline{O_f M}$ is the position vector of M.

The normal as a straight line may be represented by the direction vector $\mathbf{N}$ and the respective vector moment $\mathbf{m}_N$. Thus, we consider as given

$$\mathbf{N}, \qquad \mathbf{m}_N = \mathbf{r} \times \mathbf{N} \tag{C.5.4}$$

If $(\mathbf{N}, \mathbf{m}_N)$ belongs to Pluecker's linear complex, the following equation must be observed:

$$\mathbf{m}_{\omega} \cdot \mathbf{N} + \mathbf{m}_N \cdot \boldsymbol{\omega} = 0 \tag{C.5.5}$$

Equations (C.5.3) to (C.5.5) yield

$$-\left(\mathbf{E} \times \boldsymbol{\omega}^{(2)}\right) \cdot \mathbf{N} + (\mathbf{r} \times \mathbf{N}) \cdot \boldsymbol{\omega} = 0 \tag{C.5.6}$$

After transformations of equation (C.5.6), we obtain

$$\mathbf{N} \cdot \left[\left(\boldsymbol{\omega}^{(12)} \times \mathbf{r}\right) - \left(\mathbf{E} \times \boldsymbol{\omega}^{(2)}\right)\right] = \mathbf{N} \cdot \mathbf{v}^{(12)} = 0 \tag{C.5.7}$$

Equation (C.5.7) is observed since it is the equation of meshing of the contacting surfaces. This means that $\mathbf{N}$ is the null axis.

Using Pluecker's terms, we can determine the null plane as the one that passes through contact point M, contains the normal $\mathbf{N}$, and is perpendicular to the moment.

$$\mathbf{m}^* = -\left(\mathbf{E} \times \boldsymbol{\omega}^{(2)}\right) + \left[-\mathbf{r} \times \left(\boldsymbol{\omega}^{(1)} - \boldsymbol{\omega}^{(2)}\right)\right] = \left(\boldsymbol{\omega}^{(1)} - \boldsymbol{\omega}^{(2)}\right) \times \mathbf{r} - \left(\mathbf{E} \times \boldsymbol{\omega}^{(2)}\right) \tag{C.5.8}$$

It is evident that $\mathbf{m}^* = \mathbf{v}^{(12)}$, which means that the null plane is perpendicular to the relative velocity $\mathbf{v}^{(12)}$.

The drawings of figure C.4.2(a) illustrate the concept of planar L_p and spatial L_s curves located on a tooth surface. Both curves are in tangency at point M and their common unit tangent is designated $\boldsymbol{\tau}$. These drawings also show the tangent plane to surface Σ at point M.

The drawings of figure C.4.2(b) represent in an enlarged scale curves L_p and L_s. For the case of the meshing of the envelope and the generating surface, spatial curve L_s is the trajectory that is traced out in relative motion by point M of the moving surface on the surface held at rest. The relative motion is a screw motion (see section C.4), and the relative velocity is directed along the tangent to the helix at point M that coincides with the unit tangent $\boldsymbol{\tau}$ shown in figure C.4.2(b). The null plane at any point M of the tangency of the envelope and the generating surface can be determined as follows. Consider as known at point M the common normal $\mathbf{N}$ to the surfaces and the vector of relative velocity or its unit tangent $\boldsymbol{\tau}$. Then, the null plane at point M is determined to be the plane that passes through normal $\mathbf{N}$ and is perpendicular to $\boldsymbol{\tau}$, the unit vector of relative velocity (fig. C.4.2(b)). We are reminded that the null plane also contains the binormal and the principal normal to the trajectory L_s at point M.

C.6 Interpretation of Equations of Meshing of Two-Parameter Enveloping Process

Introduction

In a two-parameter enveloping process, the generated surface is determined to be the envelope of the two-parameter family of surfaces (Litvin, Krylov, and Erikhov, 1975; Litvin and Seol, 1996).

The generation of an involute helical gear with a grinding worm is used to illustrate the two-parameter enveloping process. The method of grinding the spur and the helical gears with a cylindrical worm and the grinding equipment were developed by the Reishauer Corporation. The meshing of the worm with the gear being ground may be considered as the meshing of two involute helicoids with crossed axes (Seol and Litvin, 1996a).

Figure C.6.1 shows the grinding worm and the gear to be ground and the shortest distance between the axes as the extended one (for the simplification of the drawings). The machining center distance is installed as

$$E_{wg} = r_{pw} + r_{pg} \tag{C.6.1}$$

where r_{pw} and r_{pg} are the radii of the pitch cylinders of the worm and the gear. With such a center distance, points M_1 and M_2 of the pitch cylinders will coincide with each other and the pitch cylinders will be in tangency.

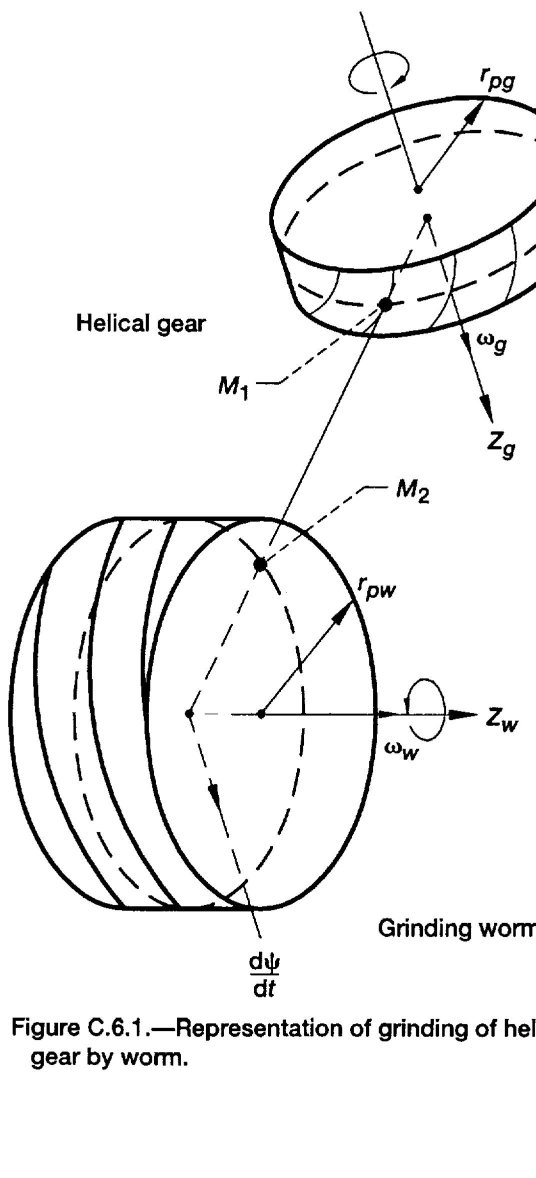


Figure C.6.1.—Representation of grinding of helical gear by worm.

Figure C.6.2 shows the coordinate systems applied to describe the generation process. The movable coordinate systems S_w and S_g are rigidly connected to the worm and the gear, respectively. Coordinate systems S_m, S_f, and S_n are fixed.

For the case when the worm and the gear are both right hand, the crossing angle γ_{wg} between the axes of the worm and the gear is represented by

$$\gamma_{wg} = \lambda_{pw} + \lambda_{pg} \tag{C.6.2}$$

where λ_{pi} $(i = w,g)$ is the lead angle on the pitch cylinder.

The process of gear generation requires two independent sets of motion:

(1) The first set of motions is executed as the rotation of the gear and the worm about axes z_g and z_w, respectively, with angular velocities $\omega^{(g)}$ and $\omega^{(w)}$ related by

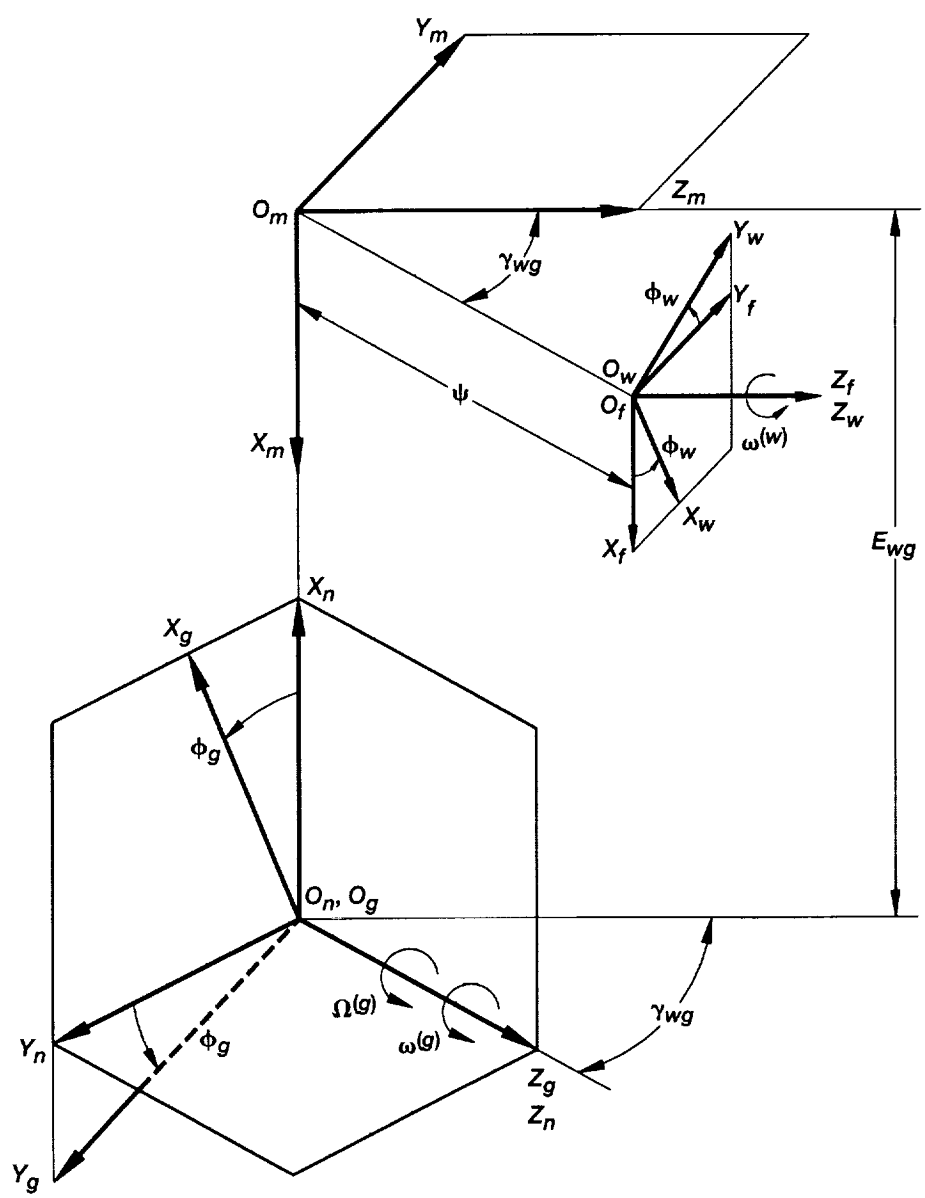


Figure C.6.2.—Coordinate systems for grinding of helical gear by worm.

$$\frac{\omega^{(w)}}{\omega^{(g)}} = N_g \qquad (C.6.3)$$

It is assumed that a one-thread grinding worm will be used; N_g is the number of the gear teeth.

(2) The second set of motions is executed as the translation of the worm with the velocity $d\psi^{(w)}/dt$ in the direction $\overline{O_m O_w}$, which is parallel to the gear axis, and the additional rotation of the gear with the angular velocity $\Omega^{(g)}$. Here,

$$\frac{d\psi^{(w)}}{dt} = p_g \Omega^{(g)} \qquad (C.6.4)$$

where p_g is the gear screw parameter ($p_g > 0$ for a right-hand gear). The second set of motions $d\psi^{(w)}/dt$ and $\Omega^{(g)}$ is required to provide the feed motion during the grinding process.

Derivation of Gear Tooth Surface

The gear tooth surface Σ_g is an envelope of the two-parameter family of worm thread surfaces and is determined by the following equations (Litvin, Krylov, and Erikhov, 1975):

$$\mathbf{r}_g(u,\theta,\phi,\psi) = \mathbf{M}_{gw}(\phi,\psi)\mathbf{r}_w(u,\theta) \qquad (C.6.5)$$

$$\mathbf{N}_i^{(w)} \cdot \mathbf{v}_i^{(wg,\phi)} = 0 \qquad (C.6.6)$$

$$\mathbf{N}_i^{(w)} \cdot \mathbf{v}_i^{(wg,\psi)} = 0 \qquad (C.6.7)$$

where (u,θ) are the surface parameters of the worm thread surface Σ_w; ϕ and ψ are the generalized parameters of motion of the two sets of independent motions mentioned above; $\mathbf{r}_g(u,\theta,\phi,\psi)$ is the vector function that determines in S_g the two-parameter family of surfaces Σ_w generated in S_g; $\mathbf{v}^{(wg,\phi)}$ is the relative velocity that is determined when the variable is the parameter of motion ϕ and ψ is held at rest; $\mathbf{v}^{(wg,\psi)}$ is the relative velocity in the case when the variable is the parameter of motion ψ and ϕ is held at rest; (C.6.6) and (C.6.7) are the equations of meshing and the subscript $i = g,n,m,f,w$ indicates that the respective scalar product of vectors is invariant to the applied coordinate system.

Equations (C.6.5) to (C.6.7) represent the sought-for tooth surface by four related parameters.

Interpretation of Equations of Meshing by Application of Pluecker's Linear Complexes

The transformation of the equation of meshing (C.6.6) is based on the following:

(1) The gear and the worm perform rotational motions about the crossed z_w- and z_g-axes with angular velocities $\boldsymbol{\omega}^{(w)}$ and $\boldsymbol{\omega}^{(g)}$, respectively (fig. C.6.2). Vectors of the scalar product are represented in the S_f coordinate system.

(2) The relative velcoity $\mathbf{v}^{(wg,\phi)}$ is represented as

$$\mathbf{v}_f^{(wg,\phi)} = \left(\boldsymbol{\omega}_f^{(w)} - \boldsymbol{\omega}_f^{(g)}\right) \times \mathbf{r}_f - \overline{O_f O_g} \times \boldsymbol{\omega}^{(g)} \qquad (C.6.8)$$

Then we obtain the following equation of meshing (C.6.6):

$$\left[\left(\boldsymbol{\omega}_f^{(w)} - \boldsymbol{\omega}_f^{(g)}\right) \times \mathbf{r}_f - \overline{O_f O_g} \times \boldsymbol{\omega}^{(g)}\right] \cdot \mathbf{N}_f^{(w)} = 0 \qquad (C.6.9)$$

where $\mathbf{r}_f$ is the position vector of the point of tangency of surfaces Σ_w and Σ_g.

We interpret the equation of meshing (C.6.9) by Pluecker's linear complex using the following procedure:

Step 1: The relative motion during generation is represented by vector $\boldsymbol{\omega}$ and moment $\mathbf{m}_\omega$ that are represented as

$$\boldsymbol{\omega} = \boldsymbol{\omega}^{(w)} - \boldsymbol{\omega}^{(g)} = \boldsymbol{\omega}^{(w,g)}, \qquad \mathbf{m}_{\omega} = -\overline{O_f O_g} \times \boldsymbol{\omega}^{(g)} \tag{C.6.10}$$

Step 2: The common normal **N** to the contacting surfaces and the moment $\mathbf{m}_N$ of **N** represented as

$$\mathbf{m}_N = \overline{O_f M} \times \mathbf{N} = \mathbf{r}_f \times \mathbf{N} \tag{C.6.11}$$

are related to $\boldsymbol{\omega}$ and $\mathbf{m}_{\omega}$ by the equation of the linear complex

$$\boldsymbol{\omega} \cdot \mathbf{m}_N + \mathbf{m}_{\omega} \cdot \mathbf{N} = 0 \tag{C.6.12}$$

Equation (C.6.12) yields the equation of meshing (C.6.9). The null plane passes through the point of tangency M, contains the normal **N**, and is perpendicular to $\mathbf{v}^{(wg,\phi)}$. The normal **N** is the null axis of the linear complex (see section C.3).

Let us now consider the interpretation of the equation of meshing (C.6.7) by application of Pluecker's linear complex. The second set of the paremeters of motion is represented by the couple of vectors $(\boldsymbol{\Omega}, \mathbf{m}_{\Omega})$, which can be obtained from equations that relate the motion of the worm with respect to the gear. Here,

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}^{(\omega)} - \boldsymbol{\Omega}^{(g)} = -\boldsymbol{\Omega}^{(g)} \tag{C.6.13}$$

since $\boldsymbol{\Omega}^{(w)} = \mathbf{0}$,

$$\mathbf{m}_{\Omega} = \frac{d\psi^{(w)}}{dt} \tag{C.6.14}$$

The other couple of vectors is represented by

$$\mathbf{N}, \qquad \mathbf{m}_N$$

where $\mathbf{m}_N$ is represented by equation (C.6.11).

Both couples of vectors are related by the equation of the linear complex:

$$\boldsymbol{\Omega} \cdot \mathbf{m}_N + \mathbf{m}_{\Omega} \cdot \mathbf{N} = 0 \tag{C.6.15}$$

that yields

$$-\boldsymbol{\Omega}^{(g)} \cdot \left(\mathbf{r}_f \times \mathbf{N}\right) + \frac{d\psi^{(w)}}{dt} \cdot \mathbf{N} = 0 \tag{C.6.16}$$

Equation (C.6.16), after simple transformations, may be represented as

$$\left(-\boldsymbol{\Omega}^{(g)} \times \mathbf{r}_f + \frac{d\psi^{(w)}}{dt}\right) \cdot \mathbf{N} = 0 \tag{C.6.17}$$

Equations (C.6.17) yields

$$\mathbf{v}_f^{(wg,\psi)} \cdot \mathbf{N}_f^{(w)} = 0 \tag{C.6.18}$$

The null plane for the second set of parameters of motion passes through the same point M and is perpendicular to $\mathbf{v}^{(wg,\psi)}$ but not to $\mathbf{v}^{(wg,\phi)}$. The common surface normal **N** is the null axis of the second linear complex.

Chapter 2

Development of Geometry and Technology

2.1 Introduction

Errors in gear alignment and manufacture may shift the bearing contact, turn it into edge contact, and cause transmission errors that, as we are reminded, are the main source of vibration. The purpose of this chapter is to present the latest developments in gear geometry and technology directed at improving bearing contact and reducing transmission errors.

The main errors of alignment and manufacture are as follows: the error of the shaft angle, the shortest center distance, the leads in the case of helical gears, the errors in machine-tool settings (errors of orientation and location of the tool with respect to the gear being generated), and the errors of the circular pitches. In addition, we have to take into account the deflection of the teeth and shafts under load. To avoid or at least to reduce such defects, it becomes necessary to substitute the line contact of the gear tooth surfaces by the point contact and then, in addition, to modify the gear tooth surfaces. The modification of gear geometry is based on the proper deviation of the gear tooth surfaces from the theoretical ones. The surface deviation can be provided (1) in the longitudinal direction with the contact path in the profile direction (the direction across the tooth surface) and (2) in the profile direction with the longitudinal direction of the contact path. In some cases, both types of deviation must be provided simultaneously, but one of them must be the dominant.

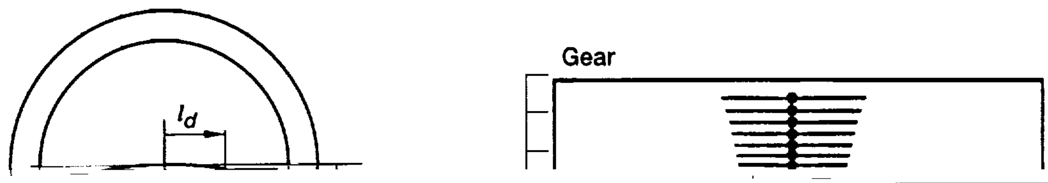
The desired modification of gear geometry becomes possible by applying inventive methods of gear technology such as (1) the mismatch of tool surfaces for the generation of spiral bevel gears and hypoid gears, (2) the varied plunge of the tool for the generation of spur and helical gears, (3) the application of an oversized hob for the generation of worm gears. Some of these examples are considered in the following sections. We emphasize that in all of such cases of gear manufacture, it is important to provide a predesigned parabolic function of transmission errors and to reduce their magnitude (see section 1.14), which will improve the conditions of the transfer of meshing while one pair of teeth is changed for the neighboring one.

This chapter summarizes the developments achieved at the Gear Research Laboratory of the University of Illinois at Chicago. Details are given in Litvin and Kin (1992); Litvin and Hsiao (1993); Litvin and Lu (1995); Litvin et al. (1995, 1996a, 1996b); Litvin, Chen, and Chen (1995); Litvin and Feng (1996, 1997); Litvin, Wang, and Handschuh (1996); Litvin and Seol (1996); Seol and Litvin (1996a, 1996b); Zhang, Litvin, and Handschuh (1995); and Litvin and Kim (1997).

2.2 Modification of Geometry of Involute Spur Gears

Localization of Contact

Spur gears are very sensitive to the misalignment of their axes, which causes an edge contact. The sensitivity of the gear drive to such a misalignment can be reduced by localizing the bearing contact. The localization of the contact as proposed in Litvin et al. (1996b) can be achieved by plunging the grinding disk in the generation of the pinion by form-grinding (fig. 2.2.1). The mating gear is generated as a conventional involute gear. The plunging means that during the pinion generation, the shortest distance between the axes of the grinding disk and the pinion will satisfy the equation (fig. 2.2.1)



l_d
Gear

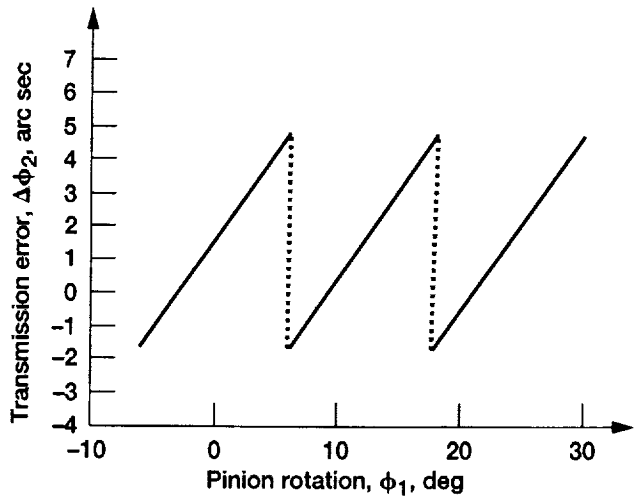


Figure 2.2.3.—Transmission errors caused by errors of pressure angle $\Delta\alpha$ = 3 arc min.

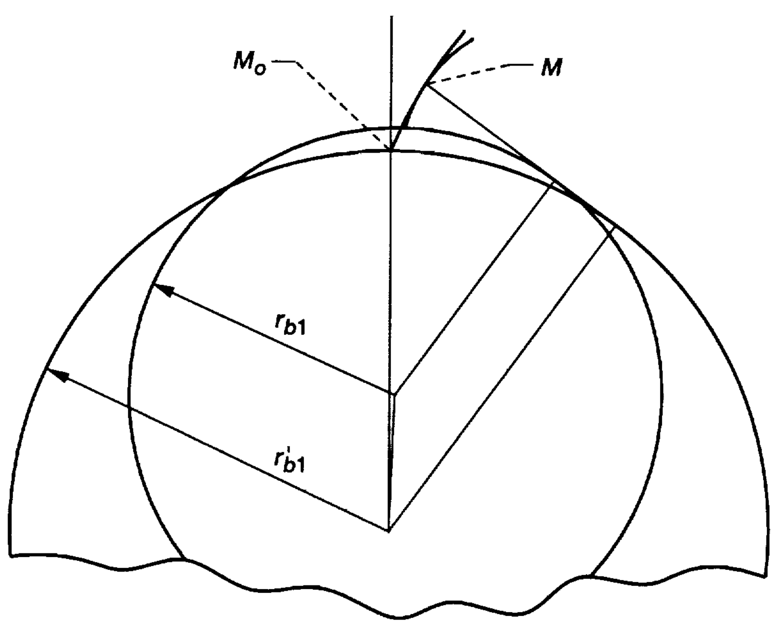


Figure 2.2.4.—Theoretical and modified profiles of spur pinion.

2.3 Modification of Involute Helical Gears

Computerized Investigation of Misaligned Conventional Involute Helical Gears

Aligned involute helical gears are in line contact at every instant, as shown in figure 2.3.1. Misalignment caused by changes in the shaft angle, the lead, and the normal profile angle of one of the mating gears causes edge contact instead of surface-to-surface tangency. Edge contact means tangency of the edge of one gear with the tooth surface of the mating gear (see section 1.11). The edge and the surface are in mesh at every instant at a point instead of on a line. An example of an edge contact caused by the change of the shaft angle $\Delta\gamma$ or the change of the pinion lead angle $\Delta\lambda_{p1}$ is shown in figure 2.3.2. The edge contact caused by $\Delta\gamma$ or $\Delta\lambda_{p1}$ is also accompanied by transmission errors, as shown in figure 2.3.3. However, the change in the normal profile angle does not cause transmission errors, only an edge contact.

In the case of a change in the center distance E, the gear tooth surfaces are still in a line contact similar to those shown in figure 2.3.1. However, an error in ΔE causes a change in the backlash and in the pressure angle of the gear drive.

There is a mistaken impression that a change in the lead is sufficient to shift the bearing contact from the edge to the central position and avoid transmission errors. Our investigation shows that a combination of $\Delta\gamma$- and $\Delta\lambda_{p1}$-errors will enable one to avoid transmission errors and obtain the favorable contact path shown in figure 2.3.4 if and only if

$$\Delta\gamma = \left|\Delta\lambda_{p1}\right| \tag{2.3.1}$$

The signs of $\Delta\gamma$ and $\Delta\lambda_{p1}$ depend on the hand of the helix.

Equation (2.3.1) must be observed with great precision because even a small difference between $\Delta\gamma$ and $|\Delta\lambda_{p1}|$ (or $|\Delta\lambda_{p2}|$) will cause an edge contact. Therefore, the change of the lead, if it is not accompanied by applying a predesigned parabolic function of transmission errors (see below), is not a convenient way to avoid an edge contact. In conclusion, we must emphasize that the computerized investigation of misaligned helical gears is a complex mathematical problem because the Jacobian of the system of equations that relate the surface parameters and parameters of motion is close to zero (see section 1.11). What is required for the computerized investigation of a misaligned gear drive is a determination of the proper initial guess.

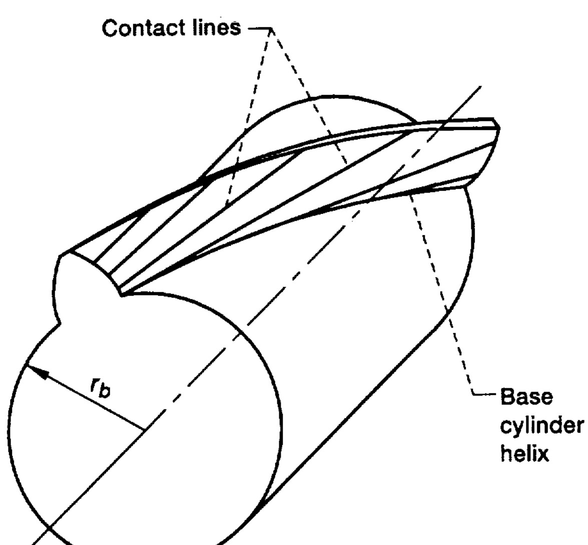


Figure 2.3.1.—Contact lines on tooth surfaces of helical gear.

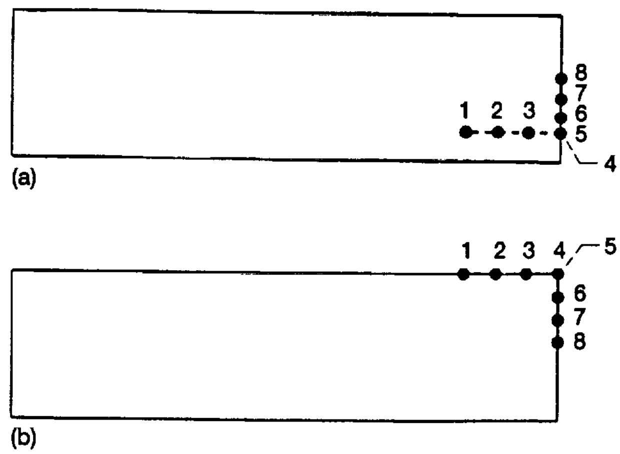


Figure 2.3.2.—Edge contact caused by $\Delta\gamma$ or $\Delta\lambda_{p1}$ = 3 arc min. (a) Pinion tooth surface. (b) Gear tooth surface.

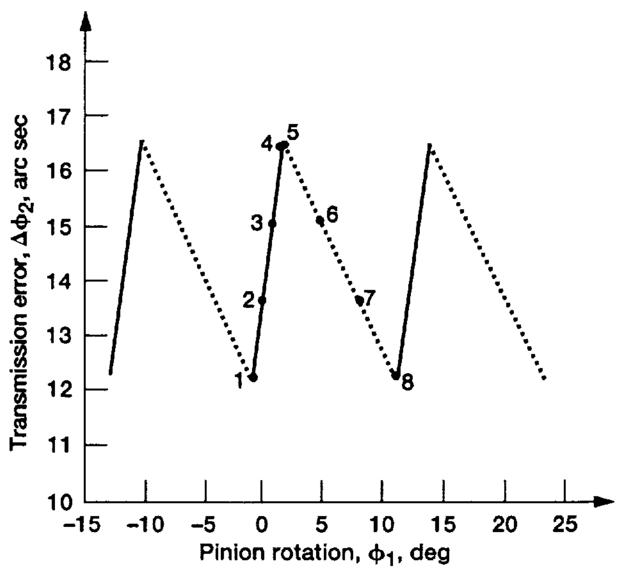


Figure 2.3.3.—Function of transmission errors caused by $\Delta\gamma$ or $\Delta\lambda_{p1}$ = 3 arc min.

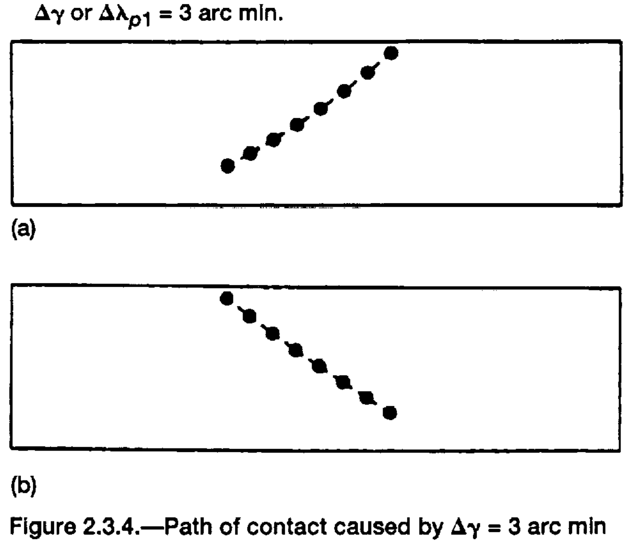
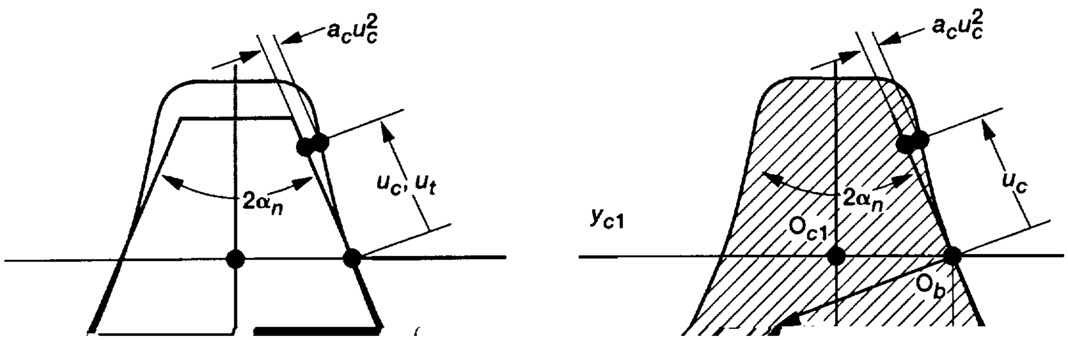


Figure 2.3.4.—Path of contact caused by $\Delta\gamma$ = 3 arc min and $\Delta\lambda_{p1}$ = −3 arc min. (a) Pinion tooth surface. (b) Gear tooth surface.



$a_c u_c^2$
u_c, u_t
$2\alpha_n$
$a_c u_c^2$
u_c
$2\alpha_n$
y_{c1}
O_{c1}
O_b

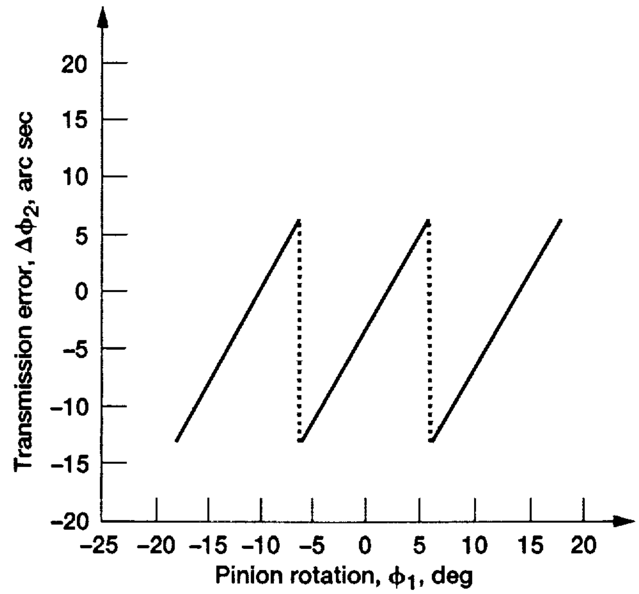


Figure 2.3.7.—Function of transmission errors for modified involute helical gear drive when $\Delta\gamma$ = 3 arc min.

Profile Modification

The sensitivity of helical gears to misalignment has led designers and manufacturers to crown tooth surfaces. The most common method of crowning is profile modification; that is, the profile of the cross section of one of the mating gears (usually of the pinion) is deviated from the conventional involute profile. This can be achieved as proposed in Litvin et al. (1995) by the application of two imaginary rack-cutters shown in figure 2.3.5. The rack-cutters are rigidly connected and generate the pinion and the gear separately. Figure 2.3.5(a) shows both rack-cutter profiles. The normal section of the pinion rack-cutter that generates the pinion space is shown in figure 2.3.5(b), and the normal section of the gear rack-cutter that generates the gear tooth is shown in figure 2.3.5(c). The deviation of the pinion rack-cutter profile from the gear rack-cutter profile is represented by a parabolic function with the parabola coefficient a_c. The tooth surfaces in the case of profile modification are in point contact, and the path of contact is a helix, as shown in figure 2.3.6.

It can be easily verified that the profile modification enables one to localize the bearing contact and to avoid an edge contact that might be caused by gear misalignment. However, the discussed modification does not allow the elimination of transmission errors caused by misalignment, as shown in figure 2.3.7. Therefore, to reduce the level of noise and vibration, it is necessary to provide a predesigned parabolic function of transmission errors in addition to profile modifications.

2.4 Development of Face-Gear Drives

Introduction

Face-gear drives have found application in the transformation of rotation between intersected and crossed axes. The Fellows Corporation invented the method of manufacturing face gears by a shaper which is based on simulating the meshing of the generating shaper with the face gear by cutting.

Face-gear drives was the subject of intensive research presented in Davidov (1950) and Litvin (1968, 1994). New developments in this area were supported by NASA Lewis Research Center and McDonnell Douglas Helicopter Systems. An important application of face-gear drives in helicopter transmissions is based on the concept of torque split (fig. 2.4.1). There are other examples of the successful application of face-gear drives in transmissions.

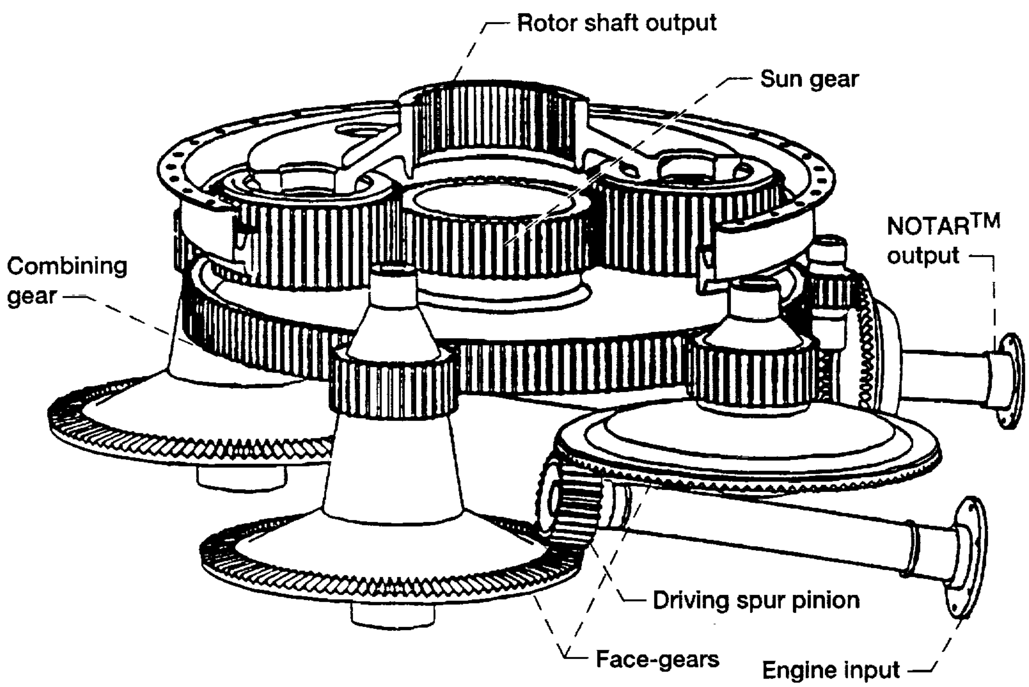


Figure 2.4.1.—Application of face-gear drives in helicopter transmissions.

Pitch Surfaces

The pitch surfaces for bevel gears are two cones (fig. 2.4.2(a)) with pitch angles γ_1 and γ_2. The line of tangency OI of the pitch cones is the instantaneous axis of rotation. The cones roll over each other in relative motion that can be represented as rotation about OI. The pitch surfaces in a face-gear drive are the pinion pitch cylinder of radius r_{p1} and the face-gear pitch cone with the pitch angle γ (fig. 2.4.2(b)). The tooth element proportions in bevel gears are related by the application of the pitch line OI (fig. 2.4.2(a)) as the middle line of the teeth. The tooth height in face-gear drives is constant, and the middle line of the teeth is the line of tangency $O'M$ of the pinion pitch cylinder of radius r_{p1} and the gear pitch cone with apex angle γ. It will be shown in the next section that since $O'M$ does not coincide with the instantaneous axis OI, the face-gear teeth become sensitive to undercutting and pointing.

Generation of Face Gears

The process of generation by a shaper is shown in figure 2.4.3. Grinding and hobbing by a worm is a recently developed process for face-gear generation. Methods for determining the surface of the worm (fig. 2.4.4) and its dressing were developed by researchers at the University of Illinois at Chicago and at McDonnell Douglas Helicopter Systems.

Avoidance of Undercutting and Pointing

The tooth of a face gear in three-dimensional space is shown in figure 2.4.5. The fillet surface is generated by the top edge of the shaper tooth. Line L^* is the line of tangency of the working part of the tooth surface and the fillet. Undercutting may occur in plane A and pointing in plane B. The tooth surface is covered by lines of tangency L of the face gear with the generating shaper.

The tooth of a face gear may be undercut and pointed in the process of generation by a shaper. These defects can be avoided by properly designing the tooth length of the face gear. Dimensions L_1 and L_2 determine the zone that is free of undercutting and pointing. The equations for computing L_1 and L_2 are represented in Litvin (1994). The length of teeth ($L_1 - L_2$) with respect to the diametral pitch P_d may be represented by the unitless coefficient

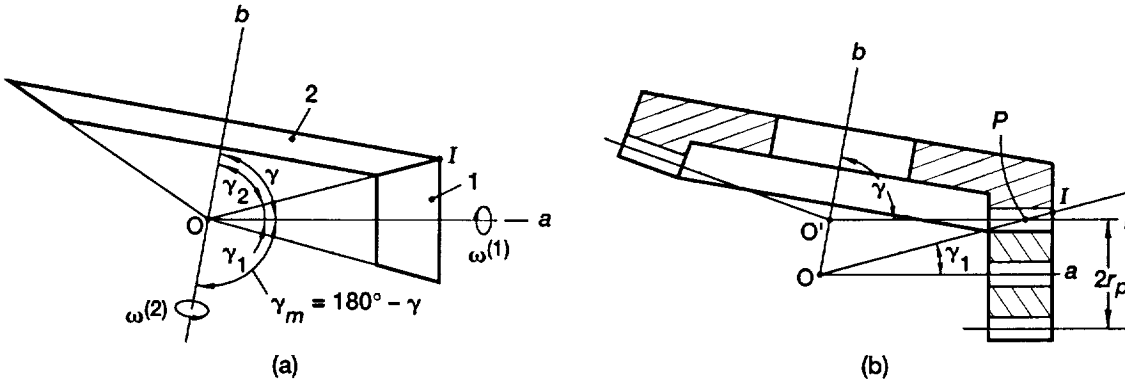
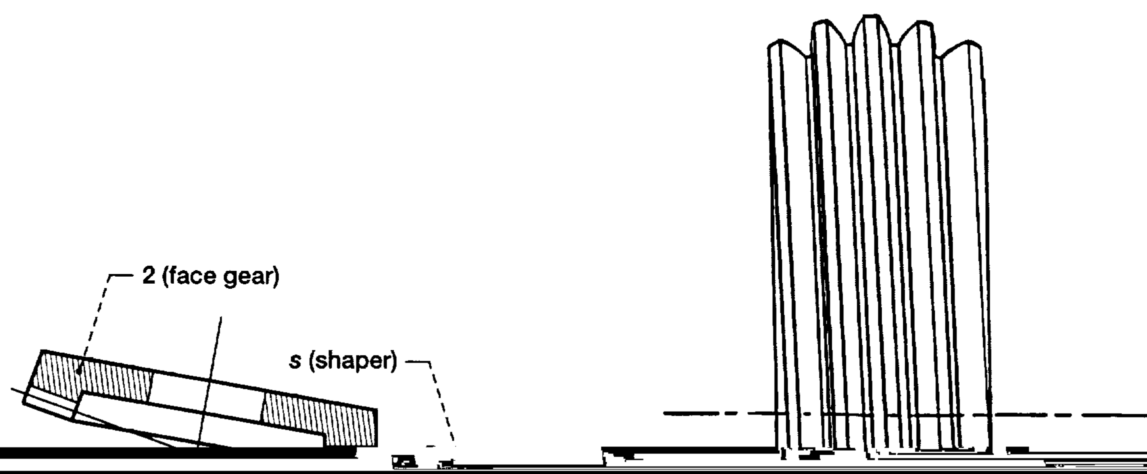


Figure 2.4.2.—Pitch surfaces of bevel gears and gears of face drive. (a) Bevel gears. (b) Face gears.



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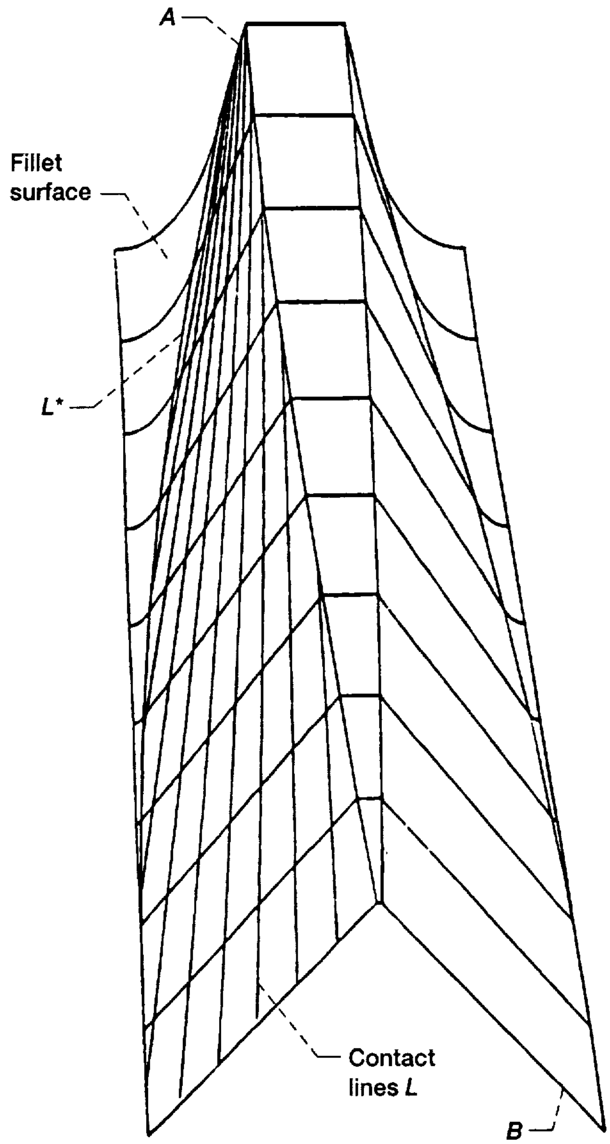


Figure 2.4.5.—Tooth of face gear in three-dimensional space.

$$c = (L_2 - L_1)P_d$$

whose value depends on the gear ratio $m_{12} = N_2/N_1$ of the face gear. It is recommend that $m_{12} > 4$ be used to obtain $c > 10$.

Localization of Bearing Contact

When the number of shaper teeth equals the number of pinion teeth, misalignment may cause the separation of the face-gear teeth and the pinion and then the loss of their edge contact. To avoid this defect, it becomes necessary to localize the bearing contact between the pinion and the face gear. This localization can be achieved by applying a shaper with the tooth number $N_s > N_p$ so that $N_s - N_p = 1$–3, where N_p is the pinion tooth number. Another approach is based on varying the tool plunging (grinding disk or cutter) in the pinion generation process (see section 2.2). In such an approach, $N_s = N_p$.

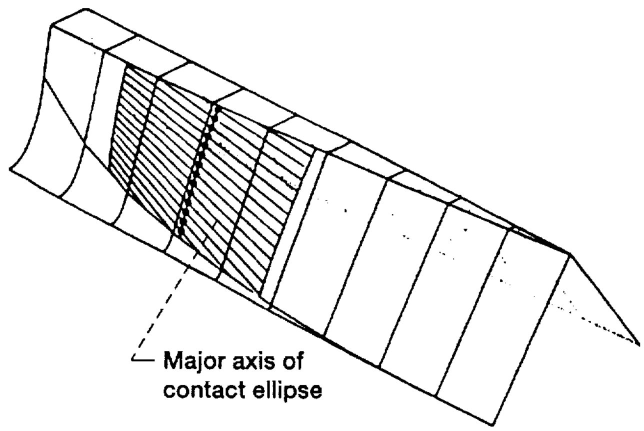


Figure 2.4.6.—Localized bearing contact; $\Delta N = 3$; $\Delta E = 0.1$ mm.

Results of Tooth Contact Analysis (TCA)

A computer program has enabled researchers to investigate the influence of the following alignment errors: ΔE, the shortest distance between axes that are crossed but are not intersected; Δq, the axial displacement of the face gear; $\Delta\gamma$, the change in the shaft angle formed by intersected axes. It was discovered that such alignment errors do not cause transmission errors, which is the great advantage of using a face-gear drive with an involute pinion. However, such errors cause a shift in the bearing contact, as shown in figure 2.4.6. If the pinion is generated by a plunged tool (see sec. 2.2), there is a good possibility of compensating for this shift due to the axial displacement of the pinion.

2.5 Development of Geometry of Face-Milled Spiral Bevel Gears

Introduction

An important contribution of The Gleason Works engineers is the development of spiral bevel gear drives and hypoid gear drives with localized bearing contact and parabolic-type transmission errors (Stadtfeld, 1993, 1995). The research conducted at the Gear Research Laboratory of the University of Illinois at Chicago (Litvin et al., 1996b; Litvin, Wang, Handschuh, 1996; Zhang, Litvin, and Handschuh, 1995) was directed at the modification and improvement of the existing geometry of spiral bevel gears. The developed projects covered two types of face-milled spiral bevel gears: uniform teeth and tapered teeth.

Modification of Geometry of Spiral Bevel Gears With Uniform Teeth

Theoretically, ideal spiral bevel gears with zero transmission errors can be generated if the following conditions are observed:

(1) Two imaginary generating surfaces are rigidly connected to each other and separately generate the pinion and gear tooth surfaces. The generating surfaces are mismatched but they are in tangency along a line. The generating surfaces produce spiral bevel gears with a constant tooth height.

(2) Two types of bearing contact are provided by this method of generation:

(a) The mismatched generating surfaces Σ_{t1} and Σ_{t2} are two cones in tangency along a common generatrix (fig. 2.5.1). The difference in mean radii R_p and R_g determines the mismatch of the generating surfaces. The bearing contact is directed across the tooth surfaces.

(b) The mismatched generating surfaces Σ_{t2} and Σ_{t1} are a cone and a surface of revolution in tangency along a circle of radius $R_g = R_p$ (fig. 2.5.2). The axial section of the surface of revolution is a circle of radius R_1. The bearing contact of the generated spiral bevel gears is directed along the tooth surfaces.

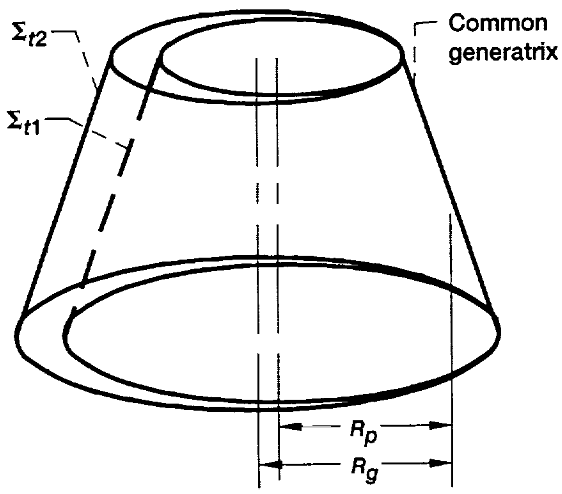


Figure 2.5.1.—Application of two generating cones.

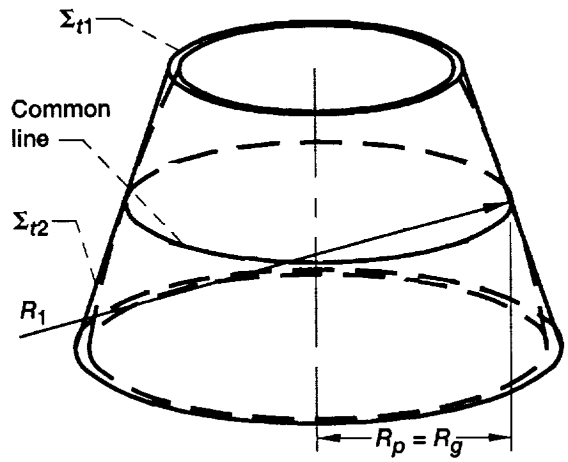


Figure 2.5.2.—Application of generating cone and generating surface of revolution.

These generation methods provide conjugated tooth surfaces that are in point tangency at every instant. However, in reality such methods cannot be applied because the generated spiral bevel gears are sensitive to misalignment that will cause a shift in bearing contact and the transmission errors of the type shown in figure 1.14.1(b). Such transmission errors are the source of vibrations.

Low-Noise Spiral Bevel Gears With Uniform Tooth Height

Litvin, Wang, and Handschuh (1996) show that the defects in gearing just discussed can be avoided by the application of the generation method which requires that a couple of the generating surfaces be in point contact and that the surfaces be properly mismatched (fig. 2.5.3). The gear generating surface is a cone, and the pinion generating surface is a surface of revolution. The gear cutting blades are straight-line blades (fig. 2.5.4); the pinion cutting blades are circular arcs (fig. 2.5.5). The method of local synthesis developed in Litvin (1994) and applied for spiral bevel gear generation enables us to determine those design parameters of the generating

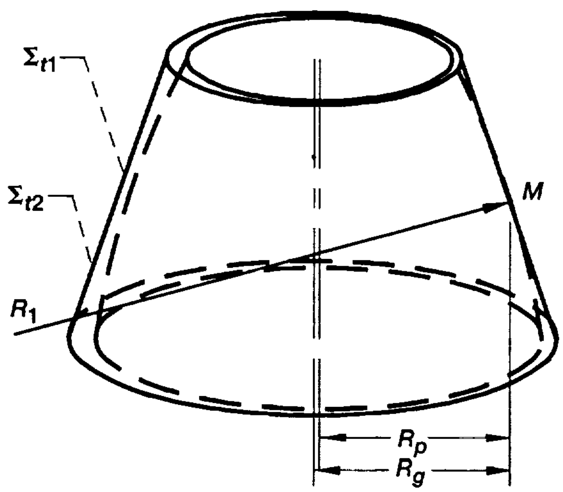


Figure 2.5.3.—Mismatched generating surfaces in point contact.

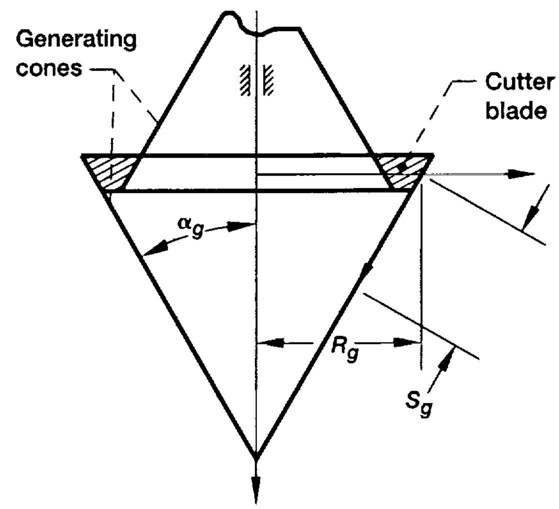


Figure 2.5.4.—Gear generating cones.

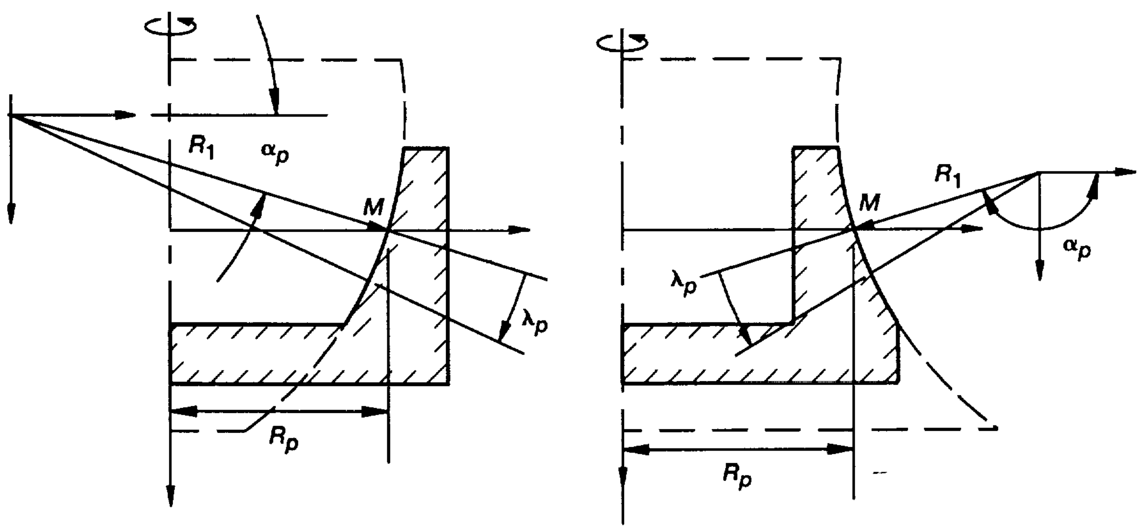


Figure 2.5.5.—Convex and concave sides of generating blades and pinion generating surfaces of revolution.

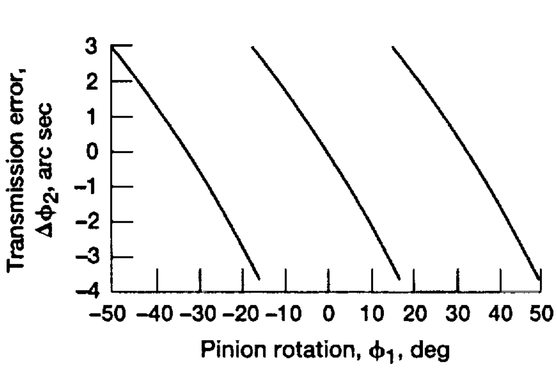


Figure 2.5.6.—Transmission errors for misaligned gear drive; $\Delta\gamma$ = 3 arc min.

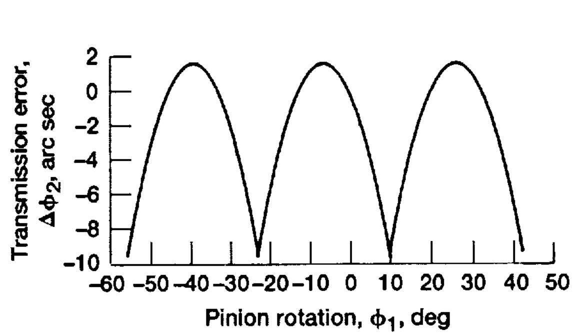


Figure 2.5.7.—Transmission errors for misaligned gear drive with mismatched gear tooth surfaces; $\Delta\gamma$ = 3 arc min.

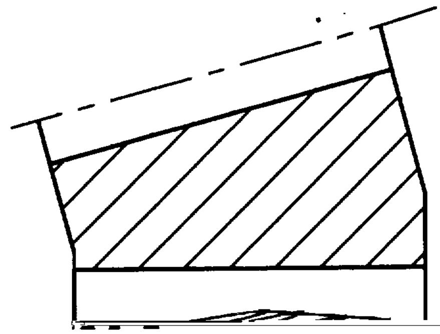


Figure 2.5.8.—Longitudinal bearing contact for misaligned gear drive; $\Delta\gamma = 3$ arc min.

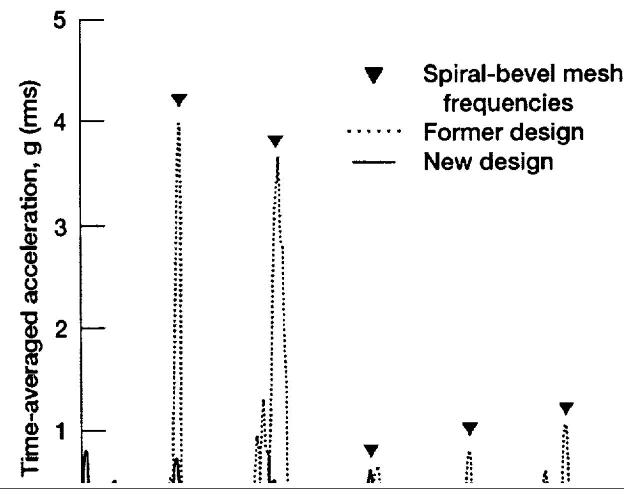


Figure 2.5.9.—Results of noise reduction.

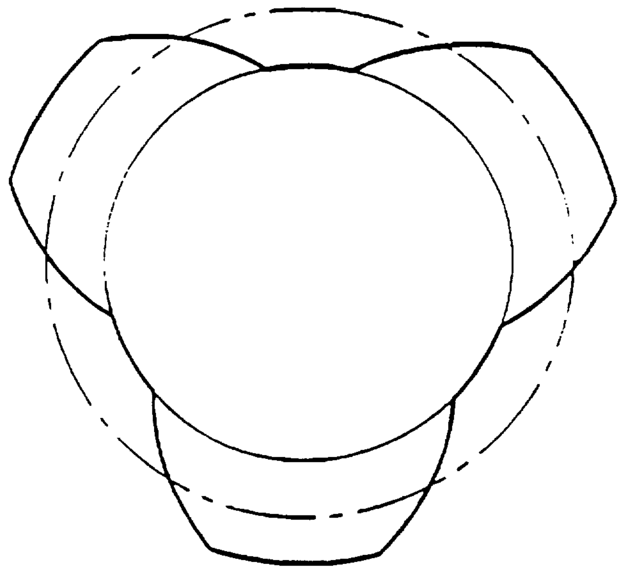
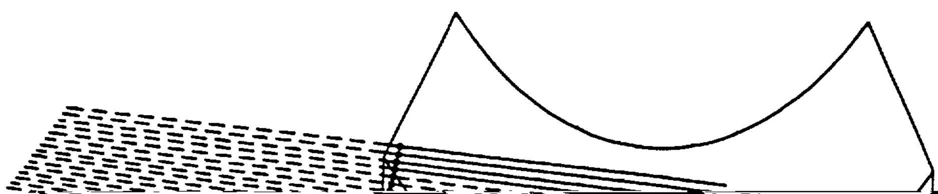


Figure 2.6.1.—Cross section of three-thread worm.



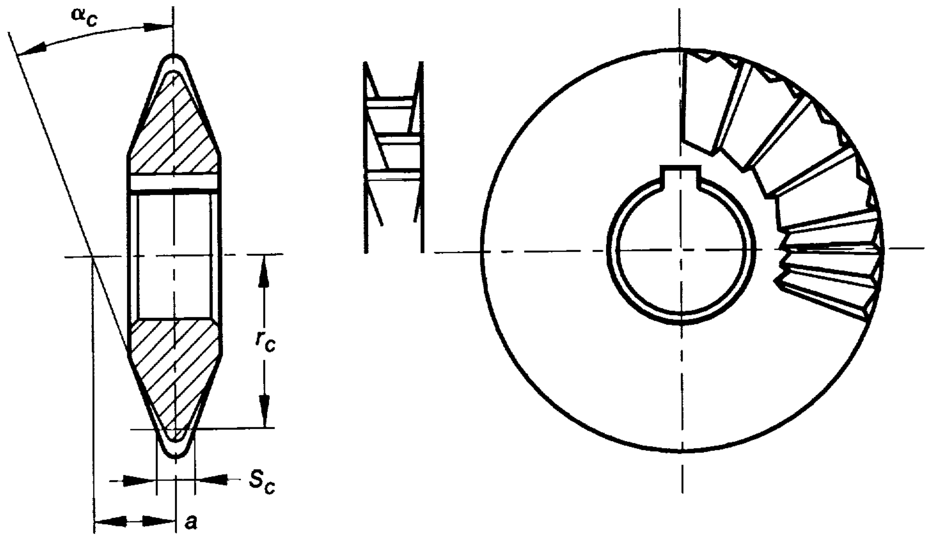


Figure 2.6.5.—Tool for generation of ZK-worms.

The generation of the ZF-worms (Flender worms) is based on the application of a grinding disk whose axial section is represented in figure 2.6.7(a). The axial profile of the tool is a circular arc. The worm in the process of generation performs a screw motion about its axis. The line of tangency between the surfaces of the grinding disk and the worm surface is a spatial curve. However, this line of tangency might be a planar curve that coincides with the axial profile α–α (fig. 2.6.7(b)) if special machine-tool settings are provided (proposed in Litvin, 1968).

Modification of Geometry of Worm-Gear Drives

Worm-gear drives with the existing geometry are very sensitive to misalignment. Errors in misalignment cause a shift in the bearing contact, as shown in figure 2.6.2. This defect can be avoided by applying an oversized hob (Colbourne, 1993; Kovtushenko, Lagutin, and Yatsin, 1994; Litvin et al., 1996b; Seol and Litvin, 1996a, 1996b).The principles of this application follow:

(1) The oversized hob and the worm are considered as two helical gears in internal tangency (fig. 2.6.8). Conjugated surfaces of the hob and the worm can be provided because they are simultaneously in mesh with a common rack.

(2) The application of an oversized hob enables one to localize the bearing contact since the surfaces of the worm and the worm gear generated by an oversized hob are at every instant in point contact, not in line contact. The bearing contact is localized and is in the middle area of the worm-gear tooth surface (fig. 2.6.9).

The localization of bearing contact in Flender worm-gear drives does not guarantee a reduction in magnitude and a favorable shape of transmission errors (fig. 2.6.10(a)). It was shown in Seol and Litvin (1996a) that a parabolic function error for Flender worm-gear drives (fig. 2.6.10(b)) can be provided by varying the process for generating the distance between the axes of the hob and the worm gear or by modifying the shape of the grinding disk. A computerized investigation of a misaligned worm-gear drive with surfaces in line contact is a complex problem, as explained in section 1.11.

A computerized investigation of a worm-gear drive generated by an oversized hob requires two stages of computation: (1) a determination of the worm-gear tooth surface generated by the oversized hob, a case in which the hob and worm-gear surfaces are considered to be in line contact; (2) a simulation of the meshing and contact of the worm-gear tooth surface and the worm thread that are in point contact. Such computer programs were developed by Seol and Litvin (1996a, 1996b).

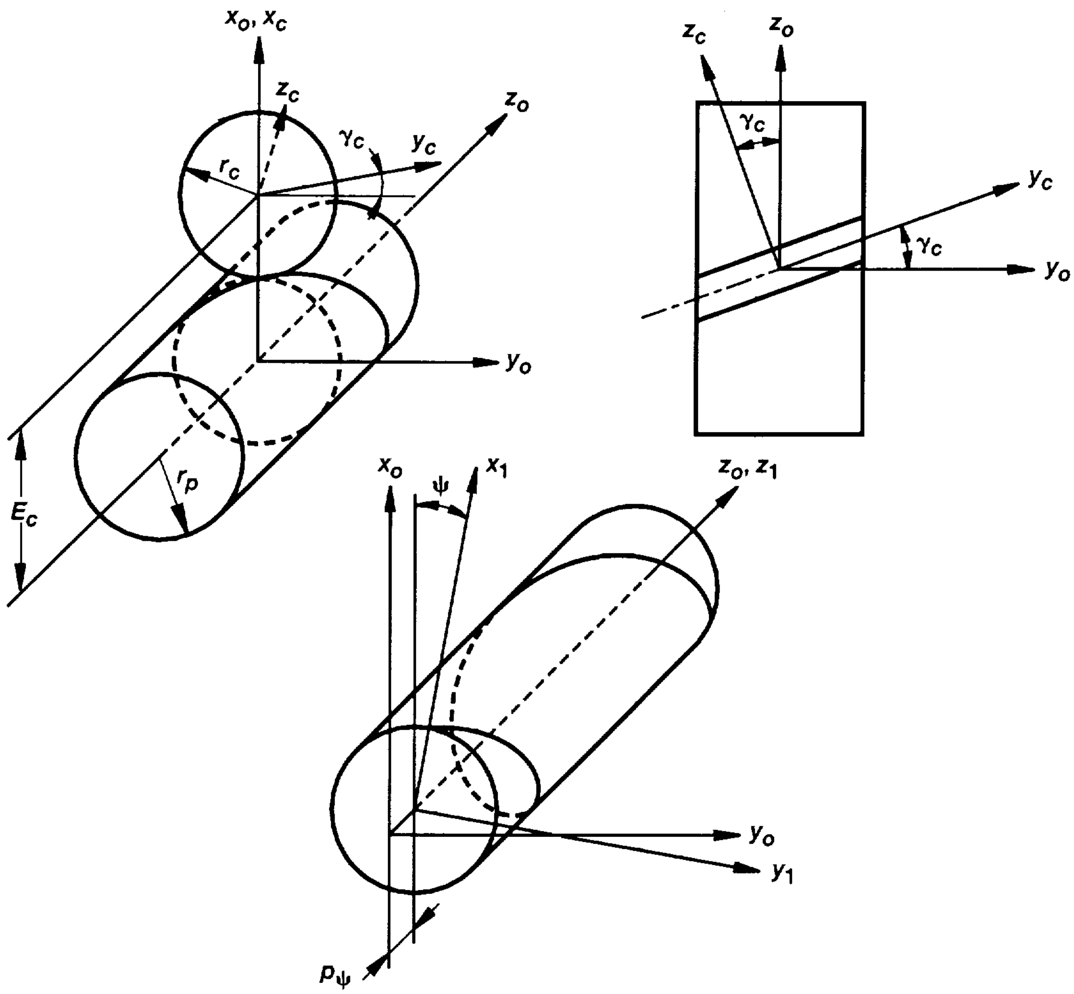


Figure 2.6.6.—Tool installment and generation of ZK-worms.

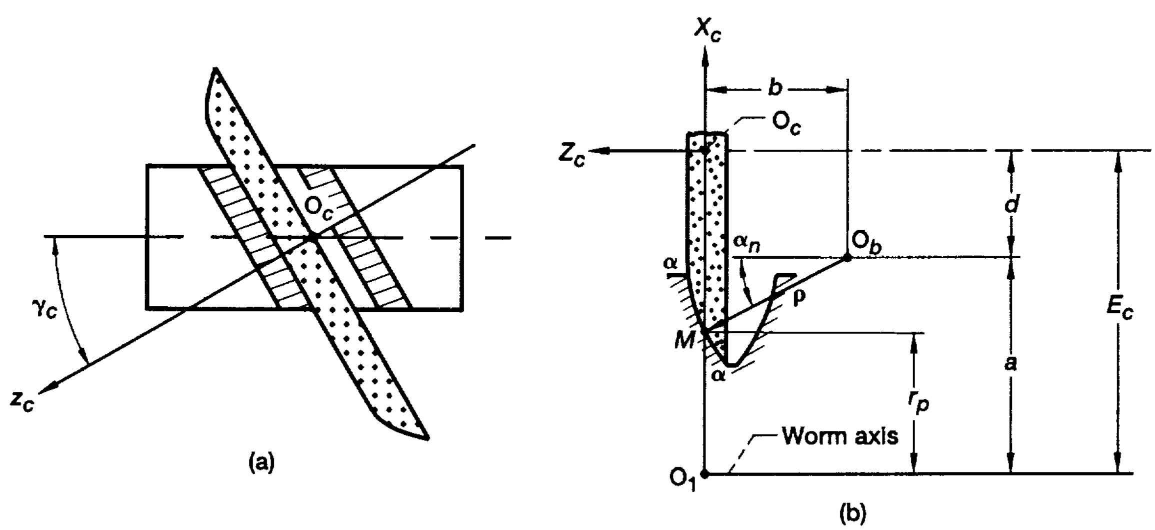


Figure 2.6.7.—Grinding of ZF-(Flender) worms. (a) Installment of grinding disk. (b) Disk axial profile.

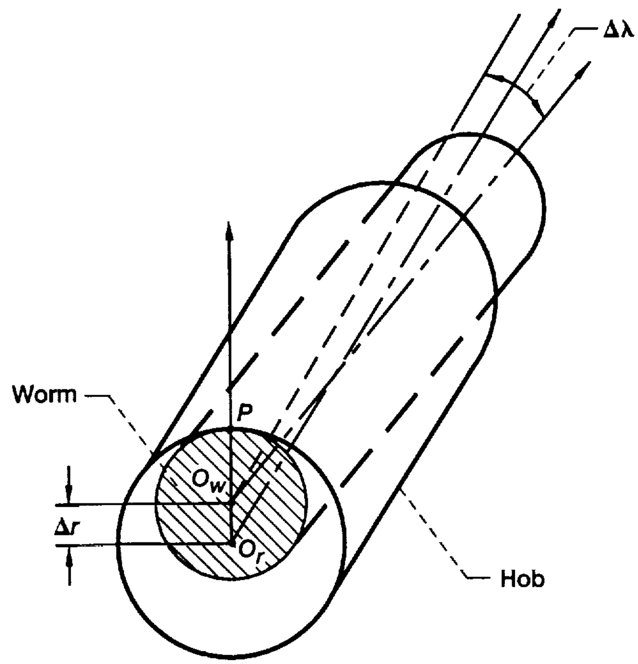


Figure 2.6.8.—Pitch cylinders of oversized hob and worm.

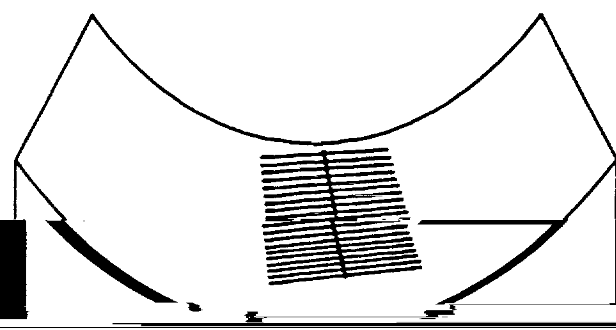


Figure 2.6.9.—Path of contact and bearing contact of misaligned ZI-worm-gear drive; ΔE = 0.1 mm; $\Delta\gamma$ = 1.0 arc min.

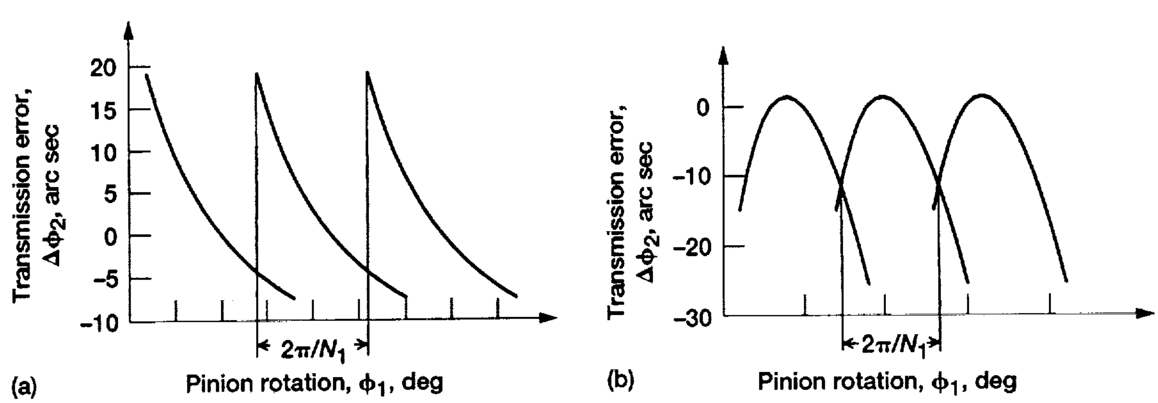


Figure 2.6.10.—Transmission errors of misaligned, modified Flender worm-gear drive. (a) No plunging. (b) Plunging (a = 0.03; ΔE = 0.1 mm; $\Delta\gamma$ = 3.0 arc min).

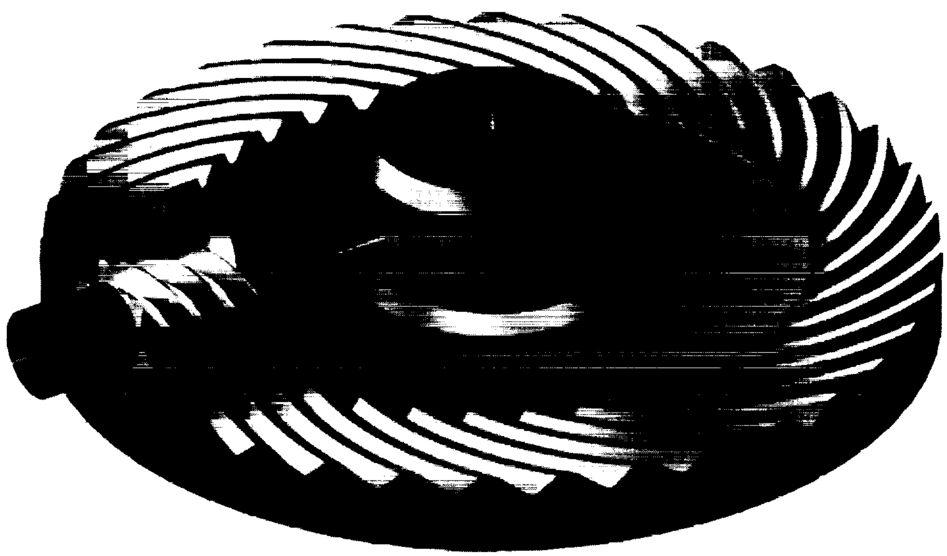


Figure 2.7.1.—Face worm-gear drive with intersected axes.

2.7 Face Worm-Gear Drives

The face-gear drives discussed in this section were formed by a cylindrical or conical worm and a face gear. These types of gear drives were invented by O.E. Saari and are described in Saari (1954, 1960). They were the subject of research conducted by the inventor and other researchers whose results are presented in Goldfarb and Spiridonov (1996), Kovtushenko, Lagutin, and Yatsin (1994), and in many other papers. This section presents the results of research conducted at the Gear Research Laboratory of the University of Illinois at Chicago by F.L. Litvin, A. Egelja, and M. De Donno. The goals of the research projects were (1) to provide a computerized design that enables one to avoid the undercutting and pointing of face worm gears, (2) to effect the localization of bearing contact, and (3) to accomplish a reduction in magnitude and a transformation of the shape of the transmission error function into a favorable one (see section 1.14). The cause of the the transmission errors was considered to be misalignment. Special attention was given to the simulation of meshing and the contact of misaligned gear drives by developed TCA (Tooth Contact Analysis) computer programs.

Saari's invention was limited to the application of ZA-worms (with straight-line profiles in the axial section) and the transformation of rotation between crossed axes only. The research of Litvin, Egelja, and De Donno was extended to the application of other types of worm thread surfaces and the transformation of rotation between intersected axes (in addition to the case of the crossed axes of rotation).

Figures 2.7.1 and 2.7.2 show respectively (1) a face worm-gear drive with a cylindrical worm and intersected axes of rotation and (2) a face worm-gear drive with a conical worm and crossed axes of rotation.

We have to emphasize that pointing is much easier to avoid in face worm-gear drives in comparison with conventional face-gear drives because of the application of screw thread surfaces with different pressure angles for the driving and coast tooth sides and relatively small lead angle values. The width of the topland of a face worm-gear varies in a permissible range and does not equal zero.

A detailed investigation to detect singularities based on the ideas presented in sections 1.4, 1.6, and 1.7 enabled us to discover that the singular points on a face worm-gear tooth surface form an envelope $\mathbf{E}_r$ to the contact lines that is simultaneously the edge of regression (fig. 2.7.3). Eliminating $\mathbf{E}_r$ from the surface of the face gear guarantees the avoidance of undercutting.

One of the important problems in designing face worm-gear drives is the localization of bearing contact and the predesign of a parabolic function of transmission errors. Based on the initial results of completed investigations, the authors of these research projects consider a promising solution to be the combination of profile and longitudinal deviations of the worm thread surface. These deviations are with respect to the

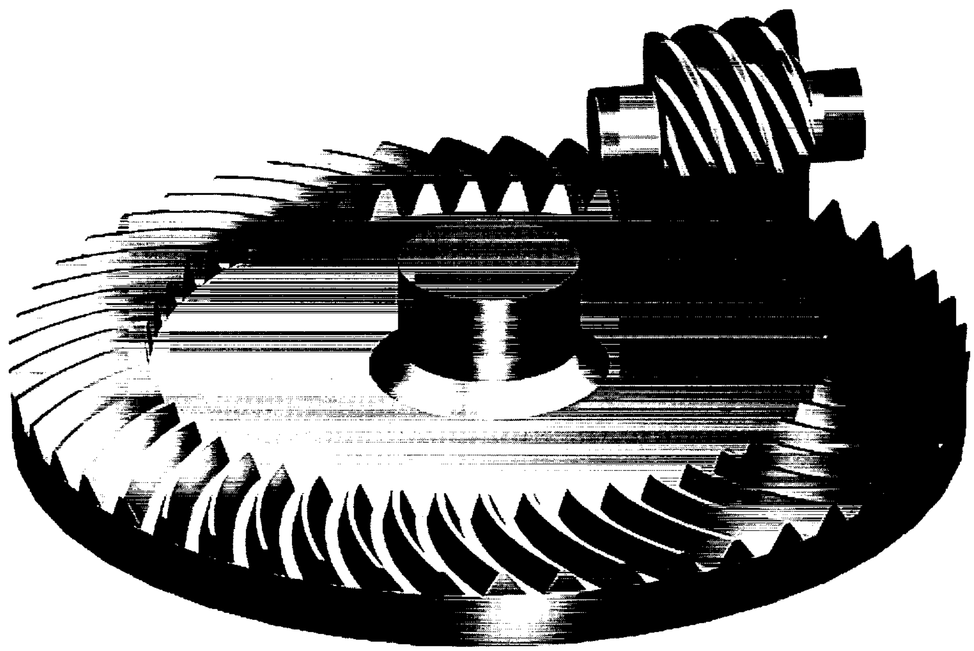


Figure 2.7.2.—Face worm-gear drive with conical worm.

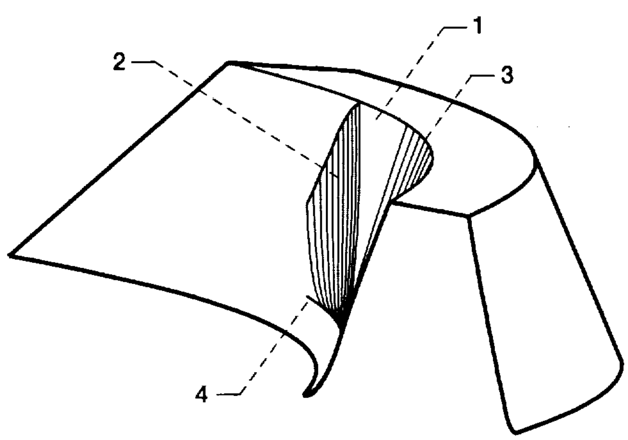


Figure 2.7.3.—Concave side of face worm-gear surface. (1) and (2) Surface branches. (3) Contact lines. (4) Envelope to contact lines and edge of regression.

theoretical worm thread surface that corresponds to the instantaneous line contact of worm and face worm-gear tooth surfaces.

One of the great advantages of the face worm-gear drive is the increased strength of the worm in comparison with the spur pinion of a conventional face-gear drive.

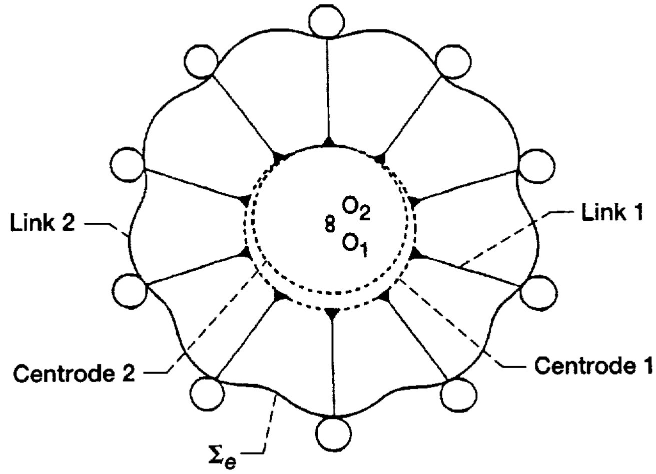


Figure 2.8.2.—Planar cycloidal gearing with external tangency of envelope Σ_e and pin teeth.

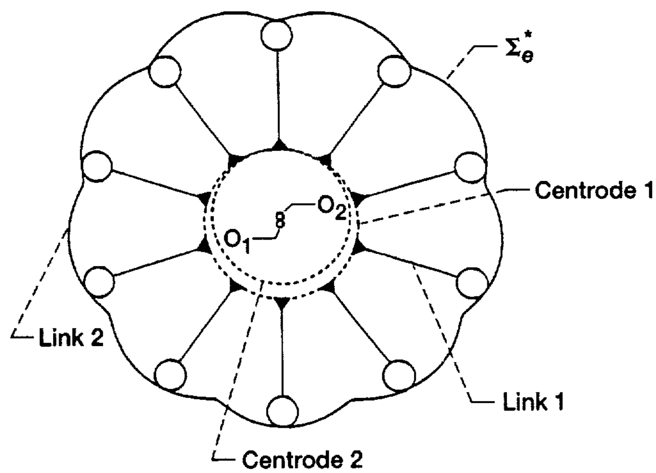


Figure 2.8.3.—Planar cycloidal gearing with internal tangency of envelope Σ_e^* and pin teeth.

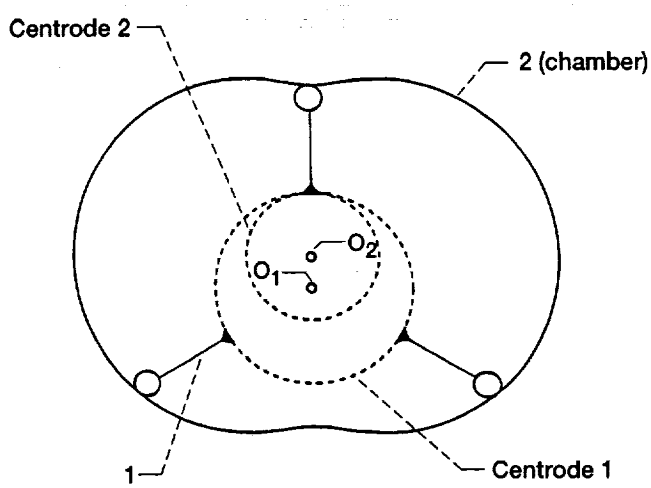


Figure 2.8.4.—Wankel's chamber and rotor.

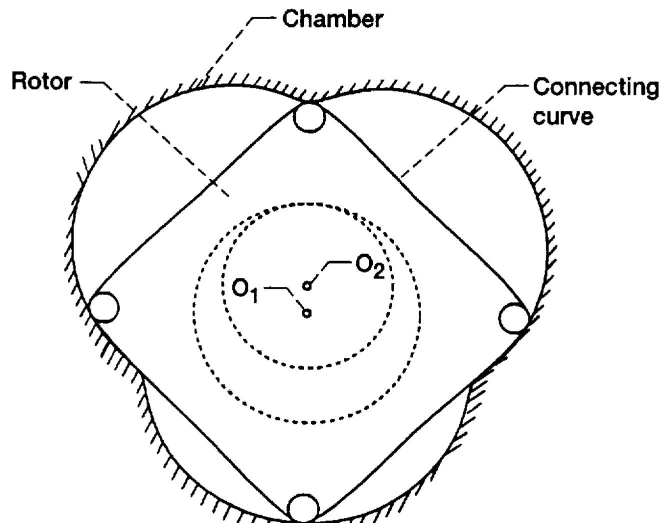


Figure 2.8.5.—Wankel's engine.

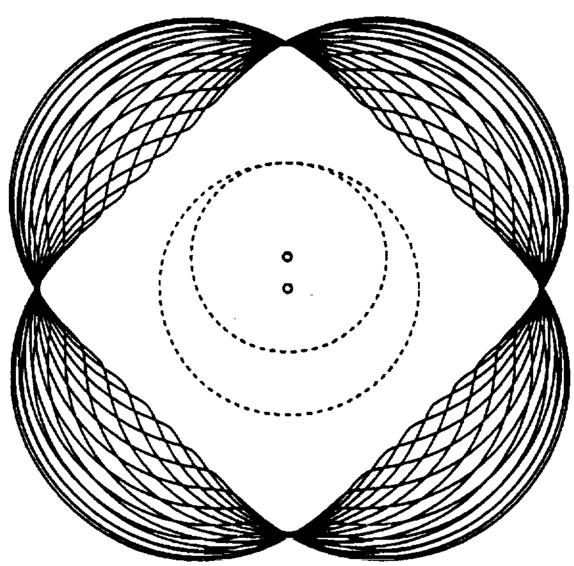


Figure 2.8.6.—Determination of space swept out in chamber by pins.

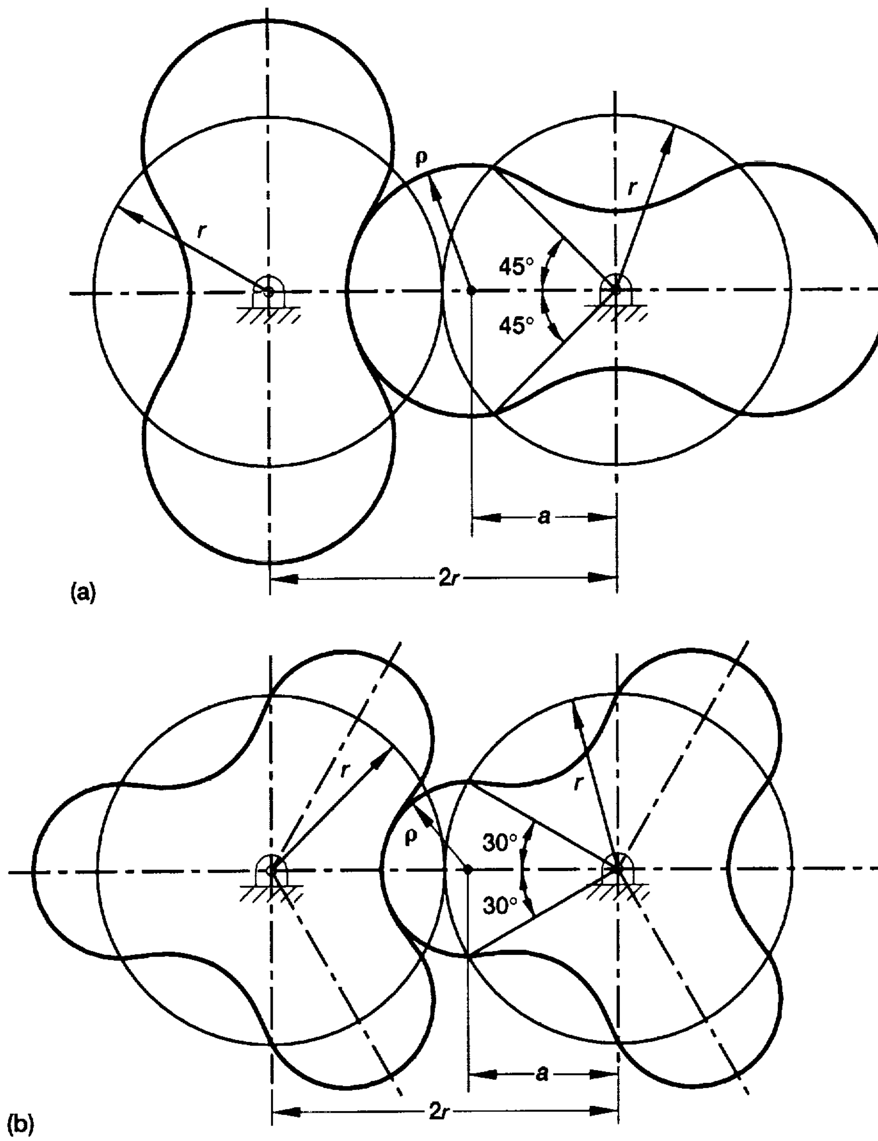


Figure 2.8.7.—Root's blower. (a) With two lobes. (b) With three lobes.

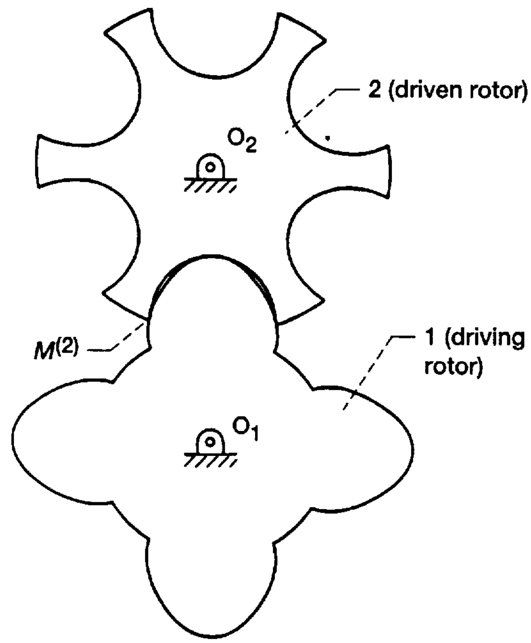


Figure 2.8.8.—Cross sections of screw rotors.

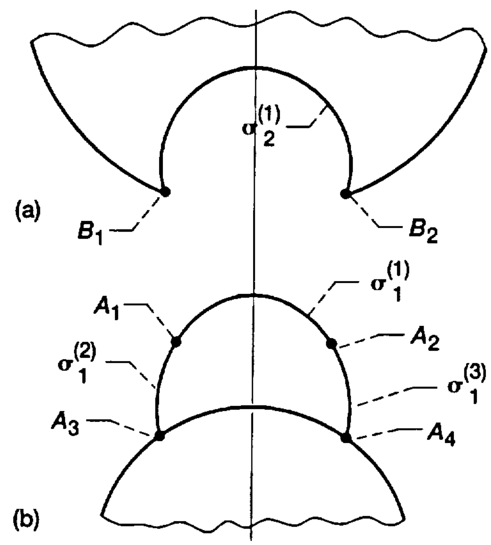


Figure 2.8.9.—Cross-sectional profiles of rotors 1 and 2. (a) Driven rotor. (b) Driving rotor.

a concave curve that is conjugated to the pin of the mating rotor. The design requires that in a certain range the parameter ratio *a*/*r* be observed to avoid singularities of the dedendum profile (Litvin and Feng, 1996).

Cycloidal Gearing of Screw Rotors of Compressors

Screw compressors, in comparison with conventional compressors that are based on the application of a crank-slider linkage, enable the compressor to operate with an increased rotor angular velocity and to obtain a higher compression. However, the application of screw compressors requires that the rotor surfaces be conjugated (Litvin and Feng, 1997).

The cross sections of screw rotors are shown in figure 2.8.8. The profiles of the rotors (fig. 2.8.9) are synthesized so that the rotor surfaces are in tangency at two contact lines. The synthesis is based on the following considerations:

(1) The profile of driving rotor 1 (fig. 2.8.9(b)) is combined by curves $\sigma_1^{(1)}$ and by symmetrically located curves $\sigma_1^{(2)}$ and $\sigma_1^{(3)}$. Points A_1 and A_2 indicate the points of tangency of $\sigma_1^{(1)}$ with $\sigma_1^{(2)}$and $\sigma_1^{(3)}$, respectively.

(2) Profile $\sigma_1^{(1)}$ is designed as an elliptical curve, and $\sigma_2^{(1)}$ (fig. 2.8.9(a)) is the profile of the driven rotor that is conjugated to $\sigma_1^{(1)}$. The meshing of the rotor surfaces with profiles $\sigma_1^{(1)}$ and $\sigma_2^{(1)}$ provides the first instantaneous line of tangency of the rotor surfaces.

(3) Curves $\sigma_1^{(2)}$ and $\sigma_1^{(3)}$ of rotor 1 are epicycloids that are generated by points B_1 and B_2 of the driven rotor (fig. 2.8.9). The second contact line of the rotor surfaces is the edge of rotor 2 (a helix of the screw surface of rotor 2). To reduce the wearing, the profile of rotor 2 may be rounded at the edge (Litvin and Feng, 1997).

Chapter 3

Biographies and History of Developments

3.1 Introduction

Visitors coming to the Gear Research Laboratory of the University of Illinois in Chicago are invited to see the Gallery of Fame, a photographic collection of gear company founders, inventors, and researchers who devoted their careers to the study and development of gears. This collection is unique because retrieving items and information for the gallery was a difficult task. Time had destroyed documents and memories of the many contributors who were deceased, requiring much detective work of the author who desired that these creative men be credited for their valuable contributions. In his effort to personalize these biographies, he sometimes had to find and then contact relatives, to search for information in libraries, and to communicate with gear company management to trace a predecessor's activities. The reader will have the opportunity to see, perhaps for the first time, the photographs of Samuel Cone, one of the inventors of the double-enveloping worm-gear drive; Ernst Wildhaber, one of the most successful inventors of gears; Dr. Chaim Gochman, the Russian scientist and founder of the gear theory; Dr. Hillel Poritsky, the American mathematician; Dr. Alexander Mohrenstein-Ertel, the founder of the hydrodynamic theory of lubrication; and many others who significantly contributed to this field.

The author hopes that the biographies of these men will give the reader an opportunity to learn more about their lives through their thoughts, successes, and tribulations.

3.2 Johann Georg Bodmer—Inventor, Designer, and Machine Builder (1786–1864)

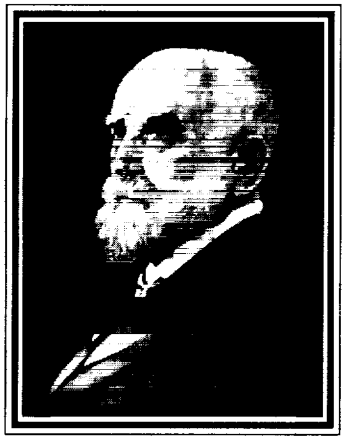
The memorial plaque on the house at 12 Muhlegasse Street in Zurich, Switzerland bears the name of a former resident and distinguished engineer and inventor, Johann Georg Bodmer. We thank the Verein für Wirtschaftshistorische Studien of Switzerland and R. Gardner (1941) for providing valuable information about the pioneering work of this remarkable man. Fortunately, Bodmer's descendants, devoted to their famous forefather, preserved his drawings and diaries for history.

Johann Georg Bodmer came from a family of skilled handymen who traced their history in Switzerland for more than 400 years. Members of this family were experienced stonecutters and textile workers. The young Bodmer graduated from the School of Art of Zurich when he was only 16, but he was highly motivated, talented, and creative and educated himself through experience. His inventions dealt with the textile industry, steam machines, locomotives, drives for ships, cannons, and various metal-cutting machines, including gear-cutting machines. Some of the machines that were designed for the production of tools used to manufacture screws and nuts were developed by successful cooperation with his son-in-law Jacob Reishauer, one of the founders of the Reishauer Company.

Of special interest to gear engineers is that Johann Bodmer designed and manufactured spur gears with the ratio of 145 to 15, bevel gears, and helical gears. His achievements in the field of gears and in other areas of the machine industry were related to his remarkable ability to design tools.

Undoubtedly, Johann Bodmer was in advance of his time and full recognition of his genius came later and not in his lifetime.

3.3 Friedrich Wilhelm Lorenz—Doctor of Engineering, h.c. and Medicine, h.c., Inventor, and Founder of the Lorenz Company (1842–1924)



The life of Friedrich Wilhelm Lorenz is best described by the title of his biography *Ein Leben für die Technik (A Life for the Technique)*, written by Dr. Susanne Pach-Franke (1990). Wilhelm Lorenz, an inventor and successful entrepreneur, was a self-made man who was born in 1842 in Geseke, a small German city. His father's position as court manager provided the family with very little income so that after his sudden death, Wilhelm could not continue his high school education and had to work when he was only 13 years old. This biography covers the four stages of his career: (1) the first years of training, (2) his work for the military branch of industry, (3) the production of Daimler's motors, and (4) the founding of a company to manufacture gear production equipment.

Wilhelm's professional training began with his apprenticeship to a blacksmith. His outstanding creativity was immediately revealed when he automated the production of nails. Later, in 1860, he obtained a position at Funcke and Hueck in Hannover where he was recognized as a gifted designer by the company's owner, Wilhelm Funcke, whose great affection for the young Lorenz paved the way for him to continue his professional training in Berlin.

For a talented man, education is like a diamond cutter who awakens the brightness and shine of the jewel. Wilhelm Lorenz worked hard to educate himself and, not surprisingly, in 1870 was hired as an engineer by Georg Egerstorff of the Percussion Cap Company. There he learned the technological principles of the machines and tools used in the military branch of industry and automated high-precision manufacturing. In 1875, he took a position at the Cartridge Case Company of Henri Ehrmann and Cie in Karlsruhe where after only 2 years he became the company's technical manager and then its owner.

Although his career in industry was very successful, his desire to create led him to consider manufacturing car engines invented by Daimler. Lorenz' description of this period of his life was short and impressive: "Daimler invented motors for cars, but I, Lorenz, gave life to the cars."

The last period of Lorenz' life will be particularly interesting to gear specialists. Initially, he was interested in and concentrated on the production of double-enveloping worm-gear drives and in 1891 invented methods to generate the worm and the gear of the drive. Having received two patents for this work, it was not a surprise that the worm-gear drive was his favorite invention and that, inspired by Julius Grundstein, Lorenz' company in Ettlingen specialized in the production of gear equipment. At the end of the 19th century, the company had become uniquely experienced in designing and producing precise metal-working machines, and in 1906 Julius Grundstein became its technical director. One of the first tests of the company's new direction was the design and production of a giant gear-cutting machine for gears having diameters up to 6 m, modules up to 100 mm, and tooth lengths up to 1.5 m.

In 1990, the Lorenz Company celebrated its 100th anniversary and is presently well known in the industry as a manufacturer of precise shaping machines for the production of external and internal involute gears, helical involute gears, chain-drive gears, and other special-profile gears (e.g., face gears) that can be generated by a shaper. The company also produces noncircular gears using CNC machines to execute the related motions of the shaper and the noncircular gear being generated (see ch. 12 in Litvin, 1994).

Wilhelm Lorenz'achievements gained him the well-earned recognition of his contemporaries. In 1910, Karlsruhe University awarded him an honorary degree of doctor of engineering, and Heidelberg University awarded him an honorary degree of doctor of medicine for his outstanding work in the manufacture of prostheses.

Dr. Lorenz's life was darkened by several tragic events. His infant son and his young wife died from tuberculosis, and his other son was killed in an accident at age 12. He devoted his life to his daughter Ada and to his pioneering work. He is survived by Ada's two granddaughters.

Although Dr. Lorenz did not want a gravestone and asked that his ashes be scattered by the wind, this remarkable man is remembered today in Geseke and Karlsruhe where streets bear his name and in Ettlingen where a school is named for him. However, the best memorial to his life in Ettlingen is his name on the company he founded.

3.4. Robert Hermann Pfauter—Inventor and Founder of the Pfauter Company (1854–1914)

The end of the 19th and the beginning of the 20th century was the "golden" era of pioneering developments in gear manufacturing. In 1897, Robert Hermann Pfauter invented the process and machine for generating spur and helical gears by hobs. Two other inventions were based on the application of shapers and rack-cutters (see secs. 3.5 and 3.6 on Erwin R. Fellows and Dr. Max Maag). These inventors were pioneers in creating gear-producing automotive machines based on the continuous relation of motions between the generating tool and the gear being generated. Such machines provide continuous indexing and generate the gear tooth surface as the envelope to the family of tool surfaces.

Robert Hermann Pfauter was born in 1854 to a farm family in Goeltzschen, a village near Leipzig in Saxony. Fortunately for the future of the gear industry, he pursued technology rather than farming because of the promising opportunities resulting from the industrial revolution. He began his professional career as a machinist and then continued his education at Hainichen Technical College near Chemnitz. He graduated with an engineering degree and went to work for the well-known machine tool companies Birnatzki and Zimmermann, Hartmann, and Reinecker.

Pfauter's employers soon recognized his talent and promoted him to higher positions, first as the principal design engineer at the Reinecker Company and then (at only age 39) as the technical executive director of the Chemnitz Knitting Machine Works (later known under the name of Birnatzki).

The process of invention is a mystery and its crowning moment like a lightning bolt. For many years, Robert Hermann Pfauter noted the disadvantages of the existing manufacturing process for spur and helical gears: (1) the necessity of indexing the gear for the generation of each tooth space and (2) the generation of the tooth space only as a copy of the tool surface. His patent for generating spur and helical gears was free of these disadvantages and although it claimed to be limited to the generation of helical gears, it could actually be applied to the generation of spur and worm gears by hobs. In this respect, Pfauter was the precursor of a group of inventors who later proposed other gear generation processes based on principles similar to those proposed in his patent.

Success in real life, unlike that in fairy tales, is a road of roses and thorns. It sounds trivial that there are no roses without thorns but this was true of Pfauter's invention. He encountered many technical and financial obstacles while trying to prove his theories and was tempted to abandon his risky invention and go back to the safe work of a technical director. He decided otherwise, built the prototype of the machine in 1889, and finally in 1905 saw the Pfauter Company become a prosperous business. From its beginning to the spring of 1913, it manufactured 2000 machines, many of which were imported to the United States.

The engineering community recognized Pfauter as a brilliant man who designed, built, and produced for industry the machine he invented. He was remembered after his death in 1914 in an article in *Chemnitzer Volksstimme* (10/14/1914) not only as a distinguished inventor but also as an employer beloved by his coworkers.

In the last 25 years, the Hermann Pfauter Company has become a multinational enterprise known as the Pfauter Group, serving the gear manufacturing community in many countries. At present it offers equipment for many gear manufacturing stages such as hobbing, shaping, hard skiving, shaving, honing, and milling. The company also produces cutting tools, noise-testing equipment, and a new generation of high-precision machines, including CNC gear grinding machines. The Pfauter Group's activity is proof that its founder successfully developed it.

3.5 Edwin R. Fellows—Inventor and Founder of the Fellows Gear Shaper Company (1865–1945)

Edwin R. Fellows, another pioneer during the "golden" era of gear manufacturing, invented a revolutionary method to generate gears by shaping. His method is based on the simulation of two gears meshing during gear rotation about two parallel axes, which is in contrast to two other inventors who based their methods on hobs and rack-cutters (see secs. 3.4 and 3.6 on Robert Hermann Pfauter and Dr. Max Maag).

The manufacture of spur gears by shaping originated at the Fellows Gear Shaper Company in Springfield, Vermont in 1896. Fellows' invention was a significant contribution because it enabled the automation of the internal and external gear manufacturing process. The automotive industry's broad application of the Fellows gear shaper process confirmed the importance of the invention. Continued development produced machines capable of generating helical and face gears.

In 1897, the Fellows Company designed and built a machine for grinding involute profiles on gear shaper cutters. This process was later recognized for its application in grinding the various tooth profile cutters that were used to shape spur, helical, and special tooth profile gears.

During the Second World War, the Fellows Company was awarded the distinguished Army-Navy Production Award for its outstanding contribution. Today, the Fellows Gear Shaper Company continues to prosper by designing and building innovative CNC machines with its latest Hydrostroke Gear Shaper, all of which attests to the importance of Edwin R. Fellows' pioneering innovations.

3.6 Max Maag—Doctor of Engineering, h.c., Inventor, and Founder of the Maag Company (1883–1960)

The son of a village teacher in Flurlingen at Zurich, Max Maag attended the Zurich Polytechnicum but later decided to study machine design at the Oerlikon Company. Soon after, he became a designer at the Kaspar Wust plant. From 1908 to 1913, Dr. Max Maag's contributions helped to distinguish him in the field of gearing. In 1908, he developed the geometry of nonstandard involute spur and helical gears, which enabled designers to avoid undercutting and to increase the tooth thickness by just modifying the installation of the tool (the rack-cutter) with respect to the gear being generated. His research led to the development of the Maag system of generating nonstandard involute gears. In this system, he applied the method and machine invented by Samuel Sunderland of Kieghley, York, in Great Britain and in 1909 received the first patent for cutting nonstandard involute gears based on the application of Sunderland's machine. The advantage in generating gears by a rack-cutter was that it was a simpler and more precise tool than a hob and a shaper.

The next 3 years were milestones in Maag's career. In 1910 he made a very courageous decision to open a small workshop to produce nonstandard involute gears with modified Sunderland's machines. The first gears cut according to the Maag method were delivered in 1911, and patents were granted in 14 countries. In 1913 he founded the Maag Gear Wheel Company Ltd. and remained its managing director until his retirement in 1927 when the company was reorganized. Today, Maag machines have a reputation for quality and high-precision manufacturing and for the production of large-dimension gears.

A significant achievement of the Maag Company was the development of the method to grind spur and helical involute gears. The grinding tool was a saucer-shaped wheel with a 15° profile angle, a prototype of which was built in 1913. This invention was important because it provided the first automatic compensation for the wear of the grinding wheel, making it the first and most famous automatic control system in the machine tool history.

Several other contributions of the Maag Company made its name well known in the gear industry. In 1950, the method of grinding spur and helical gears by a plane was invented. The originality of the idea lay in the application of two grinding planes with a zero pressure angle, each plane simulating the surface of a rack tooth with a zero pressure angle.

In recognition of his significant contributions to the design and manufacture of gears, the Swiss Federal Institute of Technology in Zurich awarded him an honorary doctorate in 1955.

Dr. Maag devoted his life and creativity to investigating gears, as illustrated by his grave marker, which shows the meshing of a rack and an involute gear.

3.7 Earle Buckingham—Professor of Mechanical Engineering, Gear Researcher, and Consultant (1887–1978)

Prof. Earle Buckingham is one of the founders of the theory of gearing and gear design and made significant contributions to this area. His monographs gained him international recognition in addition to the great respect he had already earned in English-speaking countries.

Earle Buckingham was born in 1887 in Bridgeport, Connecticut. He attended the United States Naval Academy in Annapolis, Maryland from 1904 to 1906 and then began to work in industry. In 1919, he directed his research and science interests to the field of gears, initially as a gear consultant for the Niles-Bement-Pond Company (now Pratt & Whitney) and then from 1925 to 1954 as a professor of mechanical engineering at the Massachusetts Institute of Technology in Cambridge, Massachusetts. After retirement, he continued his research activity as a gear consultant.

Prof. Earle Buckingham's books (e.g., *Analytical Mechanics of Gears*, 1963) laid the foundation for the theory of gearing and became references for at least two generations of engineers and researchers. The engineering community recognized his contributions to the design and theory of gears by presenting him these prestigious awards: the American Society of Mechanical Engineers (ASME) Worcester Reed Warner Medal, 1944; the American Gear Manufacturers Association (AGMA) Edward P. Connel Award, 1950; the American Society of Test Engineers (ASTE) Gold Medal, 1957; the Gold Medal of the British Gear Manufacturers Association

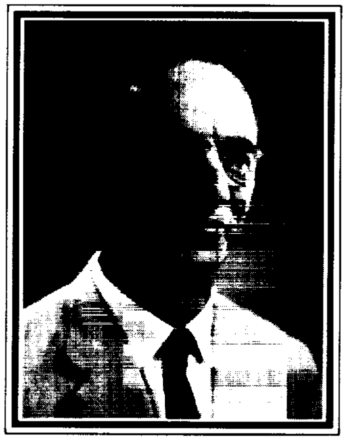
1962; and the Golden Gear Award of *Power Transmission Design* magazine in commemoration of the AGMA 50th Anniversary. Also in recognition of his contributions, Buckingham lectures are delivered at the USA Power Transmission and Gearing conferences.

3.8 Ernst Wildhaber—Doctor of Engineering, h.c., Inventor, and Consultant for The Gleason Works (1893–1979)

Ernst Wildhaber is one of the most famous inventors in the field of gear manufacture and design. He received 279 patents, some of which have a broad application in the gear industry because of his work as an engineering consultant for The Gleason Works. Dr. Wildhaber's most famous inventions are the hypoid gear drive, which is still used in cars, and the Revacycle method, a very productive way to generate straight bevel gears.

Dr. Wildhaber graduated from the Technische Hochschule of Zurich

3.9 Hillel Poritsky—Doctor of Mathematics, Professor of Applied Mechanics, and Consultant (1898–1990)



A prominent mathematician, Dr. Hillel Poritsky made significant contributions to the fields of applied mathematics, the theory and geometry of gears, stress analysis, fluid dynamics, and electromagnetics. He authored or coauthored more than 100 scientific papers. One of his major interests was gear research, about which he and Darle Dudley wrote several papers dealing with the theory, geometry, and generation of worm-gear drives and helical gears. These pioneering works (e.g., Dudley and Poritsky, 1943) were ahead of their time and made a valuable contribution to gear research. Dr. Poritsky also conducted research on the hydrodynamic theory of gear lubrication.

Hillel Poritsky is an example of an immigrant who came to the United States, pursued an education, devoted himself to his career, and contributed greatly to science and society. Born in Pinsk, Belorussia, he lost both his parents before he was 5 and was raised by older sisters in conditions of poverty. He came to the United States when he was 15, not knowing a word of English and only able to order a meal in a cafe by pointing to the lowest priced items on the menu. However, he was a person of great intellectual ability and drive and became a distinguished scientist.

After graduating from Morris High School in the Bronx in New York City, Dr. Poritsky went to Cornell University where his ability in mathematics and physics was soon recognized. Even before he received his bachelor's degree in 1920, he became an instructor. Also, during and after the First World War, he was on the staff of the United States Army Proving Ground in Aberdeen, Maryland where he used these abilities to solve problems. In 1927, he received his doctoral degree in mathematics from Cornell and in the same year went to Harvard as a National Research Council Fellow. Two years later in 1929, he joined the General Electric Company in Schenectady, New York as a senior mathematician and consultant and worked there until he retired in 1963.

Dr. Poritsky's retirement mirrored his distinguished career in industry and academia. After he left General Electric, he worked as a consultant for the United States Navy and for several companies and was also adjunct professor at Rensselaer Polytechnic Institute in Troy, New York. In 1967, he was a visiting professor at the University of Illinois Department of Theoretical and Applied Mechanics. In this same year, his achievements in mathematics and applied mechanics were recognized by his contemporaries who awarded him the prestigious Timoshenko Medal, making him the first recipient.

Dr. Poritsky was a member of several scientific societies: the American Society of Mechanical Engineers (ASME), the American Mathematical Society, and the American Institute of Electrical and Electronic Engineers. He was the chairman of the executive committee of the Applied Mechanics Division of the ASME.

3.10 Gustav Niemann—Professor and Doctor of Mechanical Engineering (1899–1982)

3.11 Meriwether L. Baxter, Jr.—Gear Theoretician (1914–1994)

Meriwether L. Baxter was born in 1914 in Nashville, Tennessee. He graduated from a Hartford, Connecticut public high school in 1931 and entered Yale University. After his graduation in 1935, he pursued his engineering and scientific work at The Gleason Works where he was the chief research engineer and the director of engineering until he retired in 1975. He then worked as a consultant.

In his early years at The Gleason Works, Meriwether Baxter spent time learning bevel and hypoid gear theory under Ernst Wildhaber's supervision, and Revacycle gears became his area of expertise. Then in the 1950's, he recognized that the computer could be a powerful tool in the area of gear theory; it could relieve engineers of the long, tedious calculation procedures needed to make machine settings for generating bevel and hypoid gears. More important, he realized that with the computer and all its power, it was practical to apply vector and matrix notations in the development of gear theory and in doing so, he pioneered a new era in gear calculation and theory (Baxter, 1961, 1973). With respect to computers, his most noted accomplishments were developing tooth contact analysis (TCA), an undercut program and an exact duplex helical machine setting calculation procedure. He also held seven patents relating to gears and gear manufacturing.

Meriwether Baxter was most concerned with education and training and spent much of his time ensuring that the methods and theory he developed be preserved and passed on to the next generation of gear engineers. To this end, Theodore Krenzer became his right-hand aid. All told, the contributions of Ernst Wildhaber, Meriwether Baxter, and Theodore Krenzer to the theory of gearing laid the foundation for important research that has continued successfully to the present.

Meriwether Baxter was a member and fellow of the American Society of Mechanical Engineers and a member of the American Gear Manufacturers Association.

3.12 Hans Liebherr—Doctor of Engineering, h.c., Designer, and

Hans Liebherr was a gifted designer and the founder of several successful industrial companies. He began his engineering career at his parents'

3.13 Alexander M. Mohrenstein-Ertel—Doctor of Engineering (1913–)

Alexander Ertel-Mohrenstein made a tremendous contribution to the theory of hydrodynamic lubrication of loaded double-curvature surfaces. His theory has been applied to determine the lubrication conditions of gear tooth surfaces and bearings.

Alexander Ertel (Oertel was the original German spelling) was born to a German family who had deep roots in Russian society. His great grandfather Vasily Ertel, a doctor at Leipzig University, settled in Russia and became a citizen in 1844. His descendants attained important positions in Russian society and became hereditary gentlemen (dvorjane).

After graduation from high school, Alexander was admitted to the renowned Lomonosov University in Moscow in 1936 where he studied theoretical physics. His research in the hydrodynamic theory of lubrication was very successful, and his two fundamental papers, upon the recommendation of academician L.S. Leibenzon, were published in 1939 and 1945 in the Proceedings of the Academy of Sciences of USSR, Department of Technical Sciences. It is remarkable that his 1939 paper was published in the Proceedings while he was still a student. He graduated in 1941 and with the endorsement of the famous physicist Peter L. Kapiza, became a researcher at the Central Scientific Research Institute of Machine Industry (ZNIITMASH) in Moscow. While he worked at ZNIITMASH, Ertel prepared to defend his Russian doctoral thesis, which summarized the results of his research in 1945 and was presented in five seminars (1944–45) at the Institute of Machine Science of the USSR Academy of Sciences. Ertel's thesis was highly acclaimed by the Russian reviewers.

At the end of 1945, Dr. Ertel's life took a dramatic turn. While under assignment to the Soviet ministry, he visited Berlin-Pankow in East Germany (controlled by the Soviet Army) and decided to cross the border to the English zone and settle in West Germany. Had it not been for the prominent English scientist Dr. Alastair Cameron, who confirmed Ertel's identity, he might not have been able to leave safely. Fortunately he did and began the second period of his research.

In West Germany, Dr. Ertel's scientific work was initially performed under the name of Alexander von Mohrenstein and later under Mohrenstein-Ertel. From 1946 to 1948, working under the supervision of Prof. Gustav Niemann at the Technical University of Braunschweig, he defended the German version of his doctoral thesis in 1947. He also conducted research at the Max Plank Institute of Gettingen and at the University of Freiburg.

A tragic car accident in 1959 interrupted Dr. Mohrenstein-Ertel's work; however, because of his wife's devotion and care, he has been able to continue in his private life.

3.14 Darle W. Dudley—Gear Consultant, Researcher, and Author (1917–)

In some cases a person's contributions to a given field are not recognized by his contemporaries but by others long after his death. Fortunately, Darle W. Dudley did not have to wait for such recognition. In the field of gearing, he has gained worldwide recognition by writing books, acting as chairman of technical committees, writing standards for gear design, and promoting gear research. Some of his books are *Practical Gear Design* (1954), *Gear Handbook* (1962), *The Evolution of the Gear Art* (1969), *Handbook of Practical Gear Design*, (1984), and *Dudley's Gear Handbook* (edited by D.P. Townsend, 1991).

Darle Dudley has written much about designing and manufacturing gear drives that are adequate for many applications. Having conducted numerous investigations of gear drives for large and small companies, he has become a "pathologist" in analyzing drives that prematurely failed.

The Dudley family can trace its origin back more than 10 centuries to the small English town of Dudley where the ruins of a medieval castle give evidence of their past activities. From the standpoint of the Industrial Revolution, the castle is noteworthy because the first piston engine was put into service in a mine beneath it.

Presently, thousands of Dudleys live in the United States and England. Darle Dudley descended from the Francis Dudley branch of the family who, in the first half of the 17th century during the English civil war, emigrated to Massachusetts seeking peace and religious freedom.

Darle W. Dudley was born on April 8, 1917 in Salem, Oregon. He graduated from Oregon State University in 1940 with a bachelor of science degree in mechanical engineering. He worked as an engineer and then as the manager of gear research and development at the General Electric Company (1940–64), as the manager of mechanical transmissions at Mechanical Technology, Inc. (1964–67), and as the chief of gear technology at Solar Turbines Inc. (1967–78). In 1978, he took an early retirement and founded the Dudley Engineering Co., Inc.

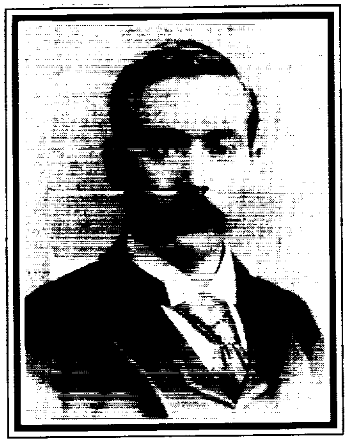
In recognition of his significant contributions to the development of gear art, Darle Dudley has received several awards in the United States and Europe. In 1958, the American Gear Manufacturers Association presented him the Edward P. Connell award. The most prestigious is the Ernst Blickle Prize awarded to him in Germany in 1995.

3.15 Oliver E. Saari—Inventor at the Illinois Tool Works (ITW) Spiroid Division (1918–)

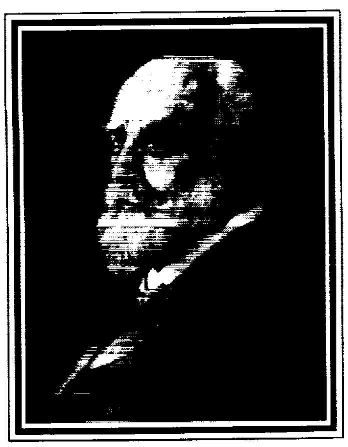
Oliver E. Saari is the creative and prolific inventor of Spiracon Roller Screws, Planoid Gears, Endicon, and new gear drives called Spiroid, Helicon, and Concurve (trademarks of ITW). Spiroid and Helicon gear drives transform rotation between crossed axes. The driving link of the Spiroid gear drive (Bohle and Saari, 1955; Saari, 1954) is a conical worm with a constant lead whereas the driving link of the Helicon gear drive (Saari, 1960) is a cylindrical worm. However, both these types differ substantially from conventional worm-gear drives: (1) the gear is a face type, and (2) the worm has different profile angles for the driving and coast sides. They offer specific advantages: one is that the worm and the gear are in line contact at every instant and the orientation of the line contact with respect to the relative velocity is favorable for the conditions of lubrication. This is especially important for drives with crossed axes where the sliding of surfaces is inevitable in all meshing zones. Another advantage is the almost constant top land of the gear helical teeth. The Concurve gear is a modification of the profiles of conventional involute spur and helical gears, which results in a constant profile curvature for the stabilization of contact stresses (Saari, 1972). Saari's inventions bear the features of an unorthodox way of thinking which resulted in original ideas that have already been applied in industry and will be widely used in the future.

The life and activity of Oliver Saari exemplify that the United States benefited as a country built by emigrants who valued education and worked exceedingly hard to attain it. He was born on March 22, 1918, in Helsinki, Finland. In 1927, his family emigrated to the United States where, over a period of 8 years, they moved from Brooklyn, New York, to Rochester, Minnesota, and finally to Minneapolis. After graduation from high school, he joined the Civilian Conservation Corps (CCC) during which time he published several science fiction stories. He then learned tool and diemaking (his father's trade) at a vocational school before entering the University of Minnesota to study mechanical engineering. While at the university, he worked nights as a shop inspector at Honeywell Inc. Upon graduation in 1943, he went to work at the Buick Motor Division of General Motors Corporation in Flint, Michigan where he became interested in gears. In 1945, he joined the Illinois Tool Works (ITW) in Chicago and at the same time attended the Illinois Institute of Technology, receiving a master's degree in mechanics. In 1974, he retired from ITW and became a gear consultant.

3.16 History of the Invention of Double-Enveloping Worm-Gear Drives



Samuel I. Cone



Friedrich Wilhelm Lorenz

The invention of the double-enveloping worm-gear drive is a breathtaking story with two dramatic personae, Friedrich Wilhelm Lorenz and Samuel I. Cone, each acting in distant parts of the world—one in Germany and the other in the United States.

Initially, the author intended to find a photograph of Samuel Cone (1865–1949), the American inventor of the drive that carries the name of Cone Drive Co., the main producer of the aforementioned drives in the United States. Unfortunately, the photograph disappeared from the company files but was finally located in the family archives with the help of Cone's granddaughter Mrs. Mary Bell Kluge (nee Taylor). The search ended happily, but it was not the end of the story because during our visit to the Lorenz Co. in Ettingen, Germany, we learned that Dr. Lorenz had invented methods to generate the worm and the gear of the double-enveloping worm-gear drive and that he had received two patents for this work in 1891.

Dr. Lorenz' invention was unknown to Samuel Cone, a modest draftsman to whom the idea of a double-enveloping worm-gear drive came independently, as seen in the following statement from the archives of the Cone Drive Co. in 1924:

> About 15 years ago, Mr. Samuel I. Cone of Portsmouth, Virginia, manufactured at the Norfolk Navy Yard the first Cone Type Worm. It is believed that this was the first instance in which *Double Throated* or *Double Enveloping* Worm Gears, having area contact, were ever made.

Even though such worm-gear drives were manufactured at the Lorenz Co., the value of Cone's contribution cannot be diminished when one considers his courage and strong will. Cone acted as a lone inventor whereas Lorenz used the skills of his many employees. Still, it took a long time and required much effort before the Cone drive technology became applicable in the United States. We must pay credit to the Lorenz invention, but the father of the American invention was Samuel I. Cone. Despite these facts, the question of who should claim exclusive rights for the double-enveloping worm-gear remains unanswered. Our opinion is that we have to credit both Lorenz and Cone.

Wilhelm Lorenz and Samuel Cone understood very well the advantages of the drives they had invented, particularly, the increased load capacity due to the higher contact ratio in comparison with that of conventional worm-gear drives. Although the geometry of the Lorenz and Cone drives differs, both types offer this advantage. Later, investigations showed that the double-enveloping worm-gear drive's higher efficiency results from the existence of more favorable lubrication conditions.

The complex geometry of the double-enveloping worm-gear drive, the specific conditions of lubrication (found later), and the formation of the worm-gear tooth surface as a two-part surface inspired many researchers to develop the analytical aspects of the meshing of the worm and the worm-gear tooth surfaces. Among such researchers are N. I. Kolchin, B.A. Gessen, and P.S. Zak in Russia, Sakai in Japan, and Litvin in the United States, whose investigation and results are presented in Litvin (1968,1994).

The primary manufacturer of double-enveloping worm-gear drives in the United States is the Cone Drive Textron Co. Presently, the Lorenz Co. does not produce these gear drives.

3.17 History of the David Brown Company

Percy Brown

The David Brown story concerns a prosperous British company that originated in the Victorian era and was successfully run by one family for 135 years. The company made significant contributions to the design and manufacture of gear transmissions and special products.

In 1860, the first David Brown (1843–1903) founded the company when he was only 17. Originally, he began his business as a patternmaker for cast gears in Huddersfield and later extended it to the manufacture of cast gears. About 1898, his sons inspired him to begin manufacturing machine cut gears, an idea that later led to his making significant contributions to the design and manufacture of gear drives.

After David Brown's death in 1903, his three sons, Ernest, Frank, and Percy, continued to run the company. Ernest took over the pattern business and Percy and Frank, the gear design and manufacture.

When Percy Brown died in 1931 and Frank in 1941, the company was then operated by David Brown (later Sir David), Frank's son and the only grandson of the first David Brown. Significant engineering contributions made by the David Brown Company are connected with the names of these outstanding engineers who worked for the company: F.J. Bostock, Arthur Sykes, H.E. Merrit (1971) (later doctor), and W.A.Tuplin (later doctor and professor). Among such contributions are the following:

Sir David Brown

1. In 1915, the invention of an involute worm-gear drive based on the application of a screw involute surface as the surface of the worm thread (proposed by F. J. Bostock and Percy Brown)
2. The ability to manufacture large gears (up to 40 ft in diam) following the design and installation in 1967 of a purposely built machine tool (later extended to 46 ft in diam with the acquisition of a Maag 1200)
3. In 1933, the design and manufacture of the Radicon worm-gear drives (the trademark "Radicon" indicates heat dissipation by radiation, conduction, and convection due to the special design proposed by Arthur Sykes.)
4. The successful design of vehicle transmissions (including the Merrit-Brown Tank Gearbox)
5. The manufacture of high-precision gears used in turbine reducers and equipment designed for the production and inspection of gears

In recognition of the David Brown Company's tremendous contribution to British industry during and after the Second World War, David Brown was knighted in 1968, and Arthur Sykes received the Order of the British Empire (O.B.E.) and a special gold medal presented by the AGMA for his services to the industry. In addition, Arthur Sykes, Dr. Merrit, and Prof. Tuplin gained wide recognition as engineering educators and textbook authors. Prof. Tuplin became the head of the postgraduate Department of Applied Mechanics at Sheffield University.

Sir David Brown could not perceive a logical successor to his family-operated company and sold it in 1990. It continues its successful activity as the David Brown Group PLC under the management of Cris Cook and Cris Brown.

3.18 History of The Gleason Works

William Gleason

Kate Gleason

William Gleason (1836–1922), founder of The Gleason Works, came to the United States to Rochester, New York from Ireland in 1851. He was a gifted inventor and a skilled mechanic. In 1874, his invention of the straight bevel gear planer for the production of bevel gears with straight teeth substantially advanced the progress of gear making. Manufacturing early straight bevel gears was a difficult process because the gears were cast in metal and then were finished by manually filing each tooth.

Other members of the Gleason family made substantial contributions to the prosperity of The Gleason Works. Noteworthy are the inventions of James E. Gleason (see below) and the active role Miss Kate Gleason, the secretary and treasurer, played in the sale of the company's products. N. Bartels (1997) has written an impressive article about her life and activity in the journal *Gear Technology*.

The early part of the 20th century was the beginning of the automotive industry, which required a broader application of bevel gears to transform rotation and power between intersected axes. In the 1920's, automotive industry designers also needed (1) a gear drive to transform motions and power between crossed axes and (2) a lower location for the driving shaft. The Gleason Works engineers met these needs with pioneering developments directed at designing new types of gear drives and the equipment and tools to generate the gears for these drives. The following patents granted to The Gleason Works proves how innovative their response was to the automotive industry's needs:

1. In 1905, James E. Gleason invented a two-tool bevel generator that enabled manufacturers to reduce the production time of straight bevel gears by 2 times because two reciprocating blades could generate the opposite sides of the tooth space.
2. In 1916, the process for generating spiral bevel gears and the machine for producing them were developed.
3. In 1927, The Gleason Works developed the method for generating hypoid gear drives.
4. In 1938, James Gleason invented the Formate cut method by which spiral bevel gears and hypoid gears were generated.

At present, The Gleason Works holds over 700 patents. A historian studying its achievements would discover that the company's activity combines the design and production of cutting tools and equipment with intensive gear research. Its major developments in gear theory follow:

1. Spiral bevel gears and hypoid gears with the gear tooth surfaces being in point contact instead of in line contact: the bearing contact can be localized as a set of instantaneous contact ellipses, reducing the sensitivity of the gear drives to misalignment.
2. Computerized methods (TCA, tooth contact analysis) that simulate the meshing and contact of hypoid and spiral gear drives: before manufacturing, these programs can be used to predict the shift in the bearing contact and the transmission errors caused by misalignment, allowing the design to be improved. Computer programs are commercially available for the gear industry.
3. Coordinate measurement of gear tooth surfaces: the data obtained can be applied to correct machine tool settings.

4. Finite-element analysis is used to determine stresses and TCA programs to simulate the meshing and contact of loaded gear drives.
5. Phoenix machines, six-axis, computer numerically controlled (CNC), for gear cutting and grinding: these machines can be used to modify the geometry of gears.

Many of the solutions to gear manufacturing problems were contributed by Edward W. Bullok, Ernst Wildhaber (1926, 1946, 1956), Meriwether L. Baxter, Jr. (1961, 1973), Theodore J. Krenzer (1981), Lowel E. Wilcox, Wells Coleman, and Dr. Hermann J. Stadtfeld, who also summarized the achievements of Gleason technology (Stadtfeld, 1993, 1995).

3.19 History of the Klingelnberg and Oerlikon Companies

Emil Georg Bührle

Gustav Adolf Klingelnberg

Klingelnberg Söhne and Oerlikon Geartec currently represent a group of companies very well known and respected for significant contributions that their engineers and managers have made to the technology and theory of gears, particularly, to the development of (1) the face-hobbed process of and the equipment for manufacturing spiral bevel gears and hypoid gears and (2) the method of grinding the worms of worm-gear drives (the worms are ground, or cut, by a disk whose surface is a cone). This method of manufacturing worms was applied by industry and resulted in a new type of worm-gear drives known as ZK-worms.

The face-hobbed process of generating spiral bevel gears and hypoid gears is one of the crowning technological achievements in this area. Its great advantage is the efficiency of continuous indexing; its disadvantage is the difficulty of grinding (in comparison with the grinding of face-milled gears). The history of the developments by Klingelnberg and Oerlikon shows that the "crown" was not just a lucky finding but the result of enthusiastic efforts of the inventors and risky decisions of the company managers, as exemplified in the following discussions.

In 1923, Klingelnberg Söhne built a machine that used the Palloid method for generating spiral bevel gears and hypoid gears with a conical hob. This method was invented in the 1920's by engineers Schicht and Preis and was later replaced by the face-milled and face-hobbed methods.

The Oerlikon company began early, in 1886, with the production of bevel gears by templates. Later developments are related to the names of inventors Brandenberger and Mamano and to Oerlikon-Bührle Company engineers who designed the Spiromatic machines and the head-cutters for them.

The Klingelnberg and the Oerlikon companies were successful because their founders were devoted and energetic. Among those we wish to mention are Dr. Gustav Adolf Klingelnberg and Emil Georg Bührle.

Under Dr. Klingelnberg's (1880–1947) leadership, Klingelnberg Söhne was transformed from a small trading company to a large manufacturer. In appreciation of his achievements and services to industry, Aachen University awarded him an honorary doctorate.

Emil Georg Bührle (1890–1956) was born in Germany and in 1924 settled in Switzerland where he became a naturalized citizen. His tremendous success there can be seen in his building the small Oerlikon Company to a group of more than 30 Oerlikon-Bührle Companies located throughout the world.

3.20 History of the Reishauer Corporation

Jacob Friedrich Reishauer

The Reishauer Corporation is presently well known in the gear industry for the development of methods and equipment for grinding high-precision involute spur and helical gears. These methods are based on the application of two types of grinding worms: (1) a cylindrical worm whose design parameters depend mainly on the normal pressure angle and the normal module of the gear to be ground and (2) a globoidal worm that is designed as the conjugate to the gear. The cylindrical worm and the gear being ground are in instant point contact whereas the globoidal worm and the gear are in line contact. Methods for dressing the grinding worms have been developed as well.

These methods and machines are a logical extension of the skill and experience of a company that has been in existence for more than 200 years. The Reishauer Corporation was founded in 1788 by Hans Jakob Daniker, a blacksmith and toolmaker. The activity of the company was continued by [illegible]

Dorothea Reishauer

1848) whose name the company received in 1824. This family operated the company over three generations: Jacob Friedrich Reishauer (1813–1862) represented the second generation and Georg Gottfried Reishauer (1840–1885), the third. After 1870, the ownership of the company was shared with other partners.

The period when the company was operated by Jakob Friedrich Reishauer deserves special attention. Jakob, a gifted engineer, married Dorothea Bodmer, the daughter of Johann Georg Bodmer (1786–1864), a distinguished inventor and engineer. Bodmer was well ahead of his time in developing tools to manufacture gears and may be considered the first pioneer in this area. The successful cooperation of Johann Bodmer and Jakob Reishauer resulted in their developing machines to produce the tools used in the manufacture of screws and nuts. They also designed and produced a lathe, a model of which still exists today.

The history of the Reishauer Corporation illustrates that the company's engineers have always been successful in applying the skill and experience they accumulated throughout the years of its existence.

3.21 The Development of the Theory of Gearing in Russia

Introduction

An die Nachgeborenen.
Wirklich, ich lebe in finstern Zeiten!

To the Descendants.
I live in dark times, indeed!

In this verse, Bertolt Brecht (Brecht, 1962, p.114), the convinced opponent of fascism, expressed with great emotion the pain he felt while living in a communist state in East Germany. Despite the Aesopian language, its meaning was clear for those who were unlucky enough to be born in the Soviet Union. Boris Pasternak, who described the same fear of communist terror in his novel *Doctor Zhivago* and was ostracized and persecuted for doing so, declared in the poem "The Nobel Prize" (1960, p. 246):

I am lost like a beast tracked down
Somewhere men live in freedom and light
But the furious chase closes in,
And I cannot break out from my plight.

For some who looked for a decent existence, safe harbor was found in their devotion to science. However, during the communist era in the former Soviet Union, all branches of science were controlled by party ideologues. The iron curtain separated Soviet scientists from the West, and the party falsified the history of developments in science and technique to further separate them. Admission to the graduate colleges and the recruitment of new researchers were controlled by the party through an insidious policy termed "the selection of personnel."

Among those who tried not to obey the dirty policy of selection were Profs. Chrisanf F. Ketov, Nikolai I. Kolchin, Vladimir N. Kudriavtsev, and Lev V. Korostelev, to whom we are indebted for the development of the theory of gearing in Russia. We call the years 1946 to 1980, the golden years of the Russian theory of

Chaim I. Gochman—Doctor of Applied Mathematics (1851–1916)

The pre-Soviet period of the theory of gearing concerns the research of Dr. Chaim I. Gochman, a distinguished Russian scientist who deserves the title "the founder of the analytical theory of gearing."

Theodore Olivier and Chaim Gochman, two great scientists, developed the foundation for the theory of gearing. Olivier introduced the idea of the generation of conjugate surfaces as the enveloping process and applied the concept of an auxiliary surface as the intermediate generating surface. In modern terms, the intermediate surface is called the tool surface. He also considered the gear tooth surfaces to be the envelopes to the auxiliary surface, and he discovered the way to provide the conditions for line contact and point contact of the generated surfaces of gears. However, Olivier insisted that the theory of gearing was exclusively the subject of projective geometry.

Gochman, while confirming the indisputable contribution of Olivier, opposed his statement relating the theory of gearing to projective geometry. By developing analytical methods, he transferred the theory of gearing to analytical and differential geometry, releasing it from the constraints of projective geometry. His additional contribution was based on the possibility of determining the instantaneous line of contact of enveloping surfaces in any reference system, not only in the system of the driven gear as was determined previously.

Gochman's pioneering work was advanced for his time and was almost forgotten until Artobolevski, a Russian academician, "discovered" it again and inspired Prof. Kolchin (see below) to seek its application.

Dr. Gochman's biography exemplifies one whose creativity is not recognized or acclaimed during his lifetime. He was born in 1851 in a small city in Grodnensk County, which at that time belonged to the western part of Russia. He graduated from Kherson gymnasium in 1871 when he was 20. His biographer Bogoliubov (1976) did not explain the reason for his late graduation from the gymnasium, but it was probably Dr. Gochman's second education, the first being a Jewish theological one. He then attended Novorossiysk University in Odessa and graduated in 1876, receiving the Golden Medal for his research project entitled "Analytical Method of the Solution to the Gearing Problems." He received a scholarship to continue his education at Novorossiysk University. In 1881, he passed the exams for a master's degree and received a 2-year scholarship to study abroad. Gochman's scientific career was proceeding well until, for some unknown reasons, he could not continue his activity at Novorossiysk University and had to work as an inspector for the Jewish Pedagogical Institute of Zhitomir. Here, he completed his master's thesis entitled "Theory of Gearing Generalized and Developed Analytically" (Gochman, 1886). He lost his job in 1887 when the Zhitomir Institute was closed, and he returned to Novorossiysk University to work as an adjunct associate professor in the mechanics department. His second period of research activity began here. In 1890, Gochman was awarded a doctor's degree in applied mathematics for his work "Kinematics of Machines (Part 1)." Unfortunately, Dr. Gochman could not finish this work, planned as a capital research project.

Chrisanf F. Ketov—Doctor of Technical Sciences and Professor of Mechanisms and Machines (1887–1948)

Chrisanf Ketov was born to a family of peasants and had no hope of an education but that made possible by the heroic efforts of his widowed mother. In 1914, the young Ketov graduated from the highly ranked St. Peterburg Institute of Technology and received a postgraduate position with a track for teaching in the Department of Applied Mechanics. During the difficult postrevolutionary period, he left St. Peterburg in 1919 to take a professorship at Turkestan University. In 1922, he returned to his alma mater as professor and head of the Department of Applied Mechanics. In 1930, after the colleges of two universities were joined, he became the head of the same department at the Leningrad Polytechnic Institute. He was then appointed dean of the College of Mechanics and Machines in 1944. He kept both positions until his death in 1948. One of his great contributions to education was in 1934 organizing, with Profs. Viakhirev and Kolchin, the Department of Automotive Machines.

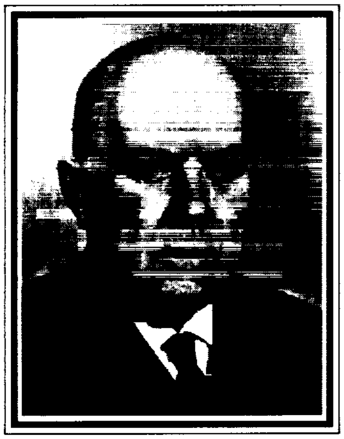
Prof. Ketov's personal contributions to gear technology and theory include a chapter in the *Technical Encyclopedia* (1927), his monograph on involute gearing (1934), and a textbook on applied mechanics (in present terms, the theory of mechanisms and machines) written with Prof. Kolchin (1939). He is well known as a distinguished engineer and a recipient of several patents. He deserves a special tribute for supervising doctoral students' scientific work in various areas, especially in the theory of gearing.

Prof. Ketov's role in the development of the theory of gearing should be recognized not only by his personal contributions but also by his influence on colleagues and students and by his founding The Leningrad Committee for Analysis of Gear Problems. The Leningrad Committee was soon recognized as the National Center of Gear Specialists and keeps the name of Prof. Ketov in memory of his work. The author of this book worked for Prof. Ketov for 11 years as an aide on the gear committee and as a professor in his department.

Chrisanf F. Ketov was a self-made man whose knowledge of literature and history was extensive. He was considered in all circles a very intellectual man. He was a very impressive interlocutor who was able to enrich conversation with historical parallels and witty definitions. He had no illusions about the cultural environment and his lifetime and cited Leo Tolstoy who said, "There are moral people and immoral people but all governments are immoral." Chrisanf Ketov meant of course *his own* government.

He faced difficult situations in his life. During the purge in the 1930's, he was arrested as a former "cadet" of the Constitution-Democrat Party, which was the most important liberal party in prerevolutionary Russia, but fortunately avoided the Gulag. Then as dean of the College of Mechanics and Machines, he was accused of resisting the policy of "selection of personnel," making his dismissal, if not his sudden death, inevitable. An ironic coincidence darkened the last day of his life when his request that a talented student be granted a position was declined because of the selection policy.

Nikolai I. Kolchin—Doctor of Technical Sciences and Professor of the Theory of Mechanisms and Machines (1894–1975)



Profs. Ketov and Kolchin were colleagues for many years. They both graduated from St. Peterburg Institute of Technology and then worked for the Leningrad Polytechnic Institute. They were coauthors of a textbook on the theory of mechanisms and machines. It was not a surprise that very often their names appeared together and that they were considered a team. In one such instance at a party, toasts were proposed to both of them, and a slightly irritated Prof. Kolchin joked, "I do not mind if you join our names, but do not expect a wedding kiss." In reality, they had quite different personalities: Prof. Kolchin did not like administrative duties so the administrative job was usually accomplished by his aids out of respect for his many merits.

Nikolai I. Kolchin was born in 1894 in Sarapul to a lower middleclass family. After graduation from high school, he was admitted to the prestigious St. Peterburg Institute of Technology. In 1918 after graduation, he was assigned to a university position having a professorship track. Initially, his interests were in thermodynamics, and he published a few papers on this subject in highly ranked journals. However, later in 1939, he chose the theory of gears as his major scientific area. As professor and scientist, he concentrated his activities in the Leningrad Polytechnic Institute from 1935 until his death in 1975 where he was the head of the department of mechanisms and machines.

Prof. Kolchin made significant contributions to the theory of gearing; thus, it would be incorrect to consider him just a follower of Chaim Gochman. He developed the geometry of bevel gears and worm-gear drives with double-enveloping and cylindrical worms and published the monograph "Analytical Investigation of Planar and Spatial Gearing" (Kolchin, 1949), which became very well known.

Prof. Kolchin devoted his life to scientific research, not only during the week but on weekends and vacations. He supervised the programs of 50 doctoral candidates and doctors of technical sciences and was always available for his students and for those who needed to consult with him.

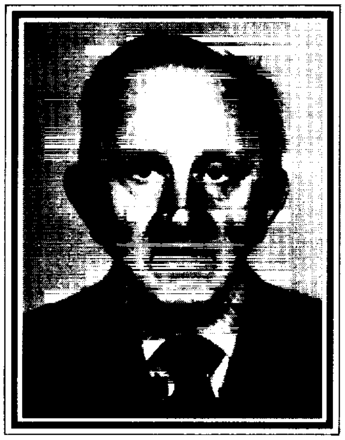
Mikhail L. Novikov—Doctor of Technical Sciences and Professor and Department Head (1915–1957)

Mikhail Novikov became very well known in his own country and in the international engineering community for the invention of a special type of helical gear. His idea was that he could overcome the barrier caused by the relations between the curvatures of the contacting surfaces when the gear tooth surfaces are in line contact. His approach is based on the following considerations: (1) the line contact of helical gear tooth surfaces is substituted by point contact; (2) the unfavorable relations between the curvatures required for line contact of the tooth surfaces do not exist any more (because of point contact), but the contacting surface line of action must be parallel to the gear axes; and (3) the designer is free to choose a small difference between the curvatures to reduce the contacting stresses.

Prof. Novikov proposed that the profiles of helical gear cross sections be chosen as circular arcs with a small difference between the radii of the circles (Novikov, 1956). Some researchers were reminded of Wildhaber, who proposed and patented a rack-cutter with a circular-arc profile (Wildhaber, 1926), and they began to call Novikov gears Wildhaber-Novikov gears. The main difference between these two inventions is that Novikov gears are in point contact and have more favorable relations between the gear curvatures. Wildhaber helical gears are in line contact and the relations between the curvatures are constrained. Novikov's invention became very popular in the former USSR and later in Russia, and he was awarded a prestigious national prize. His invention inspired many researchers, which resulted in valuable contributions being made to the theory of gearing.

Mikhail Novikov graduated in 1940 from the Military Aircraft Engineering Academy that bears the name of the famous scientist Zhukovsky. He worked at the academy as a professor and department head and was involved in intensive research work, which unfortunately ceased with his sudden death at a young age, a catastrophe for his family, colleagues, and students.

Vladimir N. Kudriavtsev—Doctor of Technical Science and Professor and Head of the Department of Machine Elements (1910–1996)



Vladimir N. Kudriavtsev was recognized by his contemporaries as a distinguished scientist, teacher, and gear drive designer. He was remarkable in that his engineering intuition enabled him to predict the behavior of gear trains under load, a talent that impressed his colleagues. The following are among his distinguished contributions to this area:

1. Methods to synthesize planetary drives
2. An effective approach to determine the efficiency of planetary trains
3. Force distribution (a) along the instantaneous contact line, (b) between teeth in the case of a contact ratio larger than 1, and (c) distribution of forces among satellites of a planetary train
4. Development (as the leading author) of the USSR standard for computing the strength of gears
5. Excellent experimental tests to determine gear drive life service and failure

The son of a physician, Vladimir Kudrjavtsev was born in Ostrogozhsk, a small city in Voronezh county. After graduation from high school, he worked in the machine industry as a designer to acquire experience while he continued his education at the Leningrad Institute of Industry. He graduated in 1938 with distinction, receiving a bachelor of science degree in engineering.

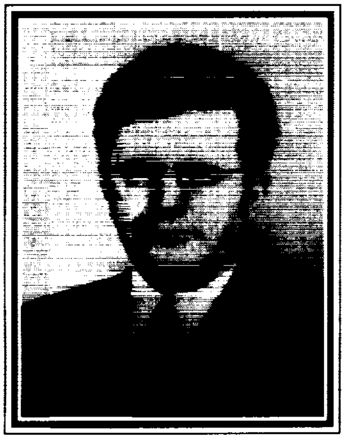
In 1942, he received a doctor's degree for his research entitled "Engineering Methods of Computation of Strength of Gear Trains." In 1952 he received the degree of doctor of technical science for his work "Investigation of Planetary Trains."

In 1946, he began his teaching career as a professor of machine elements at the Leningrad Engineering Academy of the Air Forces and then in 1958, as professor and department head of machine elements and director of the Laboratory of the Leningrad Institute of Mechanics. Books written by Prof. Vladimir N. Kudriavtsev and his colleagues became important handbooks for designers (Kudriavtsev, Kuzmin, and Filipenkov, 1993). Prof. Kudriavtsev published the well-known textbook *Machine Elements* and was an excellent teacher who supervised the scientific work of 67 doctoral candidates and 7 doctor of science programs.

In recognition of his achievements, Prof. Kudriavtsev received the prestigious title "Honored Worker of Science and Engineering."

In addition to his extensive scientific and engineering background, Prof. Kudriavtsev was a passionate lover of music and literature as evidenced by his excellent taste and knowledge of both.

Lev V. Korostelev—Doctor of Technical Sciences and Professor (1923–1978)



Lev V. Korostelev was born to a family who had been members of the intellectual class of Russian society for several generations.

He graduated from the Moscow Machine and Tool Institute in 1950 and then concentrated his research work there. He received a Ph.D. in engineering in 1954 and a doctor of technical sciences in 1964 for his thesis entitled "Geometric and Kinematic Criteria of Spatial Gearing Correlated with Loading Ability." He then successfully continued his research and made significant contributions, in recognition of which he was selected for the positions of department head and vice president of academic affairs of the university.

Dr. Korostelev belonged to the second generation of scientists who developed the theory of gearing in Russia. He published approximately 100 papers that dealt with the following subjects: the kinematic sensitivity of gears to alignment errors of the gear axes (Korostelev, 1970); the synthesis of spatial gearing by applying the concept of a generating surface performing screw motion; and the investigation of a generalized type worm-gear drive. He also received 30 patents and supervised the Ph.D. programs of 10 graduate students.

To the deep regret of his relatives, friends, and colleagues, Dr. Korostelev's very promising career was interrupted by his death in a car accident.

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This book presents recent developments in the theory of gearing and the modifications in gear geometry necessary to improve the conditions of meshing. Highlighted are low-noise gear drives that have a stable contact during meshing and a predesigned parabolic transmission error function that can handle misalignment during operation without sacrificing the low-noise aspects of operation. This book also provides a comprehensive history of the development of the theory of gearing through biographies of major contributors to this field. The author's unique historical perspective was achieved by assiduous research into the lives of courageous, talented, and creative men who made significant contributions to the field of gearing.

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