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# FUZZY SPATIAL MAP REPRESENTATION FOR MOBILE ROBOT NAVIGATION

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## Introduction

An aim of intelligent robotics research is to develop mobile robots capable of navigation in ustructured environments. Research is being pursued in industry and government organizations towards a variety of applications of such vehicles. Examples include planetary rovers, mobile security robots, military surveillance vehicles, and robotic vehicles for use in factories, hazardous environments, and on smart highways. The practicality of such vehicles is a function of their ability to utilize spatial representations of the physical environment for goal-directed navigation throughout. In many of these applications the ultimate goal is *autonomous* navigation in environments that are uncertain, unpredictable, and possibly unstructured and dynamic. As such, robots deployed in these environments must be capable of handling unexpected events and complex situations. Autonomous navigation in such real world environments requires access to information about locations of obstructions and/or landmarks. This spatial information is typically represented in the form of a map. The particular form of map representation employed by mobile robot control systems is an important issue as it directly affects real-time map-based navigation.

Traditional approaches to map representation (for autonomous vehicle navigation) have attempted to capture explicit metric information about the environment at the expense of real-time performance. Explicit world maps require significant memory and processing power for data storage and effective maintenance. For autonomous mobile robots to be useful they must maintain real-time performance and relatively low computation since, typically, limited computational resources are carried onboard. At the same time, they must deal with the numerous sources of uncertainty presented by the real world. With this in mind, a useful map representation should be space (memory) efficient and easy to retrieve and maintain [1]. Furthermore, stored information should be maintained at a high level of abstraction, e.g. as an approximate description of the actual world. While explicit spatial data may be required for applications that rely on precise information, uncertain spatial maps are sufficient for autonomous navigation.

As an alternative to traditional approaches, a map representation based on fuzzy relations is proposed that can be used to facilitate real-time autonomous navigation in real world environments. The proposed approach is motivated by the practical requirements that drive the design of mobile robot systems, namely, real-time performance, modest computation, and qualitative reasoning and modelling even at low levels of control.

## Spatial Map Representations

Autonomous mobile robots must have some cognizance of the spatial distribution of objects and places of interest in the environment in order to effect goal-directed behavior. As is the case with humans who visit a city for the first time, or office building for that matter, the requirement for spatial familiarity often necessitates access to a map (be it physical or mental). Autonomous robots, like humans, may make use of spatial information represented in a variety of forms. These can be generally classified as either geographical, topographical, or topological maps. Geographical and topographical [2] maps highlight detailed information about natural and geometrical features and locations in a physical region. They are usually employed to model large-scale outdoor regions. Topological maps specify distinct objects or places in a region and connections among them. The amount of detail is typically less than that provided by geo/topographical maps. Topological maps are suitable for representing indoor and outdoor environments [3, 4].

As alluded to above, geo/topographical maps are often quite memory intensive. In addition, processes that interpret or modify these maps tend to be computationally intensive and are subject to error or misinterpretation due to sensor and actuator inaccuracies. Consequently, for mobile robots that carry computation completely onboard, real-time response is often impractical. The qualitative nature of topological maps makes them more desirable for most practical applications. The choice of map representation for a given application involves a tradeoff between the requirements of vehicle tasks and implementation efficiency. A recent comparative study [5] suggests that if reliability (of map correctness) and accurate vehicle path tracking is important, one should opt for explicit map representations. If light computation and the ability to accommodate uncertainties is important, then qualitative topological representations should be chosen.

### Uncertain spatial map representations

Qualitative navigation requires abstract spatial representations that can account for real-world uncertainties. Robust and error-tolerant representations allow for simple but useful descriptions of complex environments. When viewed as an intelligent autonomous system the human, for example, exhibits the capability to successfully move about using some form of abstraction of the physical environment. When necessary to navigate in cluttered environments, humans are not concerned with exact information about locations or sizes of objects therein (e.g. 'the desk is 3.23 ft. ahead and extends 5.4 ft. from a wall that is oriented $23^{o}$ from the current heading, etc.'). In fact, much of the information available in an explicitly mapped environment is not used to navigate. This holds for humans and, to a smaller degree, for systems of lesser intelligence such as autonomous robots.

A suitable abstraction for use by autonomous vehicles can be determined using fuzzy logic and the underlying fuzzy set theory introduced by Zadeh [6, 7]. Spatial information can be represented in a map that reveals an approximate distribution of obstacles in some region of interest. The uncertainty of this distribution is represented as a fuzzy relation which can be thought of in this case as a "fuzzy Cartesian system" where coordinates are approximate locations of objects in the region. Figure 1 illustrates the idea of a hypothetical 2-dimensional

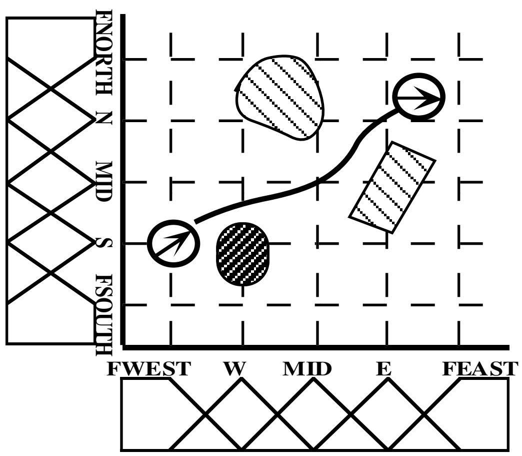


Figure 1. Fuzzy Cartesian coordinate system.

fuzzy Cartesian space. The ordinate and abscissa are partitioned using fuzzy sets with labels designating varying degrees of north, south, east, and west (the "F" indicates Far). The circle represents an autonomous vehicle with the arrow indicating its direction of travel. In this representation the darkened object, for example, is located at a fuzzy coordinate of south-west; the map does not reveal its exact location or size. Other locations can be similarly represented using any desired fuzzy set resolution on the ordinate and abscissa. We develop this idea further and in a more formal manner in the Section IV. Let us first make mention of similar approaches to uncertain spatial map representation.

**Related Work**

The notion of employing approximate reasoning techniques for representing and manipulating uncertain spatial information is not unique. A number of approaches based on probability and fuzzy logic have been recently proposed. Elfes [8] proposed a probabilistic tessellated representation called the occupancy grid. Estimates of the state of a grid cell are determined from range readings using probabilistic sensor models and updated using Bayesian estimation procedures. A state of a grid cell is a discrete random variable indicating a measure of confidence in the occupancy of the cell. Later the histogram grid [9], a variant of the occupancy grid representation, was proposed to facilitate real-time in-motion mapping for obstacle avoidance. It differs from the occupancy grid in the way that the histogram grid is built and updated, resulting in a representation more suitable for real-time execution by an onboard computer. Dutta [10] employed fuzzy logic for representing induced spatial constraints between landmarks given imprecise or incomplete information about them. A framework was developed for reasoning about relative positions and motion of objects in response to spatial queries about such relationships. Ning et al [11] proposed a fuzzy set theoretic approach to solving the weighted region problem associated with cost-based path planning. An uncertain map representation was used in which fuzzy regions of the terrain were defined by intersections of "fuzzy half-planes" (linear inequalities with triangular fuzzy numbers as coefficients). Similar approaches that utilize the spatial reasoning capability of fuzzy logic have also been suggested in the image processing arena [12-14]. These focus on relative position relationships among image objects in a scene.

Each of these uncertain spatial representations are well-suited for the problems for which they have been proposed. For a given application the most appropriate representation depends on what spatial information is relevant and how easily it can be extracted and maintained. Of the approaches mentioned, [8] shares the greatest conceptual similarity with the method presented here regarding the potential applicability to autonomous navigation. The fundamental difference is that in [8], occupancy is indicated as a probability whereas in the fuzzy relational approach it is indicated as a *possibility*. For distinctions between probability and possibility see [15] and articles therein. In mobile robot navigation systems it is sometimes beneficial if a map is general enough to represent multiple terrain attributes. In the framework of stochastic representations, Elfes [8] refers to such models as inference grids. The fuzzy relational approach offers such a general representation and is flexible in the amount of different approximate attributes that can be represented at a given spatial location.

**Fuzzy Relational Representation**

Zadeh [6] refers to a fuzzy set as a class of objects with a continuum of grades of membership. This is in contrast to the classical notion of a (crisp) set for which set membership is a discrete binary value (0 or 1). The membership function of a fuzzy set A, denoted by $\mu_A(x)$ maps the elements of some universal set X into a numerical value within the range [0,1], i.e.

$$\mu_A(x) : X \dashrightarrow [0,1] \qquad (1)$$

A membership grade of zero corresponds to an element which is definitely not a member of the fuzzy set, while a grade of one corresponds to the case where the element is definitely a member of the fuzzy set. In classical set theory, a relation is considered to be a set of tuples or ordered pairs that represents the absence or presence of association between the elements of two or more crisp sets. This concept can be generalized to allow for various degrees or strengths of relation between elements of *fuzzy* sets [16]. That is, a fuzzy relation is a fuzzy set of tuples where each tuple has a membership degree in [0, 1]. Its formal definition is

*Definition 1.* :
Let $X_1$, $X_2$, ..., $X_n$ be continuous universes of discourse (universal sets), and $\mu_R(x)$: $X_1 \times X_2 \times \ldots \times X_n$ ---> [0,1], then

$$R = \int_{X_1 \times X_2 \times \ldots \times X_n} \frac{\mu_R(x_1, x_2, \ldots, x_n)}{(x_1, x_2, \ldots, x_n)} \qquad (2)$$

represents an n-ary fuzzy relation on $X_1 \times X_2 \times \ldots \times X_n$.

The integral symbol in (2) denotes the set of all tuples of the integrand on the Cartesian product, $X_1 \times X_2 \times \ldots \times X_n$ (the integrand notation in (2) is common for representing elements of a fuzzy set and their corresponding membership grades).

For the purpose of applying this formalism to the representation of spatial information we consider a two-dimensional plane in physical space. Let X be the universe of discourse spanning the east-west dimension of the physical space of interest; let Y be the universe spanning the north-south dimension. This sets up a two-dimensional plane defined by the Cartesian product $X \times Y$. The universes, X and Y, can be partitioned by fuzzy sets (e.g. as in Figure 1) such that any location in the plane can be indicated by a "fuzzy coordinate". This fuzzy coordinate is simply an ordered pair, $(x, y) \subseteq X \times Y$, made up of the fuzzy sets $x$ and $y$ given by the projections of the location on X and Y respectively. An example is depicted in Figure 1 where the location of the darkened object is given by the ordered pair (S, W).

Now, with any such fuzzy coordinate, we can associate information revealing the degree to which that location is occupied by an obstacle of interest (for whatever reason) to an autonomous vehicle navigating within the region. In the simplest case this information could be a membership grade in a fuzzy set called OCCUPANCY, for example, defined over some universe Z with corresponding linguistic labels like EMPTY and FULL. This simple case characterizes a ternary fuzzy relation on $X \times Y \times Z$. As an example, Figure 2 depicts the uncertain spatial information that was previously represented in two dimensions in Figure 1. The third dimension represents occupancy. The shapes shown are meant to represent the degree of occupancy rather than the actual shapes of the obstacles. The highest degree ($\mu_R = 1$) of occupancy corresponds to the (fuzzy) center of the obstacles and it gradually tapers off with the associated level of uncertainty. Such a representation accounts for uncertainty in the location and size of obstacles in the region while providing sufficient information for sensor-based navigation.

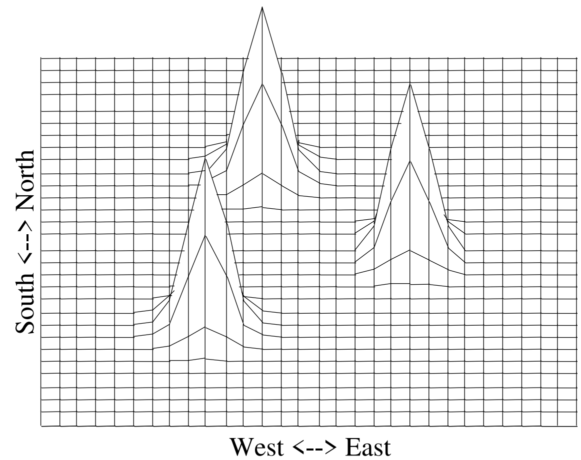


Figure 2. Fuzzy Spatial Map.

Without much difficulty the concept generalizes to the case where multiple attributes existing at a fuzzy coordinate can be specified as an n-ary fuzzy relation. For instance, in addition to occupancy, fuzzy information about terrain slope, elevation, traversability condition, etc. that may be available can be represented in the map. This could be beneficial in representing natural terrain for an outdoor mobile robot (e.g. a planetary rover or military surveillance vehicle) capable of traversing regions of varied elevation and surface characteristics. The representation is particularly convenient for use with fuzzy controllers that provide a mapping capability [17].

Finally, the uncertain spatial map data can be represented at a high level of abstraction using linguistic statements in the form of fuzzy if-then rules. This is possible since a fuzzy rule, or fuzzy rule base for that matter, is formally represented as a fuzzy relation. Using the linguistic terminology from the examples above we can write the following fuzzy rule: IF y is S and x is W THEN occupancy is FULL. A finite set of rules of this form can be formulated to represent the approximate distribution of obstacles, landmarks, etc. throughout the region of interest. Information can be easily retrieved and maintained by accessing the fuzzy rule base and manipulating the appropriate rules. In addition, only information associated with occupied areas need be represented in the spatial rule base. This allows for a concise representation of the environment, particularly for sparsely occupied regions. Thus, memory and computational efficiency are conceivable. These characteristics contribute to the suitability of the fuzzy relational representation for use in real-time autonomous navigation.

## Example Simulation

A navigation simulation was done using the fuzzy spatial map representation. The mobile robot is modeled after a differential drive vehicle of 1/2-meter diameter. Its maximum velocity is 1.5 m/s. The robot is controlled using fuzzy logic-based goal-seeking and collision avoidance behaviors.

The objective of the simulation was to navigate from the origin to a goal position designated as the coordinate (8.5m, 8.0m) within a 9mx9m region. The region of interest has the same obstacle arrangement as that shown in Figure 2. Figure 3 illustrates the result of the simulation. In the figure, the robot icon is the small circle with the radial line representing its direction. The obstacles are shown for illustration as 2-D contours where the innermost rings represent the highest possibility of occupancy in the map. A fuzzy spatial map of three rules was used to represent the obstacles (a rule for each

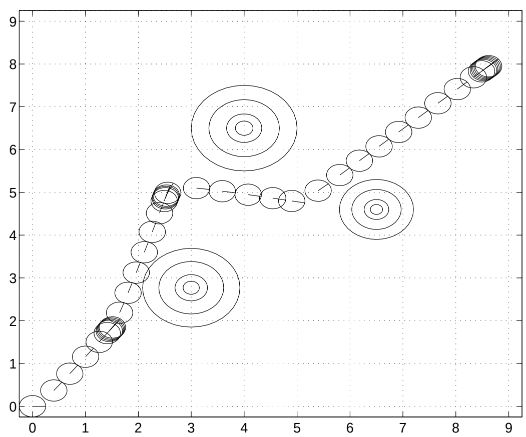


Figure 3. Fuzzy spatial map-based navigation.

obstacle) and to infer the degree of occupancy of locations in the region. Controlled by a combination of goal-seeking and collision avoidance behaviors, the robot was able to use the fuzzy spatial map to navigate to the designated goal without explicit metric information about the region.

## Summary

The form of map representation employed by autonomous mobile robot navigation systems is an important issue as it directly affects real-time map-based navigation. A method has been proposed for representing uncertain spatial information in maps used to guide autonomous motion in real world environments. The method is based on the use of fuzzy relations. It results in a concise representation that can be expressed in linguistic terminology and is easy to retrieve and maintain. A simulated navigation example demonstrated the applicability of the representation in a simple cluttered environment.

The fuzzy relational representation is suitable for storing uncertain information about the occupancy of spatial regions and can be generalized to include additional information about the state of mapped areas.

Further research is planned that will include additional simulation studies employing the fuzzy spatial map representation. This will be followed by integration of the map representation into existing hierarchical behavior control architectures based on fuzzy logic.

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