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Computerized Generation and Simulation of Meshing of Modified Spur and Helical Gears Manufactured by Shaving

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Abstract

Modification of geometry of spur and helical gears with parallel axes and helical gears with crossed axes is proposed. The finishing process of gear generation is shaving. The purposes of modification of the gear geometry are to localize and stabilize the bearing contact, and to reduce noise and vibration. The goals mentioned above are achieved by using profile crowning and plunging the shaver by a prescribed motion during pinion generation. The pinion becomes double crowned. The gear member is generated as a conventional involute gear. A tooth contact analysis (TCA) program for simulation of meshing and contact was developed and the analysis is illustrated with TCA results for spur and helical gears.

Nomenclature

α_n	Pressure angle in normal section
β_l $(l=c,\ t)$	Skew angle of rack-cutter applied for generation of pinion *c* and gear *t* shaver
β_i $(i=1,2,s)$	Helix angle of pinion *1* and gear *2,* and shaver *s*
$\Delta\lambda_2$	Error of lead angle of gear *2*
$\Delta\gamma$	Error of crossing angle γ formed between pinion and gear axes
ΔE	Error of center distance *E* between pinion and gear axes
ϕ_i $(i=1,2)$	Angles of rotation of pinion *1* and gear *2* in the process of meshing
$\Delta\phi_2\ (\phi_1)$	Function of transmission errors
$\Delta\psi_p$	Additional rotation of the pinion due to the plunging motion
γ_s	Crossing angle formed between pinion and shaver axes
Σ_l $(l=c,\ t)$	Surface of rack-cutter applied for generation of pinion shaver and gear shaver
Σ_d $(d=s,\ g)$	Surface of shaver applied for generation of pinion and the gear
Σ_i $(i=1,2)$	Surfaces of pinion *1* and gear *2*
ψ_d $(d=s,\ g)$	Angle of rotation of the shaver in the process for generation of the pinion and the gear
a	Parabola coefficient of predesigned parabolic function of transmission errors
a_c	Parabola coefficient of profile crowned rack-cutter
a_{pl}	Parabola coefficient of plunging motion of the shaver
E	Shortest distance between the axes of the pinion and gear
E_{s1}	Shortest distance between the axes of pinion and shaver
l_p	Displacement of the pinion in axial direction during the plunging motion
m_n	Normal module of gear teeth
$\mathbf{n}_f^{(i)}$ $(i=1,2)$	Surface *i* unit normal in coordinate system S_f
N_i $(i=1,2,s)$	Number of teeth of pinion *1* , gear *2*, and shaver *s*
p_1	Screw parameter of the pinion
$\mathbf{r}_f^{(i)}$ $(i=1,2)$	Position vector of surface *i* in coordinate system S_f
r_i $(i=1,2,s)$	Radii of pitch cylinders of pinion *1* , gear *2*, and shaver *s*
$\mathbf{v}^{(ld)}$ $(l=c,t)$ $(d=s,g)$	Relative velocity of contact point in coordinate system S_l with respect to coordinate system S_d

1. Introduction

Shaving was invented as a finishing process of generation of spur and helical involute gears [1]. The basic idea of shaving is based on simulation of meshing of two helical gears or a helical and spur gears that perform rotation between crossed axes. One gear of the pair is a shaver that is designed as a helical gear provided with gashed teeth to obtain cutting edges (Fig. 1). In the process of meshing of the shaver with the workpiece, due to sliding in relative motion, the cutting edges of the shaver act as blades and remove very thin pieces of material from the teeth of the workpiece.

The shaver and the workpiece are in point contact at every instant. Contact pressure is provided wherein the shaver is installed at the shaving machine and therefore the surface contact is spread over an elliptical area of small dimensions. The simulation of meshing of crossed gears in point contact permits the paths of contact on the shaver and the workpiece tooth surfaces (Fig. 2) to be obtained. There is only a narrow strip on the tooth surface of the workpiece in the neighborhood of the path of contact that is shaved by the shaver. Therefore, to shave the whole tooth surface of the workpiece, it is necessary to provide for the shaver or the workpiece translational motion along the axis of the workpiece. Such translational motion must be accompanied with additional rotation of the workpiece to provide the resulting motion as a screw one with the same parameter of the workpiece that is considered as a helicoid.

Using a conventional shaving process, one can obtain the shaved surface of the workpiece as an involute surface. Therefore, the shaved tooth surfaces of the pinion and gear will be in line contact in an aligned gear drive.

A new trend in the design of gears is directed at substitution of instantaneous line tangency of contacting surfaces by instantaneous point contact. This permits, if a proper design is accomplished, a reduction in sensitivity of the mating gears to misalignment and a reduction in the shift of the bearing contact caused by errors of alignment.

In addition to observation of point contact of shaved surfaces, it is required as well to reduce the vibration of shaved gears. This can be obtained by the reduction of magnitude of transmission errors and obtaining a favorable shape of the function of transmission errors (see below).

The shaving process has been described in a gear handbook [1] and it was a subject of research represented in [2]. Modification of helical and spur gears was a subject represented in [3].

The main goals of the paper are modification of the existing shaving process that satisfy the following requirements:

1. The shaved pinion-gear tooth surfaces are in point contact (not line contact). Therefore, the shift of the bearing contact caused by misalignment is reduced.
2. The shape of the function of transmission errors of the gear drive and the low magnitude of transmission errors reduces vibration and noise of the gear mesh.

These goals are achieved by application of an approach based on the following ideas:

1. The shaved pinion tooth surface is generated as a double-crowned surface. The term double-crowning means crowning (modification) in two directions, profile and longitudinal ones. Profile crowning of the pinion tooth surface is achieved by application of a shaver with a tooth surface that is deviated from the conventional screw involute surface usually applied for a pinion shaver. Crowning of the pinion in the longitudinal direction is accomplished by variation of the plunge of the shaver executed as variation of the shortest distance E between the axes of the shaver and the pinion. The variation of the plunge is represented by equation

$$E = E_0 - a_{pl} l_p^2 \quad (1)$$

Here: E and E_0 are the current and initial distances between the axes of the pinion and the shaver; l_p is the axial displacement in the direction of the pinion axis that is measured from the middle of pinion tooth length; a_{pl} is the parabola coefficient of parabolic function ($a_{pl} l_p^2$).

2. The gear tooth surface is not modified: it is generated as a screw involute tooth surface.

3. A parabolic function of transmission errors [4, 5] is provided for the meshing of shaved pinion and gear tooth surfaces. Such a function reduces noise and vibration of the gear drive that are caused by errors of alignment.

4. The analytical determination of the crowned pinion tooth surface and gear tooth surface is based on application of two-parameter enveloping process wherein two independent sets of parameters of motion are considered: (i) one set of parameters of motions are the angles of rotation of the shaver and the workpiece (that is the pinion or the gear), and (ii) the other set are the parameters of feed motion (including the shaver plunge wherein the pinion generation is considered).

5. The meshing and contact of shaved pinion and gear tooth surfaces is simulated by developed Tooth Contact Analysis (TCA) computer program. A TCA computer program has been developed for shaved spur gears and helical gears with parallel and crossed axes.

The developed theory is illustrated with numerical examples.

2. Derivation of Shaver Tooth Surfaces

Imaginary Rack-Cutter Surfaces. A surface of a helical gear may be considered as generated by an imaginary rack-cutter that performs translational motion wherein the helical gear performs related rotation [4]. Henceforth, we will consider two cases of derivation of shaver tooth surfaces:

Case 1: In the existing shaving process the pinion and gear are considered as conventional involute gears and pinion shaver and gear shaver are designed as screw involute gears. In this case a common rack-cutter with straight-line profiles may be applied for derivation of screw involute surfaces of the pinion and gear shavers. The disadvantages of such an approach have been mentioned above.

Case 2: The pinion tooth surface is shaved as a double-crowned one and the gear tooth surface is shaved as a conventional involute surface. In case 2 we apply two rack-cutters with mismatched profiles as shown in Fig. 3(a). The profiles are the normal sections of skew rack-cutters. One of the rack-cutters with straight-line profiles is applied for generation of gear screw involute shaver. The other rack-cutter is provided with parabolic profiles (Figs. 3(a) and 3(b)) and is applied for generation of profile crowned pinion. Application of rack-cutters shown in Fig. 3 improves the existing shaving process (see section 6).

Rack-Cutter Surfaces. We designate by Σ_c and Σ_t surfaces of rack cutters applied for generation of pinion and gear shavers, respectively.

Coordinate systems S_a and S_b (Fig. 3(b)) are rigidly connected to the parabolic profile that is the normal section of Σ_c. The origin of coordinate system S_a coincides with point Q of tangency of mismatched rack-cutter profiles (Fig. 3(a)). Axes x_a and y_a are the tangent and the normal to the parabolic profile of the rack-cutter. Axis y_a passes through point P.

Surface Σ_c of the rack-cutter is generated wherein coordinate system S_b with the parabolic profile of the rack-cutter performs translation along $\overline{O_cO_b}$ (Fig. 4). Axes z_c and z_b form angle β_c that determines the orientation of skew teeth of the rack-cutter. Parameters u_c (Fig. 3(b)) and $\theta_c = |\overline{O_cO_b}|$ (Fig. 4) are the surface parameters of the skew rack-cutter. Fig. 4 shows the right-hand skew rack-cutter.

Surface Σ_c of the rack-cutter is determined by the following matrix equation (Figs. 3(b) and 4)

$$\mathbf{r}_c(u_c,\theta_c) = \mathbf{M}_{cb}(\theta_c)\mathbf{M}_{ba}\mathbf{r}_a(u_c) = \mathbf{M}_{ca}\begin{bmatrix}u_c & -a_c u_c^2 & 0 & 1\end{bmatrix}^{\mathrm{T}} \tag{2}$$

where a_c is the parabola coefficient of parabolic function $(-a_c u_c^2)$. Equation (2) means that surface Σ_c is represented by vector function $\mathbf{r}_c(u_c,\theta_c)$ which components are

$$\begin{aligned} x_c &= u_c \cos\alpha_n + (a_c u_c + l_c)\sin\alpha \\ y_c &= [u_c \sin\alpha_n - (a_c u_c + l_c)\cos\alpha_n]\cos\beta_c + \theta_c \sin\beta_c \\ z_c &= [u_c \sin\alpha_n - (a_c u_c + l_c)\cos\alpha_n](-\sin\beta_c) + \theta_c \cos\beta_c \end{aligned} \tag{3}$$

where (Fig. 3(b))

$$l_c = \frac{\pi m_n \cos\alpha_n}{4} \tag{4}$$

Normal $\mathbf{N}_c$ to rack-cutter surface Σ_c is represented as

$$\mathbf{N}_c = \frac{\partial \mathbf{r}_c}{\partial u_c} \times \frac{\partial \mathbf{r}_c}{\partial \theta_c} \tag{5}$$

Similarly, we may represent surface Σ_t of rack-cutter of the gear shaver by vector function $\mathbf{r}_t(u_t,\theta_t)$ taking into account drawings of Fig. 3(c) and coordinate transformation from S_e to S_t. Here,

$$\mathbf{r}_c(u_t,\theta_t) = \mathbf{M}_{tk}(\theta_t)\mathbf{M}_{ke}\mathbf{r}_e(u_t) = \mathbf{M}_{te}\begin{bmatrix}u_e & 0 & 0 & 1\end{bmatrix}^{\mathrm{T}} \tag{6}$$

Normal $\mathbf{N}_t$ to Σ_t is represented as

$$\mathbf{N}_t = \frac{\partial \mathbf{r}_t}{\partial u_t} \times \frac{\partial \mathbf{r}_t}{\partial \theta_t} \tag{7}$$

Derivation of Shaver Tooth Surface. The shaver tooth surface is generated by the rack-cutter that performs translational motion $s_d = r_d\psi_d$ wherein the shaver performs rotational motion determined by rotation angle ψ_d (Fig. 5). The axodes of the rack-cutter and the shaver are represented by plane Π and cylinder r_d, respectively. Axis *P-P* is the instantaneous axis of rotation. Designation $l=c,\ t$ and $d=s,\ g$ indicate rack-cutter surfaces Σ_c and Σ_t and the shaver surfaces Σ_s and Σ_g designated for the pinion and the gear, respectively. Coordinate system S_m is the fixed one where the motions of the rack-cutters and the shavers are represented.

Surface Σ_d of the shaver is represented by the equations

$$\mathbf{r}_d(u_l,\theta_l,\psi_d)=\mathbf{M}_{dl}(\psi_d)\mathbf{r}_l(u_l,\theta_l) \tag{8}$$

$$f_{dl}(u_l,\theta_l,\psi_d)=0 \tag{9}$$

Here: matrix equation (8) represents the family of rack-cutter Σ_l ($l=c,\ t$) in coordinate system S_d . Equation (9) is the equation of meshing that may be determined considering the scalar equation

$$\mathbf{N}_l\cdot\mathbf{v}^{(ld)}=0 \tag{10}$$

or the rule that the normal to rack-cutter surface Σ_l at the line of tangency of Σ_l and Σ_d must pass through the instantaneous axis of rotation *P-P* [4].

3. Derivation of Pinion Tooth Surface Σ_1

Kinematics of Pinion Shaving. We consider two independent sets of parameters of motion that are performed during the shaving of the pinion (Fig. 6):
Set 1: Combination of related rotations ψ_s and ψ_1 of the shaver and the pinion, where $\psi_s=(N_1/N_2)\psi_1$.
Set 2: Combination of translation l_p in direction of pinion axis, additional rotation of the pinion $\Delta\psi_p$ and plunge Δs_{pl} in the direction of shortest distance E_{s1} . Here,

$$\Delta\psi_p=\frac{l_p}{p_1} \tag{11}$$

where p_1 is the screw parameter of the pinion and

$$E_{s1}=E_{s1}^0-\Delta s_{pl}=(r_1+r_s)-a_{pl}l_p^2 \tag{12}$$

Here E_{s1}^0 and E_{s1} are the initial and current magnitudes of the shortest distance; a_{pl} is the parabola coefficient of the parabolic function $(a_{pl}l_p^2)$; r_1 and r_s are the radii of the pitch cylinders of the pinion and the shaver, respectively.

In reality the second set of parameters of motion (it is the set of feed motions) becomes related with the first one after the speed of the feed is chosen. Consideration of the second set as an independent one with the respect to the first one permits determination of the shaved pinion tooth surface as independent to the feed motion.

Applied Coordinate Systems. Coordinate systems S_s and S_1 are rigidly connected to the shaver and the pinion (Fig. 7). The meshing of the shaver and the pinion is considered in the basic fixed coordinate system S_0. Rotation of S_1 is considered in coordinate system S_w that performs translational motions l_p and s_{pl} with respect to S_0 . Rotation of the shaver is considered in additional fixed coordinate system S_u .

Axes z_s and z_1 form a crossed angle γ_s determined as (Fig. 8)

$$\gamma_s=|\beta_s\pm\beta_1| \tag{13}$$

where the upper (lower) sign correspond to application of the same (opposite) directions of the helices of the shaver and the pinion.

Pinion Tooth Surface. The derivation of the pinion tooth surface is based on application of two-parameter enveloping process [4,6] and requires the following procedure:

Step 1: Considering surface Σ_s of the pinion shaver as given, we represent equations of the family of Σ_s in coordinate system S_1 rigidly connected to the pinion as follows

$$\mathbf{r}_1(u_s,\theta_s,\psi_s,l_p)=\mathbf{M}_{1s}(\psi_s,l_p)\mathbf{r}_s(u_s,\theta_s) \tag{14}$$

where ψ_s and l_p are the generalized independent parameters of motion; (u_s,θ_s) are the surface parameters of the shaver.

The normal $\mathbf{N}_s^{(s)}$ to the shaver surface Σ_s is represented in S_1 by the following equation

$$\mathbf{N}_1^s(u_s,\theta_s,\psi_s,l_p)=\mathbf{L}_{1s}(\psi_s,l_p)\mathbf{N}_s^{(s)}(u_s,\theta_s) \tag{15}$$

where $\mathbf{L}_{1s}$ is the (3×3) submatrix of (4×4) matrix $\mathbf{M}_{1s}$.

Step 2: In the case of two-parameter enveloping, we have to consider two (not one) equations of meshing that may be represented as follows

$$f_{1s}^{(1)}(u_s,\theta_s,\psi_s,l_p)=\mathbf{N}_1^{(s)}\cdot\mathbf{v}_1^{(1s,\psi_s)}=0 \tag{16}$$

where $\mathbf{v}_1^{(1s,\psi_s)}$ is the relative velocity obtained for the condition that ψ_s is varied and l_p is constant, and

$$f_{1s}^{(2)}(u_s,\theta_s,\psi_s,l_p)=\mathbf{N}_1^{(s)}\cdot\mathbf{v}_1^{(1s,l_p)}=0 \tag{17}$$

Designation $\mathbf{v}_1^{(1s,l_p)}$ means that the relative velocity is determined wherein parameter ψ_s is held at rest and l_p is varied.

Step 3: Instead of relative velocities, we may consider in equations (16) and (17) infinitesimal displacements in relative motion and represent the equations of meshing as follows

$$f_{1s}^{(1)}(u_s,\theta_s,\psi_s,l_p)=\mathbf{N}_1^{(s)}\cdot\frac{\partial\mathbf{r}_1}{\partial\psi_s}=0 \qquad (l_p=const) \tag{18}$$

$$f_{1s}^{(2)}(u_s,\theta_s,\psi_s,l_p)=\mathbf{N}_1^{(s)}\cdot\frac{\partial\mathbf{r}_1}{\partial l_p}=0 \qquad (\psi_s=const) \tag{19}$$

The partial derivatives $\frac{\partial\mathbf{r}_1}{\partial\psi_s}$ and $\frac{\partial\mathbf{r}_1}{\partial l_p}$ can be expressed by the following matrix equations

$$\frac{\partial\mathbf{r}_1}{\partial\psi_s}=\left[\frac{\partial\mathbf{M}_{1w}(\psi_1)}{\partial\psi_s}\mathbf{M}_{wo}(l_p)\mathbf{M}_{ou}(l_p)\mathbf{M}_{uj}\mathbf{M}_{js}(\psi_s)+\right.$$

$$\mathbf{M}_{1w}(\psi_1)\mathbf{M}_{wo}(l_p)\mathbf{M}_{ou}(l_p)\mathbf{M}_{uj}\frac{\partial \mathbf{M}_{js}(\psi_s)}{\partial \psi_s}\Bigg]\mathbf{r}_s \qquad (20)$$

and

$$\frac{\partial \mathbf{r}_1}{\partial l_p} = \Bigg[\frac{\partial \mathbf{M}_{1w}(\psi_1)}{\partial l_p}\mathbf{M}_{wo}(l_p)\mathbf{M}_{ou}(l_p)\mathbf{M}_{uj}\mathbf{M}_{js}(\psi_s) + \mathbf{M}_{1w}(\psi_1)\frac{\partial \mathbf{M}_{wo}(l_p)}{\partial l_p}\mathbf{M}_{ou}(l_p)\mathbf{M}_{uj}\mathbf{M}_{js}(\psi_s) + \mathbf{M}_{1w}(\psi_1)\mathbf{M}_{wo}(l_p)\frac{\partial \mathbf{M}_{ou}(l_p)}{\partial l_p}\mathbf{M}_{uj}\mathbf{M}_{js}(\psi_s)\Bigg]\mathbf{r}_s \qquad (21)$$

Matrix $\mathbf{M}_{1w}(\psi_1)$ depends not only on the parameter of motion ψ_1 but also of the plunging motion l_p because (see equation (11)) variation of the axial position of the pinion must be accomplished by the additional rotation $\Delta\psi_p$.

Step 4: Equations (14), (18), and (19), considered simultaneously, represent the shaved pinion tooth surface.

4. Derivation of Gear Tooth Surface Σ_1

We use for derivation an approach that is similar to the discussed one in section 3. Surface Σ_2 is determined as the envelope to the two-parameter family of surfaces of gear shaver Σ_g. Two independent sets of motions are used for determination of the envelope Σ_2, but in the second set of motions the plunge is taken as zero. The derivations performed yield that gear tooth surface Σ_2 is an involute one if the gear shaver surface Σ_g is a screw involute surface.

5. Tooth Contact Analysis (TCA)

The authors have developed a TCA computer program for simulation of meshing and contact of a shaved pinion and gear tooth surfaces Σ_1 and Σ_2. The purposes of the simulation are:

1. That the point contact is localized and stabilized indeed.
2. Determination of the path of contact and bearing contact of shaved pinion-gear tooth surfaces Σ_1 and Σ_2.
3. The parabolic function of transmission errors of low magnitude is provided.
4. Investigation of influence of errors of alignment: (i) on the shift of the bearing contact, and (ii) on the transmission errors.

The developed TCA computer program has been used for simulation of meshing and contact of helical and spur gears with parallel axes and helical gears with crossed axes.

In the cases of spur and helical gears with parallel axes, the pinion tooth surface is shaved as a double-crowned surface.

In the case of involute helical gears with crossed axes, the pinion-gear tooth surface are already in point contact and errors of alignment such as change of center distance, change of crossing angle and lead angle do not cause transmission error. Therefore, the pinion and gear shavers are provided with screw involute surfaces and double-crowning of the pinion is not required (see section 8).

Algorithm of TCA for Simulation of Meshing. For the purpose of simplification of discussions, we consider that surfaces Σ_1 and Σ_2 and their normals $\mathbf{N}_1$ and $\mathbf{N}_2$ are represented in two-parameter form in coordinate systems S_1 and S_2.

We apply movable coordinate systems S_1 and S_2 rigidly connected to the pinion and gear, respectively, and fixed coordinate system S_f (Fig. 9). An auxiliary coordinate system S_r (Fig. 9) is used for simplification of coordinate transformation. Three types of errors are considered: change of center distance ΔE, shaft angle $\Delta\gamma$, and error $\Delta\lambda$ of the lead angle of the gear.

The algorithm of TCA is based on simulation of continuous tangency of pinion-gear tooth surfaces. Using coordinate transformation, we represent surfaces Σ_1 and Σ_2 in fixed coordinate system S_f (Fig. 9). Continuous tangency of shaved surfaces Σ_1 and Σ_2 is provided by observation of the following vector equations (Fig. 10)

$$\mathbf{r}_f^{(1)}(u_1,\theta_1,\phi_1)-\mathbf{r}_f^{(2)}(u_2,\theta_2,\phi_2)=\mathbf{0} \tag{22}$$

$$\mathbf{n}_f^{(1)}(u_1,\theta_1,\phi_1)-\mathbf{n}_f^{(2)}(u_2,\theta_2,\phi_2)=\mathbf{0} \tag{23}$$

where $\mathbf{n}_f^{(i)}(i=1,2)$ is the surface unit normal.

Vector equation (23) yields only two independent equations since $\left|\mathbf{n}_f^{(1)}\right|=\left|\mathbf{n}_f^{(2)}\right|=1$. Instead of equality of surface unit normals, we may require collinearity of surface normals using the equation

$$\mathbf{N}_f^{(1)}=\lambda\mathbf{N}_f^{(2)} \qquad (\lambda\neq 0) \tag{24}$$

Equation (22) and (23) provide a system of five nonlinear equations in six unknowns represented as

$$f_i(u_1,\theta_1,\phi_1,u_2,\theta_2,\phi_2)=0 \qquad (i=1,...,5) \tag{25}$$

The Theorem of Implicit Functions System Existence yields that we may consider one of the parameters, say ϕ_1, as the input one and obtain the solution to equation system (25) by functions

$$\{u_1(\phi_1),\theta_1(\phi_1),u_2(\phi_1),\theta_2(\phi_1),\phi_2(\phi_1)\}\in C^{(1)} \tag{26}$$

if the following conditions are observed

$$f_i\in C^1, \quad \frac{D(f_1,f_2,f_3,f_4,f_5)}{D(u_1,\theta_1,u_2,\theta_2,\phi_2)}\neq 0 \tag{27}$$

Then, we may obtain [4,5]:

(i) the paths of contact on surfaces Σ_1 and Σ_2 determined as

$$\mathbf{r}_i(u_i(\phi_1),\theta_i(\phi_1)), \qquad (i=1,2) \tag{28}$$

and

(ii) the function of transmission errors

$$\Delta\phi_2(\phi_1) = \phi_2(\phi_1) - \frac{N_1}{N_2}\phi_1 \tag{29}$$

Parabolic Function of Transmission Errors. The approach developed enables a predesigned parabolic function of transmission errors to be obtained:

$$\Delta\phi_2 = -a\phi_1^2 \tag{30}$$

for shaving spur and helical gears.

In the case of shaving of spur gears, function (30) is provided by application of a profile crowned pinion shaver. Longitudinal crowning of the pinion is necessary for localization of the bearing contact and is accomplished by the plunging of the shaver during the process of shaving of the pinion (see section 3).

In case of shaving of helical gears, parabolic function (30) is provided due to plunging of the shaver during the shaving of the pinion. Profile crowning of the pinion shaver provides localization of the bearing contact.

It has been proven that a parabolic function of transmission errors is able to absorb a discontinuous linear function of transmission errors caused by misalignment [4, 5]. Discontinuous functions of transmission errors cause vibration and noise. Therefore, absorption of discontinuous linear functions of transmission errors by application of developed parabolic function of transmission errors is a great advantage of the developed approach for shaving.

Simulation of Bearing Contact. The shaver and workpiece tooth surfaces, as well the pinion-gear tooth surfaces, are in point contact at every instant. The contact is spread over an elliptical area due to the elastic deformation of tooth surfaces. The bearing contact during the process of shaving and during the meshing of shaved tooth surfaces is formed by a set of instantaneous contact ellipses. The determination of dimensions and orientation of the contact ellipse require knowledge of principal curvatures and principal directions of contacting surfaces and the approach of contacting surfaces as the result of elastic deformation. We refer to [4, 5] for the solution of this problem.

The shaved gear tooth surface is a screw involute surface and determination of principal curvatures and directions for such a surface is a simple problem [4, 5]. However, it is a more complex problem for a double-crowned pinion tooth surface. Taking into account that the deviations of a double-crowned pinion from an involute surface are in the range of (0.01-0.05)mm, we may determine the pinion principal curvatures and directions as for an involute surface.

The contact of the shaver and the workpiece is also spread over an elliptical area. For the purpose of simplification, we will consider the contact of the shaver and the workpiece as the contact of crossed involute gears.

Numerical Example 1: Contact Ellipse for Crossed Helical Gears. We will consider two cases of crossed helical gears: (i) with the crossing angle $\lambda = 20^\circ$, and (ii) with the crossing angle $\lambda = 90^\circ$. The first case corresponds to the contact of a shaver with the workpiece. The second case corresponds to a helical gear drive with crossed axes. The numerical example illustrates the influence of the crossing angle on the dimensions of the instantaneous contact ellipse.

The design parameters are given in Tables 1 and 2.

Table 1: Design Parameters ($\gamma = 20^{\circ}$)

N_1	Tooth number of the pinion	25
N_2	Tooth number of the gear	77
b	Gear width (mm)	40
m_n	Normal module (mm)	5
α_n	Pressure angle (degree)	27.5
β_1	Helix angle of the pinion (degree)	10
β_2	Helix angle of the gear (degree)	-10
γ	Cross angle (degree)	20

Table 2: Design Parameters ($\gamma = 90^{\circ}$)

N_1	Tooth number of the pinion	25
N_2	Tooth number of the gear	77
b	Gear width (mm)	40
m_n	Normal module (mm)	5
α_n	Pressure angle (degree)	27.5
β_1	Helix angle of the pinion (degree)	45
β_2	Helix angle of the gear (degree)	-45
γ	Cross angle (degree)	90

The contact ellipses obtained under very low load are shown at three positions of meshing (Fig. 11(a) and (b)) and are represented in Tables 3 and 4. The cross-hatched contact ellipse corresponds to the contact at the pitch point. The results obtained show that the crossing angle of helical gears affects substantially the dimensions of the contact ellipse.

Table 3: Parameters of Contact Ellipse ($\gamma = 20^{\circ}$)

Major axis	10.80 mm
Minor axis	1.45 mm
Area	12.28 mm^2

Table 4: Parameters of Contact Ellipse ($\gamma = 90^{\circ}$)

Major axis	3.90 mm
Minor axis	1.91 mm
Area	5.85 mm^2

6. Output of TCA for Spur Gears with Parallel Axes

As a reminder that in the case of spur gears, the purpose of profile crowning of the pinion shaver is to provide a parabolic function of transmission errors of the gear drive. Such a function is able to absorb (see [4]) the discontinuous linear ones that are caused by the errors of alignment of the gear drive. The plunging of the shaver during the process of shaving of the pinion permits localization of the bearing contact in the middle of the tooth and avoids edge contact.

However, another type of edge contact may occur, if the point of contact reaches the top of the tooth. In case of spur gears, such an edge contact may be avoided by the displacement down of point Q of tangency of mismatched profiles of rack-cutters (Fig. 3).

The meshing of shaved spur gears has been simulated by the computer program developed. The design parameters of the simulated gear drive are given in the following table:

Table 5: Design Parameters of Spur Gears

N_1	Tooth number of the pinion	25
N_1	Tooth number of the gear	77
b	Gear width (mm)	50
m_n	Normal module (mm)	5
α_n	Pressure angle (degree)	27.5
a_c	Parabola parameter of profile crowning	4×10^{-4}
a_{pl}	Parabola parameter of plunging	1×10^{-4}

The results of TCA for the paths of contact on the gear tooth surface are shown in Figs. 12(a), 12(b), and 12(c). In all three cases the dashed line shows the path of contact for an aligned gear drive.

The influence of change $\Delta\gamma$ of the crossing angle ($\Delta\gamma = 3$ *arcmin*) on the shift of the bearing contact is shown in Fig. 12(a).

Drawings of Figs. 12(b) and 12(c) show that the shift of the bearing contact caused by $\Delta\gamma$ may be compensated by the correction of $\Delta\lambda$, the lead angle of the gear or the pinion. In the case of spur gears, the nominal value of the lead angle is $\lambda = 90^\circ$. Fig. 12(b) shows the shift of bearing contact caused by $\Delta\lambda_2 = -3$ *arc min*. Fig. 12(c) shows that the shift of bearing contact is zero for $\Delta\gamma + \Delta\lambda_2 = 0$.

The results of TCA show: (i) that the shift of contact caused by change ΔE of center distance is zero; (ii) profile crowning of the pinion shaver provides a predesigned parabolic function of transmission errors. The functions of transmission errors obtained are represented in Figs. 13(a), 13(b), 13(c), and 13(d). The drawings illustrate that the resulting functions of transmission errors (Figs. 13(b), 13(c), and 13(d)) have the same slope as the predesigned parabolic function of transmission errors shown in Fig. 13(a). The displacement of the origin of the resulting function of transmission errors with respect to the predesigned one indicates the error of angular position of the driven gear caused by misalignment.

7. Output of TCA for Helical Gears with Parallel Axes

In the case of helical gears with parallel axes, the localization of the bearing contact is achieved by application of profile crowned shaver for the pinion and a conventional involute shaver for the gear. The bearing contact on the shaved gear (or pinion) tooth surface of an aligned gear drive is a helix, similar to the case of N-W (Novikov-Wildhaber) gears [4].

In addition to localization of the bearing contact of the shaved pinion and gear, it is necessary to provide a parabolic function of transmission errors designated for the absorption of transmission errors caused by misalignment of the gear drive. A predesigned parabolic function of the transmission errors of the gear drive is provided due to the parabolic plunge of the pinion shaver during the pinion shaving.

The developed TCA (Tooth Contact Analysis) computer program has been applied for simulation of a helical gear drive with the following design parameters (Table 6):

Table 6: Design Parameters of Helical Gear Drive

N_1	Tooth number of the pinion	25
N_2	Tooth number of the gear	77
b	Gear width (mm)	40
m_n	Normal module (mm)	5
α_n	Pressure angle (degree)	27.5
β_1	Helix angle of the pinion (degree)	20
β_2	Helix angle of the gear (degree)	20
γ	Cross angle (degree)	0
a_c	Parabola parameter of profile crowning	1.4×10^{-3}
a_{pl}	Parabola parameter of plunging	8×10^{-5}

The influence of three types of errors of alignment has been investigated: change of center distance ΔE , shaft angle $\Delta\gamma$, and change $\Delta\lambda_2$ of the lead angle of the gear.

The results of the investigation are illustrated with the drawings of Figs. 14(a), 14(b), 14(c), and 14(d) that show the paths of contact on the gear tooth surface.

Fig. 14(a) corresponds to the case wherein only the profile crowning of the pinion shaver has been accomplished and the parabolic plunge of the pinion shaver has not been applied. The path of contact on the tooth surface of an aligned gear drive in such a case is a helix.

Figs. 14(b), 14(c), and 14(d) correspond to the cases wherein the pinion shaver is profile crowned and a parabolic plunge during the pinion shaving is executed. Due to the plunge, the path of contact of the gear drive deviates from a helix, even for an aligned gear drive. The magnitude of the deviation depends on the assigned magnitude of the predesign parabolic function of transmission errors.

The path of contact indicated in Fig. 14(b) by continuous line corresponds to $\Delta\gamma = 3$ *arcmin*. The dashed line in Fig. 14(b) illustrates the path of contact for $\Delta\gamma = 0$.

Similarly, 14(c) indicates paths of contact obtained for $\Delta\lambda_2 = -3$ *arcmin* and $\Delta\lambda_2 = 0$.

A combination of errors of alignment $\Delta\gamma + \Delta\lambda_2 = 0$ enables to avoid the shift of the bearing contact.

Fig. 14(d) shows that error of alignment $\Delta E = 0.05$ mm does not cause the shift of the bearing contact.

The paths of contact shown in Figs. 14(b), 14(c), and 14(d) deviate from a helix shape of path of contact as the result of a parabolic plunge of the pinion shaver. However, such a plunge is required because it provides a parabolic function of transmission errors and therefore the noise and vibration of the drive should be reduced. The existence of a parabolic function of transmission errors is confirmed by application of the developed TCA computer program.

8. Output of TCA for Helical Gears with Crossed Axes

We consider a gear drive formed by helical gears with screw involute tooth surfaces and crossed axes. The crossing angle γ is determined as

$$\gamma = |\beta_1 \pm \beta_2| \tag{31}$$

where β_i is the helix angle. The upper and lower sign in (31) corresponds to the same and opposite directions of teeth, respectively.

The investigation of meshing of helical gears with crossed axes accomplished in [4,6] shows:

(i) The gear tooth surfaces are in point contact and therefore their bearing contact is localized. The path of contact on gear tooth surface is shown in Fig. 15.

(ii) The advantage of crossed involute helical gears is that the change $\Delta\gamma$ of the crossing angle, change ΔE of the shortest distance between the axes, and change $\Delta\beta$ of the helix angle do not cause transmission errors. This statement has been proven in [6] and confirmed by application of TCA computer program developed by the authors of this paper.

(iii) However, due to small dimensions of the instantaneous contact ellipse (see Fig. 11) helical gears with crossed axes may be applied to gear drives that operate with light loads.

The specific conditions of meshing of crossed involute helical gears are illustrated with drawings of Fig. 16. The line of action of the gears is the line of intersection of two planes that are tangent to the respective base cylinders of the helical gears.

There are two lines of action that correspond to meshing of both sides of tooth surfaces. Both lines of action in an aligned gear drive intersect each other at a common point *P* that lies on the shortest center distance. In case of a misaligned gear drive, the lines of action are still tangents to the base cylinder but do not intersect each other at pitch point *P*.

Unlike spur and helical gear drives with parallel axes, there is no need for double-crowning of helical gears with crossed axes. As it was mentioned above, such gear tooth surfaces are in point contact, the bearing contact is localized, and the gear drive is not sensitive to errors of alignment. A shaved screw involute tooth surface can be obtained by application of a conventional involute shaver.

Numerical Example. The computer program developed has been applied for simulation of meshing and contact of a helical gear drive with the following design parameters (see Table 7):

Table 7: Design Parameters of Crossed Helical Gears

N_1	Tooth number of the pinion	25
N_2	Tooth number of the gear	77
b	Gear width (mm)	50
m_n	Normal module (mm)	5
α_n	Pressure angle (degree)	27.5
β_1	Helix angle of the pinion (degree)	45
β_2	Helix angle of the gear (degree)	-45
γ	Cross angle (degree)	90

The results of TCA have confirmed that: (i) errors of alignment $\Delta\gamma$, $\Delta\lambda$, and ΔE do not cause transmission errors of the drive, and (ii) do not shift substantially the bearing contact.

9. Conclusions

Based on the results presented in this study the following several conclusions can be made:

1. A modified geometry of spur and helical gears with parallel axes manufactured by shaving has been developed. The bearing contact of shaved gears is localized and the transmission errors caused by misalignment are of a favorable shape and low magnitude.
2. The modification of shaved pinion-gear tooth surfaces of spur and helical gears with parallel axes is based on application of a profile crowned shaver for the pinion and execution of a parabolic plunge of the shaver during the process of pinion shaving.
3. Simulation of meshing and contact of pinion-tooth surfaces has been performed by the computer programs developed.
4. An approach for shaving of crossed helical gears has been developed and investigated.

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[2] Kim, D.: Computerized Simulation of Meshing for Finishing Operations of Production of Spur and Helical Gears, Ph. D. Thesis, University of Illinois of Chicago, 1995.

[3] Patent: Low Noise, Reduced Stresses, High Precision of Helical and Spur Gears Design and Manufactured with Double-Crowned Tooth Surfaces, Inventors: Litvin, F. L. and others, US Patent Serial No: 09/340.824 (pending).

[4] Litvin, F. L.: Gear Geometry and Applied Theory, Prentice Hall, 1994.

[5] Litvin, F. L.: Development of Gear Technology and Theory of Gearing, NASA Reference Publication 1406, ARL-TR-1500, 1998.

[6] Litvin, F. L.: Theory of Gearing, 2nd edition, Nauka, Moscow, 1968, (in Russian).

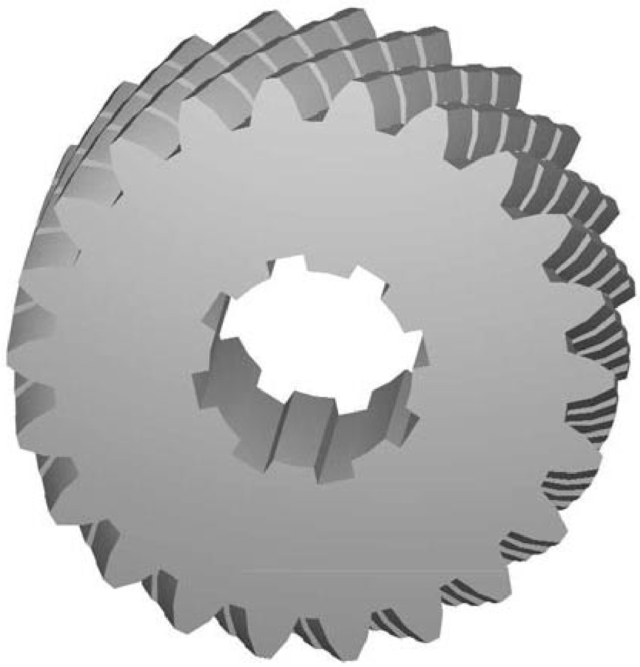

Figure 1.—Illustration of a shaver as a helical gear with gashed teeth.

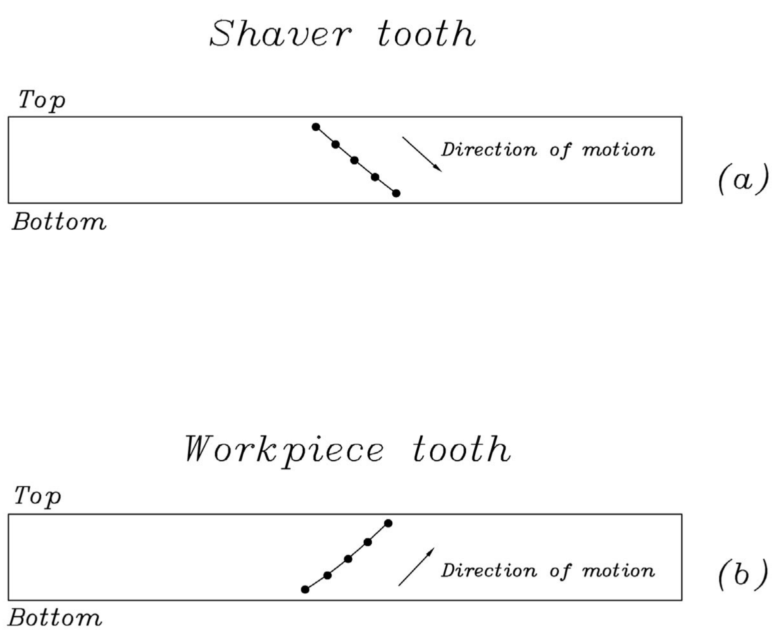

Figure 2.—Contact paths on shaver and workpiece tooth surfaces.

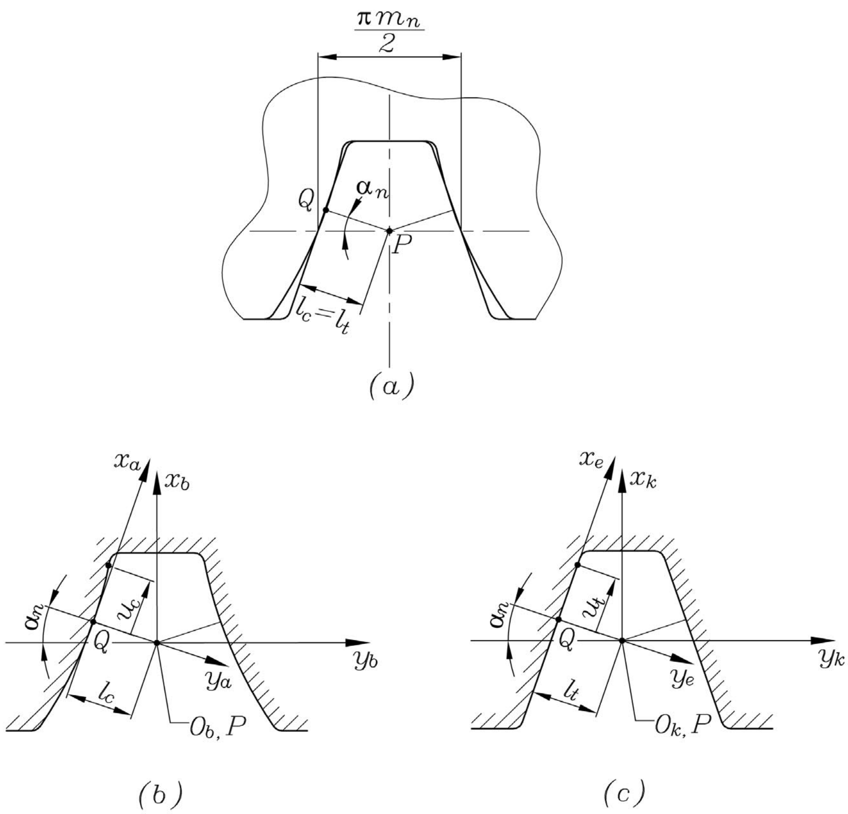

Figure 3.—Profiles of rack-cutters in normal section. (a) mismatched normal profiles of two rack-cutters. (b) parabolic profiles of rack-cutter. (c) straight-lined profiles of rack-cutter.

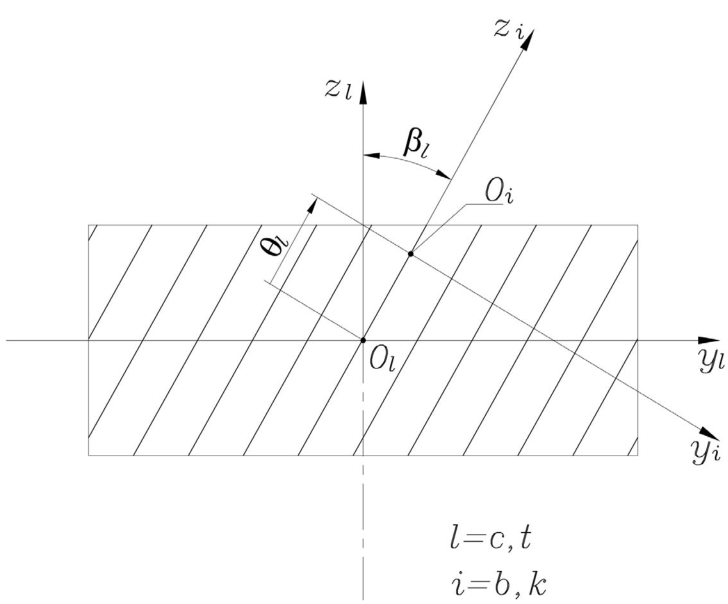

Figure 4.—For derivation of rack-cutter surfaces Σ_c and Σ_t.

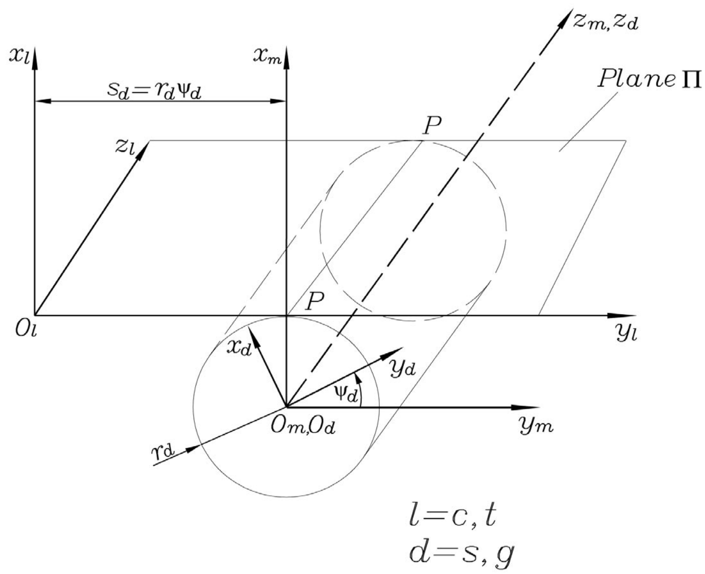

Figure 5.—Generation of a shaver tooth surface Σ_d.

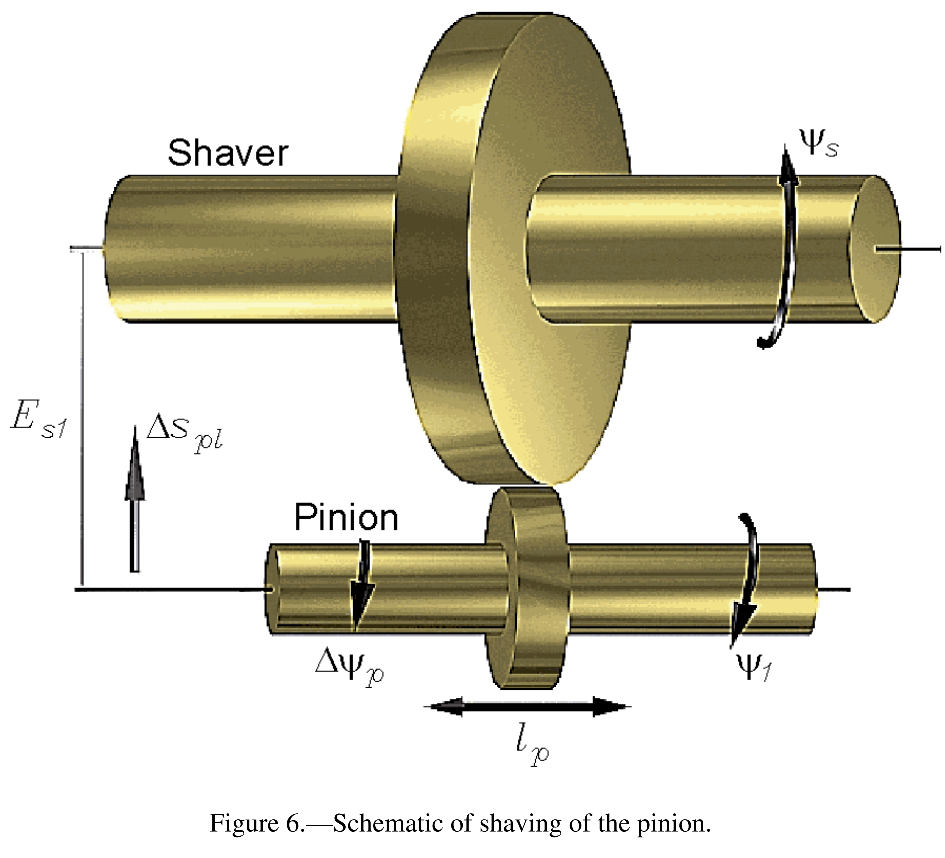

Figure 6.—Schematic of shaving of the pinion.

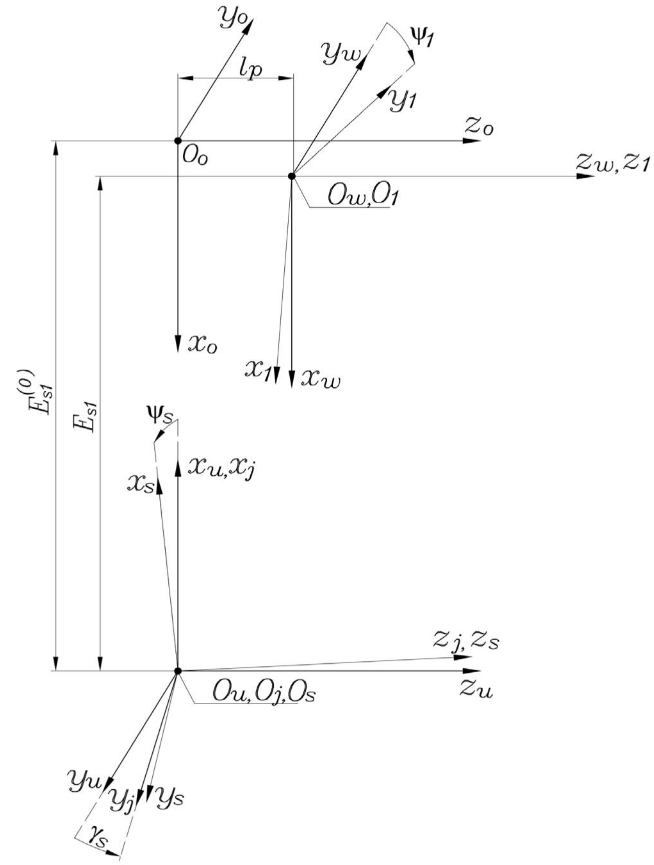

Figure 7.—Coordinate systems applied for pinion shaving.

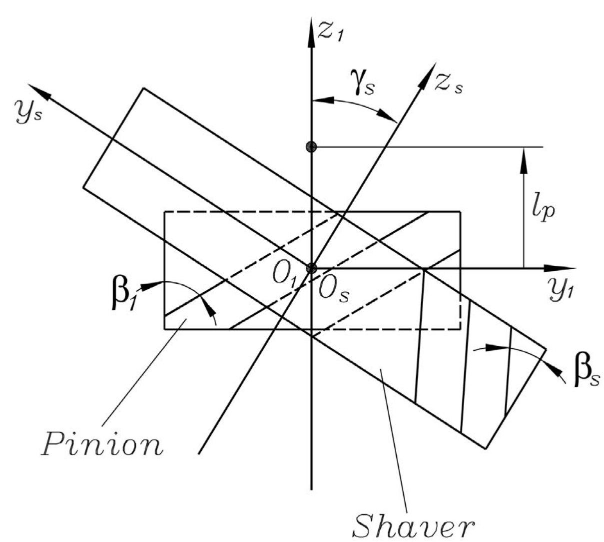

Figure 8.—Determination of crossing angle γ_s.

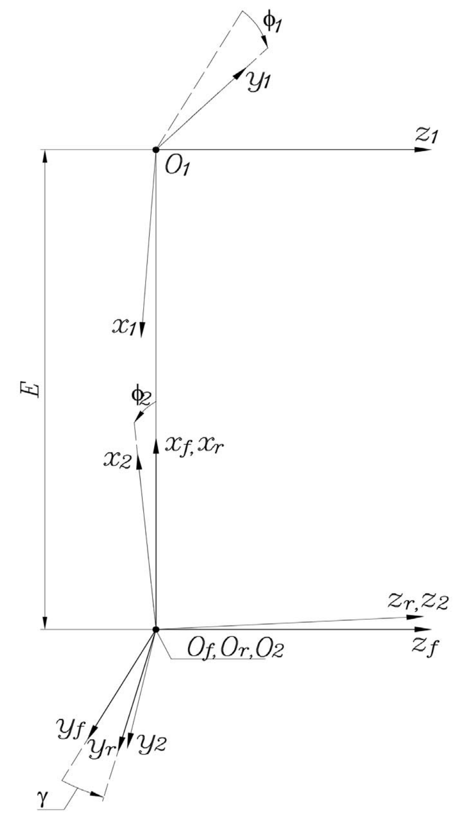

Figure 9.—Coordinate systems applied for TCA.

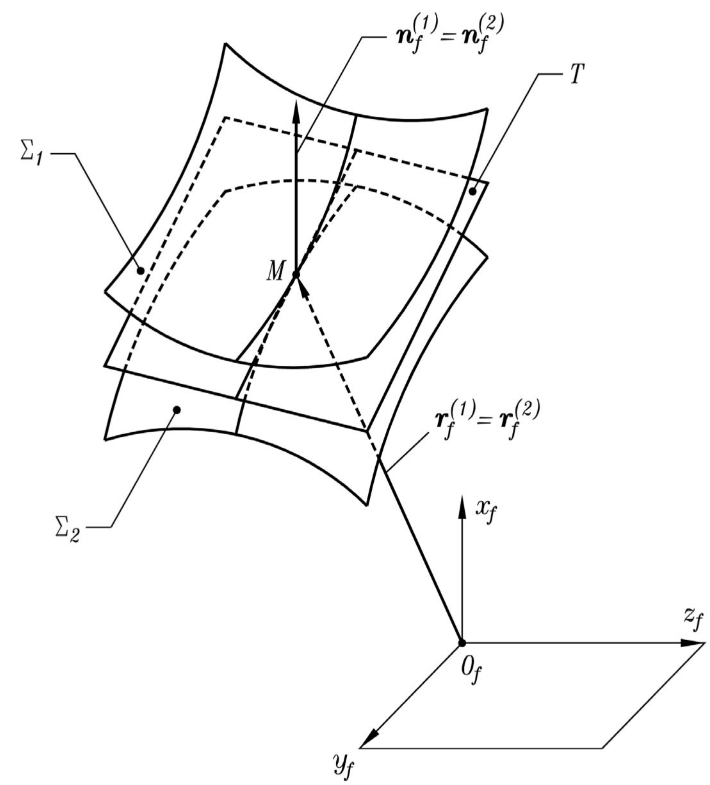

Figure 10.—Tangency of surfaces in ideal gear train.

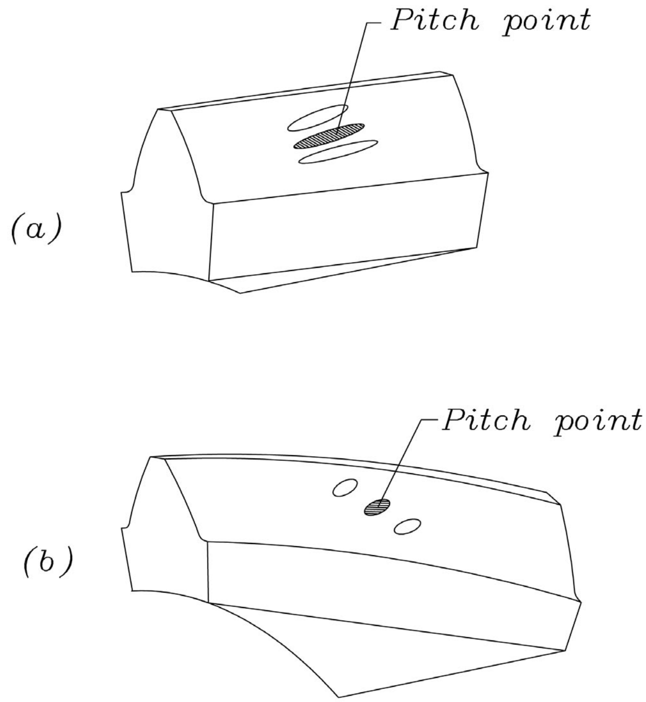

Figure 11.—Contact ellipses for helical gears with crossed axes. (a) ($\gamma = 20°$) and (b) ($\gamma = 90°$).

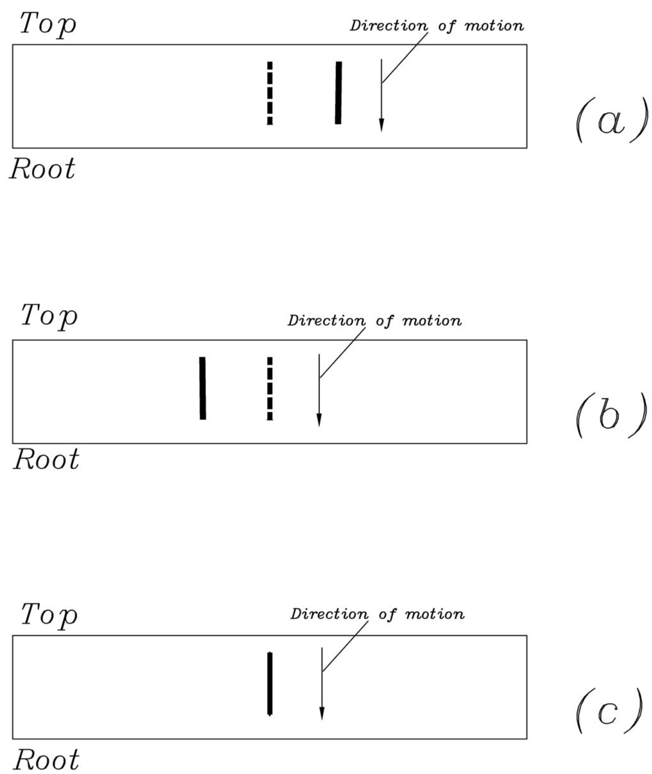

Figure 12.—Spur gears: path of contact on gear tooth surface for aligned and misaligned gear drives.

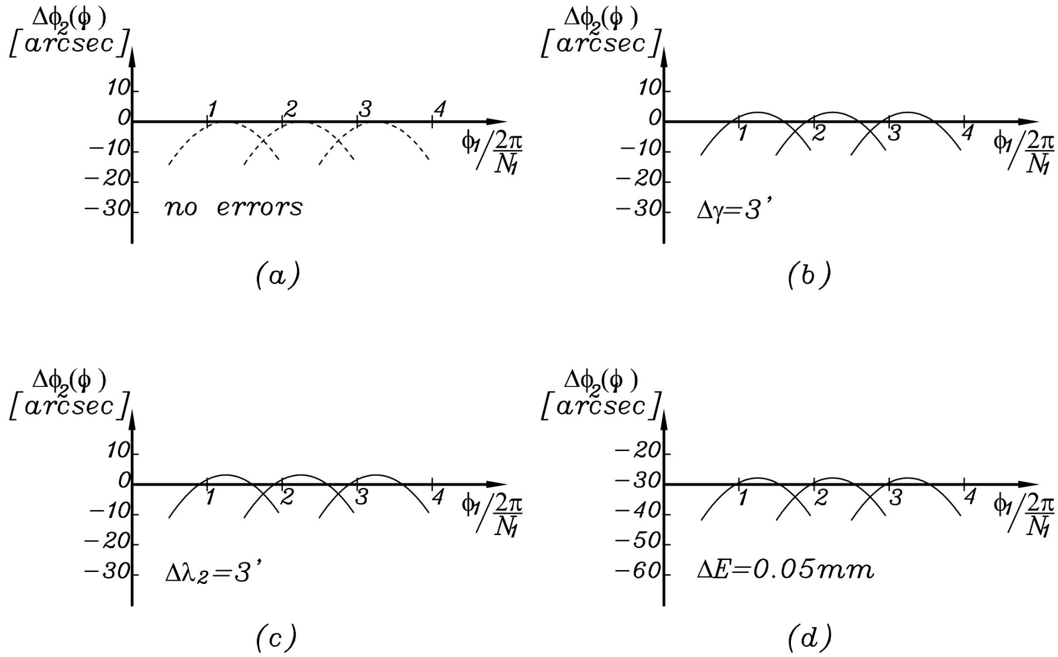

Figure 13.—Functions of transmission errors for spur gears with parallel axes. Errors of alignment are $\Delta\gamma$, $\Delta\lambda_2$ and ΔE.

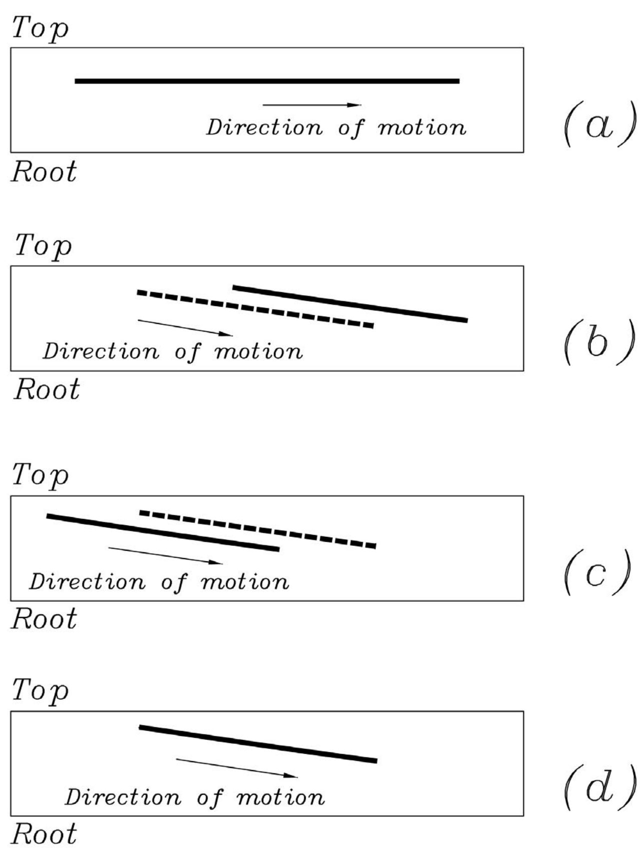

Figure 14.—Paths of contact on the gear tooth surface for aligned and misaligned helical gears with parallel axes.

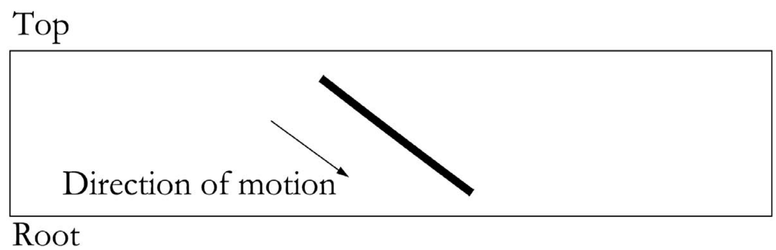

Figure 15.—Path of contact on the gear tooth surface for helical gears with crossed axes.

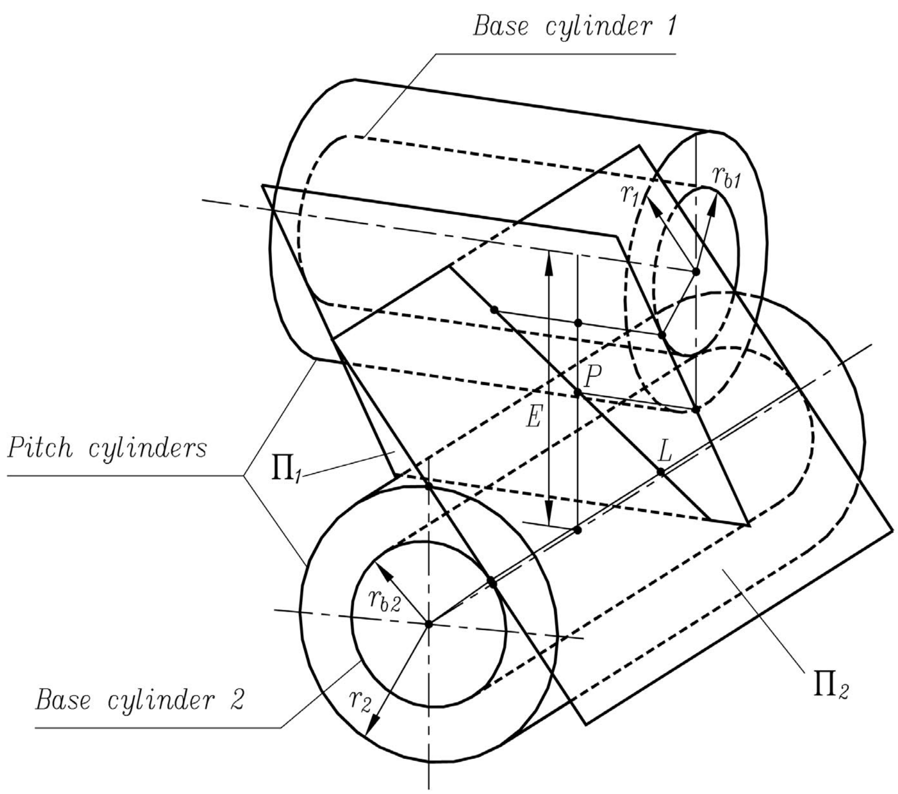

Figure 16.—Formation of line of action in a helical gear drive with crossed axes.

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13. ABSTRACT ***(Maximum 200 words)***

Modification of geometry of spur and helical gears with parallel axes and helical gears with crossed axes is proposed. The finishing process of gear generation is shaving. The purposes of modification of the gear geometry are to localize and stabilize the bearing contact, and to reduce noise and vibration. The goals mentioned above are achieved by using profile crowning and plunging the shaver by a prescribed motion during pinion generation. The pinion becomes double crowned. The gear member is generated as a conventional involute gear. A tooth contact analysis (TCA) program for simulation of meshing and contact was developed and the analysis is illustrated with TCA results for spur and helical gears.

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