4. Constraints

The minimization of cost in WinDS is subject to a large number of different constraints, involving limits on wind resources, transmission constraints, national growth constraints, ancillary services, and pollution. Unless specifically noted otherwise (see, for example, Constraint 1 below), these constraints apply to new wind turbines and storage (e.g., electrolyzers/hydrogen storage) facilities built in the time period being optimized.

After the constraint number, the constraint name is shown with the subscripts over which the constraint applies. For example, in Constraint 1 immediately below, the parenthetical (c,i) immediately following the name of the constraint implies that this constraint is applied for every class of onshore wind c and every region i. Because there are 358 regions i and five classes of wind c, this first type of constraint is repeated 1,790 times (358x5).

The constraints are listed in the order they occur in the model. Following the equation for the constraint, we define those constants and variables that are first introduced by this constraint.

<u>**Constraints On Wind**</u>

Constraint 1
WIND_RES_UC(c,i): For every wind class c and wind supply region i, the sum of all onshore wind capacity installed in this and preceding time periods must be less than the total onshore wind resource in the region (**$WRUC_{c,i}$**)

$$\begin{aligned}&\sum_{wscp} WturN_{i,wscp} * class_{c,i} + WturTN_i * classT_{c,i}\\&+ Wtur\_inregion_{c,i} + WturO_{c,i} + WTturO_{c,i}\\&\leq WRUC_{c,i}\end{aligned}$$

Where:

> **$WturO_{c,i}$** is existing ("O"ld) (from the preceding time period) class c wind transmitted on existing transmission lines [MW] from region i
> **$WTturO_{c,i}$** is existing class c wind transmitted on new transmission lines [MW] from region i

Constraint 2
WIND_RES_UC_ofs(c,i): For every wind class c and wind supply region i, the sum of all shallow offshore wind capacity installed in this and preceding time periods must be less than the total shallow offshore wind resource in the region (**$WRUCofs_{c,i}$**)

$$\sum_{wscpofs} WturNofs_{i,wscpofs} * classofs_{c,i} + WturTNofs_i * classTofs_{c,i}$$
$$+ Wtur\_inregionofs_{c,i} + WturOofs_{c,i} + WTturOofs_{c,i}$$
$$\leq WRUCofs_{c,i}$$

Where:

> **$\mathbf{WturOofs_{c,i}}$** is existing (from the preceding time period) shallow offshore wind on existing transmission lines (MW)
> **$\mathbf{WTturOofs_{c,i}}$** is existing shallow offshore wind on new transmission lines (MW)

Constraint 3
WIND_RES_UC_ofd(c,i): For every wind class c and wind supply region i, the sum of all deep offshore wind capacity installed in this and preceding time periods must be less than the total deep offshore wind resource in the region (**$\mathbf{WRUCofd_{c,i}}$**)

$$\sum_{wscpofd} WturNofd_{i,wscpofd} * classofd_{c,i} + WturTNofd_i * classTofd_{c,i}$$
$$+ Wtur\_inregionofd_{c,i} + WturOofd_{c,i} + WTturOofd_{c,i}$$
$$\leq WRUCofd_{c,i}$$

Where:

> **$\mathbf{WturOofd_{c,i}}$** is existing (from the preceding time period) deep offshore wind on existing transmission lines (MW)
> **$\mathbf{WTturOofd_{c,i}}$** is existing deep offshore wind on new transmission lines (MW)

The next three constraints ensure that the amount of wind that can access existing transmission is less than or equal to the amount of wind available in each step of the supply curve for the cost of connecting to the grid.

Constraint 4

WIND_supply_curves(c,i,wscp): New onshore wind of class c in region i at interconnection cost step wscp must be less than the remaining onshore wind resource in that cost step ($\mathbf{WR2G_{c,i,wscp}}$)[6]

$$WturN_{i,wscp} * class_{c,i} \leq WR2G_{c,i,wscp}$$

Constraint 5

WIND_supply_curves_ofs(c,i,wscpofs): New shallow offshore wind of class c in region i at inteconnection cost step wscpofs must be less than the remaining shallow offshore wind resource in that cost step ($\mathbf{WR2Gofs_{c,i,wscpofs}}$)

$$WturNofs_{i,wscpofs} * classofs_{c,i} \leq WR2Gofs_{c,i,wscpofs}$$

Constraint 6

WIND_supply_curves_ofd(c,i,wscpofd): New deep offshore wind of class c in region i at inteconnection cost step wscpofd must be less than the remaining deep offshore wind resource in that cost step ($\mathbf{WR2Gofd_{c,i,wscpofd}}$)

$$WturNofd_{i,wscpofd} * classofd_{c,i} \leq WR2Gofd_{c,i,wscpofd}$$

The next six constraints ensure that the new wind capacity on transmission lines from a region are less than the new turbine capacity built in the region

Constraint 7

Wind_2_Grid(c,i): The new class c onshore wind transmitted from a region i on existing lines to all regions j must be less than or equal to the total amount of new onshore region i class c wind used from the onshore class c wind supply curve for existing lines.

$$\sum_{j} WN_{i,j} * class_{c,i} \leq \sum_{wscp} WturN_{i,wscp} * class_{c,i}$$

---

[6] Wind Transmission Pre-Calculation

A preliminary optimization is performed outside and prior to the main model to construct a supply curve for onshore wind, shallow offshore wind and deep offshore wind for each wind class c and region i. This supply curve is comprised of four quantity/cost pairs ($WR2G_{c,i,wscp}$ / $WR2GPTS_{c,i,wscp}$). The "curve" provides the amount of class c wind $WR2G_{c,i,wscp}$ of each type l (onshore, shallow offshore, and deep offshore) that can be connected to the existing grid for a cost between $WR2GPTS_{c,i,wscp-1}$ and $WR2GPTS_{c,i,wscp}$. This "pre-LP" optimization is described in more detail in Appendix B. The quantity $WR2G_{c,i,wscp}$ is reduced after each period's LP optimization by the amount of wind used in the time period from that cost step.

Constraint 8

Wind_2_Grid_OFS(c,i): The new class c shallow offshore wind transmitted from a region i on existing lines to all regions j must be less than or equal to the total amount of new shallow offshore region i class c wind used from the shallow offshore class c wind supply curve for existing lines.

$$\sum_{j} WNofs_{i,j} * classofs_{c,i} \leq \sum_{wscpofs} WturNofs_{i,wscpofs} * classofs_{c,i}$$

Constraint 9

Wind_2_Grid_OFD(c,i): The new class c deep offshore wind transmitted from a region i on existing lines to all regions j must be less than or equal to the total amount of new deep offshore region i class c wind used from the deep offshore class c wind supply curve for existing lines.

$$\sum_{j} WNofd_{i,j} * classofd_{c,i} \leq \sum_{wscpofd} WturNofd_{i,wscpofd} * classofd_{c,i}$$

Constraint 10

Wind_2_New(c,i): The new class c onshore wind transmitted from a region i on new lines to all regions j must be less than or equal to the total amount of new onshore region i wind used from the class c wind supply curve for new lines.

$$\sum_{j} WTN_{i,j} * classT_{c,i} \leq \sum_{wscp} WturTN_{i,wscp} * classT_{c,i}$$

Constraint 11

Wind_2_New_OFS(c,i): The new class c shallow offshore wind transmitted from region i on new lines to all regions j must be less than or equal to the total amount of new shallow offshore region i class c wind used from the shallow offshore class c wind supply curve for new lines.

$$\sum_{j} WTNofs_{i,j} * classTofs_{c,i} \leq \sum_{wscpofs} WturTNofs_{i,wscpofs} * classTofs_{c,i}$$

Constraint 12

Wind_2_New_OFD(c,i): The new class c deep offshore wind transmitted from a region i on new lines to all regions j must be less than or equal to the total amount of new deep offshore region i class c wind used from the deep offshore class c wind supply curve for new lines.

$$\sum_{j} WTNofd_{i,j} * classTofd_{c,i} \leq \sum_{wscpofd} WturTNofd_{i,wscpofd} * classTofd_{c,i}$$

Constraint 13
WIND_EXISTTRANS_BALANCE(i): Constraint to set the value of the amount of new wind on existing transmission lines taken from the onshore wind supply curve

The new wind from all steps on the supply curve must equal the sum of new wind sent from region i to all regions j. WNSC will be used in the objective function for the class of wind being considered this period in region i to determine the cost of reaching the grid.

$$\sum_{wscp} WNSC_{i,wscp} = \sum_{j} WN_{i,j}$$

Constraint 14
WIND_EXISTTRANS_BALANCE_ofs(i): Constraint to set the value of the amount of new wind on existing transmission lines taken from the shallow offshore wind supply curve

The new wind from all steps on the supply curve must equal the sum of new wind sent from region i to all regions j. WNSCofs will be used in the objective function for the class of wind being considered this period in region i to determine the cost of reaching the grid.

$$\sum_{wscpofs} WNSCofs_{i,wscpofs} = \sum_{j} WNofs_{i,j}$$

Constraint 15
WIND_EXISTTRANS_BALANCE_ofd(i): Constraint to set the value of the amount of new wind on existing transmission lines taken from the deep offshore wind supply curve

The new wind from all steps on the supply curve must equal the sum of new wind sent from region i to all regions j. WNSCofd will be used in the objective function for the class of wind being considered this period in region i to determine the cost of reaching the grid.

$$\sum_{wscpofd} WNSCofd_{i,wscpofd} = \sum_{j} WNofd_{i,j}$$

Constraint 16
RPSconstraint: This allows the model to include a national Renewable Portfolio Standard (RPS), wherein the total national annual wind generation must exceed a specified fraction of the national electricity load or a penalty must be paid on the shortfall (**RPS_Shortfall**).

$$
\begin{aligned}
&\sum_{\substack{c,i,j,m \\ I_i = I_j}} WN_{i,j} * class_{c,i} * CF_c * CF\_corr_{c,i,m} \\
&+ WNofs_{i,j} * classofs_{c,i} * CFofs_c * CF\_corrofs_{c,i,m} \\
&+ WNofd_{i,j} * classofd_{c,i} * CFofd_c * CF\_corrofd_{c,i,m} \\
\\
&+ WTN_{i,j} * classT_{c,i} * CF_c * CF\_corr_{c,i,m} \\
&+ WTNofs_{i,j} * classTofs_{c,i} * CFofs_c * CF\_corrofs_{c,i,m} \\
&+ WTNofd_{i,j} * classTofd_{c,i} * CFofd_c * CF\_corrofd_{c,i,m} \\
\\
&+ WO_{c,i,j} * CFO_{c,i} * CF\_corr_{c,i,m} \\
&+ WOofs_{c,i,j} * CFOofs_{c,i} * CF\_corrofs_{c,i,m} \\
&+ WOofd_{c,i,j} * CFOofd_{c,i} * CF\_corrofd_{c,i,m} \\
\\
&+ WTO_{c,i,j} * CFTO_{c,i} * CF\_corr_{c,i,m} \\
&+ WTOofs_{c,i,j} * CFTOofs_{c,i} * CF\_corrofs_{c,i,m} \\
&+ WTOofd_{c,i,j} * CFTOofd_{c,i} * CF\_corrofd_{c,i,m} \\
\\
&+ \sum_{c,escp,j} WELEC\_inregion_{c,escp,j} * CF_c * CF\_corr_{c,j,m} \\
&+ WELEC\_inregionofs_{c,escp,j} * CFofs_c * CF\_corrofs_{c,j,m} \\
&+ WELEC\_inregionofd_{c,escp,j} * CFofd_c * CF\_corrofd_{c,j,m} \\
&+ RPSPenalty \\
&\geq \\
&RPSFrac * \sum_{m,n} L_{m,n}
\end{aligned}
$$

Where:

$CFO_{c,i}$ is the average capacity factor of all existing (at the start of the current period) class c onshore wind on existing (at the start of the analysis time frame) lines in region i
$CFofs_c$ is the capacity factor for new shallow offshore wind at a class c site

$CFofd_c$ is the capacity factor for new deep offshore wind at a class c site
$CFOofd_{c,i}$ is the average capacity factor of all existing (at the start of the current period) class c deep offshore wind on existing (at the start of the analysis time frame) lines in region i
$CFOofs_{c,i}$ is the average capacity factor of all existing (at the start of the current period) class c shallow offshore wind on existing (at the start of the analysis time frame) lines in region i
$WO_{c,i,j}$ is the existing (from the preceding time period) class c onshore wind on existing (at start of the simulation) transmission lines from region i to region j (MW)
$WOofd_{c,i,j}$ is the existing (from the preceding time period) class c deep offshore wind on existing (at start of the simulation) transmission lines from region i to region j (MW)
$WOofs_{c,i,j}$ is the existing (from the preceding time period) class c shallow offshore wind on existing (at start of the simulation) transmission lines from region i to region j (MW)
$WTO_{c,i,j}$ is the existing (at start of this time period) class c onshore wind on new transmission lines from region i to region j (MW)
$WTOofd_{c,i,j}$ is the existing (at start of this time period) class c deep offshore wind on new transmission lines from region i to region j (MW)
$WTOofs_{c,i,j}$ is the existing (at start of this time period) class c shallow offshore wind on new transmission lines from region i to region j (MW)

Constraints 17 and 18
WIND_GROWTH_TOT and WIND_GROWTH_BIN(G): These two constraints allocate new wind capacity (MW) to bins that have turbine costs associated with them over and above the turbine costs of the wind farms themselves. The bins are defined as a fraction of the national wind capacity (MW) at the start of the period

$$\begin{aligned}
&\sum_{c,i} Wtur_inregion_{c,i} + Wtur_inregionofs_{c,i} + Wtur_inregionofd_{c,i} \\
&+ \sum_{c,i}\sum_{wscp} WturN_{i,wscp} * class_{c,i} + \sum_{wscpofs} WturNofs_{i,wscpofs} * classofs_{c,i} + \sum_{wscpofd} WturNofd_{i,wscpofd} * classofd_{c,i} \\
&+ \sum_{c,i} WturTN_i * classT_{c,i} + WturTNofs_i * classTofs_{c,i} + WturTNofd_i * classTofd_{c,i} \\
&\le \sum_{g} WCt_g
\end{aligned}$$

$$WCt_g \le Gt_g * BASE\_WIND$$

Where:

Gt_g is a fractional multiplier of the national wind (MW) capacity
BASE_WIND is the national wind capacity (MW) at the start of the period

Constraints 19 and 20
WIND_GROWTH_INST(i) and WIND_GROWTH_BIN_INST(ginst,i): These two constraints allocate new wind capacity (MW) to bins that have costs associated with them over and above the installation cost of the wind farms themselves. The bins are defined as a fraction of the region

i wind capacity (MW) at the start of the period. These constraints capture the increase in installation prices with high growth in a single region. The first 200 MW don't count toward the installation cost increase.

$$
\begin{aligned}
&\sum_{c} Wtur\_inregion_{c,i} + Wtur\_inregionofs_{c,i} + Wtur\_inregionofd_{c,i} \\
&+ \sum_{c} \sum_{wscp} WturN_{i,wscp} * class_{c,i} + \sum_{wscpofs} WturNofs_{i,wscpofs} * classofs_{c,i} + \sum_{wscpofd} WturNofd_{i,wscpofd} * classofd_{c,i} \\
&+ \sum_{c,i} WturTN_{i} * classT_{c,i} + WturTNofs_{i} * classTofs_{c,i} + WturTNofd_{i} * classTofd_{c,i} \\
&- 200 \\
&\leq \sum_{g} WCtinst_{ginst,i}
\end{aligned}
$$

$$WCtinst_{ginst,i} \leq Gtinst_{ginst} * BASE\_WINDinst_{i}$$

Where:

$Gtinst_{ginst}$ is a fractional multiplier of the regional wind (MW) capacity
$BASE\_WINDinst_{i}$ is the region i wind capacity (MW) at the start of the period

Constraint 21
WIND_GROWTH_2000: This constraint ensures that the wind installed by 2000 is consistent with historical installations totaling 3,125 MW

$$
\begin{aligned}
&\sum_{c,i} Wtur\_inregion_{c,i} + Wtur\_inregionofs_{c,i} + Wtur\_inregionofd_{c,i} + \sum_{wscp} WturN_{i,wscp} * class_{c,i} + WturO_{c,i} \\
&\leq 3125
\end{aligned}
$$

Constraint 22
WIND_GROWTH_2002: This constraint ensures that the wind installed by 2002 is consistent with historical installations totaling 4,500 MW

$$
\begin{aligned}
&\sum_{c,i} Wtur\_inregion_{c,i} + Wtur\_inregionofs_{c,i} + Wtur\_inregionofd_{c,i} + \sum_{wscp} WturN_{i,wscp} * class_{c,i} + WturO_{c,i} \\
&\leq 4500
\end{aligned}
$$

Constraint 23
WIND_interregion_trans(j): Due to existing transmission capacity usage and other limitations, the amount of wind power able to be transported on existing lines is limited. This constraint limits the wind imports and exports on existing lines to some fraction (**a_j**) of the capacity **(TRj)** of the transmission lines crossing the boundaries of supply region j

$$\sum_{\substack{i \neq j \\ I_i = I_j \\ c}} WN_{i,j} * class_{c,i} + WO_{c,i,j} + WN_{i,j} * class_{c,j} + WO_{c,i,j}$$
$$+ \sum_{\substack{i \neq j \\ I_i = I_j \\ c}} WNofs_{i,j} * classofs_{c,i} + WOofs_{c,i,j} + WNofs_{i,j} * classofs_{c,j} + WOofs_{c,i,j}$$
$$+ \sum_{\substack{i \neq j \\ I_i = I_j \\ c}} WNofd_{i,j} * classofd_{c,i} + WOofd_{c,i,j} + WNofd_{i,j} * classofd_{c,j} + WOofd_{c,i,j}$$
$$\leq a_j * TR_j$$

Where:

$\boldsymbol{a_j}$ is the fraction of existing transmission lines available to wind

$TR_j = \sum_{k \in j} T_k$ is the sum of the capacity of all existing lines crossing the region boundary

Constraint 24
WIND_BALANCE_PCAS(n): This constraint is a transmission capacity balance that defines $\mathbf{WT_{n,p}}$, the transmission capacity needed to handle wind transmission between PCAs. This transmission capacity required for wind is combined with that required by conventional generation to identify bottlenecks between PCAs through Constraint 55. The left-hand side of the constraint is the sum of all wind generation transmitted into the PCA plus all that generated within the PCA. The right-hand side is the sum of all the wind generation consumed in the PCA plus all that transmitted from the PCA.

$$\sum_{\substack{n \prec p \\ I_n = I_p}} WT_{n,p} + \sum_{\substack{i \in n \\ I_i = I_j \\ c,j}} WN_{i,j} * class_{c,i} + WO_{c,i,j} + \sum_{\substack{i \in n \\ I_i = I_j \\ c,j}} WNofs_{i,j} * classofs_{c,i} + WOofs_{c,i,j} + \sum_{\substack{i \in n \\ I_i = I_j \\ c,j}} WNofd_{i,j} * classofd_{c,i} + WOofd_{c,i,j}$$
$$= \sum_{\substack{n \prec p \\ I_n = I_p}} WT_{n,p} + \sum_{\substack{j \in n \\ I_i = I_j \\ c,i}} WN_{i,j} * class_{c,i} + WO_{c,i,j} + \sum_{\substack{j \in n \\ I_i = I_j \\ c,i}} WNofs_{i,j} * classofs_{c,i} + WOofs_{c,i,j} + \sum_{\substack{j \in n \\ I_i = I_j \\ c,i}} WNofd_{i,j} * classofd_{c,i} + WOofd_{c,i,j}$$

Constraint 25
WIND_DEMAND_LIMIT(j,m): This constraint defines $\mathbf{WS_{j,m}}$ to be the maximum of zero and the difference between the wind-generated electricity consumed in region j in time slice m and all the electricity consumed in region j (i.e., $WS_{j,m}$ is non-zero only if the wind power consumed in region j is greater than the total demand in time slice m. This can occur in off-peak time slices if large amounts of wind are sent to region j to meet the demand in other time slices). $WS_{j,m}$ is then subtracted from the wind contribution to meeting the LOAD_PCA constraint for time slice m. In effect, these two constraints impose a penalty on excessive shipments of wind to an individual region j by not counting the wind power that exceeds the demand in any individual

time slice. This precludes the model from shipping wind to a region near the wind production region and then shipping the wind generation out with conventional generation to other PCAs using conventional lines, i.e. without taking account of the fact that any transmission reserved for wind will only be used when the wind is blowing.

$$
\begin{aligned}
& WS_{j,m} \geq \\
& (\sum_{\substack{c,i \\ j \in n \\ I_i = I_j}} WN_{i,j} * class_{c,i} * CF_c * CF\_corr_{i,c,m} * (1 - IWSurplusMar_{c,i,in}) \\
& + WNofs_{i,j} * classofs_{c,i} * CFofs_c * CF\_corrofs_{c,i,m} * (1 - IWSurplusMar_{c,i,in}) \\
& + WNofd_{i,j} * classofd_{c,i} * CFofd_c * CF\_corrofd_{c,i,m} * (1 - IWSurplusMar_{c,i,in})) * (1 - TWLOSS * dis_{i,j}) \\
& \\
& + (\sum_{\substack{c,i \\ j \in n \\ I_i = I_j}} WTN_{i,j} * classT_{c,i} * CF_c * CF\_corr_{c,i,m} * (1 - IWSurplusMar_{c,i,in}) \\
& + WTNofs_{i,j} * classTofs_{c,i} * CFofs_c * CF\_corrofs_{c,i,m} * (1 - IWSurplusMar_{c,i,in}) \\
& + WTNofd_{i,j} * classTofd_{c,i} * CFofd_c * CF\_corrofd_{c,i,m} * (1 - IWSurplusMar_{c,i,in})) * (1 - TWLOSS * dis_{i,j}) \\
& \\
& + (\sum_{\substack{c,i \\ j \in n}} WO_{c,i,j} * CFO_{c,i} * CF\_corr_{c,i,m} * (1 - IWSurplusOld_{in}) \\
& + WOofs_{c,i,j} * CFOofs_{c,i} * CF\_corrofs_{c,i,m} * (1 - IWSurplusOld_{in}) \\
& + WOofd_{c,i,j} * CFOofd_{c,i} * CF\_corrofd_{c,i,m} * (1 - IWSurplusOld_{in})) * (1 - TWLOSS * dis_{i,j}) \\
& \\
& + (\sum_{\substack{c,i \\ j \in n}} WTO_{c,i,j} * CFTO_{c,i} * CF\_corr_{c,i,m} * (1 - IWSurplusOld_{in}) \\
& + WTOofs_{c,i,j} * CFTOofs_{c,i} * CF\_corrofs_{c,i,m} * (1 - IWSurplusOld_{in}) \\
& + WTOofd_{c,i,j} * CFTOofd_{c,i} * CF\_corrofd_{c,i,m} * (1 - IWSurplusOld_{in})) * (1 - TWLOSS * dis_{i,j}) \\
& \\
& + \sum_{\substack{C \\ j \in N}} \sum_{escp} WELEC\_inregion_{c,escp,j} * CF_c * CF\_corr_{c,j,m} * (1 - IWSurplusMar_{in}) \\
& + WELEC\_inregionofs_{c,escp,j} * CFofs_c * CF\_corrofs_{c,j,m} * (1 - IWSurplusMar_{in}) \\
& + WELEC\_inregionofd_{c,escp,j} * CFofd_c * CF\_corrofd_{c,j,m} * (1 - IWSurplusMar_{in}) \\
& \\
& - \mathrm{Re}\, gDmd_{j,m} * loadgrowth_{n \subset j}^{\ curyear - 2000} \\
& \\
& - (\sum_{j \in n} hfdiselec_j * H2\_loadprofile_m + hfdiselec\_2\_fcell_{j,m}) / H_m / CHEFF_{distributed\_electrolyzer} \\
& - \sum_{j \in n} hfdiselec\_2\_fcell_{j,m} * CHEFF_{storage\_at\_city} / H_m \\
& - (\sum_{j \in n} grid\_2\_welectrolysis_{j,m} + grid\_2\_welectrolysis\_inregion_{j,m} + old\_grid\_2\_welectrolysis_{j,m}) / H_m
\end{aligned}
$$

Where:

$CFTO_{c,i}$ is the average capacity factor of all existing (at the start of the current period) class c onshore wind on new (built in this period) lines in region i

$CFTOofd_{c,i}$ is the average capacity factor of all existing (at the start of the current period) class c deep offshore wind on new (built in this period) lines in region i

$CFTOofs_{c,i}$ is the average capacity factor of all existing (at the start of the current period) class c shallow offshore wind on new (built in this period) lines in region i

$RegDmd_{j,m}$ is the electric load in each hour of time slice m in the year 2000 in region j

$loadgrowth_n$ is the annual rate of growth of load in the PCA n that contains region j

cur_year is the first year of the two-year period for which the optimization is being performed

Conventional Transmission Constraints

Constraint 26

CONV_TRAN_PCA(m,n,p): Ensures that there is sufficient transmission capacity between contiguous PCAs n and p within the same grid interconnect to transmit wind generation and conventional generation in each time slice m. Transmission capacity added this period is included in both directions p-to-n and n-to-p because transmission lines are bidirectional.

$$WT_{n,p} + convt_{m,n,p} \leq TPCAN_{n,p} + TPCAN_{p,n} + TPCAO_{n,p}$$

Where:

$TPCAO_{n,p}$ is the transmission capacity between n and p that existed at the start of this period

Constraint 27

CONTRACT_TRAN_PCA(m,n,p): Ensures that there is sufficient transmission capacity between contiguous PCAs n and p within the same grid interconnect to transmit wind generation and contracted conventional capacity. Transmission capacity added this period is included in both directions p-to-n and n-to-p because transmission lines are bidirectional.

$$WT_{n,p} + CONTRACTCAP_{n,p} \leq TPCAN_{n,p} + TPCAN_{p,n} + TPCAO_{n,p}$$

Constraints 28 and 29

TPCA_GROWTH_TOT and TPCA_GROWTH_BIN(TPCA_G): These two constraints allocate new transmission capacity (MW) to bins that have costs associated with them over and above the cost of the transmission lines themselves. The bins are defined as a fraction of the national transmission capacity at the start of the period.

$$\sum_{\substack{n,p \\ n \prec p \\ n \in I_p}} TPCAN_{n,p} * dis_{n,p} + \sum_{\substack{c,i,j \\ i \neq j}} (WTN_{i,j} * classT_{c,i} + WTNofs_{i,j} * classTofs_{c,i} + WTNofd_{i,j} * classTofd_{i,j}) * dis_{i,j}$$

$$\leq \sum_{TPCA\_G} TPCA\_Ct_{tpca\_g}$$

$$TPCA\_Ct_{tpca\_g} \leq TPCA\_Gt_{tpca\_g} * BASETPCA$$

Where:

$TPCA\_Gt_{tpca\_g}$ is a fractional multiplier of the national transmission (MW) capacity BASETPCA

Constraint 30
TPCA_GROWTH_2000: This constraint ensures that the transmission capacity added nationwide in 2000 is consistent with historical additions in 2000 of 2,585,158 MW-miles

$$\sum_{\substack{n,p \\ n \prec p \\ n \in I_p}} TPCAN_{n,p} * dis_{n,p} + \sum_{\substack{c,i,j \\ i \neq j}} WTN_{i,j} * classT_{c,i} * dis_{i,j}$$

$$\leq 2585158$$

Constraint 31
TPCA_GROWTH_2002: This constraint ensures that the transmission capacity added nationwide in 2002 is consistent with historical additions in 2002 of 2,070,715 MW-miles

$$\sum_{\substack{n,p \\ n \prec p \\ n \in I_p}} TPCAN_{n,p} * dis_{n,p} + \sum_{\substack{c,i,j \\ i \neq j}} WTN_{i,j} * classT_{c,i} * dis_{i,j}$$

$$\leq 2070715$$

Constraint 32
TPCA_GROWTH_2004: This constraint ensures that the transmission capacity added nationwide in 2004 is consistent with historical additions in 2004 of 2,766,900 MW-miles

$$\sum_{\substack{n,p \\ n \prec p \\ n \in I_p}} TPCAN_{n,p} * dis_{n,p} + \sum_{\substack{c,i,j \\ i \neq j}} WTN_{i,j} * classT_{c,i} * dis_{i,j}$$

$$\leq 2766900$$

System Constraints

Constraint 33

LOAD_PCA(m,n): This constraint ensures that the load (MW) in time period m in PCA n is met with imports from PCAs contiguous to PCA n ($CONVT_{m,n,p}$) decremented for transmission losses and generation from conventional sources and wind, also reduced by transmission line losses. The wind output is also decreased by the amount of surplus wind that blows when there is no use for it because loads are low. Output from fuel cells, either at a wind site or distributed within the grid, also contributes to meeting peak loads. Finally, any wind generation that exceeds the demand in the time slice is lost (this prevents wind from simply being shipped to the nearest contiguous region in excess of the demand for power in that region at that time).

The load is expressed in terms of MW that must be provided during each hour of the time slice m. The load is comprised of the direct load, Lmn, plus the power needed by the distributed electrolyzers/compressors and the grid power provided to the wind-sited electrolyzers.

$$\sum_{\substack{p \prec n \\ I_p = I_n}} CONVT_{m,n,p} * (1 - TLOSS * dis_{p,n}) + \sum_{q} CONVGEN_{m,n,q} + \sum_{\substack{q \in Btech \\ m' \in ptime}} CONVP_{m',n,q}$$

$$+ \Big(\sum_{\substack{c,i \\ j \in n \\ I_i = I_j}} WN_{i,j} * class_{c,i} * CF_c * CF\_corr_{c,i\,m} * (1 - IWSurplusMar_{c,i,in})$$

$$+ WNofs_{i,j} * classofs_{c,i} * CFofs_c * CF\_corrofs_{c,i\,m} * (1 - IWSurplusMar_{c,i,in})$$

$$+ WNofd_{i,j} * classofd_{c,i} * CFofd_c * CF\_corrofd_{c,i,m} * (1 - IWSurplusMar_{c,i,in}) \Big) * (1 - TWLOSS * dis_{i,j})$$

$$+ \Big(\sum_{\substack{c,i \\ j \in n \\ I_i = I_j}} WTN_{i,j} * classT_{c,i} * CF_c * CF\_corr_{c,i,m} * (1 - IWSurplusMar_{c,i,in})$$

$$+ WTNofs_{i,j} * classTofs_{c,i} * CFofs_c * CF\_corrofs_{c,i,m} * (1 - IWSurplusMar_{c,i,in})$$

$$+ WTNofd_{i,j} * classTofd_{c,i} * CFofd_c * CF\_corrofd_{c\,i,\,m} * (1 - IWSurplusMar_{c,i,in}) \Big) * (1 - TWLOSS * dis_{i,j})$$

$$+ \Big(\sum_{\substack{c,i \\ j \in n}} WO_{c,i,j} * CFO_{c,i} * CF\_corr_{c\,i,\,m} * (1 - IWSurplusOld_{in})$$

$$+ WOofs_{c,i,j} * CFOofs_{c,i} * CF\_corrofs_{c\,i,\,m} * (1 - IWSurplusOld_{in})$$

$$+ WOofd_{c,i,j} * CFOofd_{c,i} * CF\_corrofd_{c\,i,\,m} * (1 - IWSurplusOld_{in}) \Big) * (1 - TWLOSS * dis_{i,j})$$

$$+ \Big(\sum_{\substack{c,i \\ j \in n}} WTO_{c,i,j} * CFTO_{c,i} * CF\_corr_{c,i,m} * (1 - IWSurplusOld_{in})$$

$$+ WTOofs_{c,i,j} * CFTOofs_{c,i} * CF\_corrofs_{c,i,m} * (1 - IWSurplusOld_{in})$$

$$+ WTOofd_{c,i,j} * CFTOofd_{c,i} * CF\_corrofd_{c,i,m} * (1 - IWSurplusOld_{in}) \Big) * (1 - TWLOSS * dis_{i,j})$$

$$+ \sum_{\substack{C \\ j \in n}} \sum_{escp} WELEC\_inregion_{c,escp,j} * CF_c * CF\_corr_{c,j,m} * (1 - IWSurplusMar_{in})$$

$$+ WELEC\_inregionofs_{c,escp,j} * CFofs_c * CF\_corrofs_{c,j,m} * (1 - IWSurplusMar_{in})$$

$$+ WELEC\_inregionofd_{c,escp,j} * CFofd_c * CF\_corrofd_{c,j,m} * (1 - IWSurplusMar_{in})$$

$$+ \Big(\sum_{s \subset m} fcelldest_{n,s} + fcelldestold_{n,s} + \sum_{\substack{c \\ j \in n}} fcell\_inregion_{c,j,s} \Big) / H_m * ptime_m$$

$$+ \sum_{j \in n} \sum_{s \subset m} \Big(\sum_{mm \in s} hfdiselec\_2\_fcell_{j,mm} * optime_{mm} \Big) * ptime_m * CHEFF_{fuel-cell} / H_m$$

$$- \sum_{j \in n} WS_{j,m}$$

$$\geq L_{m,n} + \sum_{p|n} CONVT_{m,n,p}$$

$$+ \Big(\sum_{j \in n} hfdiselec_j * H2\_loadprofile_m + hfdiselec\_2\_fcell_{j,m} \Big) / H_m / CHEFF_{distributed\_electrolyzer}$$

$$+ \sum_{j \in n} hfdiselec\_2\_fcell_{j,m} * CHEFF_{storage\_at\_city} / H_m$$

$$+ \Big(\sum_{j \in n} grid\_2\_welectrolysis_{j,m} + grid\_2\_welectrolysis\_inregion_{j,m} + old\_grid\_2\_welectrolysis_{j,m} \Big) / H_m$$

Where:

TLOSS is the fraction of conventional power lost in each mile of transmission

$\mathbf{CF\_corr_{c,i,m}}$ is the correction to the annual capacity factor for class c onshore wind in region i and time slice m

$\mathbf{CF\_corrofs_{c,i,m}}$ is the correction to the annual capacity factor for class c shallow offshore wind in region i and time slice m

$\mathbf{CF\_corrofd_{c,i,m}}$ is the correction to the annual capacity factor for class c deep offshore wind in region i and time slice m

$\mathbf{IWSurplusMar_{c,in,j}}$ is the fraction of wind generation lost from the next unit of wind installed in region i because there is no remaining load to be met by the wind in interconnect in.

$\mathbf{IWSurplusOld_{in}}$ is the fraction of wind generation lost from all the wind installed to date in interconnect in because there is no remaining load to be met by the wind in interconnect in.

TWLOSS is the fraction of wind power lost in each mile of transmission

$\mathbf{fcelldestold_{n,s}}$ is the fuel cell output from wind-sited fuel cells built in previous periods that ship power to PCA n in season s

$\mathbf{ptime_{m}}$ is a binary constant equal to one when m is a peak load time slice, zero otherwise

$\mathbf{L_{m,n}}$ is the load (MW) in time slice m in PCA n

$\mathbf{H2\_loadprofile_{m}}$ is the fraction of annual hydrogen production from nonwind production technologies that occurs in time slice m

$\mathbf{old\_grid\_2\_welectrolysis_{i,m}}$ is the electricity from the grid in region i in time slice m consumed by wind-sited electrolyzers built in previous periods

Constraint 34

RES_MARG_NERC(r): Ensures that the conventional and wind capacity (MW) and generation from storage (e.g., fuel cell generation) from both distributed grid-powered and wind-sited generators (e.g., fuel cells) during the peak summer period is large enough to meet the peak load plus a reserve margin and any storage input (e.g., electrolysis) requirements. The wind-site fuel cell generation (MWh) is converted to capacity by dividing by CFc, but is constrained by the availability of transmission to a capacity value of 1-WCVmar (i.e., the combined capacity of the wind turbines and wind-sited fuel cells is one). The capacity value of distributed fuel cells is determined as the hydrogen input (kg) to the fuel cells during the summer peak time slice times the conversion efficiency (kg to MWh) divided by the number of hours of operation during the summer peak time slice. Peak-load requirements in NERC region r can also be met by contracting for capacity located in other NERC regions. The peak-load requirements are increased by the summer peak time slice grid input to electrolyzers either distributed at load sites or located at wind sites, divided by the number of hours in the summer peak time slice.

$$
\begin{aligned}
&\sum_{\substack{q\\ n\in r}} CONVCAP_{n,q} * CONVCAPC_q \\
&+\sum_{\substack{c,i\\ j\in r}} (WN_{i,j} * class_{c,i} + WNofs_{i,j} * classofs_{c,i} + WNofd_{i,j} * classofd_{c,i} \\
&+ WTN_{i,j} * classT_{c,i} + WTNofs_{i,j} * classTofs_{c,i} + WTNofd_{i,j} * classTofd_{c,i}) * WCVMAR_{c,i,r} * (1 - TWLOSS * dis_{i,j}) \\
&+\sum_{\substack{c,i\in r\\ escp}} (WELEC\_inregion_{c,escp,i} + WELEC\_inregionofs_{c,escp,i} + WELEC\_inregionofd_{c,escp,i}) * WCVMAR_{c,i,r} \\
&+\sum_{\substack{c,i\\ j\in r}} (WO_{c,i,j} + WTO_{c,i,j} + WOofs_{c,i,j} + WTOofs_{c,i,j} + WOofd_{c,i,j} + WTOofd_{c,i,j}) * WCVold_r * (1 - TWLOSS * dis_{i,j}) \\
&+\sum_{c,i} (1 - WCVmar_{c,i,r}) / (1 - CF_c) * classT_{c,i} * fcell_{i,r,s} / numpeakhoursbyseason_s \\
&+\sum_{c,i\in r} (1 - WCVmar_{c,i,r}) / (1 - CF_c) * fcell\_inregion_{c,i,s} / numpeakhoursbyseason_s \\
&+\sum_{j\in r} hfdiselec\_2\_fcell_{h3,j} * CHEFF_{fuel-cell} / numpeakhoursbyseason_s \\
&+\sum_{n\in r} \sum_{\substack{p\prec n\\ i_p = I_n}} CONTRACTCAP_{n,p} * (1 - TLOSS * dis_{p,n}) - CONTRACTCAP_{n,p} \\
&\geq \sum_{n\in r} P_n * (1 + NERCRm_r) \\
&+\sum_{i\in r} (grid\_2\_welectrolysis_{h3,i} + grid\_2\_welectrolysis\_inregion_{h3,i} + old\_grid\_2\_welectrolysis_{h3,i}) / H_{h3}
\end{aligned}
$$

Where:

$CONVCAPC_q$ is the effective load-carrying capability of conventional capacity type q
$WCVmar_{c,i,r}$ (wind capacity value – marginal) is the effective load-carrying capacity in NERC region r of one MW at a new wind farm at a class c site in region i. $WCVmar_{c,i,r}$ is derived in detail in the later section on Wind Intermittency Parameters.
$WCVold_r$ is the effective load-carrying capacity of all the wind capacity installed in previous periods whose generation is transmitted to NERC region r. $WCVold_r$ is derived in detail in the later section on Wind Intermittency Parameters.
$NERCRm_r$ is the reserve margin requirement in NERC region r
H3 designates the peak time slice in the summer season

Constraint 35

OPER_RES(m,r): Ensures that the interruptible load, spinning reserve, quick-start capacity, and fuel cell capacity are adequate to meet the normal operating reserve requirement and that imposed by wind. The summer output is used to define the fuel cell capacity under the assumption that the maximum annual fuel cell output in each NERC region occurs in the summer peak time slice.

$$\sum_{n \in r} IL_n + \sum sr_{n,m,q} + QS_{n,q} * F_q$$

$$+ \sum_{n \in r} \sum_{q \in qck} (fcelldest_{n,s} + fcelldestold_{n,s}) / (numpeakhoursbyseason_s * (1 - CF_{class4}))$$

$$+ \sum_{j \in r} \sum_{c} fcell\_inregion_{c,j,s} / (numpeakhoursbyseason_s * (1 - CF_c))$$

$$+ \sum_{j \in r} hfdiselec\_2\_fcell_{h3,j} * CHEFF_{fuel-cell} / numpeakhoursbyseason_s$$

$$+ \sum_{n \in r} \sum_{t \in FCretper} DISFCELL\_CAP\_OLD_{n,t}$$

$$\geq resconfint * (NOR_r + \sum_{c,i} WORmar_{c,i,r}$$

$$* \sum_{\substack{j \in r \\ I_i = I_j}} WN_{i,j} * class_{c,i} + WNofs_{i,j} * classofs_{c,i} + WNofd_{i,j} * classofd_{c,i}$$

$$+ \sum_{\substack{j \in r \\ j \neq i}} WTN_{i,j} * classT_{c,i} + WTNofs_{i,j} * classTofs_{c,i} + WTNofd_{i,j} * classTofd_{c,i}$$

$$+ \sum_{\substack{c \\ j \in r}} WORmar_{c,j,r} *$$

$$\sum_{escp} WELEC\_inregion_{c,escp,j} + WELEC\_inregionofs_{c,escp,j} + WELEC\_inregionofd_{c,escp,j}$$

$$+ WORold_r * \sum_{\substack{c,i \\ j \in r}} WO_{c,i,j} + WOofs_{c,i,j} + WOofd_{c,i,j} + WTO_{c,i,j} + WTOofs_{c,i,j} + WTOofd_{c,i,j})$$

Where:

F_q is the forced outage rate for generation type q
FCretper is the period during which the older remaining (i.e. not yet retired) fuel cells were constructed
resconfint is the confidence interval multiplier applied to the operating reserve standard deviation to ensure a high probability that the operating reserve will be available
NOR_r is the normal operating reserve standard deviation in NERC region r
$WORmar_{c,i,r}$ is the operating reserve requirement induced by the marginal addition of one MW of wind in region i that is consumed in NERC region r

WOROld$_r$ is the operating reserve requirement induced by all wind installed in previous periods that contributes to NERC region r

Constraints 36 and 37
IL_PENETRATION_TOT and Interruptible_Load_BIN(ILG): These two constraints implement an interruptible power supply curve. The second constraint is actually implemented as an upper bound, not as a constraint

$$IL_n \le \sum_{ILG} ILt_{ILG,n}$$

$$ILt_{ILG,n} \le ILGt_{ILG} * PCAdmdPK_n * loadgrowth_n^{cur\_year-2000}$$

Where:

ILGt$_{ILG}$ is the fraction of peak demand in step ILG of the supply curve
PCAdmdPK$_n$ is the peak demand in PCA n in 2000

Conventional Generator Constraints

Constraint 38
SPIN_RES_CAP(m,n,q): ensures that the useful generation (CONV) from a conventional plant of type q comprises at least a minimum fraction (MINSR) of the total generation (CONV +SR) in time slice m in PCA n

$$CONVGEN_{m,n,q} \ge MINSR_q * (SR_{m,n,q} + CONVGEN_{m,n,q})$$

Where:

MINSR$_q$ is the fraction of each type of plant q that must be on line and loaded in order to serve as spinning reserve
SR$_{m,n,q}$ is the spinning reserve capacity from technology q during time period m in PCA n.

Constraint 39
CAP_FO_POa(m,n,q): Ensures that the capacity (MW) in PCA n of type q derated by the average forced outage rate for type q generators is adequate to meet the load, quick-start, and spinning reserve required in time slice m.

$$CONVGEN_{m,n,q} + CONVPGEN_{m,n,q} * btech_q + SR_{m,n,q} + QS_{n,q} \le CONVCAP_{n,q} * (1 - fo_q)$$

Where:

$btech_q$ is a binary variable that is 1 if q is a base-load technology and 0 otherwise
fo_q is the forced outage rate for generator type q

Constraint 40
B_peak_12b(q-baseload technologies,m-peak time slices,n): To prevent unrealistic cycling, base-load plants are constrained in peak time slices to generate no more electricity than the average of that which is generated in the shoulder time slices.

$$CONVGEN_{m,n,q} \leq (CONVGEN_{m',n,q} + CONVGEN_{m'',n,q})/2$$

Where:

m is the peak time slice within each season (summer, winter, spring, and fall)
m' and **m"** are the shoulder time slices within each season (summer, winter, spring, and fall)

Constraint 41
HYDRO_ENERGY(n): Restricts the energy available from hydroelectric capacity to conform to the historical availability of water (He_n).

$$\sum_m CONVGEN_{hyd,m,n} * H_m \leq He_n$$

<u>**Environmental Constraints**</u>

Constraint 42
LOWSULCOAL(n,q): This constraint essentially adds all the coal used in the different time slices throughout the year into the variable **$coalowsul_q$.**

$$coalowsul_q \leq \sum_{\substack{m \\ q \in coaltech}} H_m * CONVGEN_{m,n,q} + \sum_{\substack{m \in ptime \\ q \in coaltcch}} H_m * CONVPGEN_{m,n,q}$$

Constraint 43
LOWSULCOAL_LIMIT(q,n): This constraint precludes any unscrubbed existing (before 2000) coal plant that has made the capital investment to use low-sulfur coal from switching back to high-sulfur coal in this time period. (This constraint implicitly presumes that any existing coal plants adapted for low-sulfur coal use will retire in the same proportions as those that have not been adapted. It also implicitly presumes that new coal plants capable of burning low-sulfur coal will be built in the same proportion.)

$$coalowsul_{n,q} \geq (lowsulcoalold_{n,q} / Coal\_old\_prev_{n,q}) * CONVCAP_{n,q}$$

Where:

$lowsulcoalold_{n,q}$ is the amount of electricity (MWh) generated from low sulfur coal in PCA n by coal technology q in the previous 2-year time period.
$Coal\_old\_prev_{n,q}$ is the capacity (MW) of coal-fired generation in PCA n of type q in the previous 2-year period.

Constraint 44
LOWSULCOAL_LIMIT2(n,q): This constraint prevents the fraction of low-sulfur coal used in all existing coal generators from decreasing from the level used in the previous period. The constraint is needed because coal plants can switch from unscrubbed (coal-old-1) to scrubbed (coal-old-2) coal plants.

$$
\begin{aligned}
& coalowsul_{coal-old-1,n} + coalowsul_{coal-old-2,n} \\
& \geq ((lowsulcoalold_{coal-old-1,n} + lowsulcoalold_{coal-old-2,n}) \\
& /(Coal-old-prev_{coal-old-1,n} + Coal-old-prev_{coal-old-2,n}) \\
& *(CONVCAP_{coal-old-1,n} + CONVCAP_{coal-old-2,n})
\end{aligned}
$$

Where:

$coalowsul_{coal-old-1,n}$ is the total conventional generation from coal-fired generation with scrubbers that existed before the analysis time frame in PCA n using low-sulfur coal
$coalowsul_{coal-old-2,n}$ is the total conventional generation from coal-fired generation without scrubbers that existed before the analysis time frame in PCA n using low-sulfur coal
$Coal\_old\_prev_{coal-old-1,n}$ is the capacity of coal-fired generation with scrubbers that existed before the analysis time frame in PCA n that was still operating at the end of the previous 2-year period.
$Coal\_old\_prev_{coal-old-2,n}$ is the capacity of coal-fired generation without scrubbers that existed before the analysis time frame in PCA n that was still operating at the end of the previous 2-year period.

Constraint 45
SCRUBBER(n): The combined capacity of the scrubbed and unscrubbed coal plants must be greater than the total of the two from the last period minus retirements. This allows the unscrubbed to become scrubbed, i.e., the unscrubbed capacity can decrease.

$$
\begin{aligned}
& CONVCAP_{coal-old-1,n} + CONVCAP_{coal-old-2,n} \\
& \geq CONVOLD_{n,coal-old-1} - CONVRET_{n,coal-old-1} \\
& + CONVOLD_{n,coal-old-2} - CONVRET_{n,coal-old-2}
\end{aligned}
$$

Where:

$CONVRET_{n,q}$ is the capacity in PCA n of generation type q retired in this period. See the later section on retirements of conventional capacity.

Constraint 46

EMISSIONS(pol): Ensures that the national annual emission of each pollutant (CO_2, SO_2, Nox, Hg) by all generators and hydrogen production technologies is lower than a national cap (LP)

$$
\begin{aligned}
&\sum_{n}\sum_{q,m} CONVGEN_{m,n,q} * H_m * CONVpol_{pol,q} * cheatrate_q \\
&+\sum_{n}\sum_{\substack{q\in btech \\ m\in ptime}} CONVPGEN_{m,n,q} * H_m * CONVpol_{pol,q} * cheatrate_q \\
&-coallowsulpolred * \sum_{n}\sum_{\substack{q\in coaltech \\ pol\in SO2\,pol}} CONVpol_{pol,q} * cheatrate_q * coalowsul_{n,q} \\
&+\sum_{j} hfsteamref_j * steam\_ref\_emiss_{pol} / CHEFF_{NG\_reformer} \\
&\leq LP_{pol}
\end{aligned}
$$

Where:

$CONVpol_{pol,q}$ is the emission of pollutant pol (tons) from a million Btu of fuel consumed by generator type q
coallowsulpolred is the delta (tons) in SO_2 emissions per MWh between high sulfur and low sulfur coal
$steam\_ref\_emiss_{pol}$ is the emissions (tons) of pollutant pol per MMBtu of input gas to steam methane reforming
LP_{pol} is the national annual cap on pollutant pol (tons/year).

Storage Constraints (constraints on hydrogen production from wind)

Constraint 47

ELEC_and_H2_inregion(c,i,s): Constrains the hydrogen produced (i.e. stored wind energy) and the electricity generated for use within region i in season s to be less than or equal to the seasonal output from new onshore turbines in the region.

$$
\begin{aligned}
&\sum_{escp} WELEC\_inregion_{c,escp,i} * CF_c * CF\_corrs_{c,i,s} * numhoursbyseason_s \\
&+ wind\_2\_electrolysis\_inregion_{c,i,s} \\
&\leq Wtur\_inregion_{c,i} * CF_c * CF\_corrs_{c,i,s} * numhoursbyseason_s
\end{aligned}
$$

Where:

$CF\_corrs_{c,i,s}$ is a correction factor to the capacity factor to account for variations in the output of wind with a season compared to the annual average capacity factor CF_c.
$numhoursbyseason_s$ is the number of hours in season s

Constraint 48
ELECTROLYSIS_INPUT_INREGION(i,s): Ensures that the wind-generated electricity designated for input to the storage-conversion process (e.g., electrolyzers/hydrogen storage) within region i is greater than or equal to the energy needed to produce the stored energy (e.g., hydrogen) for transport fuel or on-peak electricity production within region i as well as the electricity required to operate the storage/compression. The ratio, $numhoursbyseason_s/8760$, at the end of the constraint apportions the stored energy (e.g., hydrogen) produced throughout the year to the season s.

$$\sum_c wind\_2\_electrolysis\_inregion_{c,i,s} + \sum_{m \in s} grid\_2\_welectrolysis\_inregion_{i,m}$$
$$\geq \sum_c fcell\_inregion_{c,i,s} / CHEFF_{fuel-cell} * (1/CHEFF_{electrolyzer} + CHEFF_{storage-at-wind})$$
$$+ \sum_{c,hscp} hf\_inregion_{c,hscp,i} / CHEFF_{electrolyzer} * (numhoursbyseason_s / 8760)$$

Constraint 49
ELECandH2_FROM_WIND(c,i,s): Ensures that the new electricity to the grid and the wind-generated electricity to the new wind-sited electrolyzers are less than or equal to the output from the new class c wind turbines in region i in season s.

$$(\sum_j WN_{i,j} * class_{c,i} * CF_c * CF\_corrs_{c,i,s} + \sum_j WTN_{i,j} * classT_{c,i} * CF_c * CF\_corrs_{c,i,s}$$
$$\sum_j WNofs_{i,j} * classofs_{c,i} * CFcofs_c * CF\_corrsofs_{c,i,s} + \sum_j WTNofs_{i,j} * classTofs_{c,i} * CFcofs_c * CF\_corrsofs_{c,i,s}$$
$$\sum_j WNofd_{i,j} * classofd_{c,i} * CFcofd_c * CF\_corrsofd_{c,i,s} + \sum_j WTNofd_{i,j} * classTofd_{c,i} * CFcofd_c * CF\_corrsofd_{c,i,s})$$
$$* numhoursbyseason_s + wind\_2\_electrolysis_{c,i,s}$$
$$\leq numhoursbyseason_s * ($$
$$CFc_c * CF\_corrs_{c,i,s} * (\sum_{wscp} WturN_{i,wscp} * class_{c,i} * CF\_corrs_{c,i,s} + WturTN_i * classT_{c,i})$$
$$+ CFcofs_c * CF\_corrsofs_{c,i,s} * (\sum_{wscpofs} WturNofs_{i,wscpofs} * classofs_{c,i} * CF\_corrsofs_{c,i,s} + WturTNofs_i * classTofs_{c,i})$$
$$+ CFcofd_c * CF\_corrsofd_{c,i,s} * (\sum_{wscpofd} WturNofd_{i,wscpofd} * classofd_{c,i} * CF\_corrsofd_{c,i,s} + WturTNofd_i * classTofd_{c,i}))$$

Where:

$CF\_corrsofs_{c,i,s}$ is a seasonal adjustment to the capacity factor for new shallow offshore class c wind in region i
$CF\_corrsofd_{c,i,s}$ is a seasonal adjustment to the capacity factor for new deep offshore class c wind in region i

Constraint 50
ELECTROLYSIS_INPUT(i,s): Ensures that the electricity input to the storage conversion process (e.g., electrolyzer) is greater than or equal to the energy needed to produce the stored energy (e.g., hydrogen) for subsequent on-peak electricity production, and for transport fuel and compression or liquefaction of that fuel before transport.

$$\begin{aligned}
&\sum_{c} wind_2_electrolysis_{c,i,s} + \sum_{m \in s} grid_2_welectrolysis_{i,m} \\
&\geq \sum_{r} fcell_{i,r,s} / CHEFF_{fuel-cell} * (1/CHEFF_{electrolyzer} + CHEFF_{storage-at-wind}) \\
&+ hfs_i (1/CHEFF_{electrolyzer} + CHEFF_{h2-transportation}) * (numhoursbyseason_s / 8760)
\end{aligned}$$

Where:

$CHEFF_{h2\text{-}transportation}$ is the electricity (MWh/kg) required to compress or liquefy the hydrogen before transporting it between regions.

Constraint 51
TRANSMIT_2_ELECTROLYZER(i,m): Ensures that the grid electricity used in the wind-sited electrolyzer can be transmitted on the transmission lines built to the wind site.

$$\begin{aligned}
&grid_2_welectrolysis_{i,m} / h_m \\
&\leq \sum_{j} WN_{i,j} * class_{c,i} + \sum_{j} WTN_{i,j} * classt_{c,i} \\
&+ \sum_{j} WNofs_{i,j} * classofs_{c,i} + \sum_{j} WTNofs_{i,j} * classtofs_{c,i} \\
&+ \sum_{j} WNofd_{i,j} * classofd_{c,i} + \sum_{j} WTNofd_{i,j} * classtofd_{c,i}
\end{aligned}$$

Constraint 51a
TRANSMIT_2_ELECTROLYZER_INREGION(i,m): Ensures that the grid electricity used in the wind-sited electrolyzer can be transmitted on the transmission lines built from the load center in the same region to the wind site.

$$grid\_2\_welectrolysis\_inregion_{i,m} / h_m \leq \sum_{c,escp} WELEC\_inregion_{c,escp,i}$$

Constraint 52
GRID_LIMIT(i,s): This ensures that the grid power will only fill in behind wind in operating the electrolyzers. If it does more, it needs to be added to the electrolyzer capacity (which is computed as the difference between turbines and wind to grid). It's critical to separate out seasons, otherwise it will use all grid power in a single season's peak. Dividing by CF yields total capacity; multiplying by 1-CF yields the amount that can be filled in by grid electricity.

$$\sum_{m \in s} grid\_2\_welectrolysis_{i,m} + grid\_2\_welectrolysis\_inregion_{i,m}$$
$$\leq \sum_{c} (wind\_2\_electrolysis_{c,i,s} + wind\_2\_electrolysis\_inregion_{c,i,s})$$
$$*(1 - CF_c * CF\_corrs_{c,i,s}) / (CF_c * CF\_corrs_{c,i,s})$$

Constraint 53
ELECTROLYZER_CAPACITY(i): Defines electrolyzer capacity (MW) as the difference between wind turbine capacity and the capacity of the wind used for power generation (as opposed to the wind capacity used for hydrogen production). Another way to think about this is that the electrolyzer capacity is assumed to be the difference between the wind turbine capacity and the capacity of the transmission lines available to move the wind generated electricity to load.

$$ELE_i \geq \sum_{c}$$
$$(\sum_{wscp} WturN_{i,wscp} * class_{c,i} + WturTN_{c,i} * classT_{c,i} + Wtur\_inregion_{c,i}$$
$$+ \sum_{wscpofs} WturNofs_{i,wscpofs} * classofs_{c,i} + WturTNofs_{c,i} * classTofs_{c,i}$$
$$+ \sum_{wscpofd} WturNofd_{i,wscpofd} * classofd_{c,i} + WturTNofd_{c,i} * classTofd_{c,i})$$
$$-$$
$$(\sum_{j} WN_{i,j} * class_{c,i} + \sum_{j} WTN_{i,j} * classT_{c,i} + \sum_{escp} WELEC\_inregion_{c,escp,i}$$
$$+ \sum_{j} WNofs_{i,j} * classofs_{c,i} + \sum_{j} WTNofs_{i,j} * classTofs_{c,i}$$
$$+ \sum_{j} WNofd_{i,j} * classofd_{c,i} + \sum_{j} WTNofd_{i,j} * classTofd_{c,i})$$

Constraint 54
FUEL_CELL_PEAK_OUT(i,r,s): Ensures that peak fuel cell output (MWh) from a wind site fits on the transmission line leaving supply region i for NERC region r along with the direct wind electricity (MWh).

$$
\begin{aligned}
& fcell_{s,i,r} \leq numpeakhoursbyseason_s * \\
& (\sum_c (1 - CF_c * CF\_corrps_{c,i,s}) * \sum_{n \in r} (\sum_{\substack{j \in n \\ j \in I_i}} WN_{i,j} * class_{c,i} + \sum_{\substack{j \in n \\ j \neq i}} WTN_{i,j} * classT_{c,i}) \\
& + \sum_c (1 - CFofs_c * CF\_corrpsofs_{c,i,s}) * \sum_{n \in r} (\sum_{\substack{j \in n \\ j \in I_i}} WNofs_{i,j} * classofs_{c,i} + \sum_{\substack{j \in n \\ j \neq i}} WTNofs_{i,j} * classTofs_{c,i}) \\
& + \sum_c (1 - CFofd_c * CF\_corrpsofd_{c,i,s}) * \sum_{n \in r} (\sum_{\substack{j \in n \\ j \in I_i}} WNofd_{i,j} * classofd_{c,i} + \sum_{\substack{j \in n \\ j \neq i}} WTNofd_{i,j} * classTofd_{c,i}))
\end{aligned}
$$

Where:

$CF\_corrps_{c,i,s}$ is the correction to the annual capacity factor for class c onshore wind in region i for the peak time slice in each season s
$CF\_corrpsofs_{c,i,s}$ is the correction to the annual capacity factor for class c shallow offshore wind in region i for the peak time slice in each season s
$CF\_corrpsofd_{c,i,s}$ is the correction to the annual capacity factor for class c deep offshore wind in region i for the peak time slice in each season s

Constraint 55
FUEL_CELL_PEAK_IN(n,s): Ensures that the peak fuel cell output (MWh) entering a PCA from a wind-sited fuel cell fits on the transmission line entering that PCA from the same wind site.

$$
\begin{aligned}
& fcelldest(n,s) \leq numpeakhoursbyseasons * \sum_{\substack{i \\ j \in n}} \\
& (\sum_c (1 - CF_c * CF\_corrps_{c,i,s}) * (WN_{i,j} * class_{c,i} + WTN_{i,j} * classT_{c,i}) \\
& + \sum_c (1 - CFofs_c * CF\_corrpsofs_{c,i,s}) * (WNofs_{i,j} * classofs_{c,i} + WTNofs_{i,j} * classTofs_{c,i}) \\
& + \sum_c (1 - CFofd_c * CF\_corrpsofd_{c,i,s}) * (WNofd_{i,j} * classofd_{c,i} + WTNofd_{i,j} * classTofd_{c,i}))
\end{aligned}
$$

Constraint 56
FUEL_CELL_PEAK_INREGION(c,i,s): Ensures that peak fuel cell output (MWh) from a wind site fits on the transmission line leaving the wind site along with the direct wind electricity (MWh) to be used in the same region.

$$fcell\_inregion_{c,i,s} \leq numpeakhoursbyseason_s *$$
$$((1 - CF_c * CF\_corrps_{c,i,s}) * \sum_{escp} WELEC\_inregion_{c,escp,i}$$
$$+ (1 - CFofs_c * CF\_corrpsofs_{c,i,s}) * \sum_{escp} WELEC\_inregionofs_{c,escp,i}$$
$$+ (1 - CFofd_c * CF - \_corrpsofd_{c,i,s}) * \sum_{escp} WELEC\_inregionofd_{c,escp,i})$$

Constraint 57
FUEL_CELL_BALANCE(r,s): Sets the fuel cell output (MWh) from wind supply regions going to NERC region r equal to the fuel cell input (MWh) to the NERC region in each season. It doesn't have to be done for each time slice, because such transfers are allowed only during peak time slices of each season. This is done to reduce the number of variables associated with the fuel cells

$$\sum_i fcell_{i,r,s} = \sum_{n \in r} fcelldest_{n,s}$$

Constraint 58
FUEL_CELL_CAPACITY_LOW(i,s): Sets a lower bound on the fuel cell capacity needed by translating the fuel cell generation into a capacity estimate. Because it doesn't know whether the new wind capacity at the wind farms where the fuel cell is located is on new or existing transmission lines, to be conservative, it uses the higher (because lower-class wind may be competitive with higher-class wind that must also pay for transmission capacity) capacity factor of new wind on new transmission lines (conservatively leaves less room for the fuel cell output).

$$Fcellcapacity_i \geq \sum_r fcell_{i,r,s} / (numpeakhoursbyseason_s * (1 - \sum_c CF_c * classT_{c,i}))$$
$$+ \sum_c fcell\_inregion_{c,i,s} / (numpeakhoursbyseason_s * (1 - CF_c))$$

The next 12 constraints increase the price of hydrogen and wind technologies over their costs to reflect rapid growth constraints.

Constraints 59 and 60
ELECTROLYZER_GROWTH_TOT and ELECTROLYZER_GROWTH_BIN(hebp): These two constraints allocate new electrolyzer capacity (both distributed and wind-sited) (MW) to bins that have costs associated with them over and above the direct cost of the electrolyzers

themselves. The bins are defined as a fraction of the national electrolyzer capacity (MW) at the start of the period

$$\sum_j HF\_DISELEC\_CAP_j + ELE_j \leq \sum_{hebp} HEGBIN_{hebp}$$
$$HEGBIN_{hebp} \leq HEGBINCAP_{hebp} * BASE\_ELEC$$

Where:

$HEGBINCAP_{hebp}$ is a fractional multiplier of the national electrolyzer capacity
BASE_ELEC is the national electrolyzer capacity (MW) at the start of the period

Constraints 61 and 62
SMR_GROWTH_TOT and SMR_GROWTH_BIN(hsmrbp): These two constraints allocate new SMR capacity (kg/year) to bins that have costs associated with them over and above the direct cost of the SMR themselves. The bins are defined as a fraction of the national SMR capacity (kg/year) at the start of the period

$$\sum_j HF\_STEAMREF\_CAP_j \leq \sum_{hsmrbp} HSMRGBIN_{hsmrbp}$$
$$HSMRGBIN_{hsmrbp} \leq HSMRGBINCAP_{hsmrbp} * BASE\_SMR$$

Where:

$HSMRGBIN_{hsmrbp}$ is a variable for new national steam methane reformer capacity in growth bin hsmrbp
$HSMRGBINCAP_{hsmrbp}$ is a fractional multiplier of the national SMR capacity
BASE_SMR is the national SMR capacity (kg/year) at the start of the period

Constraints 63 and 64
FUELCELL_GROWTH_TOT and FUELCELL_GROWTH_BIN(hfcbp): These two constraints allocate new fuel cell capacity (both distributed and wind-sited) (MW) to bins that have costs associated with them over and above the direct cost of the fuel cells themselves. The bins are defined as a fraction of the national fuel cell capacity (MW) at the start of the period

$$\sum_j Fcellcapacity_j + \sum_n DISFCELL\_CAP_n \leq \sum_{hfcbp} HFCGBIN_{hfcbp}$$
$$HFCGBIN_{hfcbp} \leq HFCGBINCAP_{hfcbp} * BASE\_FCELL$$

Where:

$HFCGBINCAP_{hfcbp}$ is a fractional multiplier of the national fuel cell capacity
BASE_FCELL is the national fuel cell capacity (MW) at the start of the period

Hydrogen Fuel Constraints

Constraint 65
H2FUEL_MARKET_BALANCE(j): Ensures that the hydrogen fuel shipped into a region and the fuel supplied by the region is balanced by the fuel consumed in the region and the fuel shipped out of the region.

$$\sum_{i \prec j} hf_{i,j} + hfs_j = \sum_{i \prec j} hf_{j,i} + hfd_j$$

where the symbol $\prec$ indicates contiguity

Constraint 66
H2FUEL_INTER_REGION(j): Ensures that hfs is not used in region; because, if it were, the transport distance would be zero and therefore the cost of transport would be zero. In-region use can occur through the inregion variable with the cost of transport taken from the supply curve for inregion use.

$$hfs_j \le \sum_{i \prec j} hf_{j,i}$$

Constraint 67
HF_DEMAND(j): Ensures the intraregion transport of hydrogen fuel is accounted for. Demand can be met by wind-sited electrolyzers, distributed electrolyzers, or natural gas steam methane reformers – hfdold includes all wind-generated H2, but not H2 from diselec or steam reforming as these vary each period. Includes demand escalation over time.

$$\sum_{c,hscp} hf\_inregion_{c,hscp,j} + hfd_j + hfdiselec_j + hfsteamref_j$$
$$\le hfdemand_j * hfdemand\_escal^{ordyear} - hfdold_j$$

Where:

hfdemand$_j$ is the maximum annual demand for hydrogen as a transportation fuel for light-duty vehicles in region j in the base year
hfdemand_escal is the annual escalation in the demand for light-duty vehicle fuels.
ordyear is the year of the optimization minus 2000 (i.e., the number of years of demand growth)
hfdold$_j$ is the hydrogen transportation fuel supplied by all remaining hydrogen production facilities built in prior periods

Constraint 68
HF_STEAMREF(j): Ensures that the hydrogen (kg) produced by steam methane reformers (SMR) is less than the capacity (kg per year) of the steam methane reformers built in this period and those built in previous period that are not yet retired.

$$hfsteamref_j \le (HF\_STEAMREF\_CAP_j + \sum_{t \ge SMRretper} HF\_STEAMREF\_CAPOLD_{j,t}) * H2\_prodnhours / 8760$$

Where:

SMRretper is the period during which the older remaining (i.e. not yet retired) SMR were constructed
Hf_steamref_capold$_{j,t}$ is the SMR capacity (kg/year) built in region j in period t
H2_prodnhours is the number of hours that the conventional hydrogen production is operated each year

Constraint 69
HF_DISELEC(j): Ensures that the hydrogen produced by distributed electrolyzers (not sited at wind farms) is less than the capacity (MW) of distributed electrolyzers built in this period and those not yet retired.

$$hfdiselec_j + \sum_m hfdiselec\_2\_fcell_{j,m}$$
$$\le (HF\_DISELEC\_CAP_j + \sum_{t \ge DEretper} HF\_DISELEC\_CAPOLD_{j,t}) * h2\_prodnhours * CHEFF_{distributed-electrolyzer}$$

Where:

DEretper is the period during which the older remaining (i.e. not yet retired) distributed electrolyzers were constructed
HF_DISELEC_CAPOLD$_{j,t}$ is the distributed electrolyzer capacity built in region j in period t