Improving the Constraints on Self-Interacting Dark Matter (SIDM) from Galaxy Cluster Observations

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Abstract

Galaxy cluster X-ray and lensing observations can be used to constrain the collisional nature of cold dark matter. Current constraints are limited by the lack of a statistically meaningful , fully cosmological prediction for the evolving density profiles of SIDM cluster halos for different elastic scattering cross sections. We seek here to remedy that situation. We have developed a semi-analytic model that describes the relaxation of cosmological SIDM halos toward cored profiles. Our model is calibrated and verified by *N*-body simulations with Monte Carlo collision algorithm, for individual halos in both isolated and cosmological environments, respectively. With this model, we are able to predict the **SIDM halo profiles** and their **statistical distribution,** for hard-sphere and Yukawa-like velocity-dependent collisional cross sections, for ~1000 clusters in a large (600 Mpc)^3 collisionless, *N*-body simulation of ΛCDM, without running computationally prohibitive Monte Carlo + *N*-body simulations of such a large volume. If, e.g. X-ray/lensing data at z~0 show that NFW profiles apply *without* cores, down to core radius r_c~20 kpc for M > ~10^{14} solar masses, then σ < 0.1 cm^2/g for velocity-independent scattering, but σ(v=0) = 5 cm^2/g is allowed for Yukawa-like cross section if velocity cut-off v_c < ~200 km/s.

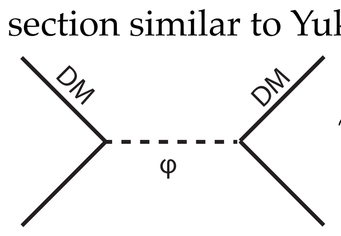
1. Introduction

- CDM predicts singular halo density profiles
- Observed rotation curves of DM dominated dwarf & LSB galaxies prefer density profiles with constant cores
- SIDM hypothesis: elastic scattering flattens core profile if SIDM cross section σ > ~1 cm^2/g
- But σ big enough to solve cuspy core crisis for galaxies may be *too big* for clusters:
  - SIDM clusters are more spherical & cored than observed (e.g. Yoshida 2000)
  - SIDM theory suggests SIDM relaxation may be less efficient for clusters than galaxies, however, due to finite mean free path for much larger σ (Ahn & Shapiro 2005)
  - But, merging cluster DM too fluid-like to explain segregation of DM from baryons in "bullet cluster" if σ too large (Clowe et al 2006; Randall 2007, σ < ~1 cm^2/g)
  - Perhaps σ is velocity dependent: σ(cluster) << σ(galaxy).
- Previous SIDM *N*-body simulations only presented a few illustrative halos
  - Need larger, statistically meaningful theoretical sample to quantify observational limit
  - Individual halo relaxation demands high numerical resolution
  - We use high-res Monte Carlo + *N*-body SIDM simulations of individual halos to model relaxation of many clusters in a large-volume collisionless CDM *N*-body simulation.

2. Scattering Cross Section Model

We use a velocity dependent cross section similar to Yukawa interaction.

$$\sigma = \frac{\sigma_0}{(1+(v/v_c)^2)^2}$$



$$v_c = \frac{m_\phi}{m_{DM}} c$$

- This includes the velocity independent (hard sphere) cross section in the limit $v_c >> v$
- This velocity dependence often appears from the propagator of mediator boson in quantum field theory calculations. Cutoff velocity v_c is related to the mass ratio of mediator boson and the dark matter particle
- We assume isotropic scattering in center-of-mass frame (neglecting angular dependence of scattering cross section for simplicity).

3. Relaxation of Isolated NFW halo

We modified Gadget N-body code (Springel et al 2001) to include Monte Carlo elastic scattering of SIDM to simulate relaxation of single, isolated NFW halo,

$$\rho(r) = \frac{\rho_0}{r/r_s(1+r/r_s)^2}$$

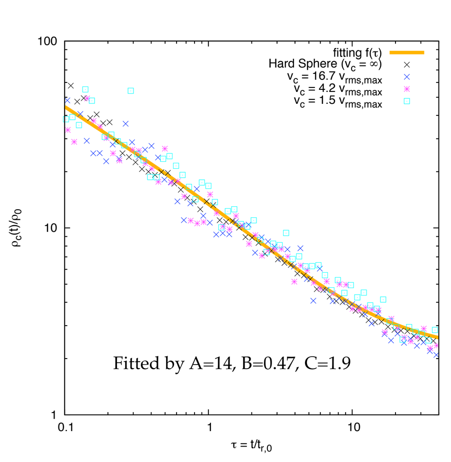


Fig 1. Central density of isolated NFW halo as a function of time. The time evolution is cross section independent in units of SIDM mean-free-collision-time $t_{r,0}$ (A function of ρ_0 and r_s). See also Fig. 4.

- Time dependence can be fitted by an empirical formula,

$$\frac{\rho_c(t)}{\rho_0} = f(\tau) \equiv \frac{A}{\sqrt{\tau + B\tau^2}} + C; \quad \tau = t/t_{r,0} \qquad \textbf{(Eq. 1)}$$

- We use this to model the relaxation of *cosmological* SIDM halos, too, as follows

References

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4. Evolution of Cosmological SIDM Halos

Semi-analytic SIDM relaxation model

- Cosmological halos evolve through a sequence of NFW profiles as mergers and infall grow the mass
- Central density, ρ_c, of isolated SIDM halo, (eq. 1), follows an ordinary differential equation (ODE),

$$\frac{1}{\rho_0}\frac{d\rho_c(t)}{dt} = \frac{1}{t_{r,0}} F(\rho_c(t)/\rho_0); \quad F(x) \equiv \frac{df}{d\tau}(f^{-1}(x))$$

Generalize — Time dependent halo

$$\boxed{\frac{1}{\rho_0(t)}\frac{d}{dt}\rho_c(t) = \frac{1}{t_{r,0}(t)} F\left(\frac{\rho_c(t)}{\rho_0(t)}\right)} \qquad \textbf{(Eq. 2)}$$

- We solve this time dependent ODE using NFW parameter history, {$\rho_0(t)$,$r_s(t)$}, extracted from collisionless *N*-body simulations.
- This model explains the evolution of SIDM halos very well (see below).

Cosmological SIDM *N*-body Simulation

We run muli-resolution SIDM *N*-body simulations, zooming in to a single halo (grafic2 package, Bertschinger) to verify our relaxation ODE model.

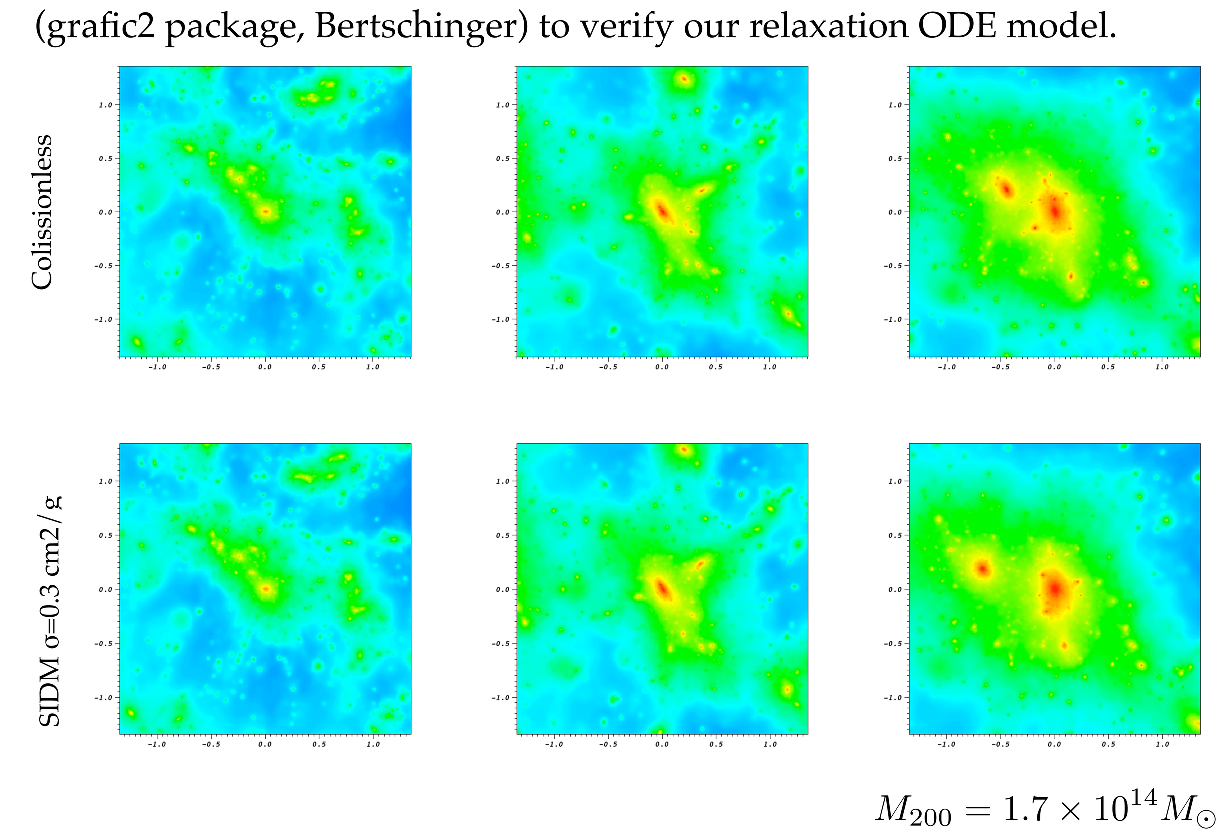


Fig 2. Cosmological *N*-body simulation of a cluster with (lower panels) and without (upper panels) SIDM scattering at redshift 1.8, 0.78, 0.23 from left to right, respectively.

Relaxation Model v.s. Monte Carlo SIDM *N*-body

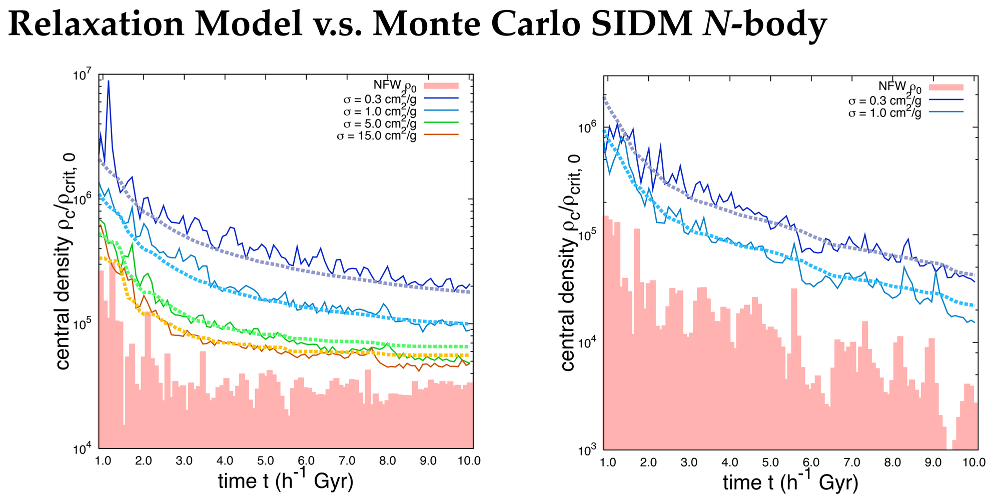


Fig 3. Evolution of central density in cosmological SIDM *N*-body simulations (*solid*) and our relaxation model (*dotted*) for several velocity independent cross sections. Left is a Milky-way size halo and right is a cluster size halo.

* **Our model tracks the central density very well.**

6. SIDM Density Profile: "Cored NFW Profile"

Partially-relaxed SIDM density profiles (ρ_c>5ρ_0) can be fitted by a "**cored NFW profile**",

$$\boxed{\rho(r) = \frac{\rho_0}{[(r_c/r_s)^2 + (r/r_s)^2]^{1/2}(1+r/r_s)^2}} \qquad \textbf{(Eq. 3)}$$

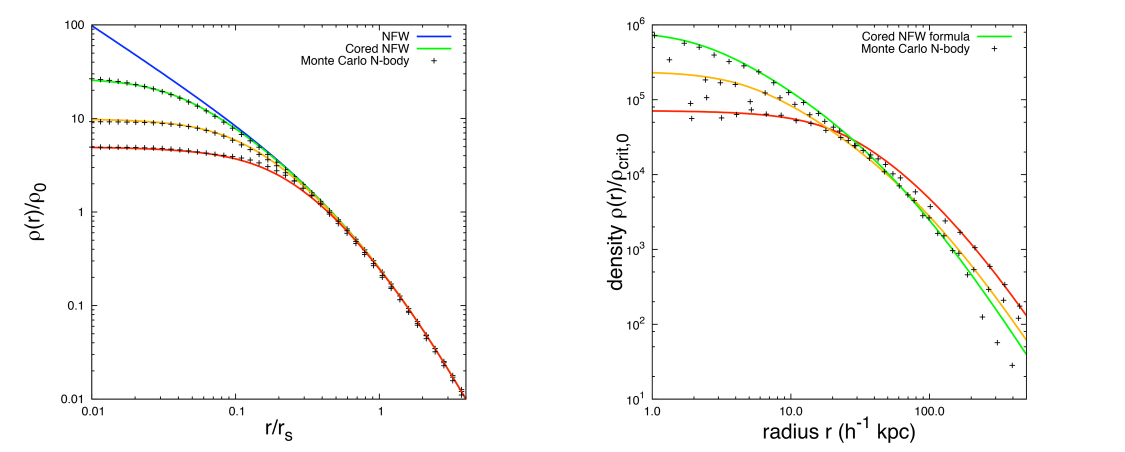
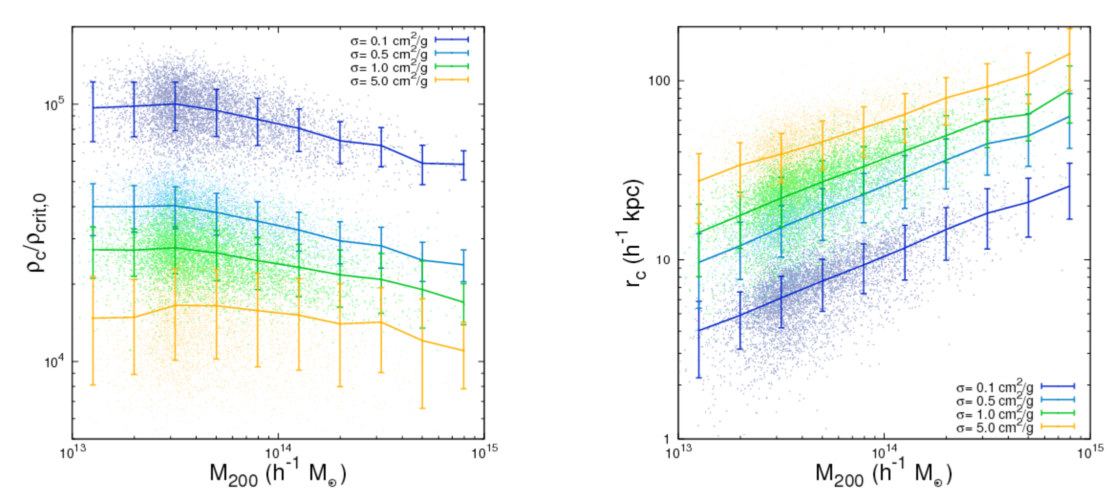


Fig 4. Cored NFW fit to SIDM simulation of isolated (left) and a cosmological cluster (right). Profile relaxes to a non-singular isothermal profile with ρ_c = 2 ρ_0 as t →∞

7. Result: Statistical Distribution of SIDM halos

Hard sphere cross section (velocity independent)



Yukawa-like cross section with v_c = 200 km/s

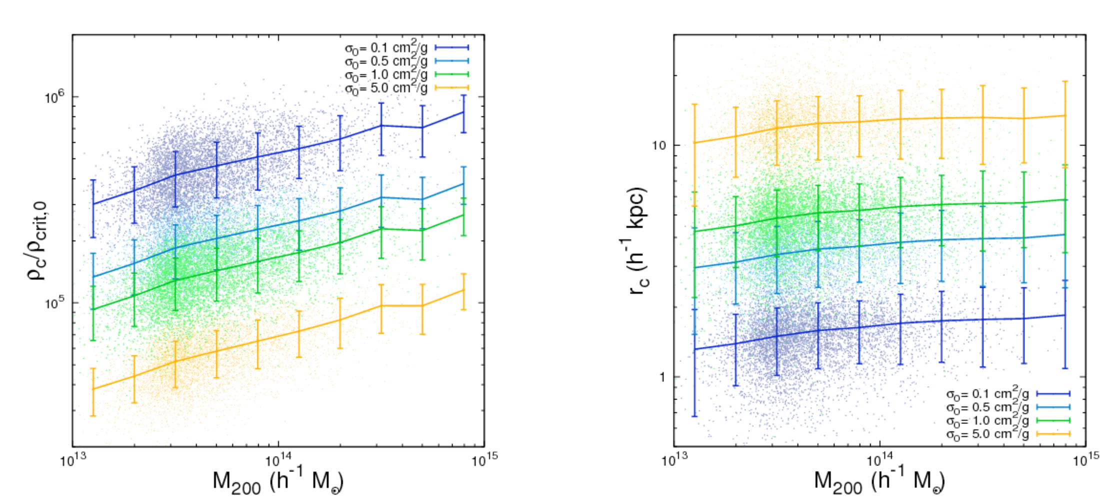


Fig 5. Distribution of SIDM halo parameters - central density ρ_c and core radius, $r_c = r_s\, \rho_0 / \rho_c$, using relaxation ODE model (eq. 2) for ~1000 cluster histories extracted from (600 Mpc)^3 box collisionless *N*-body ΛCDM simulation. Lines & error bars are means and standard deviations for each σ.

- **Eq. 3 can be used to fit observed halo profiles (i.e. X-ray/lensing data) to constrain σ by comparing with these distributions**
- If, e.g., a cluster has cored NFW profile with r_c < ~20 kpc (effective radius of BCG, below which mass is dominated by baryon, e.g. Lewis et. al. 2003) then,
  - Hard sphere cross section **σ<0.1 cm^2/g** (upper right panel)
  - Yukawa-like cross section with σ_0=5 cm^2/g must have **v_c<200 km/s** (lower left)

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