To: Students in AS-551 General Plasma Physics I.

Subject: Additional explanation of problem on last Plasma Physics Homework.

This note provides some additional explanation of the solution to problem 1 on the last homework (#10, due Dec. 15). The written solution that was handed out are fairly good, but had some inaccuracies which should be corrected a little bit.

The best analogy for this problem is to a button with tightly stretched threads going through the button's holes and connecting to two parallel walls (an analogy suggested by one of the students). Twisting the button causes the threads to twist, and there will be a restoring force which will try to untwist the button.

We are to assume that the disk is rotated slowly and by a small angle (thus we can linearize for small changes). In the (z, θ), the magnetic field will be straight lines given by:

$$\theta = \theta_0 + \Delta\theta(1 - |z|/L) \text{for } r < b \tag{1}$$

where $\Delta\theta$ is the angle by which the disk has been rotated, $2L$ is the length of the cylinder, and θ_0 is the initial position in θ of a field line. This is like slowly pulling on a guitar string: the string will form straight lines from your finger to where it is attached to the guitar. Note that for $r > b$, the field lines are unchanged, and are just straight in the z direction. I.e., a field line in the (z, θ) plane is a straight line give by:

$$\theta = \theta_0 \text{ for } r > b \tag{2}$$

In ideal MHD with viscosity and resistivity ignored, such strong perpendicular gradients are perfectly acceptable.

This is all true linearly for small angle rotations. To rigorously satisfy force balance (the disk was rotated slowly, so we assume everything is in equilibrium), we must have some additional small changes in the B field. Neglecting plasma pressure gradients, force balance requires:

$$0 = \frac{\vec{j} \times \vec{B}}{c} = -\nabla \frac{B^2}{8\pi} + \frac{1}{4\pi} \vec{B} \cdot \nabla \vec{B} \tag{3}$$

because B_θ went up for $r < a$ and not for $r > a$, we now have radial gradients of B^2 which must get balanced by other small changes in the magnetic field to all be in equilibrium. However, this is a nonlinear effect, and I was interested only in the linear effects for small angle rotations.

If the disk was rotated rapidly, it would launch shear Alfven waves along the z axis. In order for the fields to always be in quasi-equilibrium, the disk must be rotated slow enough so that $\omega L \gg v_A$, where v_A is the Alfven wave speed.

In the limit as the walls of the cylinder go to infinity, a rotating disk will launch shear Alfven waves which propagate along z and cause more and more of the plasma to spin. This takes angular momentum out of the disk, which will eventually slow down.