Optimal Postseismic GPS Deployment Strategy

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Geodetic observations made after the M7.9 Denali earthquake have the potential to reveal the mechanism(s) of postseismic relaxation at a variety of spatial and temporal scales. Postseismic mechanisms proposed in previous studies of large dominantly strike-slip earthquakes include:
(i) shallow and/or deep afterslip,
(ii) poroelastic rebound, and
(iii) viscoelastic relaxation of the lower crust and upper mantle.

Here we present a simple sensitivity analysis of potential deployment sites in order to ascertain which regions are best suited to detect specific postseismic processes. The method outlined below applies to any postseismic model which can be described with a linear dependence upon perturbations in model parameters with respect to an a-priori model. Results will be presented for the suggested deployment of 3-component GPS sites to detect viscoelastic relaxation beneath the (elastic) upper crust governed by a transient rheology.

Method

Let $\{\hat{\mathbf{r}}_i \mid i=1,\cdots,I\}$ represent the set of 3-component GPS sites under consideration, and let $\mathbf{m}=\{m_j \mid j=1,\cdots,J\}$ be a collection of model parameters. The postseismic displacement field at $\hat{\mathbf{r}}_i$ accumulated between times t_1 and t_2 may be written as $\mathbf{u}(\mathbf{m};\hat{\mathbf{r}}_i;t_1,t_2)$. Assuming that departures of the actual velocity field are linearly dependent upon perturbations in model parameters δm_j from an a-priori model $\mathbf{m}_0$, we have

$$\mathbf{u}(\mathbf{m}_0 + \delta\mathbf{m};\hat{\mathbf{r}}_i;t_1,t_2) = \mathbf{u}(\mathbf{m}_0;\hat{\mathbf{r}}_i;t_1,t_2) + \sum_{j=1}^{J} \frac{\partial \mathbf{u}}{\partial m_j} \Big|_{\mathbf{m}_0} \times \delta m_j \tag{1}$$

If horizontal and vertical GPS observations are used, then there are a total of $N=3\times I$ observations. We wish to fit N postseismic observations at the I GPS sites in terms of a best-fitting model $\mathbf{m}_0 + \delta\mathbf{m}$. This can be solved through weighted least squares inversion for $\delta\mathbf{m}$ based on equation (1) and an assignment of a data covariance matrix $\mathbf{C}$ to the observations.

We assume uncorrelated errors for horizontal and vertical components at all sites. Let $\{\hat{x}_k \mid k=1,2,3\}$ define local Cartesian unit axes at a given location. Define $u_{3i+k} = \mathbf{u}(\mathbf{m};\hat{\mathbf{r}}_i;t_1,t_2)\cdot\hat{x}_k$ and $G_{nj} = \frac{\partial u_n}{\partial m_j}(\sigma_n)^{-1}$, where σ_n is the standard deviation of the nth observation. Let the reduced singular value decomposition of $\mathbf{G}$ be given by

$$\mathbf{G} = \mathbf{U}\,\Lambda\,\mathbf{V}^T \tag{2}$$

where Λ is the $J\times J$ diagonal matrix of singular values and $\mathbf{U}^T\mathbf{U} = \mathbf{V}\mathbf{V}^T = \mathbf{I}$ (the $J\times J$ identity matrix). Then data importances $\{p_n \mid n=1,\cdots,N\}$ are given by

$$p_n = \sum_{j=1}^{J} U_{nj}U_{nj} \tag{3}$$

We adopt the data importances as a measure of the sensitivity of the data to particular parameters m_j or to a combination of these parameters.

In the case of time-dependent postseismic deformation (i.e., viscoelastic relaxation), the data importances will also depend upon the time period (t_1, t_2).

Sensitivity to Transient Rheology

It has long been thought that a transient rheology applies to Earth's mantle (e.g., Weertman, 1978; Peltier et al., 1981; Karato, 1989). Observations supporting this idea are:

- The absorption band model of seismic wave attenuation implies a series of material relaxation components, each corresponding to a very short material relaxation time.
- Creep experiments on rocks at constant applied stress are typically characterized by a phase of transient (or primary) relaxation followed by steady-state relaxation.

Further support for transient rheology is derived from postseismic geodetic observations. The simplest representation of a transient rheology is a Burgher's body, which consists of a dashpot in series with a standard linear solid (Peltier et al., 1981). Postseismic crustal deformation observed after the 1992 Landers and 1999 Hector Mine earthquakes can be interpreted with a Burgher's body for the lower crust (Ivins, 1996) and the upper mantle (Pollitz, 2002), respectively.

Figure 1 summarizes the model of postseismic relaxation derived by Pollitz (2002) for the epicentral region of the 1999 Hector Mine earthquake. It is characterized by a univiscous lower crust (Maxwell fluid) underlain by a biviscous upper mantle (Burgher's body). The material parameters imply two relaxation times ~0.07 and ~2 years for the upper mantle. The effective shear modulus after relaxation of the transient component μ' is one-half the steady state shear modulus, meaning that 50% of the total relaxation is accommodated during the transient relaxation phase alone (Kelvin element), and the remaining 50% by the steady-state relaxation (Maxwell element). We adopt the material parameters of Figure 1 as the a-priori model for the epicentral region of the 2002 Denali earthquake, with the modifications that the upper crust - lower crust boundary is shifted from 15 to 20 km and the crust-mantle boundary is shifted from 30 km to 38 km.

Data importances have been calculated at observation sites situated on a regular grid surrounding the Denali fault rupture. Postseismic deformation for the viscoelastic models has been calculated using an 8-plane approximation to the preliminary slip model presented by researchers at the University of Tokyo (http://wwweic.eri.u-tokyo.ac.jp/EIC/EIC_News/021103AL-e.html). Anticipating that postseismic vertical deformation measurements will contribute to the inference of rheology, composite data importances $p_{\overline{n}}$ are calculated for site #m according to

$$p_{\overline{m}} = \frac{I}{J} \sum_{n=3m+1}^{3m+3} p_n \tag{4}$$

In other words, data importances are summed for the horizontal and vertical components in order to assign a unique data importance to a single hypothetical GPS site. It is assumed that observation errors are identical for all horizontal and vertical observations. Note that the composite data importance defined in (4) has been normalized by the factor I/J; the average data importance over the considered sites $\{\hat{\mathbf{r}}_i \mid i=1,\cdots,I\}$ is then unity.

The distribution of $\bar{p}$ is shown in Figure 2 for various combinations of iverted model parameters for the time period 2 weeks to 1 year, and in Figure 3 for the time period 2 weeks to 3 months. The optimal sites tend to be located either to the side of the fault or are concentrated in a quadrant pattern around the region of maximum slip. Note that quite different patterns are obtained depending on the target parameters. The patterns obtained when transient mantle viscosity η_2 are included in the inversion exhibit a stronger quadrant pattern than those involving only estimation of steady state mantle viscosity η_1. The local maxima in these quadrants are located about 50 km from the fault. The introduction of lower crust viscosity η_c focusses the range of relatively large sensitivity to closer locations about ~20 km from the fault. In all cases, however, no deployment of sites directly on the fault is as effective as deployment several 10s of km away from the fault as far as constraining rheology is concerned.

References

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Figure 1. Viscoelastic structure beneath the 1999 Hector Mine epicentral area determined from continuous and campaign GPS data collected during the first 2.5 years following the earthquake (Pollitz, 2002).

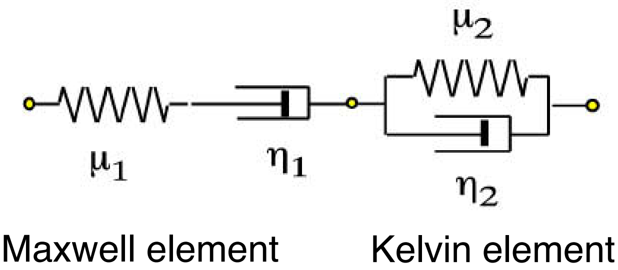
$\eta_1 = 4 \text{ x } 10^{18}$ Pa s

$\square_c = 3\text{x}10^{19}$ Pa s

$\eta_2 = 1.6 \text{ x } 10^{17}$ Pa s

$\mu' = \mu_1 \mu_2 / (\mu_1 + \mu_2) = 0.5$

$\text{-->} \; \mu_1 = \mu_2 = 70$ GPa



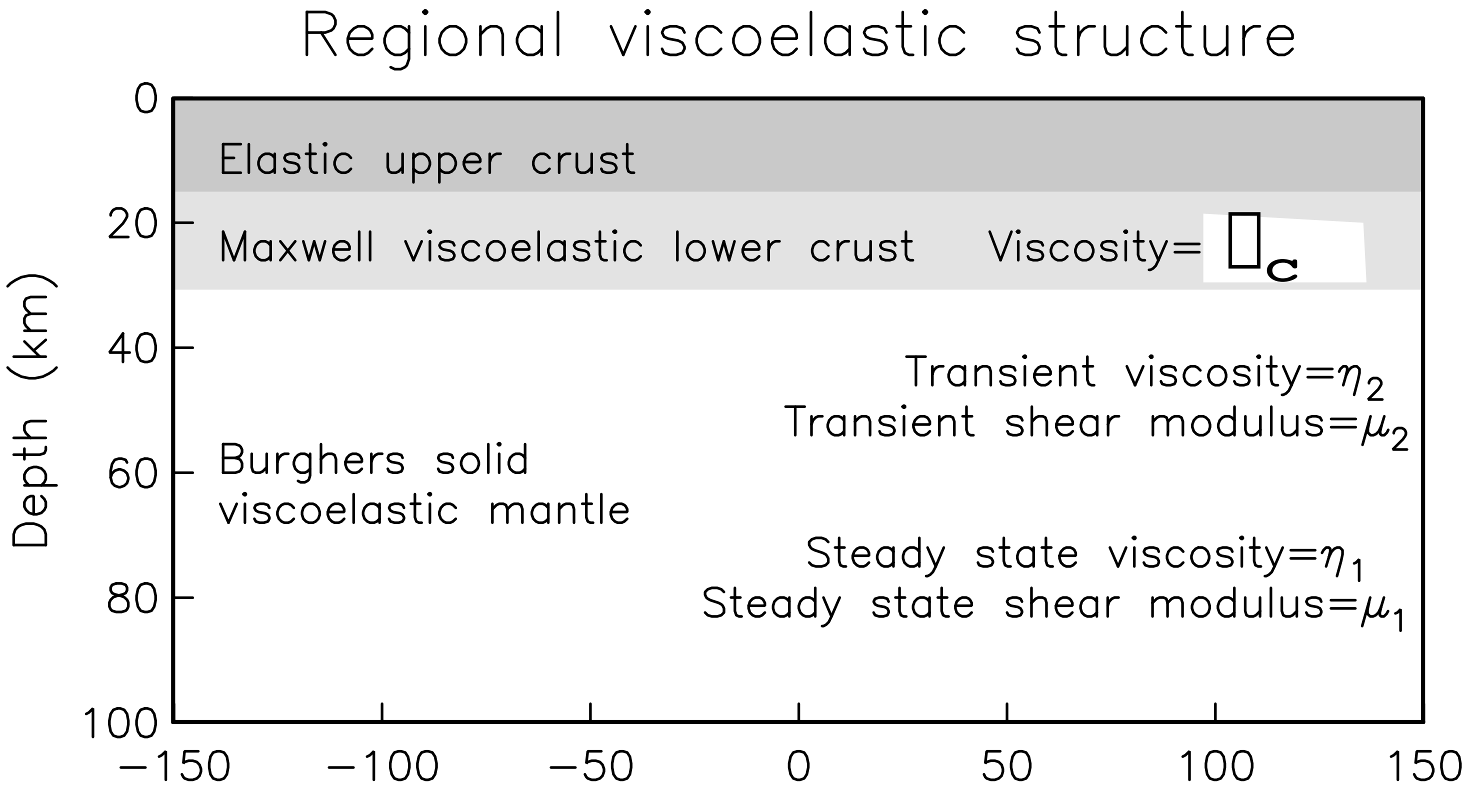


Figure 2. 2 weeks - 1 year sensitivity distribution

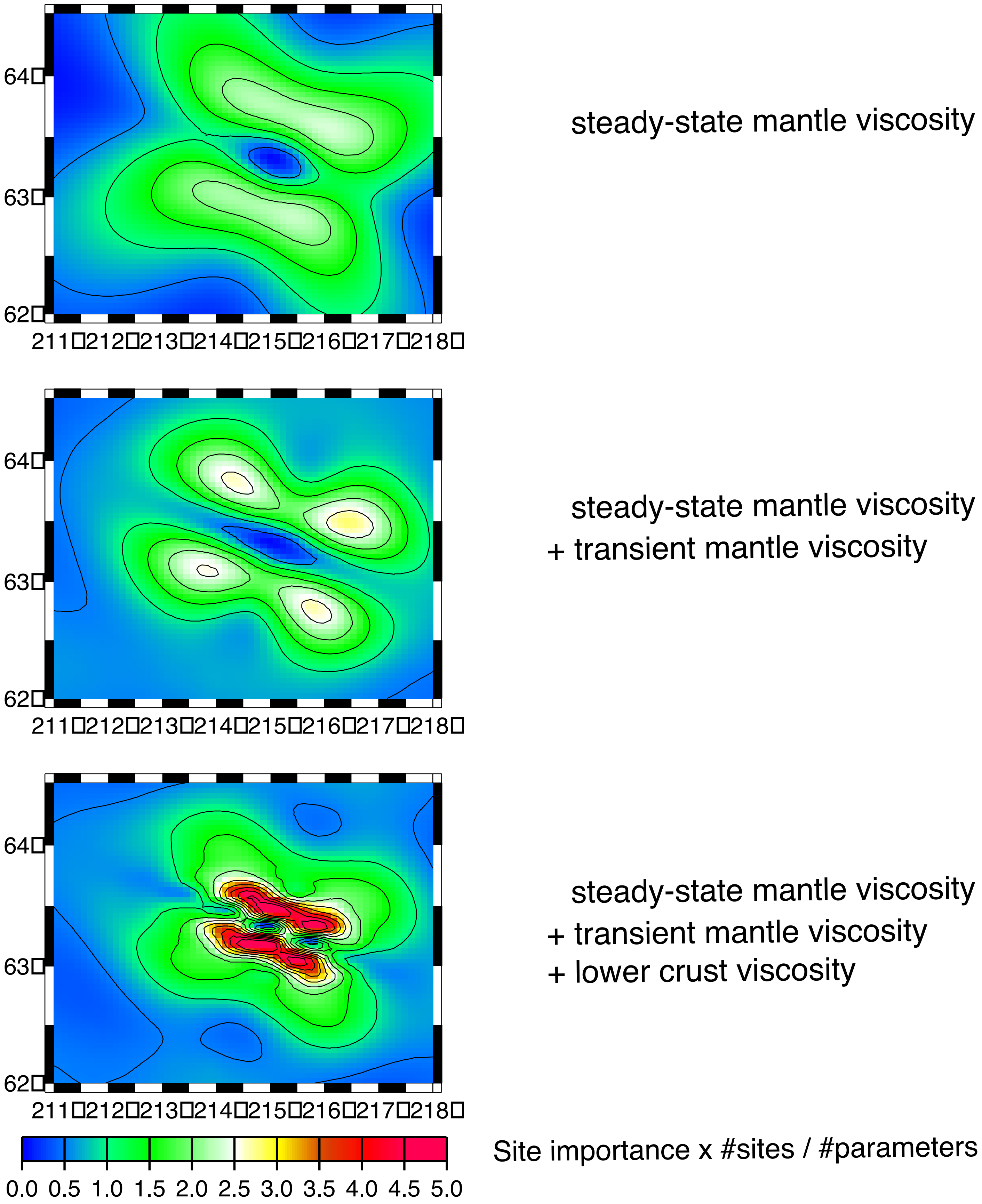


Figure 3. 2 weeks - 3 months sensitivity distribution