Clock Auctions, Proxy Auctions, and Possible Hybrids

Lawrence M. Ausubel*

University of Maryland

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***This is joint research with Peter Cramton and Paul Milgrom. Some of the methods discussed are subject to issued patents or pending applications.**

Introduction

Clock Auction

- Defining characteristic of a clock auction is that the auctioneer names prices, while bidders name only quantities
  - Auctioneer announces a price vector
  - Bidders respond by reporting quantity vectors
  - Price is adjusted according to excess demand
  - Process is repeated until the market clears
- No exposure problem in the clock auction here

Introduction

Proxy Auction

- A particular procedure for package bidding, which exhibits desirable properties
  - Bidders input their values into “proxy agents”
  - The proxy agents iteratively submit package bids, selecting the best profit opportunity according to the inputted values
  - Auctioneer selects provisionally-winning bids according to revenue maximization
  - Process continues until the proxy agents have no new bids to submit

Introduction

Hybrid Clock / Proxy Auction

- A clock auction, followed by a “final round” consisting of a proxy auction
  - Bidders directly submit bids in a simultaneous clock auction phase
  - When the clock phase concludes, bidders have a single opportunity to input proxy values for a proxy phase
  - The proxy phase concludes the auction

Introduction

Hybrid Clock / Proxy Auction

- Rules maintained throughout
  - All bids are kept “live” throughout the auction (i.e., no bid withdrawals)
  - All bids are treated as mutually exclusive (XOR)
  - The bids from the clock phase are also treated as package bids in the proxy phase
  - Activity rules are maintained within the clock phase and between the clock and proxy phases

Introduction

Advantages of Clock-Proxy Auction

- The clock phase is simple for bidders, and provides essential price discovery
- The proxy phase should be expected to yield efficient allocations and competitive revenues, while minimizing the opportunities for collusion

Part I: Clock Auctions

Simultaneous Clock Auctions

Simultaneous Clock Auction

- Practical implementation of the fictitious "Walrasian auctioneer"
  - Auctioneer announces a price vector
  - Bidders respond by reporting quantity vectors
  - Price is adjusted according to excess demand
  - Process is repeated until the market clears

Simultaneous Clock Auctions

Simultaneous Clock Auction

- Strengths
  - Relatively simple for bidders
  - Provides highly-usable price discovery
  - Yields similar outcome as current FCC format, but faster and fewer collusive opportunities
- Weaknesses
  - Limits prices to being linear
  - Therefore should not yield efficient outcomes

Simultaneous Clock Auctions

Issue 1: Treatment of bids which, if accepted, would make aggregate demand < supply

- Example: For a particular item, demand = supply, but the price of a complementary item increases. A bidder wishes to reduce his demand
  - Naive approach: Prevent the reduction
- Example: For a particular item, demand > supply, but two bidders simultaneously attempt to reduce their demands
  - Naive approach: Ration the bidders

Simultaneous Clock Auctions

Issue 1: Treatment of bids which, if accepted, would make aggregate demand < supply

- Example: For a particular item, demand = supply, but the price of a complementary item increases. A bidder wishes to reduce his demand
  - Difficulty: Creates an exposure problem
- Example: For a particular item, demand > supply, but two bidders simultaneously attempt to reduce their demands
  - Difficulty: Creates an exposure problem

Simultaneous Clock Auctions

Issue 1: Treatment of bids which, if accepted, would make aggregate demand < supply

- Example: For a particular item, demand = supply, but the price of a complementary item increases. A bidder wishes to reduce his demand
  - Our approach: Allow the reduction
- Example: For a particular item, demand > supply, but two bidders simultaneously attempt to reduce their demands
  - Our approach: No rationing

Simultaneous Clock Auctions

Issue 1: Treatment of bids which, if accepted, would make aggregate demand < supply

- "Full Flexibility" (used in EDF; advocated here)
  - After each new price vector, bidders can arbitrarily reduce their previous quantities
  - (But the bid remains "live" in the proxy auction phase)
  - Advantage: This effectively makes the clock auction a combinatorial auction. There is no exposure problem!
  - Disadvantage: There may be significant undersell. This is not a big problem, if there are frequent auctions (EDF) or if it is followed by a proxy auction (this talk)

Simultaneous Clock Auctions

Issue 2: Activity rules

- The problem is that of a bidder hiding as a “snake in the grass” until near the end of the auction, to conceal its true interests / values from opponents
- Standard approaches:
  - No activity rule (laboratory experiments)
  - Monotonicity in quantities (clock auctions in practice)
  - Monotonicity in population units (FCC)

Simultaneous Clock Auctions

Issue 2: Activity rules

- Revealed-preference activity rules (advocated here)

- Based on standard analysis in consumer theory. Compare times *s* and *t* ($s < t$). Let associated prices be p^s, p^t and let associated demands be x^s, x^t. Note:

$$v(x^s) - p^s \cdot x^s \geq v(x^t) - p^s \cdot x^t$$

and:

$$v(x^t) - p^t \cdot x^t \geq v(x^s) - p^t \cdot x^s \; .$$

Adding the inequalities yields the RP activity rule:

$$(RPAR) \qquad (p^t - p^s) \cdot (x^t - x^s) \leq 0 \, .$$

Simultaneous Clock Auctions

Issue 2: Activity rules

- Revealed-preference activity rules (advocated here)
- The bid placed by a bidder at time *t* must satisfy the RPAR inequality with respect to its prior bids at all prior times *s* (*s* < *t*):

(*RPAR*) $$(p^t - p^s)\cdot(x^t - x^s) \le 0 .$$

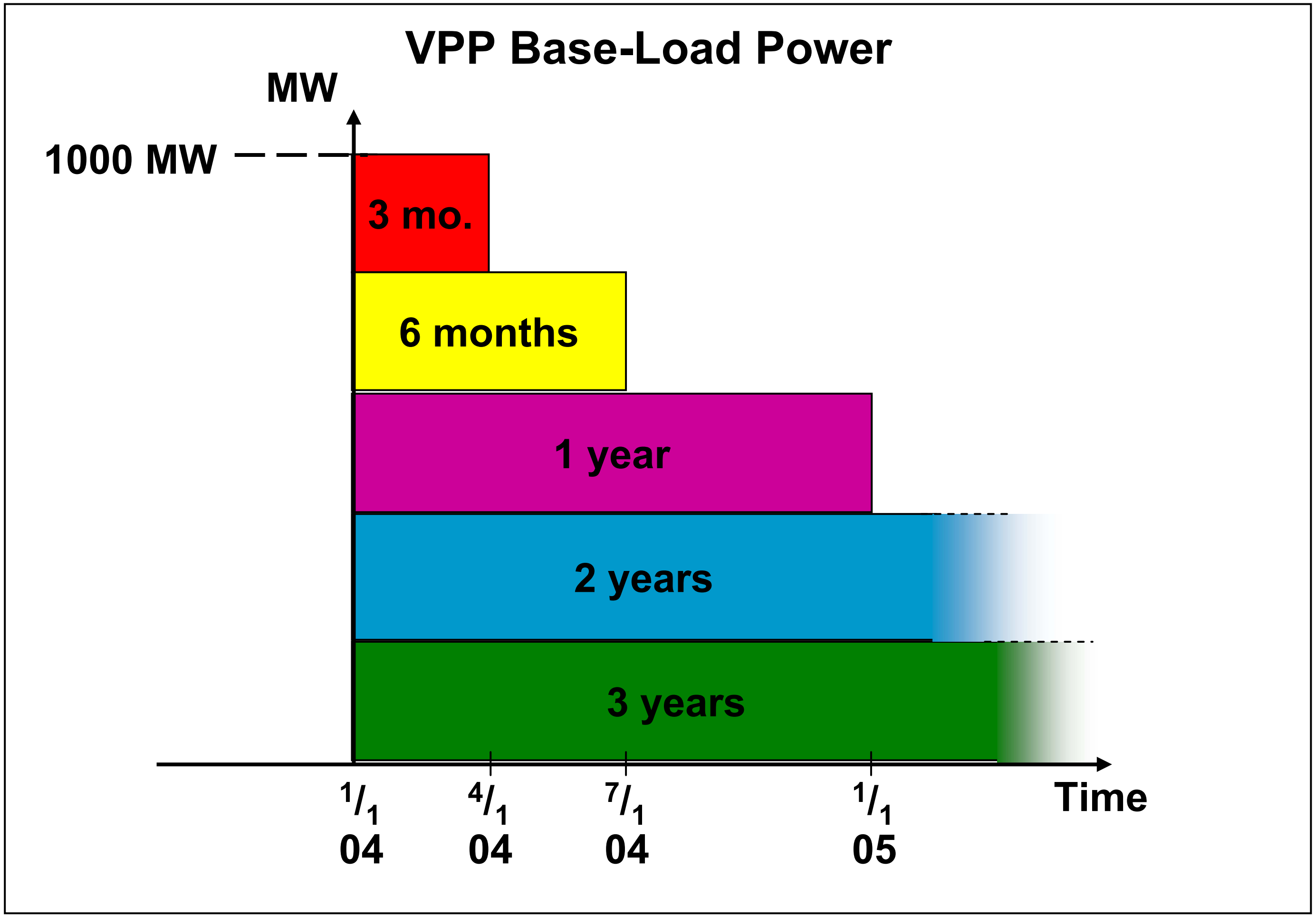
- One can also apply a "relaxed" RPAR in the proxy phase (with respect to bids in the clock phase):

(*Relaxed RPAR*) $$(p^t - p^s)\cdot(x^t - \alpha x^s) \le 0 , \qquad \alpha > 1 .$$

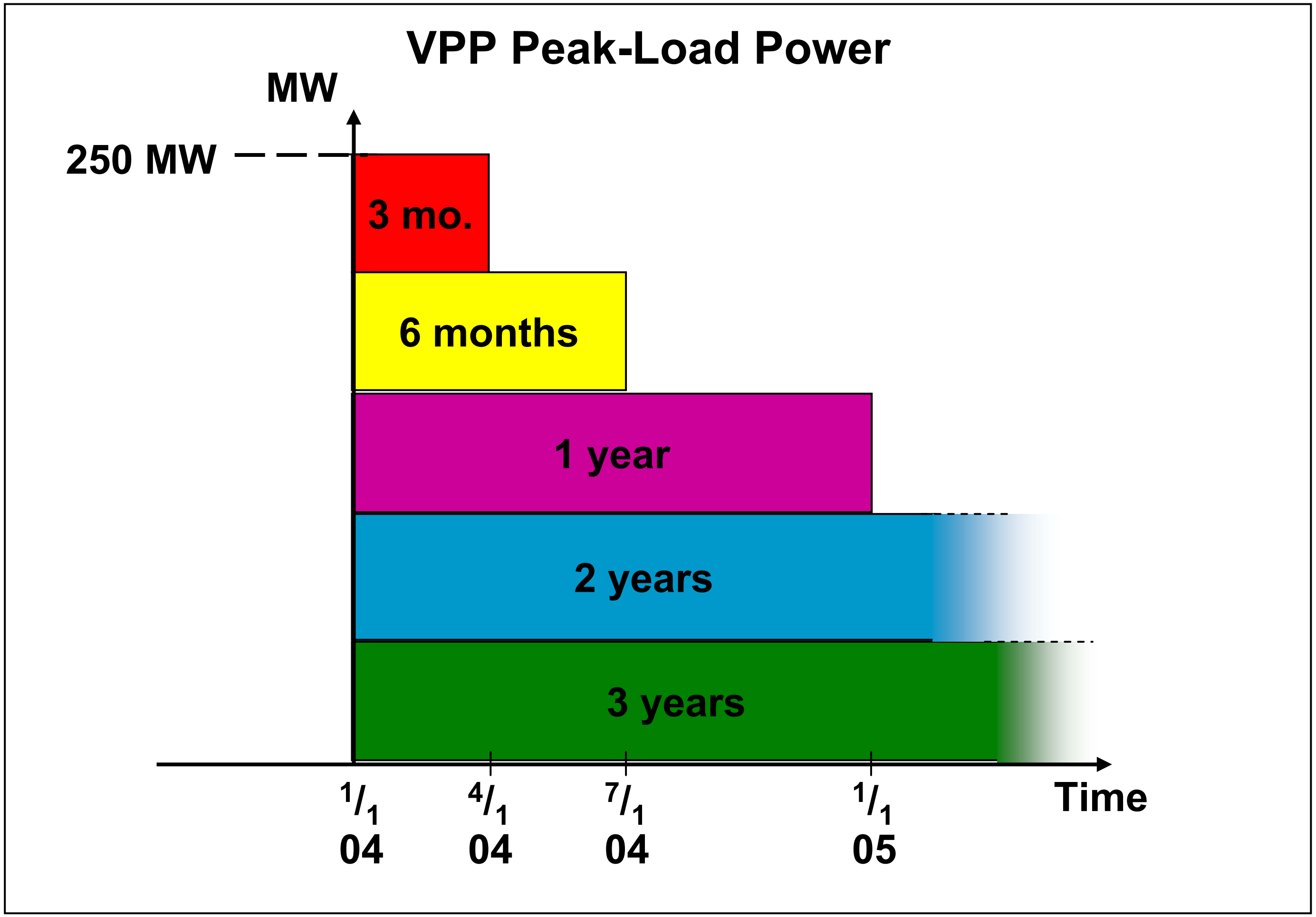
EDF Generation Capacity Auction

MDI
market design inc.

Product Group A



Product Group B



Part II: Proxy Auctions

Package Bidding

- Past FCC auctions (simultaneous ascending auction):
  - Independent bids
  - Approximately-uniform pricing
  - Bidder cannot make bid on B conditional on winning A
- Package bidding often motivated by complements
- Even without complements, package bidding may improve outcome by eliminating “demand reduction”
  - In the traditional FCC auction design, bidders have incentive to reduce their bids on marginal units in order to reduce their payments for inframarginal units

Basic Ascending Package Auction

- A set of items is offered for sale
- A bid (A,b_{jA}) by bidder *j* specifies a set of items *A* and a corresponding *bid amount*
- Bidding proceeds in a series of rounds
- After each round, *provisional winning bids* — a solution to the problem of maximizing revenues from compatible bids — are determined
- Auction ends after a round with no new bids
- All bids are treated as mutually exclusive (XOR)
- All bids are kept "live" throughout the auction

Ascending Proxy Auction

- Each bidder reports his values (and, in one version, a budget limit) to a "proxy bidder"
- The proxy bidder bids on behalf of the real bidder — iteratively submitting the allowable bid that, if accepted, would maximize the real bidder's payoff (evaluated according to his reported values)
- An ascending package auction is conducted with negligibly small bid increments
- Bidders may or may not have the opportunity to revise the values reported to their proxy agents
- Auction ends after a round with no new bids (and no further opportunities to revise values to proxy agents)

Example: Ascending Proxy Auction

- Two items, A and B; bids must be integers
- Bidder reports values of v(A) = 10, v(B) = 5, v(A,B) = 20
- Past high bids by this bidder (all “losing”) were:
  - b(A) = 4, b(B) = 3, b(A,B) = 15
- Next allowable bids are:
  - b(A) = 5 Yields profits of π = v(A) – b(A) = 10 – 5 = 5
  - b(B) = 4 Yields profits of π = v(B) – b(B) = 5 – 4 = 1
  - b(A,B) = 16 Yields profits of π = v(A,B) – b(A,B) = 20 – 16 = 4
- So the proxy bidder next places a bid of 5 on A

Example: Ascending Proxy Auction

- Two items, A and B; bids must be integers
- Bidder reports values of v(A) = 10, v(B) = 5, v(A,B) = 20
- Past high bids by this bidder (all "losing") were:
  - b(A) = 4, b(B) = 3, b(A,B) = 15
- Next allowable bids are:
  - b(A) = 5 Yields profits of π = v(A) – b(A) = 10 – 5 = 5
  - b(B) = 4 Yields profits of π = v(B) – b(B) = 5 – 4 = 1
  - b(A,B) = 16 Yields profits of π = v(A,B) – b(A,B) = 20 – 16 = 4
- Next allowable bids after that are:
  - b(A) = 6 Yields profits of π = v(A) – b(A) = 10 – 6 = 4
  - b(B) = 4 Yields profits of π = v(B) – b(B) = 5 – 4 = 1
  - b(A,B) = 16 Yields profits of π = v(A,B) – b(A,B) = 20 – 16 = 4
- So the proxy next bids 6 on A and/or 16 on {A,B}

Outcomes in the Core

- **The coalitional form game is (*L*,*w*), where…**
- **L denotes the set of players.**
  - **the seller is *l* = 0**
  - **the other players are the bidders**
- ***w*(*S*) denotes the value of coalition *S*:**
  - **If S excludes the seller, let *w*(*S*)=0**
  - **If S includes the seller, let**

$$w(S) = \max_{x \in X} \sum_{l \in S} v_l(x_l)$$

- **The *Core*(*L*,*w*) is the set of all profit allocations that are *feasible* for the coalition of the whole and *cannot be blocked* by any coalition *S***

Outcomes in the Core

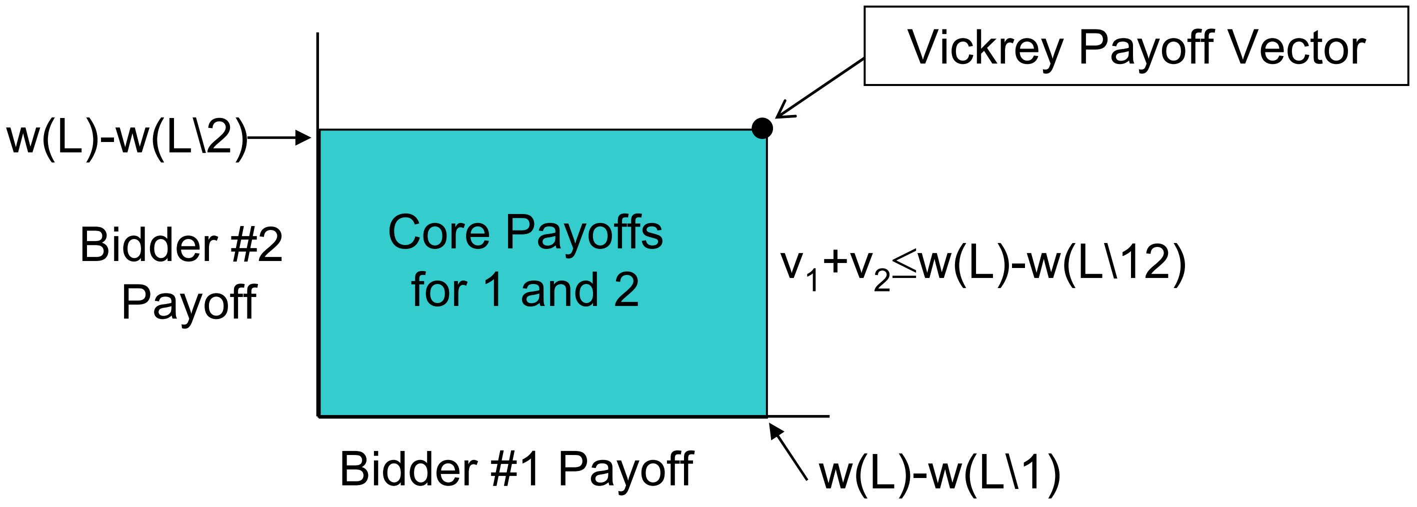
<u>Theorem (Ausubel and Milgrom, 2002)</u>. The outcome of the ascending proxy auction is a point in *Core*(*L*,*w*) relative to the reported preferences

Interpretations:

- "Core" outcome assures competitive revenues for the seller
- "Core" outcome also assures allocative efficiency, i.e., the ascending proxy auction is not subject to the inefficiency of demand reduction

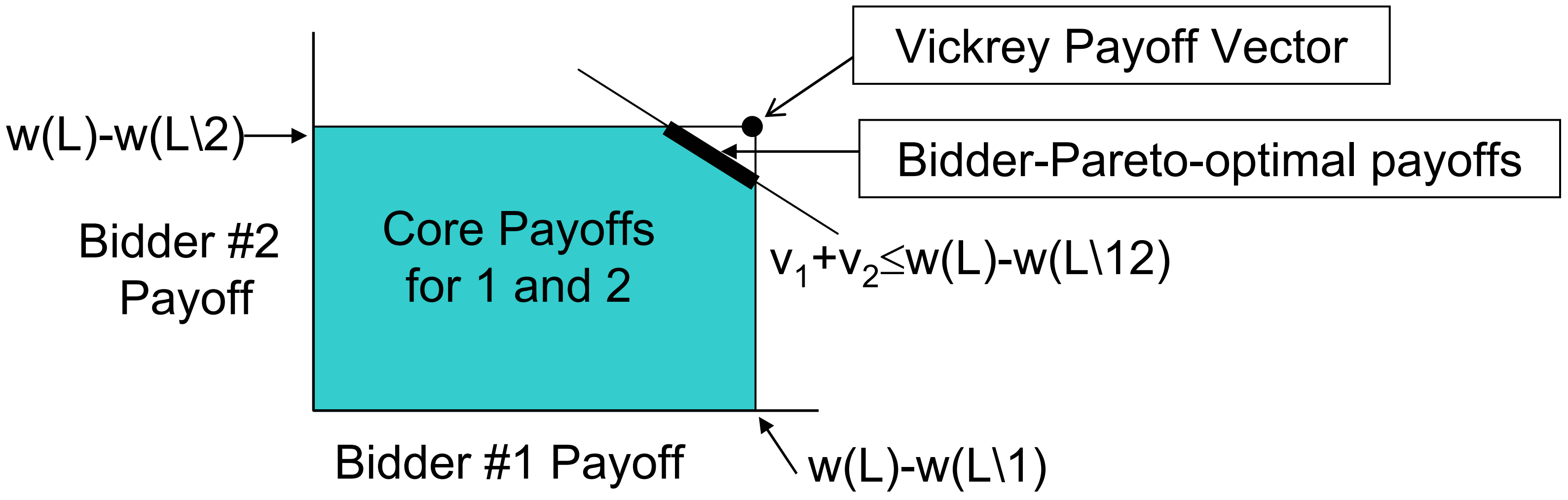
Case of Substitutes

- **If the goods are substitutes, then the Vickrey payoff profile is the bidder-Pareto-optimal point in the core, and the outcome of the ascending proxy auction coincides with the outcome of the Vickrey auction**



Case of Non-Substitutes

- **Meanwhile, if the goods are not substitutes, then the Vickrey payoff profile is not an element of the core and the ascending proxy auction yields a different outcome from the Vickrey auction (one with higher revenues)**



Outcomes in the Core

Theorem (Ausubel and Milgrom, 2002). If π is a bidder-Pareto-optimal point in *Core*(*L*,*w*), then there exists a Nash equilibrium of the ascending proxy auction with associated payoff vector π.

Note 1: This is a complete-information result

Note 2: These equilibria may be obtained using strategies of the form: bid your true value minus a nonnegative constant on every package

Monotonicity and Revenue Issues

- Example: Two identical items, A and B; three bidders
  - Bidder 1 values the pair only: $v_1(A,B)$ = $2 billion
  - Bidder 2 wants a single item only: $v_2(A)$ = $2 billion
  - Bidder 3 wants a single item only: $v_3(B)$ = $2 billion
- The Vickrey auction awards each bidder his incremental value:
  - Bidders 2 and 3 each win one item
  - Social value with Bidder 2 = $4 billion; without Bidder 2 = $2 billion
  - Prices in the Vickrey auction equal zero!
- The problem in this example is a failure of monotonicity:
  - Adding Bidder 3 reduces Vickrey revenues from $2 billion to zero
  - The Vickrey outcome lies outside the core
- The proxy auction avoids this problem: Revenues = $2 billion

The Loser Collusion Problem

- **Example: Two identical items, A and B; three bidders**
  - **Bidder 1 values the pair only: $v_1(A,B)$ = \$2 billion**
  - **Bidder 2 wants a single item only: $v_2(A)$ = \$0.5 billion**
  - **Bidder 3 wants a single item only: $v_3(B)$ = \$0.5 billion**
- **The losing Bidders 2 and 3 have a profitable joint deviation in the Vickrey auction: bidding \$2 billion each**
  - **This converts it into the previous example**
  - **Bidders 2 and 3 each win one item at prices of zero**
  - **The Vickrey auction is unique in its vulnerability to collusion even among *losing* bidders**
- **The proxy auction avoids this problem: Bidders 2 and 3 can overturn the outcome of Bidder 1 winning only by jointly bidding \$2 billion**

The Shill Bidding Problem

- Example: Two identical items, A and B; two bidders
  - Bidder 1 values the pair only: $v_1(A,B)$ = $2 billion
  - Bidder 2 has $v_2(A)$ = $0.5 billion; $v_2(A,B)$ = $1 billion
- The losing Bidder 2 can set up a bidder under a false name ("shill bidder"). Each of Bidder 2 and the shill Bidder 3 can bid $2 billion each
  - This again converts it into the first example
  - Bidder 2 wins two items and pays zero!
- The Vickrey auction is vulnerable to shill bidding

Part III: Hybrid Auctions

Clock-Proxy Auction

- A simultaneous clock auction is conducted, with a revealed-preference activity rule imposed on bidders, until (approximate) clearing is attained
- A proxy auction is conducted as a "final round".
  - Bids submitted by proxy agents are restricted to satisfy a (relaxed) revealed-preference activity rule ($\alpha > 1$) relative to all bids submitted in the clock phase. The value of α is chosen based on competitive conditions
  - The bids from the clock phase are also treated as "live" package bids in the proxy phase
  - All package bids (clock and proxy) are treated as mutually exclusive, and the auctioneer selects as provisionally-winning the bids that maximize revenues

Why Not Use the Proxy Auction Only?

- Clock auction phase yields price discovery
- The feedback of linear prices is extremely useful to bidders
- The existence of the clock phase makes bidding in the proxy phase vastly simpler
  - Focus decision on what is relevant
  - See what you don't need to consider
  - See what looks like good possibilities

Why Not Use the Clock Auction Only?

- Proxy auction ends with core outcome
  - Efficient allocation
  - Competitive revenues
- No demand reduction
- Collusion is limited
  - Relaxed activity rule means allocation still up for grabs in proxy phase

Advantages of the Clock over the SAA

- The clock auction is a fast and simple process (compared to the simultaneous ascending auction)
  - Only provide information relevant for price and quantity discovery (excess demand)
  - Takes advantage of substitutes (one clock for substitute licenses)
  - Example:
    - proposed 90 MHz of 3G spectrum in 5 blocks: 30, 20, 20, 10, 10
    - clock alternative: 9 or 18 equivalent blocks per region
  - Fewer rounds
    - Get increment increase for all items, rather than having to cycle through over many rounds
    - “Intra-round bids” allow larger increments, but still permit expression of demands along line segment from start-of-round price to end-of-round price

Advantages of the Clock over the SAA

- Clock auction limits collusion (compared to the simultaneous ascending auction)
  - Signaling how to split up the licenses greatly limited
    - No retaliation (since no bidder-specific information)
    - No stopping when obvious split is reached (since no bidder specific information)
  - Fewer rounds to coordinate on a split

Advantages of the Clock Phase

- No exposure problem (unlike SAA)
  - As long as at least one price increases, bidder can drop quantity on other items
  - Bidder can safely bid for synergistic gains
  - Bid is binding only as full package
- No threshold problem (unlike SAA with package bids)
  - Clocks controlled by auctioneer: no jump bids; large bidder cannot get ahead
  - Linear pricing: small bidders just need to meet price on single item

Hybrid Clock/Proxy Auction

- Combines advantages of
  - Clock auction
  - Proxy auction
- Excellent price discovery in clock phase simplifies bidder decision problem
- Proxy phase enables bidders to fine-tune allocation based on good price information