Ascending Auctions with Package Bidding

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Package Bidding

- Past FCC auctions:
  - Independent bids
  - Approximately-uniform pricing
  - Bidder cannot make bid on B conditional on winning A
- Package bidding may:
  - Reduce demand reduction
  - Solve the problems of complementarity
    - “exposure” problem: risks in bidding
    - “pricing” problem: non-existence of CE prices

Package Auctions:
Some Formats

Vickrey Auction

- A.k.a "pivot mechanism" or "VCG mechanism"
- One or more goods of one or more kinds
- Each bidder *i* makes bids $b_i(x)$ on all bundles
- Auctioneer chooses the feasible allocation $x^* \in X$ that maximizes the total bid accepted
  - X can incorporate policy rules
- Vickrey ("pivot") payments for each bidder *i* are:

$$p_i = \max_x \sum_{j \neq i} b_j(x_j) - \sum_{j \neq i} b_j(x_j^*)$$

subject to $x \in X$

Basic Ascending Package Auction

- A set of items is offered for sale
- A bid (A, b_{jA}) by bidder j specifies a set of items A and a corresponding *bid amount*.
- Bidding proceeds in a series of rounds
- Auction ends after no new bids
  - Bids are all mutually exclusive and all are retained
- By contrast, in FCC Auction 31 design:
  - Bids are only mutually-exclusive between rounds
  - Only some bids are retained

Ascending Proxy Auction

- A (Multi-Stage) Direct Revelation Game
  - Each bidder reports a valuation function (and budget limit) to a "proxy agent"
- The proxy… (with one stage only)
  - *makes no new bid when the proxy has a provisional winner*
  - calculates the "potential profit"—what each bid would earn if it wins
  - makes the feasible, acceptable bid with the highest potential profit
- Dual purpose of model
  - Possibly models behavior "late" in experiments
  - May be a practical design because it…
    - eliminates certain retaliatory strategies
    - runs quickly compared to multi-round auctions
    - is adaptable to a multi-round version

Bases of Evaluation

- Mechanism Performance: Theory/Lab/Real-World
  - Ease of bidding
  - Efficiency
  - Revenues
  - Resistance to collusion
- Robustness to Various Conditions
  - Value conditions
    - Substitutes only, no budget constraints
    - Some complements or budget constraints
  - Information conditions
    - Complete and incomplete information
    - Private and common value elements
  - Competitive conditions

Evaluating Revenues

Looking Ahead: Vickrey is distinguished from the ascending proxy auction <u>only</u> by its handling of the “threshold problem”—a source of inefficiency.

A Competitive "Benchmark"

- Consider a "cohesive" TU game (N,w).
- Construct an economy in which brokers bid for the players' services.
- A competitive equilibrium is a price vector and allocation such that
  - No positive profit opportunities: $w(S) \leq \sum_{I \in S} \pi_I$
  - No losses incurred: $w(N) = \sum_{I \in N} \pi_I$
- Proposition: A value allocation π is in the core if and only if π is a competitive equilibrium price vector.
  - So, the core identifies "competitive" pricing.

Vickrey Auction Payoffs

- Theorem. Each bidder's Vickrey payoff is $v_i = w(N) - w(N \backslash i) = \max\{\pi_i \mid \pi \in Core(N,w)\}$.
- Proof.

$$v_i = b(x_i^*) - p_i = b(x_i^*) - \left(\max_x \sum_{j \neq i} b_j(x_j) - \sum_{j \neq i} b_j(x_j^*)\right)$$

$$= w(N) - w(N \setminus i)$$

- If $\pi_i > v_i$ and π is feasible, then coalition $N \setminus i$ gets $w(N) - \pi_i < w(N \setminus i)$, so π is not in the core. For the converse, observe that the profile in which i gets v_i; the seller gets $w(N) - v_i$; and others get zero is in the core. **QED**

Vickrey and Submodularity

- Definition. The coalitional value function w is *submodular for bidders* if for all coalitions S and T,

$$0 \in S \cap T \Rightarrow w(S) + w(T) \geq w(S \cap T) + w(S \cup T)$$

- Theorem. The following statements are equivalent:
  - The coalitional value function is submodular for bidders.
  - For every coalition S, there is a unique point in Core(S,w) that is Pareto best for the bidders.
  - For every coalition S, the corresponding Vickrey payoff vector is in $Core(S,w)$, that is,

$$v_i^S = w(S) - w(S \setminus i)$$

$$v^S \in Core(S,w)$$

Proof

- Suppose the value function is submodular, let 0 denote the seller; $S_n = \{0,1,\ldots,n\}$, and $S=S_k$. Then,

$$\sum_{j=0}^{n} v_j^S = w(S) - \sum_{j=n+1}^{k} v_j^S = w(S) - \sum_{j=n+1}^{k} \big(w(S) - w(S \setminus j)\big)$$

$$\geq w(S) - \sum_{j=n+1}^{k} \big(w(S_j) - w(S_{j-1})\big) = w(S_n)$$

But the ordering of players was arbitrary…

- Conversely, if *w* is not submodular, then for some S and i,j$\in$S,

$$w(S \setminus i) - w(S \setminus ij) < w(S) - w(S \setminus j)$$

$$\therefore \sum_{k \in S \setminus ij} v_k^S = w(S) - \left(v_i^S + v_j^S\right) =$$

$$w(S) - \Big[\big(w(S) - w(S \setminus j)\big) + \big(w(S) - w(S \setminus i)\big)\Big] < w(S \setminus ij)$$

so $S \setminus ij$ blocks the Vickrey allocation. **QED**

Substitutes

- Suppose bidder preferences are quasi-linear. Let
  - **P** = set of possible bidder valuations.
  - $\mathbf{P}_{sub}$ = set of valuations for which goods are substitutes
  - $\mathbf{P}_{add}$ = set of additive valuations
- Theorem. Suppose that there are at least 3 bidders and $\mathbf{P}_{add} \subset \mathbf{P}$. Then the following four are equivalent:
  - $\mathbf{P} \subset \mathbf{P}_{sub}$
  - For every profile of bidder valuations drawn from $\mathbf{P}^{N \setminus 0}$, w is submodular for bidders.
  - For every profile…, $v \in Core(N,w)$.
  - For very profile…, competitive equilibrium goods prices exist.

Ascending Proxy Auctions as Deferred Acceptance Algorithms

- Simplifying assumptions
  - Negligibly small bid increments
  - Pre-determined tie-breaking rule

Deferred Acceptance Algorithms

- Marriage problem: the Gale-Shapley algorithm
  - Process involves deferred acceptance
  - Outcome is a "stable match," and best such match for the side that makes the offers.
  - Truthful reporting is a dominant strategy for offering side
- English auctions
  - Process involves deferred acceptance
  - Outcome is in the core of the economy: best point for bidders
  - Truthful reporting to proxy bidder is a dominant strategy
- Others
  - Medical resident matching program
  - Kelso-Crawford labor markets model
  - **Ascending proxy auction (*<u>even w/ complementarities</u>*!)**

Package Auction as DAA

- Is the package auction a DAA?
  - Process involves deferred acceptance
    - » But offers may not be made in order of preference
    - » Introduce straightforward bidding to guide the analysis
  - Is outcome in core of the economy?
    - » Yes!
    - » Core point is one at which the seller's revenue is minimized.
  - Is truthful bidding a dominant strategy?
    - » Yes, if goods are substitutes and offers are restricted. Else, still Nash equilibrium strategies.
  - Do bidders in experiments bid "straightforwardly" or to the core?
    - » Should be investigated

Truthful Outcomes

- Theorem. If each bidder reports truthfully to its proxy and treating bid increments as negligible, the outcome of truthful reporting is a point in Core(N,w) that is not Pareto-dominated for bidders by any other point in Core(N,w).
- Notes:
  - Unique among deferred acceptance algorithms because it uses no "substitutes" condition. The single seller replaces the substitutes condition in the formal arguments.
  - Not yet an equilibrium result, so not yet to be applied to the revenue issue.

Proof

- The insight: Follow the progress of the algorithm in utility/payoff space.
  - At round t, each bidder makes all package bids with potential profit of at least π_i^t.
  - At each round, π^t is unblocked.
  - Auction ends when π^t is feasible: all "losing bidders" have zero profits.
- Seller's revenue at round t is given by:

$$\begin{aligned}
\pi_0^t &= \max_{x\in X} \sum\nolimits_{l\neq 0} B_l^t(x_l) \\
&= \max_{x\in X} \sum\nolimits_{l\neq 0} \max\left(0, f_l(x_l) - \pi_l^t\right) \\
&= \max_{S\subset L} \left[\max_{x\in X} \sum\nolimits_{l\in S\backslash 0} f_l(x_l) - \pi_l^t \right] \\
&= \max_{S\subset L} \left[w(S) - \sum\nolimits_{l\in S\backslash 0} \pi_l^t \right] \\
&\therefore (\forall S)\, w(S) \leq \sum\nolimits_{l\in S} \pi_l^t
\end{aligned}$$

- *The argument generalizes to accommodate budget constraints using NTU core.*

Truthful Equilibria

- Theorem. The following statements are equivalent:
  - Truthful reporting is an ex post Nash equilibrium of the ascending proxy auction (and leads to Vickrey outcomes)
  - The Vickrey outcome satisfies $v \in \text{Core}(N,w)$,
- When $v \notin \text{Core}(N,w)$, the ascending proxy auction contains an implicit demand-bargaining protocol among bidders over points in $\text{Core}(N,w)$.

General Valuations

- Theorem. Let π be a Pareto-undominated point for the bidders in *Core*(*N*,*w*). Then there is a Nash equilibrium in which each bidder i with actual package values $p_i(\cdot)$ reports to its proxy that its values are $p_i(\cdot)-\pi_i$.
- Observations about this equilibrium.
  - Corresponds to Roth's observations about equilibrium in matching models.
  - Corresponds to Bernheim-Whinston bidding strategies in their "menu auction."
  - Selected as an "undiscouraged bidder equilibrium."
  - "Coalition-proof" provided undiscouraged bidder condition is consistently applied.

Technology Neutrality

- Suppose that the values are as follows.

Bidder	East	West	Package
1	40	0	40
2	0	40	40
3	0	0	50

- By merging and coordinating technologies, bidders 1 and 2 can create a package of value 100.
  - … but they will still find the merger unprofitable.

Budget Constraints

- Consider the problem of bidder #1 in a Vickrey auction with 2 items for sale when player #3's participation is uncertain.
  - Should #1 bid (5,30) or (25,30) below?

Bidder	1 item	2 items	Budget
1	25	50	30
2	20	25	30
3	30	30	30

  - If #3 participates, then #1 "should" to express a marginal value of at least 20 for one items.
  - Otherwise, #1 "should" express a marginal value of at least 20 for the second item.

Lessons for Auction Practice

- Bids are mutually exclusive
  - Richer language
  - Enables core outcome results
- Mandatory proxy intermediation
  - Quite useful in package bidding auctions
  - May also be useful in other auction formats
- Bid improvement rules
  - Relatively aggressive bid improvement rules are consistent with obtaining core outcomes
- Revealed-preference activity rules
  - $(p^{t'}-p^{t})\cdot(x^{t'}-x^{t}) \leq 0$

The End