ANALYSIS OF TWO-PHASE FLOW MODELS WITH TWO MOMENTUM EQUATIONS

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Abstract – *An analysis of the standard system of differential equations describing multi-speed flows of multi-phase media is performed. It is proved that the Cauchy problem, as posed in most best-estimate thermal-hydraulic codes, results in unstable solutions and potentially unreliable description of many physical phenomena. A system of equations, free from instability effects, is developed allowing more rigorous numerical modeling.*

At present, mathematical (numerical) modeling of physical processes, standard for different nuclear power plant (NPP) operational modes is an effective tool for safety analysis. The description of the thermal-dynamic behavior of two-phase steam-liquid flow in NPP primary circulation loops is known to be one of the most difficult and important tasks. In modeling two-phase steam-liquid flows, it is important to correctly describe the relative motion of the steam and liquid phases. In general, two approaches are used for this purpose: the first one is based on the drift model, where the relative velocity of the phases is defined from the empirical correlations, while the second approach involves two separate momentum equations for each phase.

It is worth saying that the modern best estimate thermal-hydraulic codes, such as RELAP-5/MOD3[1], CATHARE[2], and KORSAR[3] use two momentum equations in the framework of the standard system of multi-phase medium mechanics equations[4]. However, it is well known[5] that for certain choice of inter-phase interaction this form of system-equations could potentially break down: (1) it may result in non-hyperbolic type of system-equations; (2) the solutions could become unstable; (3) may pose an incorrect Cauchy problem.

As such, the systems of equations require preliminary mathematical analysis; otherwise the reliability (even at the qualitative level) of the numerical solutions may be in doubt. Indeed, due to the difficulties in measuring the physical thermal-hydraulic parameters of NPP's with sufficient accuracy, verification of the constructed mathematical models usually was performed using data based on scaled-model experiments. Even for the largest scale-models, the NPP/experiment ratio of thermal powers and the corresponding volumetric ratio are generally of the order of 10^2.

The thermal-hydraulic models use auxiliary fitting parameters to achieve a rather good qualitative and quantitative description of the selected experimental data. Whether these results are correct and can be extrapolated to the behavior of full-scale reactors is not at all obvious[6]. In order to extrapolate the results, it is important to insure that a self-consistent mathematical system of equations is used, which is generally not performed for the codes used for modeling operating power plants.

In the present paper we propose an analysis for a general system of equations that describes multi-velocity flows and find an admissible class of inter-phase interactions, which provide the correct description of the Cauchy problem and insure the stability of the solution.

It is shown in Ref. 5 that a non-hyperbolic system of equation describing two-velocity mixture flows with a well-posed Cauchy problem is independent on phase compressibility, and, hence, can be analyzed as a non-compressible mediums resulting in a simpler model.

In the framework of a multi-phase mixture flow model[4], the one-dimensional flow of a mixture containing n non-compressible phases is described by the following system of equations:

Mass conservation equation,

$$\frac{\partial}{\partial t}(\alpha_i \rho_i^\circ) + \frac{\partial}{\partial x}(\alpha_i \rho_i^\circ v_i) = 0, \quad (i = 1,..., n), \qquad (1)$$

where ρ_i°, α_i, and v_i are, respectively, the real density, the volume concentration and the velocity of the i-th phase;

Momentum conservation equation,

$$\frac{\partial}{\partial t}(\alpha_i \rho_i^\circ v_i) + \frac{\partial}{\partial x}(\alpha_i \rho_i^\circ v_i^2) + \alpha_i \frac{\partial p}{\partial x} = F_i + F_{mi}, \quad (i = 1,..., n), \tag{2}$$

and the identity

$$\sum_{i=1}^{n} \alpha_i = 1 . \tag{3}$$

In equations (2) F_i and F_{mi} are respectively the inter-phase and mass forces acting on the i-th phase. As will be seen below, it is very important to take into account the dependence of the inter-phase forces F_i on the i-th phase, which are in the form of spatial derivatives of thermal-hydraulic parameters.

Following Ref. 5, we restrict the model to the case when the inter-phase forces F_i depend linearly on these derivatives:

$$F_i = -D_{1i} \bullet \frac{\partial \alpha_i}{\partial x} - D_{2i} \bullet \frac{\partial v_i}{\partial x} + Fb_i\,(v_1,....v_n\,), \tag{4}$$

where D_{1i} and D_{2i} are certain coefficients, and $Fb_i(v_1,.....v_n)$ is a part of the inter-phase interaction, which describes the phase friction and does not contain the derivatives. Next, we will show, that the coefficients D_{1i} and D_{2i} in equation (4) are not arbitrary. Indeed, the sum of the equations (2) for all i's gives a full momentum conservation equation of the mixture, which does not contain the inter-phase interaction forces, i.e., the following condition shall be valid:

$$\sum_{i=1}^{n} D_{1i} \frac{\partial \alpha_i}{\partial x} + \sum_{i=1}^{n} D_{2i} \frac{\partial v_i}{\partial x} = 0 . \tag{5}$$

Since equation (5) holds for any distributions of α_i and v_i, and the coefficients D_{1i} and D_{2i} do not depend on the derivatives, we arrive at a rather strict limitation for the explicit forms of D_{1i} and D_{2i}. Adding equations (2) for non-compressible phases ($\rho_i^\circ = const_i$), we obtain

$$\sum_{i=1}^{n} v_i \frac{\partial \alpha_i}{\partial x} + \sum_{i=1}^{n} \alpha_i \frac{\partial v_i}{\partial x} = 0, \tag{6}$$

while from (3) it follows that

$$\sum_{i=1}^{n} \frac{\partial \alpha_i}{\partial x} = 0 . \tag{7}$$

If all $2n$ quantities $\frac{\partial \alpha_i}{\partial x}$ and $\frac{\partial v_i}{\partial x}$ were completely independent, we would obtain from (5) $2n$ identities $D_{1i} = D_{2i} = 0$. However, Equations (6) and (7) allow to express two quantities, $\frac{\partial \alpha_n}{\partial x}$ and $\frac{\partial v_n}{\partial x}$ through the other $2n-2$ ones. As a result, equation (5) provides $2n-2$ restrictions imposed on $2n$ functions D_{1i}, D_{2i}. Hence, we can parameterize the latter by means of two independent functions. After some algebra it is easy to derive the following parameterization:

$$D_{1i} = D_1 + v_i D_2\,, \text{ and } D_{2i} = \alpha_i D_2\,, \tag{8}$$

where D_1 and D_2 are some functions, being the same for any i.

The inter-phase forces (4) contain two functions D_1, D_2, and n functions Fb_i, which must be derived from experimental data. Unfortunately, almost all experimental data have been obtained for stationary tests for $\left(\frac{\partial}{\partial t} = 0\right)$ and homogeneous flow regimes $\left(\frac{\partial}{\partial x} = 0\right)$ conditions that allow obtaining only the values of Fb_i. At present, establishing D_1, D_2 through experiments is not feasible.

Theoretical expressions have been derived for D_1, D_2 by averaging exact micro-equations[7], but only for very simple cases. It is shown in Ref. 5 that if for two phases $D_1 = D_2 = 0$, the system of equations (1) –(2) is non-hyperbolic and the Cauchy problem is incorrect for any Fb_i. If non-zero D_1 and D_2 exist that would insure a correct Cauchy problem. A possible form of inter-phase interactions could be specified using data from an experiment with large inhomogeneous velocity distribution and phase volume concentrations (i.e., for relatively large values of the derivatives, $\frac{\partial v_i}{\partial x}$ and $\frac{\partial \alpha_i}{\partial x}$).

Previously, it was shown in Ref. 8 that for definite values of the coefficients D_1, D_2, the system of equations (1) – (2) allows describing flow flooding experiments, where the velocity of descending water becomes zero due to water entraining by ascending steam. In the present paper, instead of using this type of experimental data, the following simple steady-state flow model is used (see Fig. 1): a) the lower part of the vessel is filled with liquid and the upper part is occupied with gas, b) the gas bubbles move up through the liquid medium, while the liquid drops fall down through the gaseous medium.

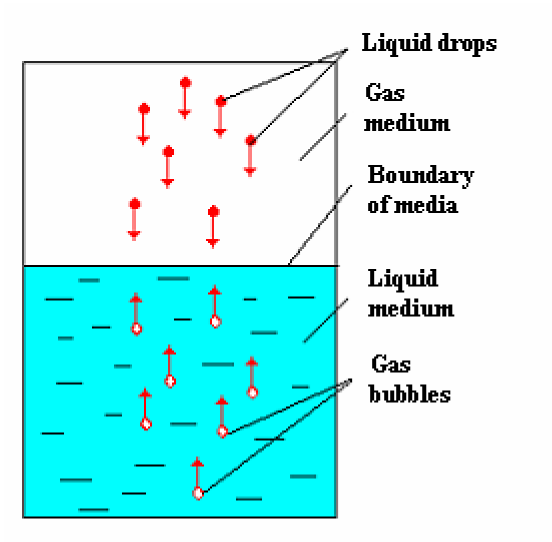

Fig. 1 Simple model of stationary two-phase flow with step-like parameter distribution

In order to insure that the water level in the vessel stays steady, the liquid from the lower part of the vessel must be evaporated that is equal to the rate of liquid drops, and, correspondingly, gas from the upper part of the vessel must outflow with the same flow rate of the bubbles. In this flow model the volumetric velocity of mixture is zero. It is clear, that such flow does exist, and at the phase boundary, the volume concentrations and velocities are discontinuous, while the boundary velocity equals zero. It is worth saying that such flow model can easily be described within the drift model, where the relative phase velocity is defined from empirical correlations.

It is easy to show that the system of equations (1)–(2), without any additional components for the interphase force that is proportional to flow parameter gradients, cannot describe discontinuity in the parameters on the boundary even in the steady-state case. From the mass and momentum conservation equations, we can easily obtain a generalized Bernoulli's equation connecting pressures and phase velocities for the steady-state conditions:

$$\frac{\partial\left(\frac{1}{2}\rho_i^{\circ} v_i^2 + p\right)}{\partial x} = \frac{F_i + F_{mi}}{\alpha_i}, \qquad (i = 1,2)$$

These equations indicate that the velocity change for both phases due to the discontinuity have the same sign, which is defined by the pressure discontinuity. On the other hand, from equation (1) it follows that by conserving phase flow through the boundary, the phase velocity changes must have different signs. This point to a contradiction indicating that the above discussed flow regime cannot be described without the derivatives in the interphase force expression and the system of equations (1)–(2) is incorrect without the additional gradient terms.

In order to examine the stability of the solution of the system of equations (1)–(4), and (8), we introduce a quantity $\vec{y} = (\alpha_1, v_1, \alpha_2, v_2, \ldots \alpha_n, v_n, p)$, which is a set of all the variables. We will look for the non-steady solution as the sum of the steady solution $\vec{y}_0$ and small disturbances $\vec{y}'$ of the type

$$\vec{y} = \vec{y}_{\circ} + \vec{y}' e^{ik(x-\lambda t)}, \qquad (9)$$

where the vector $\vec{y}'$ does not depend on time and spatial coordinates.

If equation (9) is substituted into (1)–(4), and (8), a linear equation for $\vec{y}'$ is obtained:

$$\ddot{M} \bullet (\vec{y}') = 0, \qquad (10)$$

where $\ddot{M}$ is a square matrix. The homogeneous linear equation (10) will have nontrivial solutions only when the determinant $\ddot{M}$ is zero:

$$\det(\ddot{M}) = 0. \qquad (11)$$

After some transformations the condition (11) can be written as

$$\sum_{i=1}^{n} \alpha_i \prod_{\substack{j=1 \\ j \neq i}}^{n} \left[\rho_j^{\circ} (v_j - \lambda)^2 - (D_1 + v_j D_2) + \frac{D_2 (v_j - \lambda)}{\alpha_j} \right] = 0 \qquad (12)$$

where λ is a certain characteristic velocity.

From equation (12) it is seen that there exist $2(n-1)$ values of λ, and if all v_j are different and $D_1 = D_2 = 0$, then all λ's are complex. As equation (12) has real coefficients, the set of solutions consists from $n-1$ pairs of complex conjugate values. Among them there always exist solutions with positive imaginary part, which increase exponentially when $k \to \infty$. This means that in the case of n phases, if $D_1 = D_2 = 0$, the solutions are non-stable and the Cauchy problem is incorrect. Such a statement is in full agreement with the result obtained in Ref. 4 for two phases.

The condition of that all roots of equation (12) are real does not allow to find the values of D_1 and D_2 in a unique way (since there are a wide range of values exist for D_1 and D_2 satisfying this condition).

Below for simplicity, we shall examine the two-phase case, as the majority of flow modes (film-like, mono-disperse bubble, slug, and some others) are well described when $n = 2$.

It is worth to emphasize that if the terms in equation (4) describing the inter-phase force including the derivatives are selected with arbitrary D_1 and D_2, the resulting system of differential equations may be ill-posed. For example, it may not be possible to write an appropriate momentum equations (2) allowing the definition of the flow parameters and velocities through the discontinuity from the initial equations without using additional correlations. The mixture momentum conservation law is obtained by summing equations (2) through n resulting in:

$$\begin{aligned} &\frac{\partial}{\partial t}(\alpha_1 \rho_1^\circ v_1 + \alpha_2 \rho_2^\circ v_2) \\ &+ \frac{\partial}{\partial x}(\alpha_1 \rho_1^\circ v_1^2 + \alpha_2 \rho_2^\circ v_2^2) + \frac{\partial p}{\partial x} = F_{m1} + F_{m2}. \end{aligned} \tag{13}$$

Unfortunately, it seems that no other type of conservation law can be derived from equations (2), if D_1 and D_2 are selected in an arbitrary manner.

This indicates that only certain values of D_1 and D_2 could provide the appropriate form of equations if we start from the momentum equations. For this purpose let us consider the following expression for the inter-phase forces

$$F_1 = -F_2 = -\alpha_1 \alpha_2 \frac{\partial}{\partial x}(Pf(\alpha_1) v_{12}^2) + Fb_1(v_1, v_2), \tag{14}$$

where $Pf(\alpha_1)$ is a function of the liquid phase volume concentration and $v_{12} = v_1 - v_2$. Formula (14), as written, is very general and could describe a wide range of inter-phase interactions.

Assuming the above form for the inter-phase interactions, the full system of differential equations will take the following form:

$$\begin{aligned} &\frac{\partial}{\partial t}(\alpha_1 \rho_1^\circ v_1 + \alpha_2 \rho_2^\circ v_2) + \frac{\partial}{\partial x}(\alpha_1 \rho_1^\circ v_1^2 + \alpha_2 \rho_2^\circ v_2^2) \\ &+ \frac{\partial p}{\partial x} = F_{m1} + F_{m2}, \\ &\frac{\partial v_v}{\partial x} = 0, \\ &\frac{\partial}{\partial t}(\alpha_1 - \alpha_2) + \frac{\partial}{\partial x}(\alpha_1 v_1 - \alpha_2 v_2) = 0, \\ &\frac{\partial}{\partial t}(\rho_1^\circ v_1 - \rho_2^\circ v_2) + \frac{\partial}{\partial x}(\rho_1^\circ \frac{v_1^2}{2} - \rho_2^\circ \frac{v_2^2}{2}) \\ &+ \frac{\partial}{\partial x}(Pf(\alpha_1) v_{12}^2) = \varphi, \end{aligned} \tag{15}$$

where $\varphi = \dfrac{Fb_1(v_1, v_2)}{\alpha_1 \alpha_2} + \dfrac{F_{m1}}{\alpha_1} - \dfrac{F_{m2}}{\alpha_2}$, and $v_v = \alpha_1 v_1 + \alpha_2 v_2$ is the average-volume velocity.

The system of four equations (15) contains four independent, unknown quantities, α_1, v_1, v_2, p. The structure of the first two equations is similar to the equations describing one-phase incompressible medium movement. The second equation in the equation-system (15) reflects the mixture volume velocity conservation and can be obtained by assuming incompressibility of the phases. The third equation is easily derived by combining one mass conservation equations, while the fourth equation follows from the momentum equations (2), with the inter-phase forces are given in formula (14).

It is convenient to rewrite the third and fourth equations, using new variables $\alpha = \alpha_1 - \alpha_2$ and $v_n = \rho_1^\circ v_1 - \rho_2^\circ v_2$, to obtain

$$\begin{aligned} &\frac{\partial \alpha}{\partial t} + \lambda_{11} \frac{\partial \alpha}{\partial x} + \lambda_{12} \frac{\partial v_n}{\partial x} = 0, \\ &\frac{\partial v_n}{\partial t} + \lambda_{21} \frac{\partial \alpha}{\partial x} + \lambda_{22} \frac{\partial v_n}{\partial x} = \varphi, \end{aligned} \tag{16}$$

where $\lambda_{11}, \lambda_{12}, \lambda_{21}, \lambda_{22}$ are the flow parameter functions, the explicit form of which will be specified below for the definite final form of the inter-phase interactions.

The differential part of the system of equations (16) depends on α and v_n, but do not depend on the other variables, i.e., it can be separated from equations (15) into a sub-system of equations. This allows treating the third and fourth equations in the system of equations (15) independently constructing a separate numerical solution method. The characteristics $\lambda_{1,2}$ of the system of equations (16) may be written as

$$\lambda_{1,2} = \frac{1}{2}(\lambda_{11} + \lambda_{22} \pm \sqrt{(\lambda_{11} - \lambda_{22})^2 + 4\lambda_{12}\lambda_{21}}). \tag{17}$$

In the special case when velocity distribution is stationary (corresponding to stationary distribution of v_n), the system of equations (16) becomes the well-known drift model. Indeed, if we ignore the phase inertia in the second equation of the system (16), i.e., the left-hand-side is set to zero, we get the relation

$\varphi = 0$,

from which the quantity v_{12} can easily be found and the following equation is obtained:

$$\frac{\partial \alpha}{\partial t} + \lambda_{drift} \frac{\partial \alpha}{\partial x} = 0 .$$

From the general theory of differential equations with partial derivatives it follows that, if the system of equation (16) is hyperbolic (i.e., the expression under the square root in formula (17) is positive), it will have stable solutions and is posed as a correct Cauchy problem. It is important to recognize that this condition is a necessary condition, but not sufficient. It is easy to show that the system of equation (16) has a stable solution and a correct Cauchy problem under small perturbations with linear approximation, if the value λ_{drift} is located between the characteristics λ_1 and λ_2 .

On the contrary, the solution is unstable, and the Cauchy problem is incorrect, if the value of λ_{drift} is more than the maximum or less than the minimum characteristics. If the value λ_{drift} lies between the characteristics λ_1, λ_2 and coincides with none of them, the initial step-like distribution will spread out over time. Finally, if λ_{drift} equals either λ_1 or λ_2 , the solution is stable and the initial step-like distribution retains its shape. From here an important fact follows that one of the characteristics of the system of equations (16) must coincide with the drift velocity to correctly describe the above mentioned step-like solution. This latter condition for λ_{drift} imposes very stringent restrictions on the admissible form of the function $Pf(\alpha_1)$.

As was shown above, the necessary condition for stable solutions is that the system of equations is hyperbolic. However, this is a necessary, but not sufficient condition, and additional analysis is required to determine the stability of solutions of for the specific system of equations.

The results of such analysis depend on the type of inter-phase interactions caused by viscosity effects and mass forces. To simplify the analysis, the following standard expressions will be used for the inter-phase forces Fb_1 and mass forces F_{m_1}, F_{m_2} :

$$\begin{aligned} &Fb_1(v_1, v_2) = -\alpha_1 \alpha_2 K_{Fr} v_{12} , \\ &F_{m_1} = -\alpha_1 g \cdot \rho_1^{\circ} , \\ &F_{m_2} = -\alpha_2 g \cdot \rho_2^{\circ} , \end{aligned} \tag{18}$$

where K_{Fr} is the coefficient of the inter-phase friction ($K_{Fr} > 0$), and g is the free-fall acceleration ($g > 0$).

Using (18) and making standard but rather long calculations, it can be proved, that in many cases the terms contained in the formula (14) stabilizes the system of equations (1)–(2) and insures that the system of equations (16) is hyperbolic. However, the solutions still remain unstable and, hence, the Cauchy problem is not correct. In single-phase hydrodynamics, the hyperbolic system of differential equations together with the physical “reasonableness” of closing correlations usually ensures the stability of the solution.

In the present case, even though the system of equations may be hyperbolic, the closing correlations (18) are simple and would decrease the phase velocity differences (i.e. appears physically “reasonable”), but the solution could still be unstable. In spite of defining stabilizing components, the stability of the system of equations (1)–(2) is not fully assured by insuring only hyperbolic conditions.

We have concluded earlier that one of the characteristics of the system must coincide with the drift velocity defining the form $Pf(\alpha_1)$ in formula (14) up to a constant (i.e. the quantity which does not depend on the volume concentrations of phases α_1 and α_2):

$$\begin{aligned} Pf(\alpha_1) = &\frac{-0{,}5}{\dfrac{1}{\rho_1^{\circ}} - \dfrac{1}{\rho_2^{\circ}}} - \frac{(\rho_1^{\circ} + \rho_2^{\circ})^2}{(\rho_1^{\circ} - \rho_2^{\circ})} \\ &\times \left[a + \left(\frac{\rho_2^{\circ}(\alpha_1 - 1)^2 - \rho_1^{\circ}(\alpha_1^2 - 1)}{2} \right) \cdot \frac{\rho_1^{\circ} - \rho_2^{\circ}}{(\rho_1^{\circ} + \rho_2^{\circ})^2} \right]. \end{aligned} \tag{19}$$

This result is of ultimate importance: starting from the mostly general formula for the inter-phase forces, correlation (14), we could define the only possible form of the inter-phase force (19) up to a constant term). The constant *a,* which depends on the phase densities should be defined for particular flow regimes experimentally or by averaging the micro-equations. For any fluid, if correlation (14) becomes unusable, the general method, presented in this paper, allows the construction of a differential equation for the inter-phase force.

After taking into account formula (19), the correct (divergent, hyperbolic and with stable solutions) system of equations (i.e. the last two equations in the system of equations (15)) will take the following form:

$$\frac{\partial \alpha}{\partial t}+\frac{\partial}{\partial x}\left(\frac{v_v\cdot(\rho_2^\circ-\rho_1^\circ)+v_n+\alpha\cdot(v_v\cdot(\rho_1^\circ+\rho_2^\circ)-v_n\cdot\alpha)}{\rho_1^\circ+\rho_2^\circ+\alpha\cdot(\rho_2^\circ-\rho_1^\circ)}\right)=0,$$

$$\frac{\partial v_n}{\partial t}+\frac{\partial}{\partial x}\left(\frac{v_n^2-2av_{12}^2(\rho_1^\circ+\rho_2^\circ)^2}{2(\rho_1^\circ-\rho_2^\circ)}+v_{12}^2\left(\frac{\rho_1^\circ(\alpha_1^2-1)-\rho_2^\circ(\alpha_2^2-1)}{2}\right)\right)=\varphi \tag{20}$$

For simplicity we retain in equations (18) the quantities α_1, α_2, and v_{12}, where the last variables are implied to be expressed through α, v_n, and v_v.

The first equation in (20) is simply the restatement of the well-known equation

$$\frac{\partial}{\partial t}(\alpha_1-\alpha_2)+\frac{\partial}{\partial x}(\alpha_1 v_1-\alpha_2 v_2)=0,$$

but being written with the variables α and v_n.

The system of equations (20) is written in the divergent form and can be brought to form (16) by defining the coefficients with the particular inter-phase interaction (19):

$$\lambda_{11}=\frac{\alpha_1 v_2\rho_2^\circ+\alpha_2 v_1\rho_1^\circ}{\alpha_1\rho_2^\circ+\alpha_2\rho_1^\circ},$$

$$\lambda_{12}=\frac{2\alpha_1\alpha_2}{\alpha_1\rho_2^\circ+\alpha_2\rho_1^\circ}, \tag{21}$$

$$\lambda_{21}=\frac{(\rho_1^\circ+\rho_2^\circ)^2 v_{12}^2}{(\alpha_1\rho_2^\circ+\alpha_2\rho_1^\circ)}\left(\frac{2\rho_1^\circ\rho_2^\circ-[\alpha_1(\rho_2^\circ)^2+\alpha_2(\rho_1^\circ)^2]}{2(\rho_1^\circ+\rho_2^\circ)^2}-a\right),$$

$$\lambda_{22}=\frac{\rho_2^\circ v_2-\rho_1^\circ v_1}{\rho_2^\circ-\rho_1^\circ}-\frac{a\cdot 2(\rho_1^\circ+\rho_2^\circ)^2 v_{12}}{(\rho_1^\circ-\rho_2^\circ)(\alpha_1\rho_2^\circ+\alpha_2\rho_1^\circ)}+\frac{(\rho_1^\circ\cdot(\alpha_1^2-1)-\rho_2^\circ(\alpha_2^2-1))\cdot v_{12}}{(\alpha_1\rho_2^\circ+\alpha_2\rho_1^\circ)}.$$

The system of equations (16), with the above defined coefficients (21), will have one of the characteristics coinciding with the drift velocity

$$\lambda_1=\lambda_{drift} \text{ or } \lambda_2=\lambda_{drift},$$

since exactly this condition was used in the derivation of the function Pf in form (19). Note that for the specific inter-phase and mass forces described by the correlations (18), the slip v_{12} in the drift model is independent of the volumetric phase concentrations, and

$$\lambda_{drift}=v_v+v_{12}(1-2\alpha_1).$$

The other characteristic is

$$\lambda=v_v-\frac{v_{12}[\alpha_1\alpha_2(\rho_1^\circ-\rho_2^\circ)^2+2a(\rho_1^\circ+\rho_2^\circ)^2]}{(\rho_1^\circ-\rho_2^\circ)\cdot(\rho_1^\circ\alpha_2+\rho_2^\circ\alpha_1)}. \tag{22}$$

Figure 2 shows the characteristics λ_1 and λ_2 as functions of the liquid phase volumetric concentration α_1 for the case $\rho_1^\circ=1000$, $\rho_2^\circ=10$, $v_v=0$, $v_{12}=-1$, with the constant $a=0$ in formula (19). The quantities λ_1 and λ_2 correspond, respectively, to the signs '+' and '-' in equation (17).

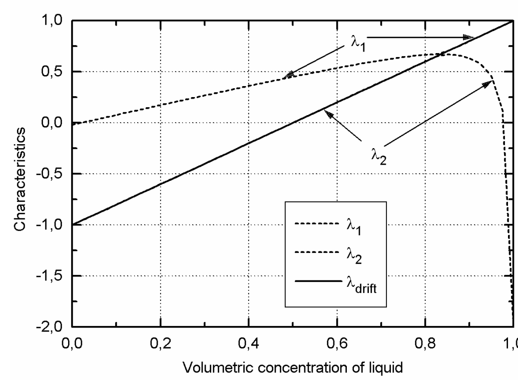

Fig. 2 Dependence of system characteristics as function of volumetric concentration of liquid

The above linear analysis was made for small perturbations, but it can be seen that the conservative non-linear equations (20) also describe steady solutions with finite, but not necessarily small, step-like distributions of phase volumetric concentrations and velocities (in particular, for the flow model shown in Fig. 1). The main ideas described in this paper, are used in the two-velocity version of the BAGIRA code (Ref. 10) and we believe that numerical calculations should rely only on such analysis.

It is of some interest to analyze the mathematical models of multi-velocity flows as used in other thermal-hydraulic codes. In general, the main differential laws of mass and momentum conservation are similar to those described here, with differences appearing primarily through the closing correlations.

In the RELAP5/MOD3 code[1], the following expression is used for the differential part of the inter-phase force F_{ijm}:

$$F_{ijm} = -k\alpha_i\alpha_j\rho\left(\frac{\partial v_i}{\partial t} + v_j\frac{\partial v_i}{\partial x} - \frac{\partial v_j}{\partial t} - v_i\frac{\partial v_j}{\partial x}\right), \quad (23)$$

where ρ is the mixture density. This means that, instead of formula (4), we have the formula

$$F_i = F_{ijm} + Fb_1(v_1...v_n)\ . \quad (24)$$

While performing the standard calculations, it can be shown that in case of two phases, the hyperbolic condition is written as:

$$\left[\frac{k\rho^2}{4} - \alpha_1\alpha_2\rho_1^\circ\rho_2^\circ\right]v_{12}^2 \geq 0\ .$$

This form can easily be transformed into:

$$\frac{(k-2)}{4}\rho^2\left(\alpha_1\rho_1^\circ - \alpha_2\rho_2^\circ\right)^2 v_{12}^2 \geq 0\ . \quad (25)$$

As seen from (25), hyperbolic conditions are reached when $k \geq 2$, however, when $k < 2$ the system is not hyperbolic, unless one of the last two factors on the right-hand-side equals zero. Reference 1 mention that the coefficient k depends on the volumetric void fraction and for all flow regimes $k < 2$ unless $\alpha = 0.5$. Thus, for all values of α, except $\alpha = 0.5$, the system of differential equations is non-hyperbolic.

In the CATHARE code[2] the differential part of the inter-phase force contains the so-called added mass term

$$F_{ijm}^{(AM)} = -k\alpha_1\alpha_2\rho\left(\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} - \frac{\partial v_j}{\partial t} - v_j\frac{\partial v_j}{\partial x}\right). \quad (26)$$

Correlation (26) differs from (23) by the convection components using the material derivative of the velocity, i.e. the derivative is in the frame of reference, connected with the given phase. Note that it has been impossible until present time, deriving a formula for "virtual mass" force in the mixture. We believe that correlation (26) seems to be a more natural generalization, than correlation (23).

In addition to equation (26), the full expression for the differential part of the inter-phase force used in the CATHARE code contains the term

$$F_{ijm}^{(1)} = -p_i\frac{\partial\alpha_i}{\partial x}\,, \quad (27)$$

where p_i is not the pressure, but some other quantity that is defined depending on the flow regime considered. For instance, for partially stratified flows

$$p_i = \frac{\alpha_1\alpha_2\rho_1^\circ\rho_2^\circ v_{12}^2}{\alpha_1\rho_2^\circ + \alpha_2\rho_1^\circ}\,, \quad (28)$$

that leads to the full differential part of the inter-phase force as

$$F_{ijm} = F_{ijm}^{(AM)} + F_{ijm}^{(1)}\ . \quad (29)$$

It is easy to show, that the hyperbolic condition of the differential equations, in case the formulas (24) and (29) are used, may be written as follows:

$$-k\rho v_{12}^2\left[\frac{\alpha_2\left(\rho_1^\circ\right)^2 + \alpha_1\left(\rho_2^\circ\right)^2}{\alpha_1\rho_2^\circ + \alpha_2\rho_1^\circ} + k\rho\right] \geq 0\ . \quad (30)$$

The above condition shows that the system of differential equations used in CATHARE, is non-hyperbolic unless $v_{12} = 0$ or $k = 0$.

It is worth mentioning that even if the coefficients k in formulas (23) and (26) satisfy the hyperbolic conditions of Equations (25) and (30), the corresponding systems of equations analogous to the system of equations (16) would have characteristics, which do not coincide with the drift velocity λ_{drift}. As a consequence, this would not allow getting a proper solution for the simple model of stationary two-phase flow with step-like parameter distribution, considered above (see Fig. 1).

The correlations for the inter-phase force used in Russian codes, including two momentum equations, are generally not analyzed, but use published correlations (for example, KORSAR[3] uses the correlation from CATHARE).

We have shown in the above analysis that modern thermal-hydraulic codes, which include two momentum equations, are generally based on non-hyperbolic systems of differential equations. Hence, the solutions of such equations are unstable, and the corresponding Cauchy problems are incorrect. This means, that the stable solutions achieved by these codes, are not the real solutions of the correspondent initial systems of equations, and their stability seems to be provided by different methods (for example, by introduction of numerical viscosity). However, such artificial methods introduce qualitative changes in the initial systems of equations.

The numerical solutions achieved using such methods correspond to equations, which are different from the initial systems leading to a loss of physical meaning of the numerical results. Two-speed effects could significantly depend on numerical methods and modeling such as numerical mesh and cell sizes. This could result in additional inaccuracies compounded when using the experimental data to verify numerical models and makes it more difficult to extrapolate the analytical results to operating reactors.

The analysis presented in this paper allows drawing the following conclusions:

- The systems of differential equations as used in the modern best-estimate thermal-hydraulic codes that describe multi-velocity flows of multi-phase mixtures are non-hyperbolic, and their solutions are non-stable.
- In order to eliminate potential inaccuracies, it is recommended to include additional components in the formulas for inter-phase forces. It is suggested to include components, as described in Equations (14) and (19), containing the spatial derivatives of velocities and phase volumetric concentrations.
- The method developed in this paper works correctly for two-phase flow cases. However, preliminary analysis indicates that if more than two phases are considered, the inter-phase force can not be constructed to insure: a) hyperbolic system of equations, b) stable solutions, and c) requiring that the basic equations have divergent form. Additional methodological developments are needed to include three or more phases.

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NOMENCLATURE

F_i – inter-phase force acting on the i-th phase
F_{ijm} – differential part of the inter-phase force
F_{mi} – mass (gravitational) force acting on the i-th phase
Fb_i – a part of the inter-phase force acting on the i-th phase, which does not contain spatial derivatives of phase velocities and volume concentrations
g – free-fall acceleration
K_{Fr} – coefficient of the inter-phase friction
n – number of phases
p – pressure
t – time
v_i – i-th phase velocity
v_n – the quantity $\rho_1^\circ v_1 - \rho_2^\circ v_2$ in the two-phase case
v_v – average volume velocity $\alpha_1 v_1 + \alpha_2 v_2$ in the two-phase case
v_{12} – relative velocity (slip) $v_1 - v_2$ in the two-phase case
x – spatial coordinate
α_i – i-th phase volume concentration
α – the difference of the phase volume concentrations $\alpha_1 - \alpha_2$ in the two-phase case
ρ_i° – i-th phase real density
ρ – mixture density, $\rho = \alpha_1 \rho_1^\circ + \alpha_2 \rho_2^\circ$
φ – the quantity $\dfrac{Fb_1(v_1, v_2)}{\alpha_1 \alpha_2} + \dfrac{F_{m1}}{\alpha_1} - \dfrac{F_{m2}}{\alpha_2}$ for the two-phase case

REFERENCES

1. RELAP5/MOD3 Code Manual, Volume 1, Code Structure, System Models and solution methods, NUREG/CR-5535 (1998).

2. D.BESTION, “General Description of CATHARE-2 VI.3, VI.3E, V1.3U”, STR/LML/EM/94-265 (1994).

3. MIGROV Yu.A. *et al.*, “KORSAR” – thermo-hydraulic code of new generation for NPP safety analysis”, *Thermal Engineering*, No. 9, p. 736 (2001).

4. NIGMATULIN R.I., *Dynamics of Multiphase Media*, Vol. 1,Hemisphere, New York (1991).

5. KLEBANOV L.A., Kroshilin A.E., Nigmatulin B.I., and Nigmatulin R.I., “On hyperbolicity, stability, and Cauchy problem correctness for system of differential equations of two-velocity two-phase media”, *Applied Mathematics & Mechanics*, **46**, No. 1, p. 83 (1982).

6. USHAKOV P.A., Sorokin A.P., and Matyukhin N.M., “Hydraulic resistance role in modeling natural

convection in tanks of fast reactor", *Thermal Engineering*, No. 5, p. 383 (2000).

7. KROSHILIN V.E., "Inter-phase force calculation in mono-disperse bubble medium with inhomogeneous bubble distribution", *Vestnik MGU*, *Mathematics & Mechanics* , No. 6, p.23 (1990) (in Russian).

8. B.L. KANTSYREV and A.A. Ashbaev, "Two-fluid hydro-dynamical model of bubble flow", *Applied Mathematics & Technical Physics*, **42**, No. 6, p. 64 (2001).

9. GODUNOV S.K. and Ryaben'ky V.S., *Raznostnye Skhemy* (*Difference Schemes*), Nauka, Moscow (1977) (in Russian).

10. VESELOVSKY A.N., Zhivotyagin A.F., Kalinichenko S.D., Kroshilin A.E., and Kroshilin V.E., "Complex of programs BAGIRA for modeling multi-phase medium thermal hydro-dynamics". *Thermal Engineering*, No. 5, p. 361 (1998).

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