Physics 139 Relativity

Problem Set 3 Due February 8, 2002

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1 Simple Twin Paradox

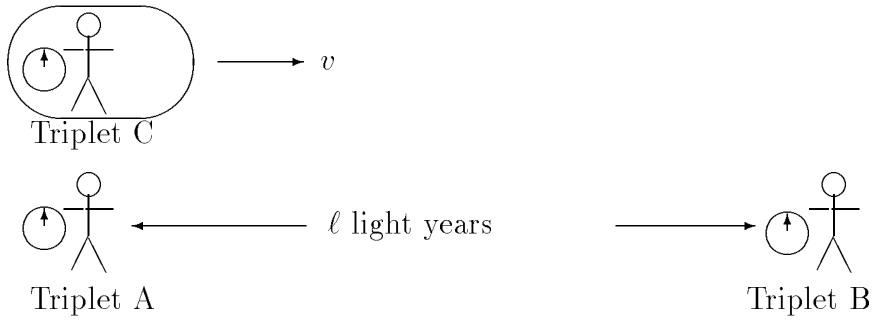
Consider a simple version of the twin paradox. Two twins, Terrence and Stella, are together on Earth in reference frame S until a spaceship (reference frame S') goes by at time and place $(x,t) = 0 = (x',t')$. Stella is instaneously beamed aboard the spaceship which is traveling with velocity v relative to Earth. The spaceship (S') travels to a nearby planet at distance L in the Earth frame. Show answers in a matrix format:
(a) How far does it appear in frame S' and how long does Stella think the journey takes. How far is it and how long does it take from the Earth frame S? Use length contraction.
(b) At the planet Stella instantaneously beams from the spaceship traveling away from the Earth with speed v to a second spaceship (reference frame S''). Again how far and how long does it take Stella to return in frame S and frame S''?
(c) Show what results you get for the same questions using Lorentz transformations and calculating proper time elapsed for Stella and Terrence.

2 StarTrek Voyager

The premiss of the StarTrek Voyager TV series was that the starship Voyager was nearly instaneously transported across the Galaxy. This is a distance of roughly 16 kpc (kiloparsec) or 50,000 light years. The Galaxy is rotating and they were transported from nearly at local rest to near local rest for a reference frame velocity difference of about 450 km/sec or $\Delta v = \beta c = 1.5 \times 10^{-3} c$.
(a) What is the difference in time that an observer near Earth (frame S) will say a clock in the Voyager's new frame S' and in Voyager's old frame S both at Voyager's new spatial location?
(b) Make a spacetime graph showing the x and ct axes normal to each other, an exaggerated angle for the new x' and ct' axes and indicate the time difference at the new Voyager location.

3 Triplet Paradox

Three identical triplets, A, B, C, find themselves in the following situation:



Triplet A and Triplet B are at rest with respect to each other in reference frame S and are ℓ light years apart with clocks synchronized at $t = 0$, when Triplet C goes by Triplet A with speed $v = \beta c$ at time $' = 0$. Put the origin of system S at Triplet A and of system S$'$ (rest frame for Triplet C) there simultaneously.

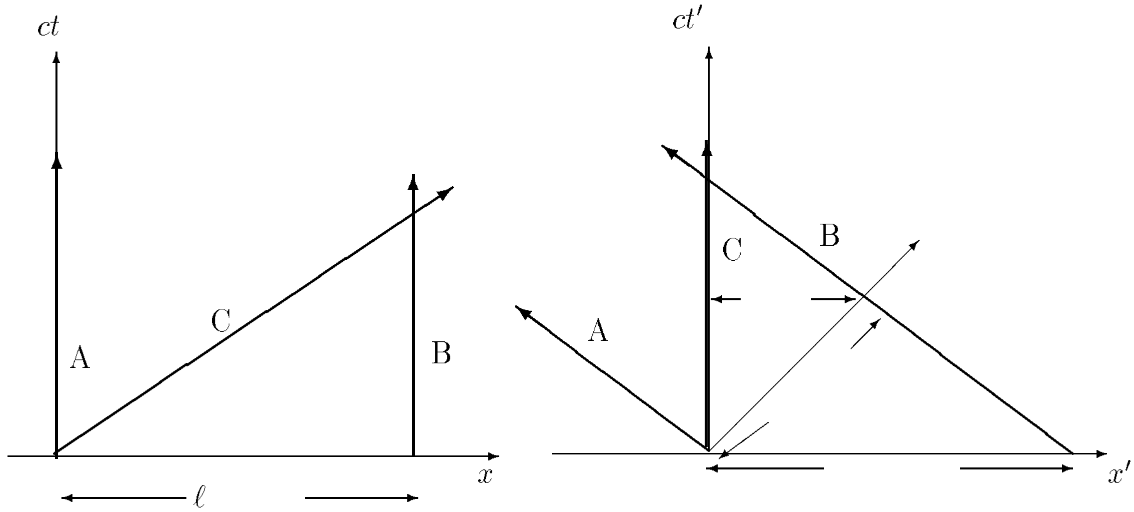
Later at time $t = \ell/(\beta c)$ years, Triplet A and Triplet B observe Triplet C pass by Triplet B's location and Triplet B sees that Triplet C's clock reads time $t' = \ell/(\gamma\beta c)$ years. Triplet B says that is just what I expect as Triplet C's clock runs slow by the factor γ, because Triplet C is moving at speed $v = \beta c$.
Do you agree? ....

Triplet A and B says that is what time that Triplet C should see on his clock, since he sees the distance between Triplet A and Triplet B contracted by the factor γ and both Triplet A and Triplet B moving at speed $-v = -\beta c$.

Triplet C expected that it would take $t' = \ell/(\gamma\beta c)$ to travel the distance $\ell' = \ell/\gamma$ between Triplets A and B as A and B are moving at speed $v = \beta c$. Furthermore, Triplet C claims that Triplet A's and B's clocks both run slower than his by a factor γ. But Triplet B's clocks should read $\ell/(\beta c)$ years.

Show by Lorentz transformation and by careful labeling of the space-time diagram how Triplet C can think that Triplet B's clock can run more slowly than his own clock and yet read a later time ($t = \ell/(\beta c)$ years) when Triplet B and Triplet C ($t' = \ell/(\gamma\beta c)$) pass each other.

Space-Time Diagram



4 Proton and Energy Units

Consider a beam of protons (mass = 0.938272 GeV/c^2) each with momentum 8.0 Gev/c. (GeV = 10^9 eV)
(a) Find the proton kinetic and total energy in GeV. (Formula & evaluate)
(b) Determine ratio of the velocity of the protons to the speed of light by formula and evaluate to three significant figures.
(c) Find the magnetic field necessary to bend them with a radius of 10 m.
(d) Find the strength of electric field needed to do the same. (Find the tangent circle or consider the protons in orbit around a large spherical charge.)

5 Equivalence of Mass and Energy

The binding energy of a hydrogen atom (a proton and an electron) is $E_b = 13.6$ electron-Volts (eV).
(a) By what fraction does the total mass of the electron plus proton change when combined into a hydrogen atom?
(b) How accurate is conservation of mass in chemical reactions?
(c) If the typical scale of nuclear reactions is of order 1 MeV (million electron volts), how accurate is conservation of mass in nuclear reactions?
(d) If the typical scale of high-energy physics reactions is of order 100 MeV (million electron volts), how accurate is conservation of mass in high energy reactions?

6 Energy and Momentum Transformation

(a) Find by formula and numerical evaluation the energy and momentum of that same beam of protons (p = 8.0 GeV/c) from the frame of reference of a proton in another beam with the same momentum heading directly towards the first beam. (i.e. colliding beams)
(b) If two protons collide head-on, what is the maximum rest mass particle that they can create. (Again show by formula and evaluate.)
(c) Why would one go to the trouble of making colliding beams rather than just accelerating the first beam up to sufficient energy to make a particle or group of particles of the same maximum mass?

7 A little more Relativistic Kinematics

The pi-zero meson (rest mass times c^2 of 134.9764 MeV) decays into two photons with a very short lifetime (8.4×10^{-17} seconds).
(a) What is the energy of each photon in the pi-zero rest frame? And what are their relative directions?
If in the laboratory the pi-zero is moving so it has a total energy twice its rest mass, and decays into two photons, for ease in calculation, that are emitted in the pi-zero rest frame at right angles to the direction of motion.
(b) What is their energy in the laboratory frame?
(c) What is the angle each makes to the pi-zero direction? E.g what would be the angle between them.
(d) How would this change if they are emitted at an angle theta to the line of flight in the rest frame? What would be the energy and angle to the original motion or opening angle between the photons as a function theta? How does your answer for the angle agree with the relativistic aberration of light formula?