Kicker Magnets for an FFAG Ring for Muons

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In[1]:= **ClearAll; Date[]**

Out[1]= {2004, 5, 5, 0, 2, 2}

■ Introduction

In this *Mathematica* notebook, I apply the packages and principles described in my report "Muon Storage Ring Design with Simple *Mathematica* Packages", CERN-SL/99-053 (AP) to the design of kicker magnets for extraction from an FFAG ring for acclerating muons. For the pulsed kicker, I use formulae of R.B.Palmer, which he presented during the FFAG Workshop at TRIUMF, Vancouver, BC, Canada, 15 to 21 April 2004. For the delay line kicker, I use formulae from the Handbook of Accelerator Physics and Engineering, p. 460-466. This notebook is available at http://keil.home.cern.ch/keil/Math/FFAG_mu-Kicker.nb.

In the section labelled "Natural Constants" I provide several natural constants with their standard names. In the section "User Input", I define the variable input parameters of the kicker magnet. For the study of parameters of another kicker magnet it suffices to change the values in this section.

■ Natural Constants

The constants.m package contains the values and standard names for the physical constants c, e, ϵ_0, μ_0, Z_0. It knows the rest voltages E_e, E_μ and E_p, the classical radii r_e, r_μ and , and the Compton wavelengths λ_e, λ_μ and λ_p for electrons, muons and protons, respectively. The Compton wavelengths λ_p and λ_μ are defined in analogy to that for electrons.

In[2]:= **Off[General::"spell"]; Off[General::"spell1"];**

In[3]:= **AppendTo[$Path, "D:\\Data_Files\\math"];**

In[4]:= **<< constants.m**

■ User Input

First, I define the particle by giving its rest voltage E_0 the value of muons. This command triggers the current definitions of classical radius r_c and Compton wavelength λ_c.

```
In[5]:= E₀ = E_μ;
```

```
In[6]:= r_c := r_p /; E₀ == E_p; λ_c := λ_p /; E₀ == E_p; r_c := r_μ /; E₀ == E_μ;
        λ_c := λ_μ /; E₀ == E_μ; r_c := r_e /; E₀ == E_e; λ_c := λ_e /; E₀ == E_e;
```

I then define the beam parameters ejection momentum ejPc, normalized emittance ϵ_{xn}, number of rms beam radii sigN that the physical aperture of the kicker magnet should provide, and the amplitude function at the kicker magnet in the plane of the kick betaT. I adjusted the value of betaT such that the kick angle kickA agrees with that given by R.B. Palmer for the extraction kicker in his 10 to 20 GeV ring.

```
In[7]:= ejPc = 20 10⁹ Volt; ϵ_xn = (10 / 3) 10⁻³ Meter; sigN = 3; betaT = 2. Meter;
```

The next batch of input data contains the input parameters of the kicker magnet, length kickL, full horizontal aperture apertX, full vertical aperture apertY, and kicker rise time riseT. I assume that the kicker deflects horizontally. For a vertical kicker it suffices to interchange apertX and apertY.

```
In[8]:= kickL = 1.5 Meter; apertX = 0.195 Meter; apertY = 0.076 Meter; riseT = 1270. 10⁻⁹ Second;
```

I assume that a full-aperture delay line ejection kicker deflects horizontally. I enter the maximum kicker voltage max-KickVolt, the kicker impedance kickZ, and make the kicker fall time equal to the kicker rise time riseT.

```
In[9]:= maxKickVolt = 9. 10⁴ Volt; kickZ = 0.75 Volt / Ampere;
```

■ Kicker Parameters

The hopefully self-explanatory table below shows the meaning, the name and the value of the kicker parameters that can be computed directly from the input parameters, marked with an asterisk in all tables. I first compute the relativistic parameters β and γ. Then I calculate kick angle kickA, assuming the optimum arrangement of kicker and septum at $\pi/2$ phase shift and a septum of zero thickness, kicker field kickB, kicker current kickI, kicker voltage kickV, kicker stored energy kickW.

```
In[10]:= γ = √( ( ejPc / E₀ )² + 1 )
```

```
Out[10]= 189.292
```

```
In[11]:= β = √( 1 - 1 / γ² )
```

```
Out[11]= 0.999986
```

$$In[12]:= \mathtt{kickA = 2\,sigN} \sqrt{\frac{\epsilon_{xn}}{\beta\,\gamma\,\mathtt{betaT}}}$$

```
Out[12]= 0.0178038
```

$In[13]:=$ **kickB = $\dfrac{\text{ejPc kickA}}{\text{c kickL}}$**

$Out[13]=$ $\dfrac{0.791828 \text{ Second Volt}}{\text{Meter}^2}$

$In[14]:=$ **kickI = $\dfrac{\text{kickB apertY}}{\mu_0}$**

$Out[14]=$ 47888.8 Ampere

$In[15]:=$ **kickV = $\dfrac{\text{kickB apertX kickL}}{\text{riseT}}$**

$Out[15]=$ 182370. Volt

$In[16]:=$ **kickW = $\dfrac{\text{kickB}^2 \text{ kickL apertX apertY}}{2\,\mu_0}$**

$Out[16]=$ 5545.76 Ampere Second Volt

■ Injection Kicker Parameters

The kickPack.m package gets the maximum kicker voltage maxKickVolt, the kicker impedance kickZ, the rise time riseT, full horizontal and vertical apertures apertX and apertY, and deflection angle kickA. It computes the integrated kicker field kickTm, makes the number of kicker modules kickModules an integer, chosen such that the kicker voltage kickVolt is smaller than the maximum value given, and computes the length of a module kickLength and the kicker field kickB, and puts all results into a table. Since kickpack.m expects to find the momentum with the name collPc, I make it equal to ejPc.

$In[17]:=$ **<< kickPack.m**

$In[18]:=$ **collPc = ejPc;**

$In[19]:=$ **kickTable[maxKickVolt, kickZ, riseT, apertX, apertY, kickA]**

$Out[19]//TableForm=$

*Maximum kicker voltage	90000. Volt
*Kicker characteristic impedance	$\dfrac{0.75 \text{ Volt}}{\text{Ampere}}$
*Kicker rise time	1.27×10^{-6} Second
*Width of kicker aperture	0.195 Meter
*Height of kicker aperture	0.076 Meter
*Deflection angle	0.0178038
Integrated kicker field kickTm	$\dfrac{1.18774 \text{ Second Volt}}{\text{Meter}}$
Length of kicker module kickLength	0.295416 Meter
Number of kicker modules kickModules	5
Kicker voltage kickVolt	72947.9 Volt
Kicker magnetic field kickB	$\dfrac{0.804114 \text{ Second Volt}}{\text{Meter}^2}$

I noticed that the length of a kicker module is proportional to the impedance kickZ, and adjusted kickZ such that the length of the delay line kicker is practically the same as that of the pulsed magnet. By increasing maxKickVolt, I found that the number of modules decreases.I adjusted both to get a kick that has about the same length as the pulsed magnet. The actual kicker voltage kickVolt is only 20% higher than the usual figure. The impedance is 0.75 Ω instead of the usual 5 Ω. Here comes the stored energy in all delay line kicker modules kickU.

$In[20]:=$ **kickU** = $\dfrac{\text{kickB}^2\,\text{kickLength}\,\text{apertX}\,\text{apertY}\,\text{kickModules}}{2\,\mu_0}$

$Out[20]=$ 5631.81 Ampere Second Volt

The kicker field and the stored energy is the same as in the pulsed magnet, as they should be. The delay line kicker has twice the voltage of the pulsed one, divided by the number of modules.

$In[20]:=$ **kickU** = $\dfrac{\text{kickB}^2\,\text{kickLength}\,\text{apertX}\,\text{apertY}\,\text{kickModules}}{2\,\mu_0}$

$Out[20]=$ 5631.81 Ampere Second Volt