AC and DC Voltages from a Josephson Arbitrary Waveform Synthesizer

Samuel P. Benz, Charles J. Burroughs, Jr., Paul D. Dresselhaus, and Laurie A. Christian

***Abstract*—We have synthesized and measured ac and dc voltages using a Josephson arbitrary waveform synthesizer. On-chip filtering has enabled the first practical operating margins for ac and arbitrary waveforms. Using a digital voltmeter, we demonstrate the operating margins and linearity of 101 synthesized dc voltages and confirm the flatness of a single voltage step. We present the first ac–dc and ac–ac voltage measurements of the synthesizer at 3.65 mV using an ac–dc thermal transfer standard. This quantum-based standard source can be used to measure the ac–dc difference of thermal transfer standards at small voltages.**

***Index Terms*—Digital–analog conversion, frequency control, frequency domain sythesis, frequency sythesizers, Josephson arrays, signal sythesis, standards, superconducting microwave devices, superconducting-normal-superconductor devices, voltage control.**

I. INTRODUCTION

WE HAVE generated the first ac voltage waveforms with practical operating margins by improving the on-chip filtering of the Josephson arbitrary waveform synthesizer. This Josephson voltage standard source is capable of synthesizing both ac and dc voltages as well as arbitrary waveforms with multiple frequencies [1]–[7]. Previous devices have demonstrated ac waveforms with improved harmonic distortion [4]–[6]. The new circuits in this paper are the first to have operating margins such that the output voltage and harmonic spectra remain unchanged for a finite range of all input parameters. Measurements of low harmonic distortion, a feature of ideal digital synthesis, in these circuits are presented elsewhere [7]. The improved margins combined with a modified termination circuit that reduces the common mode signal allows us to make the first ac measurements of the output signal using a thermal voltage converter. We present measured results from both ac and dc synthesized waveforms using three circuits with different terminations and filters.

The time-integrated area of every voltage pulse of every Josephson junction is precisely equal to $h/2e$, the ratio of Planck's constant to twice the electron charge. Digital synthesis using these perfectly quantized pulses enables the generation of voltage waveforms with unprecedented accuracy and stability. However, one of the major challenges in creating a practical and useful Josephson arbitrary waveform synthesizer is generating large voltages. In order to achieve the highest output voltage we use the bipolar synthesis method [4]–[6].

The circuit schematic for the bipolar method is shown in Fig. 1. The modulator algorithm converts the desired analog signal $V(t)$ into a "perfect" digital code; it reduces in-band distortion from the quantization process, called quantization noise, and pushes it out of band, usually to higher frequencies [8]. We use a second-order two-level delta–sigma modulator algorithm that has been modified for use with this bipolar bias technique. The periodic code of length N_S is loaded into the circulating memory of the semiconductor code generator, which attempts to reproduce the ideal digital code with its two level output. The array is biased by the combined signal of the code generator and a sine wave through a directional coupler C. The low-pass filter removes the out-of-band quantization noise and produces the desired synthesized signal $V'(t)$. For a clock (and sampling) frequency f_s , the minimum (non-dc) signal frequency is $f_1 = f_s/N_s$.

The maximum output voltage of a series array of junctions is $V = nNf/K_{J-90}$, where n is the number of quantized output pulses for each input pulse, N is the number of series junctions, f is the sine wave frequency, and $K_{J-90} = 483\ 597.9$ GHz/V is the Josephson constant. The precise area $1/K_{J-90}$ of each quantized pulse is very small, approximately 2 μV/GHz. Thus, high-voltage output requires both many junctions and a high drive frequency. The optimum operating margins occur when the sine frequency is precisely an odd half-integer multiple of the clock frequency f_s (eg., 3/2, 5/2) [5], so that higher sine wave frequencies can produce higher output voltages with a fixed maximum clock frequency. The relative phase between the clock and the sine wave is maintained with a 10-MHz reference signal.

The data described in this paper come from three different Josephson pulse quantizer circuits. Fig. 2 shows two simplified circuit schematics. In both circuits, the bipolar input drive is applied to the array of junctions on a 50 Ω coplanar waveguide transmission line. In configuration (a), the 50 Ω termination contributes a large common-mode signal. Configuration (b) solves this problem by grounding the array so the array voltage can be directly applied to a thermal converter or spectrum analyzer. All our previous circuits used configuration (a), where the filters were simply 50 Ω resistors [1]–[6]. Only the step flatness data presented below are from a resistively filtered circuit. The resistors were inadequate for preserving the uniformity of the input drive signal along the array at all frequencies. Distortions of the input signal applied to the array destroyed the operating margins for ac waveforms.

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S. P. Benz, C. J. Burroughs, and P. D. Dresselhaus are with Electromagnetic Technology Division, Electronics and Electrical Engineering Laboratory, National Institute of Standards and Technology, Boulder, CO 80303 USA.

L. A. Christian is with Measurement Standards Laboratory of New Zealand, Industrial Research, Lower Hutt, New Zealand.

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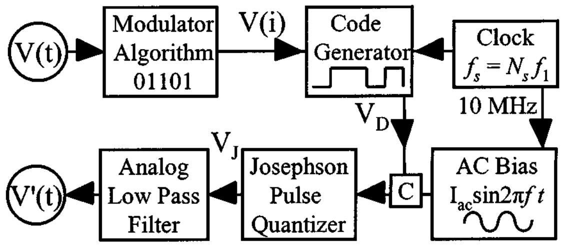


Fig. 1. Block diagram of the bipolar synthesis method for the Josephson arbitrary waveform synthesizer based on a Josephson junction pulse-quantizing array.

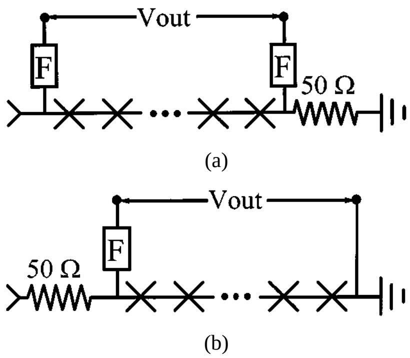


Fig. 2. Josephson pulse quantizer circuits. Each array of N junctions (junction symbol X) is embedded in a 50 Ω coplanar waveguide transmission line. The bipolar input signal drives the transmission line from the left side. (a) Typical configuration with two identically filtered output taps F where the transmission line is terminated after the array. (b) Low common-mode configuration with a single filtered output tap where the array is directly grounded.

We solved this problem by constructing low-pass filters with physically small inductors. For circuit 1, each filter in configuration (a) uses a 3-GHz low-pass filter with a pair of 2.7 nH square coil inductors that are 114 μm on a side. Circuit 1 has an array of 4096 junctions with an 8.2 mA critical current and a resistance per junction of 2.7 mΩ. This circuit is biased with a 7.5 GHz sine wave and the digital code is clocked at 3 GHz ($f/f_s = 5/2$). A high-impedance differential preamplifier is required to measure the harmonic spectra of ac waveforms for this circuit [7]. In circuit 2, the output tap of configuration (b) consists of the same 3-GHz low-pass filter close to the array followed by a second 65-MHz filter with a pair of 123 nH inductors 400 μm on each side. Circuit 2 is on the same chip as circuit 1 and has a small array of 250 junctions with a 7.2 mA critical current and a 2.8 mΩ resistance per junction. This circuit is driven with a slightly higher 10.5-GHz sine frequency and the same 3-GHz clock ($f/f_s = 7/2$).

II. DC Voltage Measurements

Next, we describe measurements of synthesized dc voltages using these two circuits. The operating range is determined by measuring the current range of dc constant voltage steps. Any dc voltage $(p-q)Nnf[K_{J-90}(p+q)]^{-1}$ can be generated using a digital code with p 1s and q 0s. The current range common to all voltages is the total operating range of the device.

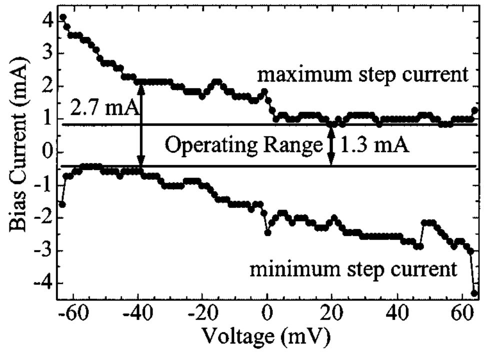
The code generator has an 8×10^6 bit pattern memory capacity so that 8 million distinct dc voltages can be generated simply by changing the digital code. Output voltage can also be tuned by changing the drive frequencies. The waveform synthesizer can generate voltages with over 22 bits of resolution for calibrating analog-to-digital (A/D) converters. We investigated a small set of 101 codes for a 100-bit pattern to demonstrate the functionality of the bipolar synthesizer and the different circuit configurations. The output voltage of each circuit was measured with a high-precision digital voltmeter on its 100-mV range.

We generated alternating positive and negative voltages starting with the maximum (all 1s) and minimum (all 0s) and decreasing toward 0 V (alternating 01 pattern). For each iteration, the positive and negative voltage patterns were changed by appending "01" to the right end of each 100-bit long code (and deleting the two left-most bits). For example, the resulting measurement sequence for circuit 1 was (+63.5, −63.5, +62.3, −62.3, ..., +1.3, −1.3, 0) mV. Each voltage can be generated using a number of different patterns; the 101 patterns measured here are only a subset of all possible patterns for these particular voltages, even within the set of 100-bit long codes.

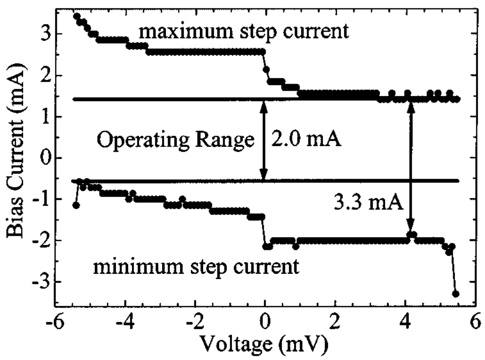
Fig. 3 shows for both circuits the current range of the flat voltage steps generated by each pattern as a function of the voltage. The step edges were determined using a search criterion of 6 standard deviations. The voltmeter averaged each voltage measurement over 10 power line cycles. For circuit 1, the smallest current range occurs for the pattern corresponding to about −40 mV. Although the current range of all steps is greater than 2.7 mA, the combined operating range for all 101 patterns is only 1.3 mA, as indicated by the horizontal lines. The main improvement of this data over previous resistively filtered circuits is that this large operating range occurs for a range of all input bias parameters. Previous circuits achieved an operating range for the 100-bit codes at only one selection of relative phase between the clock and sine drive frequencies [9]. This improvement is a result of the new nonresistive filters.

Fig. 3(b) shows the operating range for circuit 2. The voltage range is much lower because there are fewer junctions. However, each junction here is pulsing more frequently than the junctions in circuit 1 because of the higher sine frequency. This higher speed should tend to reduce the operating margins. However, both the 3.3 mA smallest current range of all patterns and the 2.0 mA total operating range are significantly larger than those found for circuit 1. The improved performance over circuit 1 is due to better microwave and junction uniformity for the smaller array, the additional lower cutoff frequency filter, and the grounded-array circuit configuration.

With regard to demonstrating the flatness of the synthesized steps, we measured a single voltage step of a resistively filtered circuit with 1000 junctions, a 9-GHz sine wave, and a 2-GHz clock frequency [9]. The flatness of the −7.4442 mV step was determined by repeatedly measuring the voltage for this particular code at different bias currents on the step. The bias current was varied over a range of 1 mA. The difference in average voltages obtained over many interleaved low, optimum, and high bias current measurements was 2.6 nV with a type A standard uncertainty of 1.6 nV. Over the measured current range, the step is flat to within 7 parts in 10^6 at a 95% level of confidence ($k = 2$). We have not yet performed an extensive flatness test with the new filtered circuits, but we anticipate that the larger operating margins will yield similar or better results.



(a)



(b)

Fig. 3. Bias current operating range versus voltage showing the current range for 101 digitally synthesized patterns of a 100-bit code. The region between the horizontal lines indicates the common operating current range of all 101 patterns. (a) Operating range for 50 Ω-terminated circuit 1 with 4096 junctions. (b) Operating range for grounded-array circuit 2 with 250 junctions.

We also measured the linearity of the 101 synthesized voltages using the two circuits with new filters. We used the 100-mV range of the digital voltmeter and fixed the bias current for all voltages in the center of the total operating current range. The sequence of measurements chosen was +V1, −V1, 0, −V1, +Vl, +V2, −V2, 0, −V2, +V2, etc. The value recorded for each voltage was the average of ten readings, each with ten power line cycles. The 0 V measurements were used to ascertain any offset voltage drift (including thermal). However, the offset drift was found to be negligible. A separate linear fit was used for the positive and negative voltages. Fig. 4 shows the resulting linearity of the voltages synthesized using circuit 2. The scatter in the data is dominated by noise in the digital voltmeter. The resulting linearity curve is within the manufacturer's 55 nV to 66 nV A/D linearity specifications.

We similarly measured the linearity on circuit 1 which produces larger voltages. Surprisingly, this circuit yielded a pattern-dependent offset voltage of 5 μV to 7 μV that peaked at the output voltages of ±32 mV, half the peak voltage range. The 100-bit patterns used were chosen for ease of programming, but have a significant frequency component at 30 MHz and its harmonics. The amplitude of the 30-MHz tone becomes a maximum at half the peak voltage. We also discovered that the digital voltmeter has a frequency dependent offset for both common-mode and differential configurations. The offset is frequency dependent and for the differential connection it peaks at 28 MHz and 56 MHz with a bandwidth of a few megahertz. In

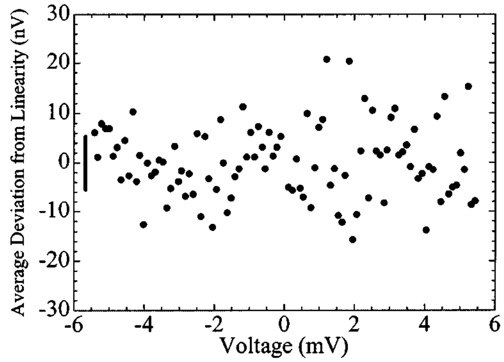


Fig. 4. Linearity measurement for 100-bit code. The deviation of each voltage measurement from the least square fit is plotted in nanovolts. The vertical bar shows a deviation of 2 parts in 10^6 of the maximum 5.4 mV.

general, the harmonic content for dc voltage waveforms can be reduced both by generating codes with the same modulator used for synthesizing ac waveforms and by using longer codes. For the ac–dc comparisons below, we have used the modulator algorithm for synthesizing codes with low harmonic content.

III. AC Voltage Measurements

We synthesized ac waveforms using circuits 1 and 2. They both demonstrated low harmonic distortion over a wide range of all bias parameters. The harmonic spectra for circuit 1 are easiest to measure because of its larger output voltage. The best result is a 60 mV peak amplitude, 2.86 kHz sine wave with second and higher in-band harmonic amplitudes all at least 94 dB below the fundamental [7]. Unfortunately, only circuit 2, with ten times lower output voltage, could be used for ac–dc comparisons because of its grounded configuration.

Sine amplitudes of 0.95 times the maximum step voltage give rms voltages of approximately 3.65 mV for circuit 2. We performed ac-dc comparisons using a commercial, amplifier-aided, thermal transfer standard that uses a differential thermal converter to provide a dc signal proportional to the rms voltage of the input signal. Five ac waveforms were synthesized from 1 kHz to 50 kHz, each with a 3 000 064 bit code length. Each sine wave was compared with +dc and −dc waveforms, each having 93 752 bits. All waveforms have nearly the same 3.65 mV rms amplitude.

The measured differences in the rms values between the five synthesized frequencies and the equivalent synthesized dc signals are shown in Table I. At 1 kHz, 3 kHz, and 10 kHz the data from both the Josephson synthesized source and a routine NIST calibration of the transfer standard agree within the uncertainties [10]. However, the data at 20 kHz and 50 kHz appear to be offset. Possibilities for this deviation at high frequencies include on-chip and off-chip transmission line errors, such as standing waves, input/output coupling, and inductance. The Type B uncertainties for the synthesized source have not yet been determined, so they are not included in the Josephson source data of Table I. Further investigation is needed to ascertain the source of the error at high frequency and the Type B uncertainties for the synthesizer.

The ac–dc difference measurements were slow because three waveforms were swapped for each frequency and because noise

TABLE I
MEASURED AC–DC DIFFERENCES OF A COMMERCIAL THERMAL TRANSER STANDARD. THE JOSEPHSON SOURCE DATA UNCERTAINTY IS TYPE A ($k = 2$). THE ROUTINE NIST CALIBRATION UNCERTAINTY INCLUDES BOTH TYPE A AND B UNCERTAINTIES ($k = 2$) [10] AT THE VOLTAGE OF 3.65 mV

Frequency (kHz)	Josephson Source (parts in 10^6)	NIST Calibration (parts in 10^6)
1	+59 ± 26	+90 ± 350
3	+135 ± 26	+100 ± 350
10	+160 ± 26	+130 ± 350
20	–195 ± 22	+140 ± 350
50	–2642 ± 20	+200 ± 350

TABLE II
MEASURED AC–AC (1 kHz) DIFFERENCES. UNCERTAINTIES ARE TYPE A ($k = 2$)

Frequency (kHz)	Directly measured (parts in 10^6)	Inferred from Table 1 (parts in 10^6)
3	+77 ± 0.7	+76 ± 37
10	+92 ± 0.9	+101 ± 37
20	–250 ± 0.8	–254 ± 34
50	–2700 ± 1.2	–2701 ± 30

on the dc measurements required many plus and minus comparisons. Fortunately, the code generator memory can rapidly switch between two long bit patterns. Thus, we made ac-ac comparisons, where the previously measured high-frequency sine waves were directly compared with the 1-kHz waveform. Table II shows the results of these directly synthesized comparisons along with the ac–ac comparison inferred from the ac–dc data of Table I. The two different methods demonstrate that the ac–ac and ac–dc comparisons are self-consistent. The ac–ac comparisons are less sensitive to thermal voltages and thermal voltage drift for these low-voltage measurements and much faster to complete using our synthesized source. We conclude from the results of Table II that the repeatability of the ac voltages is good and that thermal voltages are not a significant source of error in the ac–dc comparisons of Table I.

Finally, we tried to determine how accurately we can synthesize and resolve different rms voltages. We synthesized four 1-kHz sine waves in which the second harmonic tone added 20, 40, 60, and 100 parts in 10^6 to the total rms voltage. The difference in rms voltage of these waveforms compared to the rms voltage of the pure 1 kHz sine is shown in Table III. The measured differences are in agreement with the expected differences calculated from the Fourier transform of the digital code. These comparisons demonstrate that we have precise control of our synthesized rms voltages and their harmonic content.

TABLE III
MEASURED AC–AC DIFFERENCES FOR 1 kHz SINE WAVES WITH AN ADDED FINITE SECOND HARMONIC AMPLITUDE OF 20×10^{-6}, 40×10^{-6}, 60×10^{-6}, and 100×10^{-6}. UNCERTAINTIES ARE TYPE A ($k = 2$)

Expected rms voltage difference (parts in 10^6)	Measured rms voltage difference (parts in 10^6)
20	20 ± 3.4
40	43 ± 3.0
60	63 ± 3.6
100	101 ± 3.9

IV. CONCLUSION

We have made progress toward establishing the ac and dc accuracy of the Josephson arbitrary waveform synthesizer. Improved operating margins have enabled the first ac meteorology measurements of this device. These first measurements have demonstrated the usefulness of a stable calculable ac voltage source.

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