Progress in holographic model of QCD

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Outline

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- Energy scales
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Introduction

We don't know the holographic dual of QCD.

What about holographic dual of large N QCD? Even that is problematic! Strings can be solved in the regime where there is no asymptotic freedom (coupling doesn't have a chance to run to small values)

We will consider such a model. [Sakai,Sugimoto]

Introduction

Why bother? Many questions in QCD are hard to answer because the theory is strongly coupled in the IR. Sometimes lattice is hard to do (transport properties, chemical potential). String theory provides an independent technology!

On the other hand, gauge/string duality works both ways: gravity/string physics in the bulk is related to gauge theory physics. Can understand gravity/strings better?

Perhaps can get universal or nearly universal quantities?

Holographic model of QCD

Brane construction: N $D4$-branes spanning $x^0 \dots x^4$ $N_f \ll N_c$ $D8 - \bar{D}8$ pairs spanning every coordinate but $\tau = x^4 \in [0, 2\pi R_4)$. The x^4 circle has antiperiodic boundary conditions for fermions.

Theory at energies much smaller then $1/l_s$: large N 5d Yang-Mills with N_f 4d quarks. Theory at energies much smaller then $1/R_4$: large N QCD with quarks. No adjoint fields, conformal symmetry or supersymmetry!

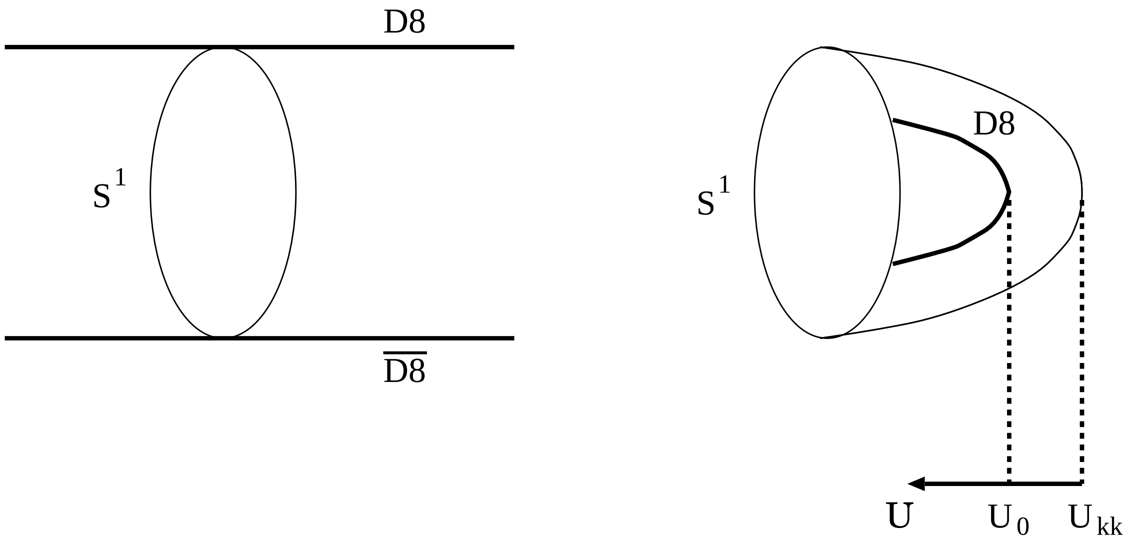
Holographic model of QCD

Parameters: $\lambda = g_{YM}^2 N = g_s l_s$; 4-dimensional t'Hooft coupling at scale $1/R_4$ $\lambda_4 = \lambda/R_4$; brane-antibrane asymptotic separation L.

$$\Lambda_{QCD} = \frac{1}{R_4} e^{-\frac{1}{\lambda_4}}$$

so we have good approximation to QCD in the regime $\lambda_4 \ll 1$. Holographic (stringy) dual is solvable in the opposite regime $\lambda_4 \gg 1$. Yang-Mills=gravity; quarks=Dirac-Born-Infield (DBI) action for D8 branes

Holographic model of QCD



Gravity dual has a form $R^{3,1} \times S^1_{x^4} \times R_{+(U)} \times S^4$. In the $U - x^4$ plane geometry has a cigar-like form. $D8 - \bar{D}8$ branes connect in the IR (small U), hence chiral symmetry is broken.

Holographic model of QCD

Metric:

$$ds^2 = \left(\frac{U}{R}\right)^{\frac{3}{2}} \left((dx_\mu)^2 + f(U)(dx^4)^2\right) + \left(\frac{U}{R}\right)^{-\frac{3}{2}} \left(\frac{dU^2}{f(U)} + U^2 d\Omega_4^2\right)$$

where $f(U) = 1 - U_K^3/U^3$.
$U = U_K$ bounds U from below; determined via
$2\pi R_4 = 4\pi R^{3/2}/3U_K^{1/2}$

Dilaton is $e^\Phi = \left(\frac{U}{R}\right)^{\frac{3}{4}}$
Similarly to AdS_5, $R^3 = \pi\lambda_5$

Holographic model of QCD; matter

Fundamental matter comes from N_f $D8 - \bar{D}8$ pairs with asymptotic separation L. Their shape is determined by solving DBI equations of motion.

Results: branes are connected at $U = U_0$; chiral symmetry is broken; pion lagrangian can be derived. U_0 and L are related: $L = R^{3/2}/U_0^{1/2}$

Fluctuations of the flavor branes give rise to scalar and vector mesons. Baryons are described by the $D4$ branes wrapping the S^4. Strings are streched to the flavor branes and source the electric field dual to the baryon current.

Energy scales

Spectrum of mesons is studied using DBI action. At high energy, WKB can be used to show

$$m_n = \frac{\mathcal{K} n}{L}$$

Glueball masses are set by $1/R_4$. Anticipate: $1/R_4$ is the scale of confinement/deconfinement and $1/L$ of chiral phase transition.

Finite temperature; Yang-Mills

Compactify euclidean time t_E with asymptotic circumference $\beta = 1/T$. There are 2 geometries which asymptote to this:

- (1) $(x^4, U > U_K)_{cigar} \times t_E$
- (2) $(t_E, U > U_T)_{cigar} \times x^4$

The Lorenzian version of (2) is a black hole (BH); it has a horizon. The geometries really are the same; smaller cigar dominates partition function.

For $T < 1/2\pi R_4$ (1) dominates; gluon confining state
For $T > 1/2\pi R_4$ (2) (BH) dominates; gluon deconfinement

Finite temperature; matter

In the gluon confining phase $D8$ and $\bar{D}8$ branes connect and chiral symmetry is broken. In the following we consider gluon-deconfining phase. There are two possibilities:

- They connect as before (curved)
- The go straight into the BH horizon (straight)

curved branes= chiral symmetry is broken; straight branes=chiral symmetry is restored.

Analyse DBI action to find thermodynamically preferred configuration. 1st order phase transition occurs at $T = 1/L$. [A.P., D.Sahakyan; O.Aharony etal]

Some formulas

$$S = 2\int dU U^{\frac{5}{2}} \sqrt{1 + f_T(U)\left(\frac{U}{R}\right)^3 (\partial_U X^4)^2}$$

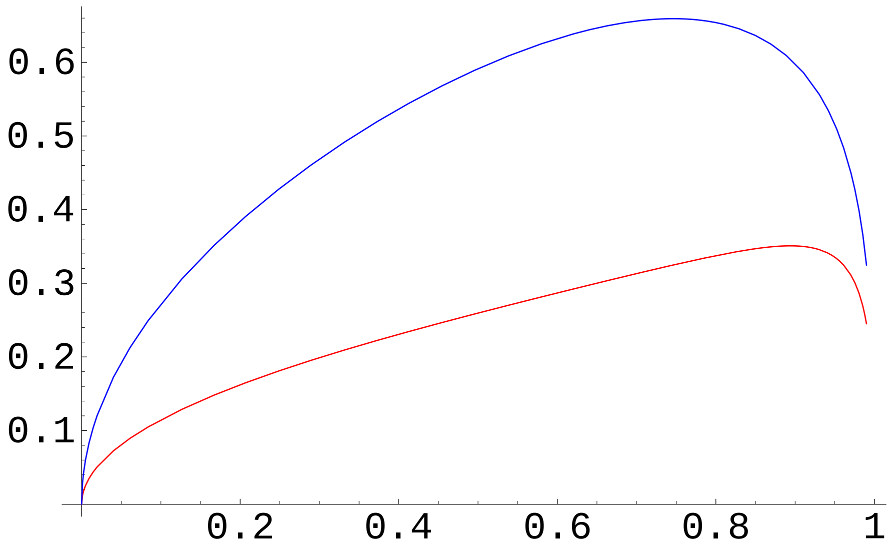
where $f_T(U) = 1 - U_T^3/U^3$, $U_T \sim \lambda T^2$
Equation of motion:

$$(\partial_U X^4)^2 = \frac{U_0^8 R^3 f_T(U_0)}{U^{11} f_T^2(U) - U_0^8 U^3 f_T(U_0) f_T(U)}$$

$U = U_0 \sim \lambda_5/L^2$ is the “turning point” of the brane,

$U > U_0 > U_T$

Calculation of LT



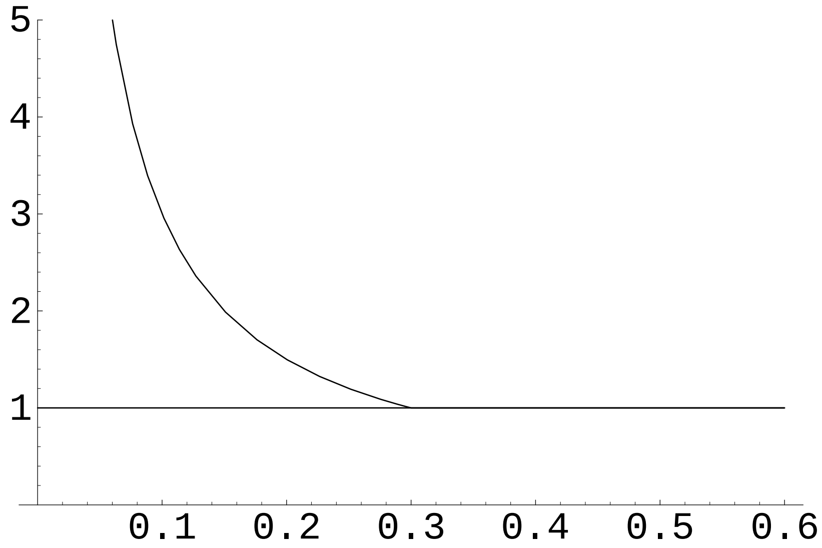
This is $2\pi LT/3$ as a function of (roughly) the position of the brane tip, U_0. Red line $\mu_I = 0$, blue line $\mu_I \to \infty$.

There're two curved solutions, but the one which connects to the vacuum always dominates. (But the straigt brane dominates for large enough LT)

Results at finite T

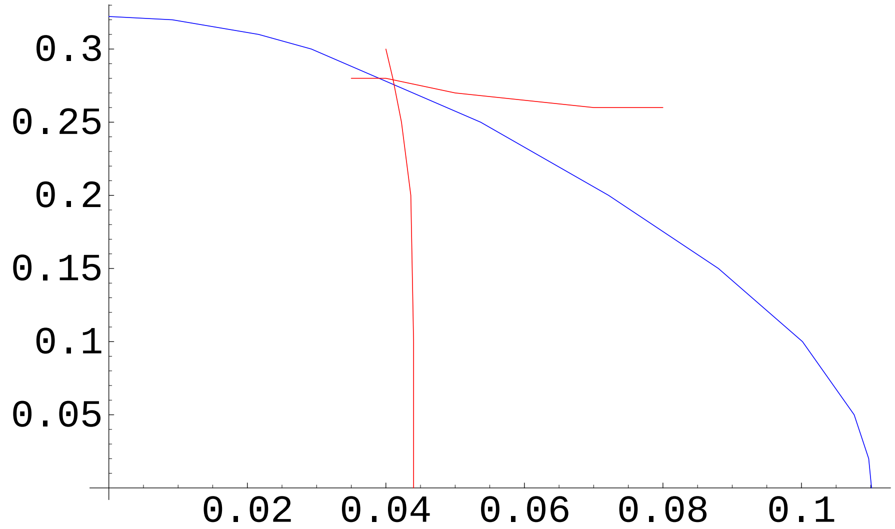
First order chiral phase transition happens at

$$T_c = \frac{0.15}{L} = \frac{0.3}{2\pi R_4}\frac{1}{L/\pi R_4}$$



$2\pi R_4 T_c(x)$ where $x = L/\pi R_4$

Finite T and μ_B



Baryon density corresponds to electric flux on the branes. Both quarks and baryons can be the sources. There are vacuum, quark and baryon matter phases at finite T and μ_B [Horigome, Tanii; A.P., D.Sahakyan; O.Bergman etal]

Instantons and θ-dependence

Field theory: in the confining phase

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_\mu \eta')^2 + \frac{N^2}{2}\chi_g \left(\frac{\theta}{N} + \frac{1}{N}\frac{2\sqrt{N_f}}{f_\pi}\eta' \right)^2$$

consistent with θ/N dependence and Veneziano-Witten formula

$$m_{\eta'}^2 = \frac{4N_f}{f_\pi^2}\chi_g$$

Instantons and θ-dependence

In the deconfined phase

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_\mu \eta')^2 + a\left(\theta + \frac{2\sqrt{N_f}}{f_\pi}\eta'\right)^2$$

the potential term is generated by instantons:

$$a \sim \mathcal{O}(e^{-N})$$

Instantons and θ-dependence

In the holographic model the situation is the same! Theta angle is defined as

$$\theta = \int_{S^1} C_1 \; mod \; 2\pi k$$

In the confining phase Stokes theorem leads to $\chi_g \sim \mathcal{O}(1)$. [Witten]

In the deconfining phase $\chi_g \sim 0$; instantons are well defined objects; $m_{\eta'} \sim e^{-N}$ [O.Bergman, G.Lifschytz]

Conclusions:

- Holographic model of QCD describes chiral symmetry breaking/restoration, baryons
- Phase diagram of holographic model exhibits similarities and differences with that of QCD
- Confinement/deconfinement phase transition leads to melting of the instantons and change in θ-behavior