PQCD APPLICABILITY TO EXCLUSIVE PROCESSES AT THE INTERMEDIATE ENERGIES

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ABSTRACT: The PQCD applicability to exclusive processes still remains as one of the main issues in upgrading the continuous electron beam energy at Thomas Jefferson National Accelerator Facility (TJNAF). We discuss the correct criterion to define the "legal" PQCD contribution to the exclusive processes and point out the important contribution from highly off-energy-shell gluons at equal light-cone time. This indicates that the criterion based on gluon four-momentum-square cut may have excluded too much of the "legal" PQCD contributions.

It has long been discussed whether the perturbative QCD (PQCD) is applicable to exclusive processes at currently available experimental energies [1–6]. While the qualitative agreement is remarkable between the quark counting rule predicted by the PQCD and the available experimental data of various hadronic form factors with Q^2 larger than a few GeV^2/c^2, the pertinent critique is mainly due to a seeming large quantitative disagreement between the PQCD prediction and the normalization of these form factors. In view of the current trend of upgrading the electron beam energy at TJNAF[7], therefore this issue needs to be further clarified.

The typical criticism on the PQCD applicability can be found in the example of pion and proton form factors[2]. For the main critique, it has been argued that in order for the perturbation to be consistent, the invariant mass (*i.e.* the four momentum square) of the exchanged gluon, k_g^2, has to be larger than a typical hadronic scale $\mu^2 \lesssim 1$ GeV$^2/c^2$. Thus, the "legal" PQCD contribution was defined by requiring $k_g^2 > \mu^2$. If μ^2 was taken to be 1 GeV$^2/c^2$, then the "legal" PQCD contribution was too small to compare with the currently available experimental data[2]. However, there is not yet a consensus on which value of μ^2 should be taken to define the "legal" PQCD. Furthermore, the ambiguity of the scale $\tilde{Q}^2$ for the argument of the QCD running coupling constant $\alpha_s(\tilde{Q}^2)$ and the renormalization scheme dependence in the PQCD expansion add more uncertainty to the criticism on PQCD [5, 8]. These issues are quite delicate because the hard scattering amplitude in the leading order PQCD are very sensitive to the values of μ^2 and $\tilde{Q}^2$.

In order to analyze this issue more clearly, we note that the factorization of the covariant hard scattering amplitude from the non-perturbative quark distribution amplitude is originated from the lightcone quantization method of the QCD Fock state expansion [1]. It is well known that the covariant Feynman diagram corresponds to the sum of lightcone time-ordered diagrams. In fact, the hard scattering amplitude T_H in the factorization formular of form factor can be derived from the sum of leading twist lightcone time-ordered diagrams as shown in Ref.[9] for the pion form factor. Including the higher twist effects, however, the usual factorization is no longer applicable and thus one has to rely on the lightcone time-ordered perturbation for the short-distance scattering amplitude of quarks and gluons. The lightcone bound-state equation should also be used for the wavefunction of hadron. For the pion form factor, in the Drell-Yan-West frame[10], it is given by

$$F_\pi(Q^2) = \int dx dy d^2\vec{k_\perp} d^2\vec{l_\perp} \psi(x,\vec{k_\perp}) T(x,y,\vec{k_\perp},\vec{l_\perp},\vec{q_\perp}) \psi(y,\vec{l_\perp}), \qquad (1)$$

where $\psi(x,\vec{k_\perp})$ is the lightcone wavefunction of the two-body Fock state and $T(x,y,\vec{k_\perp},\vec{l_\perp},\vec{q_\perp})$ is obtained by the two-body irreducible diagrams [9]. The main issue here is, however, the criterion of short-distance physics in the lightcone time-ordered perturbation. The scattering amplitudes in the lightcone time-ordered perturbation consist of the numerators from the matrix elements of interaction Hamiltonian for the scattering and the lightcone energy denominators from the intermediate states. Thus, in order for the perturbation to work, not only the order parameter α_s of numerators should be small but also the energy denominators should be large enough for the perturbative expansion to converge. In fact, the energy denominators correspond to the energy uncertainty of the intermediate states,*e.g.* $(\Delta E)_{LC}$. From the time-energy uncertainty relation, the uncertainty in the lightcone time τ is given by $\Delta\tau \sim \frac{1}{(\Delta E)_{LC}}$. Thus, the large $(\Delta E)_{LC}$ corresponds to the small $\Delta\tau$ or the short τ evolution of the system. This is consistent with the observation that the system can be treated perturbatively while the system didn't evolve too much in time. The short τ evolution also corresponds to the short lightcone distance scale. Furthermore, the numerator of scattering amplitude can be expanded with the order parameter of the frozen coupling constant[11] which is free from the divergence at the small momentum transfer region. Therefore, the essential criterion for the short-distance physics should be the large off-shellness of the lightcone energy rather than the large four-momentum square of exchanged gluon.

In the explicit example of the pion form factor calculation using the lightcone perturbation theory [9], the change of criterion for the "legal" PQCD from $k_g^2 > \mu^2$ to $P^+(\Delta E)_{LC} > \mu^2$ makes a large difference in the Q^2 domain saturated by the "legal" PQCD. As shown in Ref.[9], the large difference between the four-momentum square cut and the lightcone energy cut comes from the contribution in the region of $1 - \frac{\mu}{Q} < x \sim y < 1$. While this region is certainly near to the end points of the quark distribution amplitudes, the gluons in this region are highly off-energy-shell. More recently, Li and Sterman [4] has shown that by including the Sudakov effect, PQCD calculation can be made even more self-consistent at much lower Q^2 values.

However, by now, it seems that at least one consensus regarding on the normalization of pion elastic form factor has been reached, *i.e.* the PQCD prediction for $F_\pi(Q^2)$ is smaller than the present available data[5, 6] even though how small is still not yet completely agreed within the community. We recently used the BLM method[8] to fix the renormalization scale of the QCD coupling in exclusive hadronic amplitudes such as the pion form factor and the photon-to-pion transition form factor at large momentum transfer[5]. The commensurate scale relation connecting the heavy quark potential, as determined from lattice gauge theory, to the photon-to-pion transition form factor was in excellent agreement with $\gamma e \to \pi^0 e$ data assuming that the pion distribution amplitude is close to its asymptotic form $\sqrt{3} f_\pi x(1-x)$. We also reproduced the scaling and normalization of the $\gamma\gamma \to \pi^+\pi^-$ large momentum transfer data. Because the renormalization scale is small, we argue that the effective coupling is nearly constant, thus accounting for the nominal scaling behavior of the data. However, the normalization of the space-like pion form factor $F_\pi(Q^2)$ obtained from electroproduction experiments was somewhat higher than that predicted by the corresponding commensurate scale relation. This discrepancy in normalization may be due to systematic errors introduced by the extrapolation of the $\gamma^* p \to \pi^+ n$ electroproduction data to the pion pole.

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