

Pentaquark in K^+d total cross section data

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An analysis of K^+d total cross section data is undertaken to explore possible effects of the recently observed resonance in the $S=+1$ hadronic system with mass around 1.55 GeV. It is found that a structure corresponding to the resonance is visible in the data. The width consistent with the observed deviation from background is found to be 0.9 ± 0.3 MeV and the mass is 1.559 ± 0.003 GeV/ c^2 for spin parity $\frac{1}{2}^+$ and 1.547 ± 0.002 GeV/ c^2 for $\frac{1}{2}^-$. The errors are one standard deviation and statistical only.

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I. INTRODUCTION

The theoretical study of the structure of a resonant state of five quarks largely began with a paper by Strottman [1]. He studied systems in which all quarks are in an s state and found the lowest lying state with strangeness +1 to have spin parity $\frac{1}{2}^-$ and a mass of around 1.7 GeV with an estimated error on the mass of 50 MeV. If more modern values of parameters (in particular the strange quark mass around 150 MeV instead of 279 MeV) were used, the estimate of the pentaquark mass could be smaller.

Recently, Diakonov, Petrov, and Polyakov [2] were able to use the presumed identification of a known nucleon excited state in the antidecuplet to predict the mass of the isosinglet member with strangeness +1 to be about 1.530 GeV. They suggested that this resonance would have a small width (≤ 15 MeV). This prediction led to a number of experimental studies which, in turn, led to the apparent discovery [3–8] of a particle with about the right mass, strangeness +1, and very probably isoscalar. It remains to identify the spin and parity of the observed particle, expected to be $\frac{1}{2}^+$ from this prediction. The validity of the soliton model used in this prediction has been questioned [9].

The question is naturally raised as to why this particle was missed in the searches that were done decades ago in the direct scattering of positive kaons from hadronic systems. Since one must use an incident beam of K^+ mesons and a neutron target (to have strangeness +1 and isospin zero) K^+d scattering is studied.

The answer to the question probably lies, at least partly, in the very small width of the particle which appears to be emerging from studies. The discovery experiments mentioned above are limited by their experimental resolution so that the best they can say is that the width is less than 9 MeV [4]. Nussinov [10] estimated from the Fermi momentum of the deuteron that the resonance must have a width of less than 6 MeV in order not to have been observed. Arndt *et al.* [11] searched the data base and concluded that the resonance must have a width of the order of 1 MeV or less to have escaped notice.

Cahn and Trilling [12] calculated the width (from the discovery experiment with a K^+ beam which observed the resonance in the charge exchange channel [4]) to be 0.9 ± 0.3 MeV. They also compared a linear background with

the same total cross section to be used later in this work and observed a two standard deviation excess, but simply interpreted this as an upper limit on the width of 0.8 MeV. They used a Hulthén form of the deuteron wave function to calculate the Fermi momentum correction and did not consider the effect of double scattering or interference with the background phase shift.

The present work seeks to investigate carefully the signal to be expected in the K^+d total cross section data, given the above information. It is found that, once the proper corrections are taken into account to give the expected background, the signal is indeed observed and independent determinations of the width and mass can be obtained.

The two principal corrections necessary are the inclusion of K^+ double scattering and the neutron Fermi momentum in the deuteron. Until now, double scattering corrections for the extraction of K^+ amplitudes from the deuteron have been used only at higher energies [13], even though its importance at low energies has been known for some time [14]. Section II treats this subject. The Fermi momentum of the nucleon in the deuteron has been measured [15] and hence can be dealt with rather accurately. Section III treats the averaging of the amplitude over this momentum spread. Section IV deals with the extraction of the background phase shifts from the proton and deuteron target data.

The studies presented here will be treated in the usual isospin formalism and the reader is reminded of the relation between the charge and isospin amplitudes:

$$A_{K^+p} = T_1, \quad A_{K^+n \rightarrow K^+n} = \frac{1}{2}(T_1 + T_0), \quad A_{K^+n \rightarrow K^0p} = \frac{1}{2}(T_1 - T_0). \quad (1)$$

II. DOUBLE SCATTERING

Double scattering has a special role in its contribution to the total cross section for scattering from a multinucleon system at low energies because of unitarity constraints in the zero-energy limit. To illustrate this point, we first look at scattering from a simple two-body system, not very different from the deuteron.

A. Low energy–weak scattering limit

Consider double scattering from a lightly bound two-body system, ignoring the possibilities of spin and charge exchange. Take the the phase shifts to be represented by their low-energy limiting form

$$\delta_a(k) = ak, \quad \delta_b(k) = bk. \quad (2)$$

Since we are also considering weak scattering, a and b are also considered to be small.

In the single scattering approximation (which one might think is appropriate for small a and b) and setting the bound-state form factor to unity since we are considering the low-energy limit, the amplitude for scattering is

$$f = \frac{1}{2ik}(e^{2iak} - 1 + e^{2ibk} - 1) \rightarrow a + b + ik(a^2 + b^2) + \dots, \quad (3)$$

so the elastic amplitude and cross section are, in the threshold limit

$$f_e = a + b, \quad \sigma_e = (a + b)^2. \quad (4)$$

The integral of the elastic cross section gives the total cross section

$$\sigma_T = 4\pi(a + b)^2 = 4\pi(a^2 + b^2) + 8\pi ab, \quad (5)$$

since only elastic scattering is possible below the breakup threshold.

From the optical theorem, the total cross section is

$$\sigma_T = \frac{4\pi}{k} \text{Im} f(0) = 4\pi(a^2 + b^2). \quad (6)$$

We see that the bilinear term in a and b in Eq. (5) is missing and the optical theorem might appear to break down. It is, however, the single scattering assumption which is at fault and the resolution of this seeming discrepancy is through double scattering.

The double scattering amplitude is given by [16]

$$f_D(\mathbf{k}, \mathbf{k}') = \frac{1}{2\pi^2} \int \frac{d\mathbf{q} f_b(\mathbf{q}, \mathbf{k}') f_a(\mathbf{k}, \mathbf{q})}{q^2 - k^2 - i\epsilon} z\left(\frac{1}{2}(\mathbf{k} + \mathbf{k}') - \mathbf{q}\right), \quad (7)$$

where \mathbf{k} and \mathbf{k}' are the initial and final (on-shell) momenta of the scattering meson, $z(\mathbf{p})$ is the two-body form factor, and $f(\mathbf{k}, \mathbf{q})$ and $f(\mathbf{q}, \mathbf{k}')$ are half-off-shell basic scattering amplitudes.

For s -wave scattering we write the off-shell dependence of the amplitude as

$$f(\mathbf{q}, \mathbf{q}') = f_0 v(q) v(q'), \quad v(k) = 1, \quad (8)$$

where the form

$$v(q) = \left(\frac{k^2 + \Lambda^2}{q^2 + \Lambda^2} \right)^2 \quad (9)$$

is assumed. For the limit we are considering in this section the form is irrelevant but it is needed in the following section.

In the lowest order in a and b , the contribution of double scattering to the forward amplitude becomes

$$f_D(\mathbf{k}, \mathbf{k}) \rightarrow \frac{ab}{2\pi^2} \int \frac{d\mathbf{q} v^2(q)}{q^2 - k^2 - i\epsilon} z(\mathbf{k} - \mathbf{q}), \quad (10)$$

so its contribution to the total cross section is

$$\begin{aligned} \sigma_T^D &= \frac{2ab}{k\pi} \text{Im} \int \frac{d\mathbf{q} v^2(q)}{q^2 - k^2 - i\epsilon} z(\mathbf{k} - \mathbf{q}) \\ &= \frac{2ab}{k\pi} \pi \frac{k^2}{2k} \int d\Omega \rightarrow 4\pi ab \end{aligned} \quad (11)$$

since $z(0) = 1$. With the factor of 2 which comes from the two orders of scattering, the missing bilinear term is found. The result is independent of the off-shell form factor; only the on-shell scattering is needed.

We see that for a and b equal, the double scattering contributes half of the total cross section at threshold. Even when they are not exactly equal the contribution remains a significant fraction of the cross section.

B. Realistic case

For the application to the present case this analysis needs several corrections. First, the scattering lengths are small, but not so small that corrections can be neglected. Hence, the amplitude is not purely real which means that the principal value of the integral gives a contribution and the off-shell form factor plays a role. Second, charge exchange ($K^+n \rightarrow K^0p$ and its inverse) is possible. While single charge exchange does not lead to elastic scattering, and so does not contribute to the forward amplitude, double charge exchange does. The double charge exchange is not a small correction, the factor $2ab$ being replaced by

$$2f_p f_n - f_x^2 \quad (12)$$

where f_p , f_n , and f_x are the proton, neutron, and charge exchange scattering amplitudes. The factor of 2 comes from the two orders of scattering, the charge exchange having only one possible order. The minus sign is due to the isospin zero nature of the deuteron.

Included also is the sp -wave ($S_{11} \times P_{01}$) double scattering (on shell only) which contributes a small negative correction at the upper end of the momentum range in question. The notation L_{I2J} , where L is either S or P , I is isospin, and J is total angular momentum of the partial wave, is used. The charge exchange considerations are the same as above.

Figure 1 shows the s -wave– s -wave part of the double scattering as used in this analysis for three typical values of Λ as well as the purely on-shell contribution. While the differences due to the off-shell form factor are visible, the result is not very sensitive to the value of Λ chosen.

III. CORRECTION FOR FERMI MOTION IN THE DEUTERON

The momentum distribution of the nucleon in the deuteron has been measured [15], and these data have been parametrized [17] as

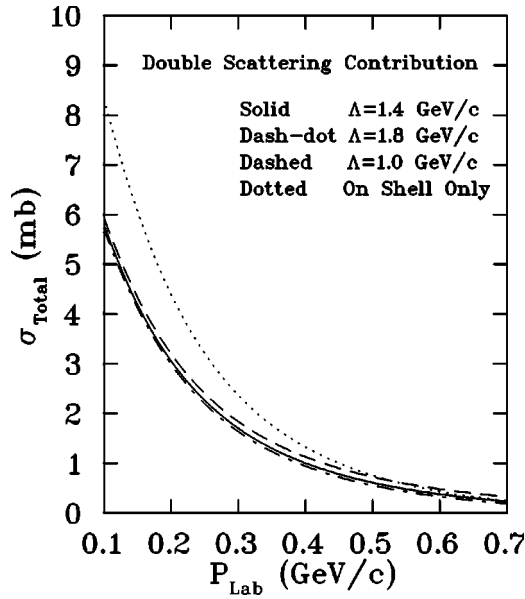


FIG. 1. Double scattering contribution to K^+d scattering. The common value of Λ used for the dipole form factor employed in this work is around 1.4 GeV/c.

$$f(p) = 1.7716 \times 10^{-5} p^2 \{ e^{-0.00063p^2} + 0.201 e^{-0.026p} + 0.0119 e^{-(p-77.7)/38.8} \} \quad (13)$$

with p in MeV/c. Here, $f(p)$ is the probability distribution function of the magnitude of the momentum, p . Notice that the p^2 from the volume element is included in $f(p)$ so that

$$\int_0^\infty dp f(p) = 1. \quad (14)$$

Figure 2 shows the data (renormalized to have integral

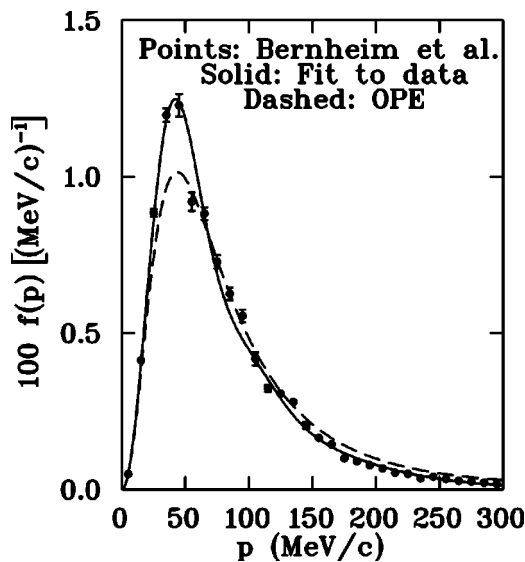


FIG. 2. The measured Fermi momentum distribution compared with the parametrization and that predicted from the one-pion-exchange deuteron.

unity) compared with the parametrization. Also shown is the prediction of the square of the momentum-space wave function of the deuteron obtained from the solution of the Schrödinger equation with a one-pion-exchange potential [18]. This deuteron wave function has been shown to reproduce to a good approximation all of the low-energy observables [19,20].

In order to evaluate the scattering matrix in the case in which there is both a background and a resonant phase, we take the following form:

$$S^b(\epsilon) = e^{2i\delta_b(\epsilon)}, \quad S^R(\epsilon) = \frac{\epsilon - M - i\Gamma/2}{\epsilon - M + i\Gamma/2} = e^{2i\delta_R(\epsilon)},$$

$$\delta_R(\epsilon) = \tan^{-1} \frac{\Gamma}{2(M - \epsilon)}, \quad (15)$$

where $\epsilon = \sqrt{s}$ and $S^b(\epsilon)$ and $S^R(\epsilon)$ are the background and resonant forms of the S matrix.

The total S matrix including both the background and the resonance is written as

$$S(\epsilon) = S^b(\epsilon)S^R(\epsilon). \quad (16)$$

While this is the form standardly used, a discussion of the representation of the S matrix in a product form may be found in Ref. [21].

For a kaon with momentum \mathbf{k} incident on a neutron in the deuteron with Fermi momentum \mathbf{p} , the square of the invariant mass of the kaon-neutron system is given by

$$s = (\sqrt{\mu^2 + k^2} + \sqrt{m^2 + p^2})^2 - (\mathbf{k} + \mathbf{p})^2$$

$$= \mu^2 + m^2 + 2\sqrt{\mu^2 + k^2}\sqrt{m^2 + p^2} - 2\mathbf{k} \cdot \mathbf{p} \quad (17)$$

where μ and m are the kaon and neutron masses. Due to axial symmetry, s is independent of the azimuthal angle and $\mathbf{k} \cdot \mathbf{p} \equiv kpx$.

For the case of a given isospin I and only s and p waves, we may write the total cross section as

$$\sigma_I(\epsilon) = \frac{2\pi}{k_{c.m.}^2} \text{Re}[(1 - S_{I_{s1/2}}) + (1 - S_{I_{p1/2}}) + 2(1 - S_{I_{p3/2}})]$$

$$\equiv \frac{2\pi}{k_{c.m.}^2} g_I(\epsilon). \quad (18)$$

The average over the Fermi momentum distribution will give the observed cross section

$$\langle \sigma_I(k) \rangle = \frac{\pi}{k_{c.m.}^2} \int_{-1}^1 dx \int_0^\infty dp f(p) g_I[\epsilon(k, x, p)]. \quad (19)$$

The slowly varying factor $1/k_{c.m.}^2$ has been factored out.

It is assumed that only one term will be resonant in Eq. (18). Its contribution to the total cross section will be

$$\frac{2\pi}{k_{c.m.}^2} \{1 - \cos 2[\delta_b(\epsilon) + \delta_R(\epsilon)]\}, \quad (20)$$

which will be zero for some value of ϵ when the sum of the phase shifts is zero or π . Since the background phase will normally have a magnitude smaller than π , if it is negative, the zero will come before the true mass (when the sum is

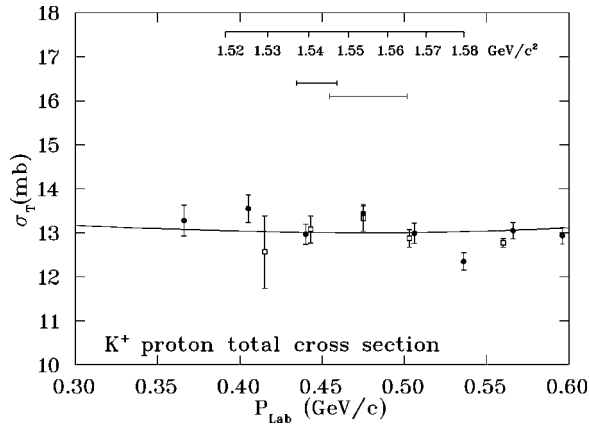


FIG. 3. K^+ proton total cross section compared with the fit used here. The solid points are from Bowen *et al.* [24] and the open points are from Carroll *et al.* [25].

zero) and if it is positive the zero will come after the true mass (when the sum is π). Thus, for a negative background phase the visible peak will occur at a higher energy than the mass and with a positive background phase it will occur at a lower value. While the Fermi averaging will smooth this behavior so that there is no longer a zero, a shift of the peak from the true value of the mass remains and is increased due to the positive and negative interference of the two phase shifts above and below the resonance.

IV. BACKGROUND PHASE SHIFTS

The phase shifts to be used in the analysis were obtained by fitting data with the corrections discussed above included

with an eye to what has been previously obtained in the literature. We will need $I=1$ and $I=0$ phase shifts for s and p waves. The phase shifts are expected to be very smooth (aside from the resonance, of course) so that the scattering length–scattering volume [$\delta(k)=vk^3$] forms are used in all cases.

A. $I=1$ phase shifts

The $I=1$ phase shifts are obtained directly from K^+p data. The total cross section is shown in Fig. 3. It is seen that its value is very nearly constant. There is no indication of a resonance, in agreement with the determination in Ref. [6] that the observed resonance is isoscalar. A nearly energy dependent cross section is in reasonable agreement with the simple representation of the phase shifts in Eq. (2). The cross section calculated with this form drops slightly below the data at the highest end of the current momentum range indicating that a small amount of p -wave contribution is needed. In Ref. [22], experiments measuring angular distributions in this region were reported. They found a very nearly isotropic angular distribution, aside from the Coulomb peak at forward angles. They were able to give an estimate of p -wave strengths, although inclusion of p waves did not decrease the χ^2 of their fit in almost all cases.

The results of the present analysis are shown in the top three panels of Fig. 4. The solid lines give the phase shifts used here. The scattering lengths and volumes are given in Table I. The s -wave phase shifts agree very well with Ref. [22] whose points are shown. Also shown are the results from the analysis of Hyslop *et al.* [23].

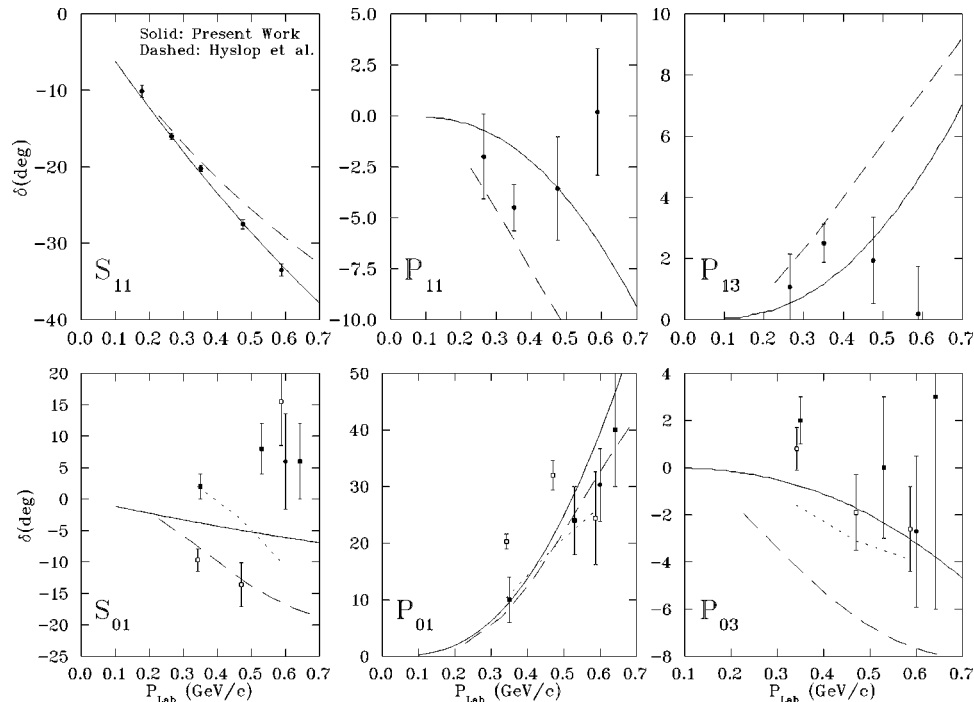


FIG. 4. Phase shifts obtained in this work from the s -wave fit (solid curves) compared with previously obtained values. The solid points for $I=1$ are from Ref. [22]. For $I=0$, the solid circles are from Ref. [28], the open circles are from Ref. [27], the solid squares are from Ref. [29], and the dotted curve is from the “C” fit by Ref. [30].

TABLE I. Scattering lengths and volumes for the s -wave fit. The p -wave fit values are in parentheses. Only the S_{01} and P_{01} values are different.

	$S_{1/2}$ (fm)	$P_{1/2}$ (fm ³)	$P_{3/2}$ (fm ³)
$I=1$	-0.328	-0.02	0.015
$I=0$	-0.06(0.00)	0.123(0.127)	-0.010

In summary, the $I=1$ s -wave scattering phase shifts are very well determined in this energy range and the p waves, although poorly determined, are small. They have often been entirely neglected in previous analyses. Aside from the s -wave scattering length, important for double scattering, the only part needed from the $I=1$ phases is the proton total cross section which could be taken directly as a parametrization of the data.

B. $I=0$ phase shifts

The $I=0$ phase shifts must be inferred from analysis of scattering from the deuteron, and hence are sensitive to the corrections introduced in Secs. II and III. The values determined here, and a summary of previous values, are shown in the lower panels of Fig. 4.

The most relevant data are the total cross sections [24–26] and the charge exchange differential cross sections [27]. The polarization data of Ray *et al.* [28] permit the determination of the sign of the p -wave phase shift. The data of Stenger *et al.* [29] were taken from angular distributions in a bubble chamber and are not as sensitive to the double scattering correction as the total cross section measurements.

For the total cross section data the eight points of Bowen *et al.* [24] in this momentum range are the most accurate in terms of individual errors with a precision of (1–2)%. The Carroll *et al.* [25] data are slightly less precise. While the Krauss *et al.* [26] data have larger error bars, they were taken with a view to obtaining the ratio to other nuclei and hence the normalization was a more important consideration. Comparing the three data sets it is seen that there is a normalization discrepancy among them. In order to bring the normalization into agreement with the Krauss *et al.* data, without changing the shape, the Bowen *et al.* data were renormalized by 1.06. This has almost no effect on the determination of the mass and width of the structure obtained later but does affect the value of the $I=0$ s -wave phase shift.

The dominant $I=0$ phase shift is in the P_{01} partial wave. It is reasonably well determined from the total cross section data at the upper end of the range considered here.

The single energy values of the S_{01} phase shift determined previously are scattered. Note that several of them (see Fig. 4 lower left panel) are zero or positive while others are significantly negative.

To determine the phase shifts to be used here, the double scattering and Fermi corrections were applied to proposed phase shifts derived from scattering lengths and volumes and then these scattering lengths and volumes were adjusted to fit the data at the high (top three points) and low (lowest two points) ends of the data set, avoiding the intermediate region

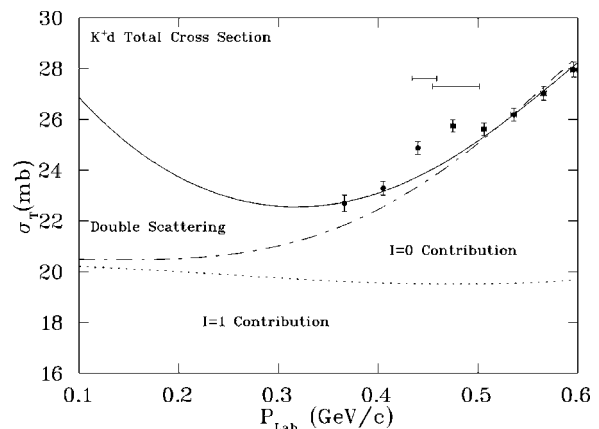


FIG. 5. Background fit used in this work. The highest and lowest momentum points were used. The solid points are from Ref. [24]. The dotted curve is only the $I=1$ contribution, the dash-dotted curve includes that plus the $I=0$ contribution, and the solid curve includes the double scattering as well. The horizontal bars indicate the range of masses from Refs. [7,8]. The ranges from all of the experiments can be found on Fig. 7.

where the resonance is expected. The Fermi correction was applied only to the scattering-volume form of the P_{01} wave since the P_{03} phase shift is very small and the S_{01} wave gives an energy-independent cross section and hence is not affected by Fermi averaging. The result and the various contributions are shown in Fig. 5. The solid curve is very similar to that obtained by Garcilazo from a fully relativistic Faddeev calculation [14].

The effect of the double scattering is to raise the cross section at low momenta, lessening the contribution from the s wave. The value of the scattering length used here is $a_0 = -0.06$ fm (0.00 in the case of the p -wave fit; see below) so it is nearly zero, more in agreement with the single energy values mentioned above.

The P_{01} partial wave obtained here agrees reasonably well with the result of Hyslop *et al.* [23]. The single energy values are fairly scattered. The P_{03} partial wave is not well determined but is very small.

V. RESULTS

It can be seen that the expected cross section obtained in the previous section (the solid curve in Fig. 5) falls well below (5σ) the data in the region where the resonance has been observed in Refs. [3–8], indicating the existence of a possible resonance effect.

The calculation of the cross section for the expected resonance is now made as a function of (a) the partial wave in which it should appear, (b) the width assumed, and (c) the mass assumed. Only partial waves S_{01} and P_{01} are considered. When the P_{01} partial wave was calculated, the fit to the background had to be redone so that the high and low momentum points were fitted with the resonance since the effect of the interference extends much further. A calculation for the P_{03} partial wave would give essentially the same result as for the S_{01} wave with a factor of 2 smaller width. If the

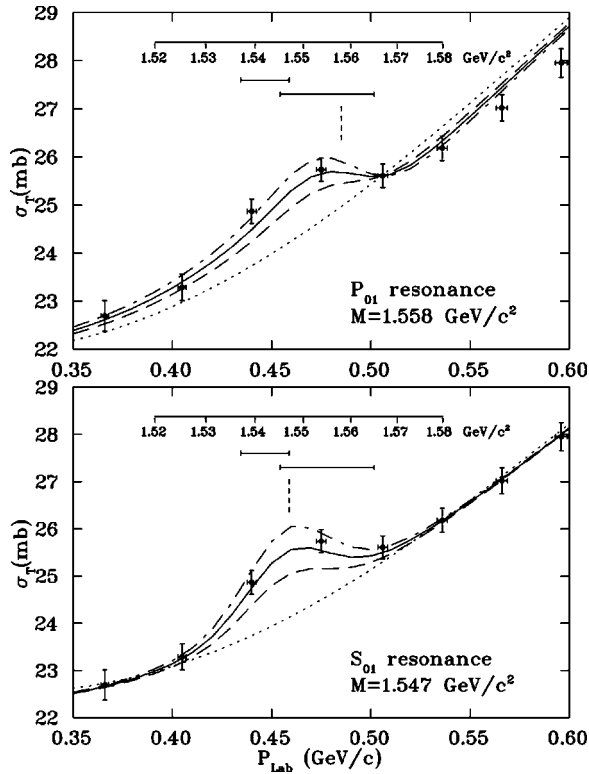


FIG. 6. Comparison of Fermi motion corrected resonances with the data. The dash-dotted, solid, and dashed curves correspond to widths of 1.2, 0.9, and 0.6 MeV, respectively. The dotted curve is the background fit. The horizontal bars are the same as in Fig. 5. The vertical dashed line in each panel shows the input value of the mass. For the s -wave resonance the theoretical peak occurs almost at this value while, for the p -wave resonance, there is a noticeable change due to the fact that the background phase shift is considerably larger.

particle is indeed the one predicted in Ref. [2], it should be seen in the P_{01} partial wave although there is no *a priori* reason why a particle in the S_{01} state could not exist. Indeed, it might be identified with the one predicted in Ref. [1].

Figure 6 shows results for Γ around the value of 0.9 MeV expected [12]. The mass assumed is given on the figure and marked with the vertical dashed line. It is seen that the experimental deviation from the background curve is in good agreement with the expectation for the mass chosen. The two assumptions for partial wave lead to equally good fits so, from these considerations, one cannot distinguish between them with the present data.

We see that the peak of the case for P_{01} is shifted to lower values of the mass than the input value while for the S_{01} partial wave there is essentially no shift (since the phase shift is very small). In principle, this effect might be used as a method to distinguish between the partial waves if an accurate determination of the mass is made by other means. The error bands for the mass for two of the most recent experiments (Refs. [7,8]) are given in the figure.

One can now determine the global best fit parameters for width and mass. Figure 7 shows χ^2 contour plots for the S_{01} (dotted) and P_{01} (solid) partial waves. The χ^2 values were calculated by comparison of the theoretical curves (similar to

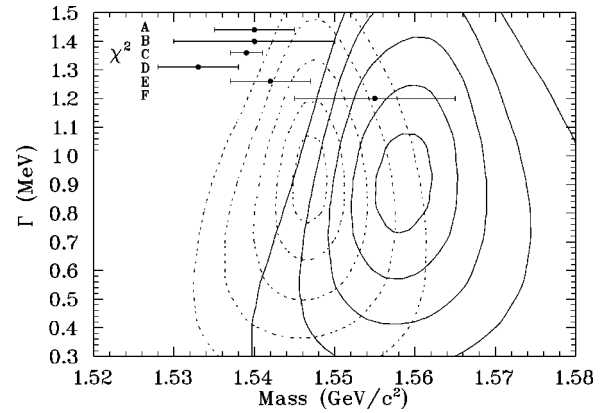


FIG. 7. χ^2 contour plots as a function of Γ and mass. The inner contour corresponds to one standard deviation, the next to two standard deviations, etc. The solid curves are for the $\frac{1}{2}^+$ case (P_{01} partial wave) and the dashed curves are for $\frac{1}{2}^-$ (S_{01} partial wave). The points at the top correspond to mass estimates given by the six discovery experiments cited in the introduction (A–F correspond to Refs. [3–8]). The vertical placement of these points has no significance.

those shown in Fig. 6) with the eight points of Bowen *et al.* [24]. The input values of the mass and width were varied as shown on the axes of Fig. 7. The inner curve in each case is one standard deviation from the minimum (center), the next concentric curve is two standard deviations, etc. The values and one sigma errors are read directly from the figure. Using the full range of variation in the two fits a single value of Γ can be obtained as 0.9 ± 0.2 MeV, in agreement with Cahn and Trilling [12]. It has been assumed that the background fit previously is correct, i.e., it was not allowed to vary. By adjusting the background to pass through the extremes of the error bars, a shift of 0.2 MeV was seen. Combining the two uncertainties in quadrature the value $\Gamma = 0.9 \pm 0.3$ MeV is obtained.

The masses obtained are 1.559 ± 0.003 GeV/c^2 (P_{01}) and 1.547 ± 0.002 GeV/c^2 (S_{01}) for the two cases, where the errors are one standard deviation only. The change in the background has negligible effect on the masses or their errors. Aside from the statistical errors quoted, systematic errors in the experiment (or the analysis) will contribute as well. For example, the beam momentum was used as given. To move the mass from 1.547 MeV/c^2 (obtained for s -wave scattering) to the nominal value of the mass obtained from the discovery experiments, 1.540 MeV/c^2 , would require a reduction of beam momentum of 3.7%. To move 1.559 MeV/c^2 (the value for the p -wave scattering) to 1.540 MeV/c^2 requires a reduction of 9.2%. A loss in momentum of the order of 3% from the beam to the center of the target is not uncommon.

Estimates for the mass from the discovery experiments cited in the Introduction are also given in Fig. 7. The letters A–F correspond to Refs. [3–8] in order. We see that there are some differences in the mass determinations.

VI. CONCLUSION

It has been seen that the strangeness +1 resonance recently observed in several experiments can also be seen in

K^+d total cross section measurements. The χ^2 contours given in Fig. 7 indicate that the value of the background phase influences the mass extracted to a considerable extent. Hence, it may be possible to infer the parity of the state from a comparison of mass values.

The question of the parity is a very important one. Both Karliner and Lipkin [31] and Jaffe and Wilczek [32] have proposed models in which the small width can be explained by the partitioning of the structure into two clusters which move relative to each other in a p wave, requiring an overall positive parity. On the other hand, the lowest lying states are most often those with all constituents in the s wave such as treated by Strottman [1]. Some modern calculations also show a theoretical preference for the negative parity state from both QCD sum rules [33,34] and lattice calculations [35,36] (although Ref. [37] finds a positive parity). It has also been argued that the model of Jaffe and Wilczek should have a lower lying negative parity state [38].

It is tempting to say from Fig. 7 that the negative parity state (S_{01} wave) is closer to the centroid of the masses determined from other measurements and hence is favored over the $\frac{1}{2}^+$ state. While this may be true, it would be premature to draw that conclusion. There is a spread among the masses of the discovery experiments and one should expect that the errors will be reduced as further work is done. The most

recent experiment [8] had a very small error on the mass from the statistics alone, the large error bar shown being due to possible systematic errors. If this error were reduced (without changing the central value) then this mass would agree with the $\frac{1}{2}^+$ case. A second reason for caution involves the product form of the s matrices, Eq. (16). While this form is commonly used, and probably incorporates the major part of the physics correctly, the sensitivity of the shift of the resonance peak to this assumption merits further study. More accurate total cross section data would be very valuable to better establish the background and to make a more precise determination of the mass(es). While one cannot *determine* the partial wave in which a resonance occurs from total cross section data alone, one can potentially *eliminate* one on the basis of an interference which would occur in a given partial wave and which is not observed in the data. It is the possibility of the elimination of the P_{01} partial wave that is considered here.

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