Dear journalists:
I will begin by noting that I send my comments to the newspaper journalists who are, at least in part, responsible for giving Alec Rawls a public forum for promoting his-in my educated opinion, extremely questionable-"error theory." I tried to address some general issues recently, but as many of you saw, in the end Alec continues to resort to name calling, rather than logic: he now also characterizes me as having "breathtaking incompetence and/or dishonesty." I suspect that at some point his Thesaurus will run out of synonyms to describe me! I said that I was not going to devote more time to this controversy, but have decided to make one last attempt to clarify my perspective. Alec states that I deliberately misrepresented his statements. He does have a "Thus" connecting the sentences in question, and I did try to find out earlier if he was talking about azimuthal projections, which he claims is obfuscation (his word). My current conclusion is the same as my initial impression, namely that he is talking about azimuthal projections:

Azimuth is a mathematical concept defined as the angle, usually measured in degrees, between a reference plane and a point.

True north-based azimuth: in navigation, the reference plane is typically true north and is considered $0^{\circ}$ azimuth. Moving clockwise, a point due east would have an azimuth of $90^{\circ}$, south $180^{\circ}$, and west $270^{\circ}$. Some navigation systems use south as the reference plane. However, any direction can serve as the plane of reference, as long as it is clearly defined for everyone using that system.

Alec should have stated the above in some clear, simple, concise, and precise way, rather than: "The shortest distance between points on the opposite sides of the northern hemisphere takes a short cut over the North Pole." This would have clarified the situation for me at least. Otherwise, the angle in question becomes debatable. Raisz (1962, p. 186) shows that the angle is a function of the selected projection, which can be orthogonal or oblique. Figure 1 illustrates the situation. If point $A$ in Figure 1a is shifted westward (a la Figure 1b), then the angle ABC increases, and accordingly can take on very many values.


Figure 1. Left (a) triangle on a spherical surface. Right (b): 3-dimensional axis rotation.

But this is not the critical point. Although Alec accuses me of "search[ing] for ways to misunderstand that will allow [me] to reach [my] desired conclusion," he continually ignores the single issue I originally was asked to assess: does he have his claimed mathematical proof of his conjecture (this is the source of his contention that I "vehemently denied that [I] had said that the crescent did not point to Mecca"; what I said was that I was unconvinced, and that he does not have a mathematical proof). Throughout the last 18 months I have stated that he has failed to convince me that he has a mathematical proof; today I am certain that he does not. I have avoided doing the difficult and tedious mathematics of showing the absence of a proof, but have spent the past 2 days undertaking this task (perhaps I should have done this sooner). I will note here that I have not been reading Alec's every word (he produces continual, and hence numerous, revisions of his essays), and am not necessarily aware of changes that he made rephrasing materials, which nevertheless have occurred after my initial evaluation (i.e., I could not have know about them when I undertook my assessment). And, my ensuing analysis does not rely on whether or not Alec was clear about the azimuthal projection used.


Figure 2. Left (a): the intersection of a plane and a sphere. Middle (b): the basic trigonometry for determining the arc lengths. Right (c): the set of infinite arcs connecting locations A and B.

Consider the intersection of a plane and a sphere (Figure 2a). If this plane goes through the center of the sphere, then it defines what is known as great circles. A great circle can extend from some point A, tracing a track across the spherical surface, until at most it reaches a point just an infinitesimally small increment before a distance of $r \pi$, where $r$ is the radius of the sphere (e.g., the Earth, approximately); at exactly $\mathrm{r} \pi$ it explodes into an infinite number of great circles of length $\mathrm{r} \pi$. An infinite number of great circles exist along this arc, because each point on the line can be a beginning and/or an end point. Consequently, I have demonstrated (within the error limits established by Alec) that one could construct a great circle arc from the Flight 93 crash site to the Nazi concentration camp location at Drancy, France, as well as to the Vatican, using essentially the same arithmetic as Alec. But Alec wants a particular length: "There is only one great circle arc from the crash site to Mecca, and Griffith certainly knows that this is the only arc I am saying anything about" (again, a statement made months after my analysis). So he dismisses the complement arcs and the infinity of possible arc lengths. This alteration still fails to furnish him with a mathematical proof.

Consider the great circle arc depicted in Figure 2c as line AB. Figure 2a illustrates that a plane can cut a sphere in an infinite number of locations. Keeping this line AB fixed, the cutting
plane can be rotated about this line (somewhat analogous to the rubber band I mentioned in my earlier comments). From my personal notes, the basic trigonometry information appears in Figure 2 b . As the angle changes from line AB by either increasing or decreasing through $90^{\circ}$ in either direction, an infinite number of additional arcs can be drawn from $A$ to $B$ (which at the very least can be approximated by incomplete circles); each of these constructions is exactly the same exercise as the great circle construction and is exactly what Alec has done (at least conceptually). The full $180^{\circ}$ range occurs when a plane intersects the sphere orthogonal to the plane that goes through the center of the sphere. The length of these arcs ranges from $r \theta$, where $r$ is the radius of the Earth and $\theta$ is the angle of the cord determining the great circle arc, to $\mathrm{r} \pi \operatorname{SIN}(\theta / 2)$, which becomes the radius of the aforementioned circle. The great circle arc Alec discusses is roughly $10,693 \mathrm{KM}$ in length. The pair of lines closest to it in Figure 2c define the $2^{0}$ "margin of error" (I use this term very loosely here) Alec finds acceptable, which results in arcs of length up to approximately $10,724 \mathrm{KM}$. The maximum arc length is $14,894 \mathrm{KM}$ (i.e., the circle defined by rotating the great circle's chord $360^{\circ}$ ). All of these arcs begin at point A, and depending upon the angle used as one begins to trace a shortest distance arc to location B , follow the shortest path along the spherical surface to B , given that beginning angle. There are an infinite number of paths within the "margin of error"; there are an infinite number of paths within the circle and outside the "margin of error." For any given angle $\alpha$, the length of its corresponding arc is given by (at least approximately)

$$
r \sqrt{\operatorname{Cos}^{2}(\theta / 2)+1-2 \operatorname{Cos}(\theta / 2) \operatorname{Cos}(\alpha)}
$$

where angle $\alpha$ ranges from $0^{\circ}$ to $\theta / 2$ at the center of the sphere (which in a fixed location A covers the entire $180^{\circ}$ range). This infinity of paths allows angles to range from $0^{\circ}$ to $180^{\circ}$ (i.e., a full half-circle); their complements also are opposite-direction minimum length paths that trace corresponding arcs from A to B around the opposite side of the sphere in order to complete the circles to which they belong (but Alec apparently rejects these latter arcs).

Alec claims that he is talking about only a particular azimuthal projection, and only a particular great circle. When I repeat my analyses on the moon, Mars, any other planet, a beach ball, a soccer ball, a basketball, or any other sphere, using exactly the same two coordinate pairs and any azimuthal projection, I get exactly the same results that I report here. I have theorems, and hence proofs, to substantiate my claims. When I repeat Alec's calculation on any of these other spheres, I do not even find Mecca! In other words, Alec has an arithmetic calculation, not a mathematical proof. After two days of intensive mathematical work, I do not believe that a theorem, let alone a proof of it, exists for his conjecture. He has a calculation; in my initial assessment, I agreed and declared in writing that his calculation was approximately correct (in my opinion, he still needs to establish margins of error; e.g., $2^{\circ}$ differences can be as much as $223 \mathrm{KM})$. But a calculation does not constitute a proof. Virtually any professional mathematician will confirm that even though trillions and trillions and trillions of correct calculations have been made for the roots of the zeta function, Riemann's hypothesis remains unproven.

I am ending my analysis of Alec Rawls's conjectures-I think that now I have nothing more to say; hopefully Alec will not convince me otherwise. But in doing so, I have begun to wonder about his motivation. Did any journalist check to see why Alec "left an economics doctoral program to devote all of his time to this project"? Did he actually do this? Why is his book subtitled: "Director's cut." Why is he declaring it to be a blockbuster? Aren't books best sellers
(or here perhaps best downloaders)? I am beginning to suspect that Alec wants a movie deal! Am I standing in his way somehow? And, you can bet that I will be crucified in his February web page essay, primarily because he has been unable to bully me into agreeing with him. Alec has slandered many good people, including a number of government employees of the National Parks Service; perhaps the newspapers, who unleashed him on us, should tell the rest of the story!

Again I will qualify my discussion here. If I have made any typographical errors (which almost certainly I did; I found a number in my most recent letter to Alec), they do not constitute my lying (as Alec has claimed in the past)! And, again, I emphasize that Alec now is sending unwarranted and derogatory e-mails about me to my colleagues and administrators, prompting me to quit ignoring him and send my responses to you! Alec has every right to disagree with my analysis; he has no right to harass me! And I have every right to my own professional opinion, as do the families of the victims of Flight 93.

In conclusion, the answer to the question "Does Alec Rawls have a mathematical proof of his claim" is an emphatic and resounding "no." The answer to the question "Can Alec Rawls construct a shortest path arc-in fact, an infinite number of such arcs based upon rotating at a fixed position through a half-circle-between the Flight 93 crash site and Mecca" is "yes."

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