First Observation of Simultaneous Alpha Buckets in a Quasi-Isochronous Storage Ring

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We present the first experimental evidence confirming the theoretical predictions of alpha buckets in an electron storage ring. By controlling both the first- and second-order momentum compaction factors, we succeeded in storing electrons simultaneously in a pair of alpha buckets or in either bucket alone. The two electron bunches are separated in energy by slightly less than 1% and the energy is tunable over a narrow range. The energy difference was directly measured using synchrotron light from an undulator. Simultaneous two-color light beams from an undulator were generated. By changing the rf voltage, we were able to vary the normally fixed longitudinal bunch separation.

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The physics of nonlinear longitudinal phase space has been understood for some time in proton accelerators, where crossing the so-called transition energy (vanishing phase slip factor) during acceleration can cause a loss of beam [1]. Electron accelerators do not have this problem since the velocity dependent term in the phase slip factor is usually negligible compared to the path lengthening term characterized by the momentum compaction factor, $\alpha \equiv \alpha_1 + \alpha_2 \delta$. As a result, electron rings nearly always operate well above the transition energy. Until now, a vanishing phase slip factor has been a concept to be avoided. However, recently several attempts have been made to generate short bunches in electron storage rings by reducing the first-order momentum compaction factor for the lattice and operating close to transition in a "quasi-isochronous mode" [2–4]. As the first-order term α_1 is reduced, the effect of the second-order term becomes important in determining the physics of the electron bunches. Additional stable fixed points, so-called alpha buckets, develop where electrons could be stored [2]. We present the first experimental evidence of the existence of these alpha buckets in an electron storage ring and we explore their dependence on the radio frequency system (rf) and lattice parameters. We also demonstrate a potential application of the alpha buckets by creating two-color light beams from an undulator.

The longitudinal equations of motion for an electron of energy E in a high energy electron storage ring are given by [2]

$$\frac{d\phi}{dt} = \omega_{\rm rf}(\alpha_1 + \delta\alpha_2)\delta\,, \qquad (1a)$$

$$\frac{d\delta}{dt} = \frac{eV_{\rm rf}\sin(\phi + \phi_s) - U_0}{E_0 T_0}\,, \qquad (1b)$$

where $V_{\rm rf}$, $\omega_{\rm rf} \equiv 2\pi h/T_0$, and h are the rf voltage, frequency, and harmonic number; E_0, T_0, and U_0 are the energy, revolution period, and radiated energy loss per turn of the synchronous electron in the ring. The damping term has been neglected in Eqs. (1) as it does not play a role in the discussion which follows. ϕ and δ are the phase deviation and the fractional energy deviation of an electron from the synchronous electron with phase, ϕ_s. The momentum compaction factor is expanded to second order in δ. The fixed points of the equations of motion are given by the condition that $\dot{\phi} = \dot{\delta} = 0$.

Under normal operating conditions, only the first-order term in the momentum compaction, α_1, is considered nonzero. In this case, there is one stable and one unstable fixed point at ($\phi = 0, \delta = 0$) and ($\pi - 2\phi_s$,0), respectively. The rf bucket height is the rf energy acceptance at the stable fixed point, and it is given by

$$\delta_{\rm rf0} = \sqrt{\frac{2eV_{\rm rf}\cos\phi_s[2 - (\pi - 2\phi_s)\tan\phi_s]}{\alpha_1 \omega_{\rm rf} E_0 T_0}}\,. \qquad (2)$$

Several groups have attempted to develop short electron bunches by reducing α_1 toward zero [3,4]. As $\alpha_1 \to 0$, one must allow for a nonzero α_2 in Eqs. (1). In this case there are two additional fixed points at $(0, \delta_e)$ and $(\pi - 2\phi_s, \delta_e)$, where $\delta_e \equiv -\alpha_1/\alpha_2$. The first is unstable and the second is stable as shown in Fig. 1a. In Figs. 1a–1c, the interior of the closed curves (separatrices) corresponds to stable motion of possible stored electrons and the exterior corresponds to unstable motion of electrons that will be lost from the ring. The character of the (ϕ, δ) phase space depends critically on the values of α_1, α_2, and $V_{\rm rf}$. As α_1 is reduced two things happen: (i) the energy separation, δ_e, decreases so that the "centers" of the two buckets approach one another and (ii) the bucket heights increase proportional to $\sqrt{V_{\rm rf}/\alpha_1}$. The two buckets collide and the separatrices of the buckets merge into a single curve when $\delta_{\rm rf0} = \delta_e/\sqrt{3}$ (see Fig. 1b). After collision, the buckets are no longer oriented in an east-west direction as shown in Fig. 1a, but in a north-south orientation as shown in Fig. 1c. These north-south oriented rf buckets have been called "alpha buckets" [2]. The physics of the alpha buckets is quite different from the "normal rf buckets," since their energy acceptance depends only on $|\alpha_1/\alpha_2|$ [2], rather than $\sqrt{V_{\rm rf}/\alpha_1}$ as in Eq. (2), and their

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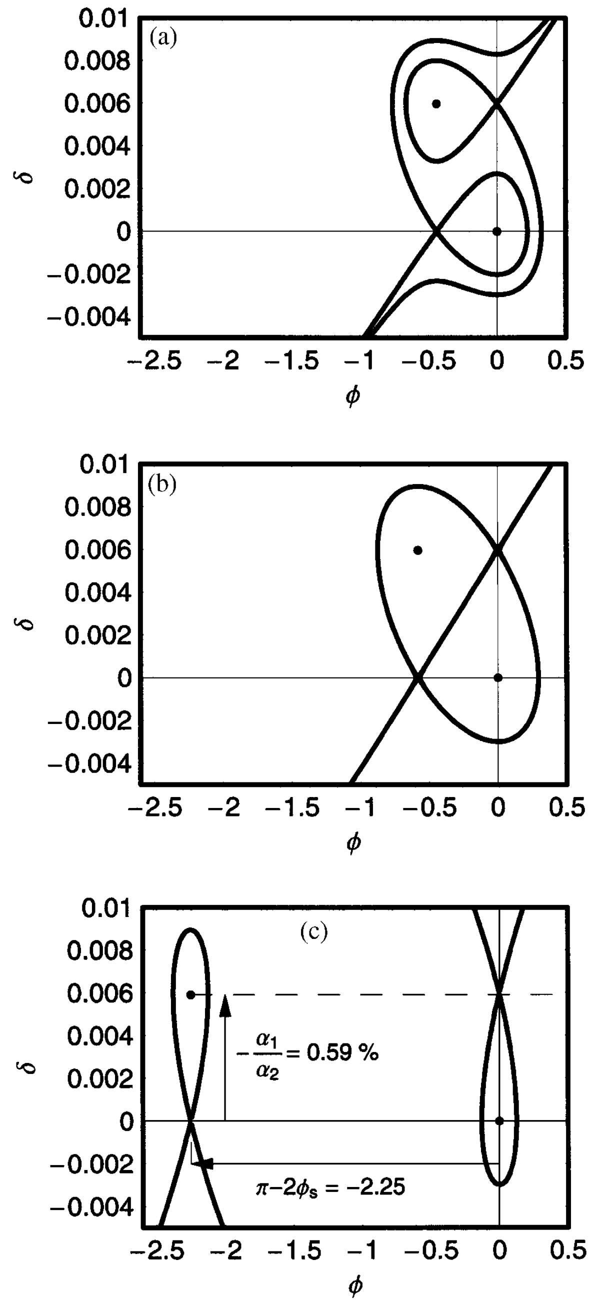

FIG. 1. Plot of the separatrices in the (ϕ, δ) phase space showing the effects of the second-order momentum compaction for $\alpha_1 = 0.0094$ and $\alpha_2 = -1.58$: (a) Two "normal" rf buckets before collision with $V_{\rm rf} = 14$ kV, (b) separatrices merge with $V_{\rm rf} = 14.28$ kV, and (c) calculated alpha buckets corresponding to our experimental parameters used to store the beam as shown in Figs. 2, 3, and 4 with $V_{\rm rf} = 31.8$ kV.

phase acceptance is given by [5]

$$\Delta\phi \approx \frac{2}{\sqrt{3}} \left| \frac{\alpha_1}{\alpha_2} \right| \sqrt{\frac{2\pi h \alpha_1 E_0}{eV_{\rm rf} \cos\phi_s}}. \tag{3}$$

In this paper we provide direct experimental proof of the existence of the paired alpha buckets in an electron storage ring. The NSLS VUV storage ring [6] normally operates as a double bend achromat lattice, with a single quadrupole family (Q3) in the dispersive region controlling the achromatic condition. By increasing the strength of Q3, the dispersion function, $\eta_x(s)$, can be made negative in part of each dipole resulting in a reduction in α_1, and eventually even a change in its sign. In order to maintain injection, the horizontal and vertical betatron tunes are restored to their nominal values using the remaining two quadrupole families. Reducing Q3 by 40% allowed us to vary α_1 over a wide range, $-\alpha_1^{(0)} < \alpha_1 < 2\alpha_1^{(0)}$, where $\alpha_1^{(0)} = 0.0235$ is the value for the achromatic lattice [7]. The value of α_1 was determined for each lattice by measuring the synchrotron frequency for $I \approx 1$ mA in a single bunch and using the fact that $f_s \propto \sqrt{\alpha_1}$. The low current was chosen to eliminate the effects of potential well distortion. By controlling α_2 using the two families of chromaticity correcting sextupoles, we were able to store beam in lattices with $f_s \approx f_{s0}/14$, a factor of 200 reduction in α_1. The values of α_2 were estimated from the measured f_s versus $f_{\rm rf}$, using the relationship [3,4],

$$f_s^4 = f_{s0}^4\left(1 - 4\,\frac{\alpha_2}{\alpha_1^2}\,\frac{\Delta f_{\rm rf}}{f_{\rm rf}}\right). \tag{4}$$

To experimentally demonstrate that the alpha buckets exist, we reduced $\delta_e \leq 1\%$ to keep the horizontal closed orbit inside the ring vacuum chamber aperture. This was achieved not by reducing α_1 to its smallest values, we used $\alpha_1 \approx \alpha_1^{(0)}/25$, but by increasing α_2 with the sextupoles. To provide adequate phase acceptance for injection, $V_{\rm rf}$ was reduced by a factor of 3–4 from the nominal value of 80 kV. Since we had previously stored beam in a stable normal bucket with $\alpha_1 < 0$, the stable phase was known for the new bucket and the injection beam phase could be shifted at will between the two stable fixed points [7].

Figure 2 shows signals from the two electron bunches as measured on a stripline pickup with an oscilloscope; also shown is the measured waveform of the 52.9 MHz rf cavity field. The time separation of the two bunches was 6.73 ns or $\Delta\phi = 2.24$ rad of the rf waveform. This spacing is $\Delta\phi = \pi - 2\phi_s$ and was tunable by changing $V_{\rm rf}$. The separation between "normal" rf buckets is fixed at 18.9 ns. Although the actual charge is not well known, the stripline signals indicate about a 2:1 ratio, with a total current of about 3 mA.

In Fig. 3, the 2D image of the two electron bunches is visible on a synchrotron radiation monitor mounted on one of the light ports of the storage ring dipole magnets. The horizontal separation of the two bunches, $\Delta x \approx 3$ mm, is proportional to their energy difference. The $\delta = 0$ bucket is on the left and the $\delta = \delta_e$ bucket is on the right, the reverse of the trace in Fig. 2. Since $\eta_x(s)$ is not well known at this beam port for this quasi-isochronous lattice, we used the measured difference between the closed orbits for beam stored in either bucket individually and the calculated $\eta_x(s)$ from a model lattice to estimate $\delta_e \approx 0.66\%$. The value of δ_e and, hence, the transverse separation of the bunches was tunable by changing α_2 with the sextupoles.

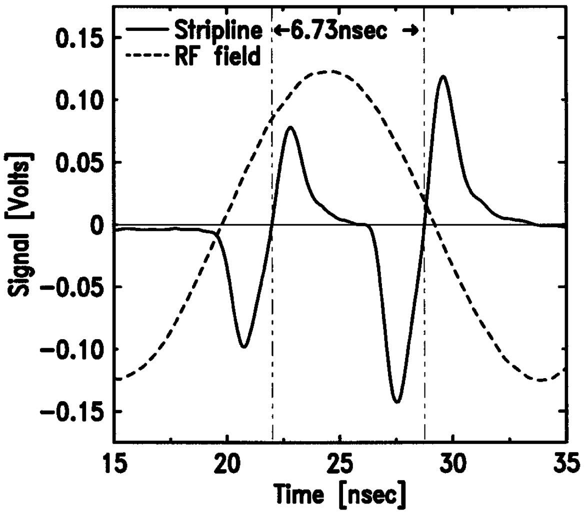

FIG. 2. Scope traces of the rf field and beam bunch signals from a stripline monitor showing electrons stored simultaneously in two alpha buckets separated by 6.73 ns. The stripline signals indicate about a 2:1 charge ratio, with a total current of about 3 mA.

It can also be seen that there is a vertical separation and tilt of the beam spots, indicating the presence of vertical dispersion and a coupling change with energy. The measured horizontal chromaticity was roughly zero ($\xi_x \approx 0$) and the vertical chromaticity was negative ($\xi_y \approx -2.5$) which can lead to a transverse head-tail instability. This may be the reason we were limited to storing at most 3 mA in the off energy bucket and 15 mA in the on energy bucket. Equalizing the current in each bunch was difficult but possible at the 6 mA total current level. If there were additional families of sextupoles, and depending on the detailed lattice structure, one could have a tunable α_2 and still correct the chromaticity. Future studies of these alpha buckets will include studies with $\alpha_1 < 0$, where the head-tail instabilities are naturally damped without the need for sextupoles, giving greater freedom to vary α_2 without the chromaticity constraint.

The energy of each alpha bucket was directly measured using synchrotron light from an undulator on the storage ring. The monochromator scan of the light beam is shown in Fig. 4 for beam stored in either bucket individually. The energy difference for the electron beam is computed as $\delta_e = \Delta E_\gamma/[2E_\gamma(0)] = 0.59\%$, where $E_\gamma(0)$ is the photon energy for the 3rd harmonic undulator radiation for an electron beam energy corresponding to $\delta = 0$. This measurement is independent of the knowledge of the dispersion at the monitor used in Fig. 3. Using this value of δ_e and the measured ratio α_2/α_1^2 from Eq. (4) yields model independent measures of the values for α_1 and α_2. Figure 1c shows the calculated alpha buckets for the measured parameters of $\alpha_1 = \alpha_1^{(0)}/25$, $\delta_e = 0.59\%$, and $V_{\rm rf} = 32$ kV. As seen in Fig. 1, the rf buckets remain in a north-south orientation for $V_{\rm rf} > 14.28$ kV. With our existing rf transmitter we cannot lower the rf voltage to much less than 30 kV, so we are confident that the stored beam is captured in a pair of alpha buckets as in Fig. 1c, rather than a pair of normal rf buckets as in Figs. 1a and 1b.

The measured energy difference of the light beams from an undulator demonstrates a possible application of these alpha buckets. The two colors of light from filled alpha buckets would allow experiments at two photon energies simultaneously (within time spacing of the two bunches) without monochromators to select the two energies. The energy difference is tunable by changing α_2 with the sextupoles and the time spacing by changing $V_{\rm rf}$. One application could be digital subtraction angiography. It should be noted that in order to store electrons in both alpha buckets in our experiments we had to modify the dispersion function throughout the ring which means that the two different energy electron beams, and, hence, the light beams, are separated horizontally in the insertion devices. For a specially designed storage ring with more superperiods, it would be possible to adjust the dispersion in some parts of the lattice to manipulate the momentum compaction and

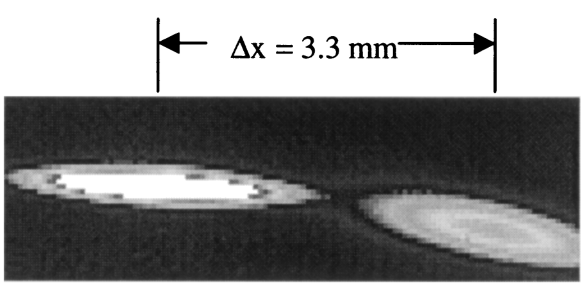

FIG. 3. Synchrotron light monitor images showing the simultaneous storage of electrons in a pair of alpha buckets. The $\delta = 0$ bucket is on the left and the $\delta = \delta_e$ bucket is on the right, the reverse of the trace in Fig. 2.

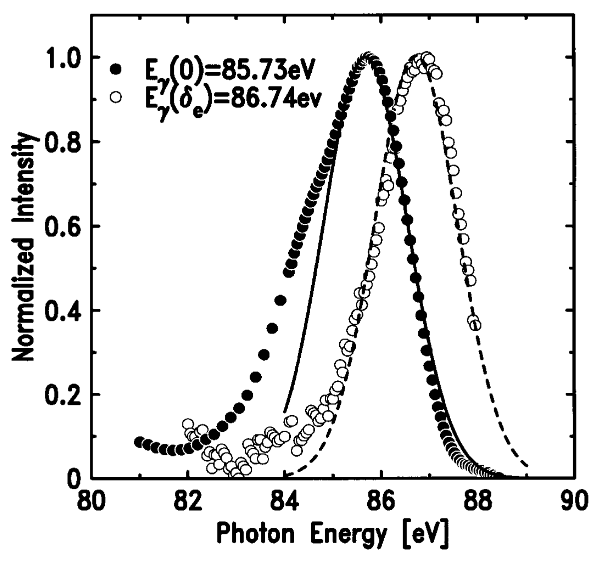

FIG. 4. Third harmonic undulator light intensity versus photon energy when an electron beam is stored in either the $\delta = 0$ (•) or the $\delta = \delta_e$ (∘) alpha bucket.

still preserve some zero dispersion straight sections to produce collinear two-color light pulses with variable time separation.

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